Optimal Tax Design and Enforcement with an Informal Sector

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ABSTRACT

An optimal commodity tax approach is taken to compare trade taxes and VATs when some commodities are produced informally. Trade taxes apply to all imports and exports, including intermediate goods while the VAT applies only to sales by the formal sector and imports. The VAT can achieve production efficiency within the formal sector, but unlike the trade tax regime, it cannot indirectly tax pure profits. Making the size of the informal sector endogenous in each regime is potentially decisive. The ability of the government to change the size of the informal sector through costly enforcement may also tip the balance in favor of the VAT.

1 Introduction

The problem of tax design for developing countries remains a vexing one. On the one hand, Emran and Stiglitz (2005) argue that, given the size of the informal sector in these countries, the conventional argument for a broad-based tax like the value-added tax (VAT) may not be valid. Since firms in the informal sector can avoid paying the VAT, but they cannot avoid paying taxes imposed on international trade administered at the border, the latter may be more effective revenue-raising devices. They show in a simple model that trade taxes can be more efficient than a VAT yielding the same government revenue despite the fact that trade taxes have a much narrower base. Related to that, Piggott and Whalley (2001) have shown that expanding the base of a VAT can reduce welfare in the presence of informality by inducing suppliers of newly taxed goods or services to move into the untaxed sector. On the other hand, Keen (2008) argues that Emran and Stiglitz underestimate the extent to which the VAT in developing countries succeeds in extracting tax revenues from the informal sector. Not only do informal firms pay the VAT on purchases from formal firms and imports (which in the case of imported inputs accomplishes the same as trade taxes), but also these countries often deploy withholding taxes that effectively impose a differential tax on the informal sector. These withholding taxes can apply on either imported inputs or on domestically produced inputs over and above the VAT. In either case, the withholding taxes can be credited by taxpaying firms against their ordinary tax liabilities, and as such constitute a tax that effectively applies only to informal firms.

Both the Emran-Stiglitz and Keen analyses are convincing in the contexts of their models, but the models themselves are somewhat restrictive. Emran and Stiglitz can ignore the payment of VAT on the inputs of informal sector firms simply because intermediate inputs are missing from the production side of the economy in their model. Indeed, the ability of the VAT to avoid distortions in production is precisely because of the crediting method it applies to intermediate inputs, something that is missing in trade tax regimes.

Keen's results are also somewhat limited since they are derived in a simple setting chosen for illustrative purposes and to generate analytically tractable results. He assumes that the informal sector produces a non-traded good that is substitutable for formal sector

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production, and uses imported intermediate inputs along with some untaxed fixed factor. The formal sector produces the same final non-traded product also using an imported intermediate input, but generates no profits or payments to an underlying fixed factor. While this generates an elegant model that in some, not unreasonable, circumstances leads to the result that trade taxes should not be used as long as withholding taxes are available alongside the VAT, some potentially important factors are missing. The assumption that the informal sector earns untaxed profits while the formal sector does not could be thought of as equivalent to assuming that profits in the formal sector can be taxed at 100 percent. If this is not the case, both the VAT and trade tax regimes would have indirect effects on untaxed profits that could affect the case for one versus the other. Similarly, restricting informal sector outputs to be non-traded rules out other avenues of difference between VAT and trade tax regimes, given that taxes on imports can also affect the price of importables produced by the informal sector, while exporters receive a refund of the VAT.

More generally, both Emran-Stiglitz and Keen assume that the number of producers in the informal sector is given, while allowing informal production to vary according to tax policies. One might expect that if the tax advantage to informality differs between the VAT and trade tax regimes, the relative number of informal producers would as well. This may be especially important if producers are systematically less efficient in the informal sector than in the formal sector. For example, Gordon and Li (2005) have argued that because informal producers cannot take full advantage of financial intermediation, they will be less efficient, and de Paula and Scheinkman (2007) argue that informal producers will be less efficient because they have limited access to capital markets. In addition, firms in the informal sector may find it more difficult to hire skilled workers and may be constrained to be small in size.

Tax enforcement should also affect the relative size of the informal sector in the economy as a whole. As noted by Emran and Stiglitz (2005), developing countries are constrained by weak tax administration capacities in raising tax revenue, with tax evasion and corruption being widespread. There is some evidence that tax administration is particularly weak in countries relying on trade taxes. Baunsgaard and Keen (2005) report that in low-income countries, revenue recovery after decreasing trade taxes is weak, with less

that 30 percent of lost revenue being recovered by alternative revenue sources. Admittedly, tax administration is difficult to improve, but it is not something to be taken as given. Indeed, the introduction of a VAT is often regarded as a step toward the overall modernization of tax administration, adopting self-assessment, a function-based administrative organization, and an effective audit program, which can subsequently extended to income taxation. Slemrod and Kopczuk (2002) argue that tax enforcement should be a part of the 'optimal taxation' problem, along with the tax rate and base structure, since it affects the elasticities of tax bases.

Our purpose in this paper is to construct a general model of optimal tax design in an economy with an informal sector. Our model encompasses the following features. Firms in both the formal and the informal sector produce goods that are tradable and that can use both importable or exportable intermediate inputs. The products produced by the two sectors can be different. The government can levy either trade taxes or VAT-type commodity taxes. In both cases, tax rates can differ across commodities and can be optimized. In the VAT regime, formal producers receive input tax credits when they use taxed commodities as inputs, whereas informal producers do not. This implies that the VAT on intermediates (as opposed to on final goods) allows the government to tax the informal sector indirectly when it uses inputs that have been taxed. In addition, all firms earn pure profits that cannot be fully taxed. Indeed, they may be untaxed, which, though extreme, is a useful benchmark to take for developing countries whose income tax systems are of limited scope. Firms are allowed to have different production functions according to whether they participate in the formal or informal sectors. While in our base case we assume, as in Emran-Stiglitz and Keen, that the scope of the informal sector is given, we also investigate the consequences of producers freely choosing their sector of production in each tax regime, and we allow the government to affect that at some administrative cost. As in these papers, we also assume that the household sector can be represented by a representative consumer so that all our analysis is based on efficiency considerations.[1]

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[1] An alternative approach is taken by Munk (2005), who models the informal sector as a form of household production. Informal producers purchase inputs from the formal sector and apply some of their own labor to produce output informally. Their output is kept within the informal

Our basic approach is to make global comparisons between a full VAT regime and a full trade tax regime rather than, say, deriving conditions for the optimal mix of commodity and trade taxes. This is obviously an ambitious task that generally leads to ambiguous results. Our purpose is to identify the considerations that tend to favor one regime over the other. To facilitate our analysis, we assume, following Emran and Stiglitz (2005), that all commodities are traded and that the economy is a small open one. This implies that world prices are fixed and that domestic taxes have a one-for-one effect on producer and consumer prices as the case may be. While this simplifies the analysis greatly, it does so at the expense of assuming away non-traded commodities in either sector. This assumption should not bias the case in favor of either trade taxes or the VAT since neither of them will apply to non-traded goods sold by the informal sector.

We begin with the case where the scope of the formal and the informal sectors is fixed, that is, where the producers in each are given. We derive conditions under which a fully differential VAT is more efficient than trade taxes. When there is no formal sector, the latter will be unambiguously preferred, but when there is a formal sector, the comparison is ambiguous, even if there is no informal sector. We then investigate the effect of changes in the size of the informal sector. First, we consider the effect on welfare of moving a producer from the informal to the formal sector. Then, producers are allowed to freely choose their sector in each tax regime. Finally, we let the government influence the size of the informal sector by increasing the resources devoted to administering the tax system.

2 Basic Setting

Consider a small open economy with $J+1$ tradable commodities denoted by $j = 0, \cdots, J$. Denote by $\boldsymbol{X}$ and $\boldsymbol{M}$ the sets of exportable and importable goods, respectively, and let $\boldsymbol{A} = \boldsymbol{X} \cup \boldsymbol{M}$ be the set of all goods. Each good j is produced by a representative producer, identified as producer j, who can operate in the formal sector (F) or the informal (shadow) sector (S). Let $\{\boldsymbol{X}^F, \boldsymbol{X}^S\}$ and $\{\boldsymbol{M}^F, \boldsymbol{M}^S\}$ be the sets of exportables and importables in the two sectors, and define the sets of goods produced in the two sectors as $\boldsymbol{F} \equiv \boldsymbol{X}^F \cup \boldsymbol{M}^F$

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sector. This model seems particularly appropriate to subsistence farming, whereas our focus is on producers who purchase and sell commodities to the market economy.

and $\boldsymbol{S} \equiv \boldsymbol{X}^S \cup \boldsymbol{M}^S$. The sizes and compositions of $\boldsymbol{F}$ and $\boldsymbol{S}$ are initially taken as given.

Consumer prices are denoted $\boldsymbol{q} = \{q_0, q_1, \cdots, q_j, \cdots, q_J\}$, while the prices faced by producers on their inputs and outputs can differ between sectors and are denoted $\boldsymbol{p}^F = \{p_0^F, p_1^F, \cdots, p_j^F, \cdots, p_J^F\}$ and $\boldsymbol{p}^S = \{p_0^S, p_1^S, \cdots, p_j^S, \cdots, p_J^S\}$. Where necessary, we shall refer to producer prices for the entire economy by the extended vector of producer prices in the two sectors: $\boldsymbol{p} \equiv (\boldsymbol{p}^F, \boldsymbol{p}^S)$.[2] World prices are normalized to be unity for all $J + 1$ commodities. Assume that commodity $j = 0$ is untaxed and tradable, so that $q_0 = p_0 = 1$.[3] There are two main types of indirect taxes: international trade taxes denoted $\boldsymbol{t} = \{t_1, \cdots, t_j, \cdots, t_J\}$ and a destination-based value-added tax (VAT) based on consumption with rates $\boldsymbol{v} = \{v_1, \cdots, v_j, \cdots, v_J\}$ that can vary across commodities. Both types of taxes on transactions can be interpreted as either per unit taxes or ad valorem taxes based on world prices. For importables ($j \in \boldsymbol{M}$), an import tax implies $t_j > 0$, whereas for exportables ($j \in \boldsymbol{X}$), an export tax means $t_j < 0$ by convention (since the tax bases are defined below to be net imports, which are negative for exportables). Trade taxes apply to all commodities whether produced in the formal or informal sector: they cannot be evaded. The VAT is paid on domestic sales by the formal sector as well as on imports. Producers in the informal sector avoid charging the VAT on goods they sell, including those for domestic consumption purposes. But, they must pay taxes on commodities purchased from the formal sector or imported, and they cannot claim a VAT rebate. This characterization of the informal sector in which producers must choose to be either in the formal or the informal sector is a simplification. In practice, informality may be less

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[2] Since producers purchase inputs from other producers, one might think that producer prices are ambiguous in an economy in which transactions are taxed. However, in our setting, domestic producers face the same price for a commodity whether they are buyers or sellers. In the case of the VAT, taxes levied on the purchase of inputs are credited, so the buyer and seller effectively face the same price. With trade taxes, no tax applies on sales within the domestic economy. Therefore, the notion of producer prices is not ambiguous.

[3] Clearly, it is arbitrary to assume that one of the traded commodities is untaxed. If not, uniform commodity taxation could be levied, which would be non-distortionary in the absence of an informal sector and would amount to a tax on pure profits. Our analysis would not be very interesting in this case. A more natural assumption would be that commodity zero is leisure, as in the optimal tax literature. This would complicate our analysis somewhat (since the relative price of leisure would be variable) without adding any additional insight. No results of substance are lost by assuming there is an untaxed tradable commodity, as in Emran and Stiglitz (2005).

clearcut and may include tax evasion. For example, producers in the formal sector who are registered in the VAT system may receive tax credits on their inputs while at the same time failing to pay all taxes on their final sales. We ignore this form of tax evasion in what follows.

We assume that income is not fully taxed either in the hands of producers or households, and this is important for our analysis. In our model, income takes the form of profits generated by producers. The source of these profits is not modeled explicitly, but presumably they could come from some fixed factor, such as labor. Let θ be the rate of tax on profits, applicable only in the formal sector, where $0 \leqslant \theta < 1$. As we shall see, the comparison between the VAT and trade tax regimes is significantly influenced by the inability to tax formal sector profits fully. The reason is that while the VAT taxes profits in the informal sector by imposing a non-refundable tax on inputs purchased from the formal sector, it does not tax profits in the formal sector. That is, formal sector producer prices under a VAT regime remain world prices, so pure profits are not affected. The VAT is ultimately fully paid by domestic consumers. On the other hand, trade taxes tax profits in both sectors to the extent that traded commodities are used as inputs since producer prices are increased by the full amount of trade taxes. If profits could be taxed fully, this advantage of trade taxes would no longer apply and the balance would tip in favor of the VAT, as argued by Keen (2008). Thus, the assumption that $\theta < 1$ is an important one.[4]

Note also that all commodities are assumed to be tradable (except any fixed factors that are responsible for generating rents in production). Non-traded goods could be added to the model but with the added complexity that their producer prices would have to be determined endogenously. Keen (2008) does assume that the output of the informal sector is non-traded, but his model is otherwise very simplified by focusing on that sector alone. To the extent that the non-traded sector is more important for informal producers, this would reduce the advantage that trade taxes have in reaching the informal sector.[5]

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[4] Gordon and Li (2005) and Auriol and Warlters (2005) also argue that seemingly inefficient revenue instruments like capital taxes, trade protection or fees on entry have the advantage of taxing rents in the formal sector, unlike the VAT.

[5] Emran and Stiglitz (2005) argue that trade taxes are not put at a disadvantage relative to the VAT when there is a non-traded sector. That is because the VAT cannot tax the informal

These assumptions imply that consumer prices are given by $\boldsymbol{q} = \boldsymbol{1} + \boldsymbol{t}$ in the trade tax regime and $\boldsymbol{q} = \boldsymbol{1} + \boldsymbol{v}$ in the VAT regime. Producer prices in the formal sector are $\boldsymbol{p}^F = \boldsymbol{q} = \boldsymbol{1} + \boldsymbol{t}$ in the trade tax regime and $\boldsymbol{p}^F = \boldsymbol{1}$ (i.e., world prices) in the VAT regime. In the informal sector, prices are somewhat more complicated. In the trade tax regime, $\boldsymbol{p}^S = \boldsymbol{q} = \boldsymbol{1} + \boldsymbol{t}$ as in the formal sector. In the VAT regime, prices facing the informal sector depend on whether the good is exportable or importable, and on whether the good is a final one or an intermediate one. In the case of exportables ($j \in \boldsymbol{X}^S$), no VAT is collected, so $v_j = 0$ and $p_j^S = 1$. Thus, exportables produced in the informal sector might be purchased by formal producers. Since no tax is paid, producers would receive no input tax credit so the price would fall to the world price. Of course, if they purchase inputs from the formal sector, a VAT is paid and a credit is recovered. On the other hand, informal producers would have to pay tax-inclusive prices on their inputs, but cannot get a tax credit. In these circumstances, producers of exportables would likely opt for the formal sector. When we later allow producers to choose in which sector to produce, it will be the case that $\boldsymbol{X}^S = \emptyset$ in the VAT regime. For importables produced in the informal sector ($j \in \boldsymbol{M}^S$), if these can be sold as final consumer goods, the price received by informal firms is $p_j^S = 1 + v_j$ since consumers are indifferent between purchasing a good at a given price from the informal and as imports. On the other hand, formal producers would not purchase intermediate importable inputs from the informal sector because they would obtain no input tax credit.

All producers act as price-takers and maximize their profits. Each produces a single output using a vector of inputs, where the latter can come from the formal or the informal sector. For producer j, profit maximization yields the profit function $R^{jF}(\boldsymbol{p}^F)$ or $R^{jS}(\boldsymbol{p}^S)$ depending on whether production is in the formal or informal sector. Production in the informal sector may be less efficient because informal producers have limited access to the banking system (Gordon and Li, 2005), capital markets (de Paula and Scheinkman, 2007), the legal system or public infrastructure, and this affects their relative profitabilities. Let $\alpha_j \geqslant 1$ represent the exogenously given advantage of producing in the formal sector. Then,

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non-traded sector indirectly in their model since there are no intermediate inputs and the VAT does not apply to imports in the informal sector.

we can write profits in the two sectors as:

$$R^{jF}(\boldsymbol{p}^F) = \alpha_j r^j(\boldsymbol{p}^F), \qquad R^{jS}(\boldsymbol{p}^S) = r^j(\boldsymbol{p}^S) \tag{1}$$

where $r^j(\cdot)$ is a common underlying profitability.[6] Aggregate profits (before income tax) are then defined as follows:

$$R(\boldsymbol{p}) \equiv R^F(\boldsymbol{p}^F) + R^S(\boldsymbol{p}^S) \equiv \sum_{j \in \boldsymbol{F}} R^{jF}(\boldsymbol{p}^F) + \sum_{j \in \boldsymbol{S}} R^{jS}(\boldsymbol{p}^S) \tag{2}$$

Using the envelope theorem, the output of producer j in sector F is denoted by $R_{p_j}^{jF} \equiv \partial R^{jF}/\partial p_j^F > 0$, and inputs of commodity k used by producer j by $R_{p_k}^{jF} \equiv \partial R^{jF}/\partial p_k^F \leqslant 0$. Similarly, for sector S, the output of producer j is $R_{p_j}^{jS} \equiv \partial R^{jS}/\partial p_j^S > 0$, and inputs of k are $R_{p_k}^{jS} \equiv \partial R^{jS}/\partial p_k^S \leqslant 0$. Note that all profit functions are homogeneous of degree one in their respective producer prices, so $R^{jF} = \sum_k p_k^F R_{p_k}^{jF}$ and $R^{jS} = \sum_k p_k^S R_{p_k}^{jS}$.

All consumers are identical so we can normalize population to unity and characterize the household sector by the representative consumer. The consumer's income comes from the profits of producers in both sectors: thus, we make no distinction between owners of formal and informal firms. Given that profits in the formal sector are taxed at the rate θ, after-tax consumer income is $R - \theta R^F$. Given this income, the consumer chooses final consumption to maximize utility given the relevant consumer prices in the two sectors. It is convenient to characterize the maximized outcome by the expenditure function $E(\boldsymbol{q}, u)$, where $\boldsymbol{q}$ is the vector of consumer prices and u is utility. By Hotelling's lemma, compensated demands for all $J + 1$ commodities are given by $E_{q_j} \equiv \partial E(\boldsymbol{q}, u)/\partial q_j$, and the consumer's budget constraint may be written:

$$E(\boldsymbol{q}, u) = R - \theta R^F \tag{3}$$

$E(\boldsymbol{q}, u)$ is homogeneous of degree one in all $J + 1$ prices, so $E(\boldsymbol{q}, u) = \sum_{j \in \boldsymbol{A}} q_j E_{q_j}$.

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[6] One could argue that for small producers, the costs of complying with tax and other regulations might outweigh the advantages of participating in the formal economy, leading to $\alpha_j < 1$. We could allow for such producers without affecting our results. Some firms with α_j sufficiently low may also fall below the VAT threshold if the value of revenues generated is not enough to offset the compliance costs, as analyzed in Keen and Mintz (2004). These firms would be indistinct from those in the informal sector.

The consumer's budget constraint can be rewritten by substituting (2) into (3) and making use of the homogeneity-of-degree-one property of the expenditure and profit functions:

$$\sum_{j \in \boldsymbol{A}} q_j E_{q_j} = \sum_{j \in \boldsymbol{F}} \sum_k p_k^F R_{p_k}^{jF} + \sum_{j \in \boldsymbol{S}} \sum_k p_k^S R_{p_k}^{jS} - \theta R^F \tag{4}$$

In the general case, where both trade taxes and the VAT may be in place, we have $q_j = 1 + v_j + t_j$ and $p_j^F = 1 + t_j$ in the formal sector. In the informal sector, $p_j^S = q_j = 1 + t_j + v_j$ with $v_j = 0$ for $j \in \boldsymbol{X}^S$. Then, (4) may be written:

$$\sum_j (R_{p_j} - E_{q_j}) = \sum_j t_j (E_{q_j} - R_{p_j}) + \sum_j v_j (E_{q_j} - R_{p_j}^S) + \theta R^F \tag{5}$$

where $R_{p_j}^S = \sum_{k \in \boldsymbol{S}} R_{p_j}^{kS}$ and R_{p_j} is the net output of j, given using (2) by:

$$R_{p_j} = \partial R / \partial p_j = \sum_{k \in \boldsymbol{F}} R_{p_j}^{kF} (\mathbf{1} + \boldsymbol{t}) + \sum_{k \in \boldsymbol{S}} R_{p_j}^{kS} (\mathbf{1} + \boldsymbol{t} + \boldsymbol{v}) \tag{6}$$

The term $\sum_j t_j (E_{q_j} - R_{p_j})$ in (5) is the revenue from trade taxes levied on the net imports of each commodity. (Recall that for exports, $t_j < 0$ and $E_{q_j} - R_{p_j} < 0$.) The term $\sum_j v_j (E_{q_j} - R_{p_j}^S)$ is the VAT revenue obtained from the final value of domestic consumption of each commodity less the net domestic production of the commodity in the informal sector, where $v_j = 0$ for $j \in \boldsymbol{X}^S$, so no tax revenue is obtained on final consumption of exportables produced in the informal sector.[7]

Tax revenue is used for some exogenously given public expenditures, denoted $\overline{G}$. The government budget constraint may therefore be written:

$$\sum_j t_j (E_{q_j} - R_{p_j}) + \sum_{j \notin \boldsymbol{X}^S} v_j (E_{q_j} - R_{p_j}^S) + \theta R^F = \overline{G} \tag{7}$$

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[7] We assume that $E_{q_n} > R_{p_n}^S$ for all importables n. This implies that for $j \in \boldsymbol{M}^S$, producers transact using consumer prices, so $p_j^S = q_j = 1 + v_j$. If the importable is a pure intermediate good, the price received by informal producers will be bid down to the world price by demand from the formal sector. Such producers will likely opt for the formal sector since there is no advantage from operating informally. More generally, if $E_{q_n} < R_{p_n}^S$ the price of good n in the informal sector will be bid down below $1 + v_n$, which complicates the analysis without adding additional insight.

3 Optimal Taxes in the Basic Model

The government chooses its tax rates to maximize the representative household's utility subject to its revenue constraint (7) and the consumer's budget constraint (5). (Note that combining (5) and (7) yields the economy's resource constraint so it is not necessary to take explicit account of the latter. In the general case where both trade taxes and VAT exist, the Lagrangian may be written:

$$\mathcal{L} = u + \lambda \left(\sum_j t_j (E_{q_j} - R_{p_j}) + \sum_{j \notin \boldsymbol{X}^S} v_j (E_{q_j} - R_{p_j}^S) - \overline{G} \right)$$

$$+ \mu \left(\sum_j (R_{p_j} - E_{q_j}) - \sum_j t_j (E_{q_j} - R_{p_j}) - \sum_{j \notin \boldsymbol{X}^S} v_j (E_{q_j} - R_{p_j}^S) + \theta R^F \right)$$

The Lagrange multiplier λ can be interpreted as the social value of an increment of revenues raised using the distorting tax system, while μ is the social value of an increment of resources available to the nation (and owned by the household). As is well-known from optimal tax theory, $\lambda > \mu$ as long as positive tax revenues are being raised (Auerbach and Hines, 2002, p. 1365). Collecting terms, we can rewrite the Lagrangian as:

$$\mathcal{L} = u + (\lambda - \mu) \left(\sum_j t_j (E_{q_j} - R_{p_j}) + \sum_{j \notin \boldsymbol{X}^S} v_j (E_{q_j} - R_{p_j}^S) + \theta R^F \right) + \mu \sum_j (R_{p_j} - E_{q_j}) - \lambda \overline{G} \quad (8)$$

where, recall, R_{p_j} is the net domestic output of commodity j produced in both sectors of the economy and E_{q_j} is its compensated domestic demand. The choice variables are u, $\boldsymbol{t}$ and $\boldsymbol{v}$, but we can focus on the latter two in what follows. To compare trade taxes and the VAT in the presence of the informal sector, we consider the two regimes separately.

The Trade Tax Regime

In this case $\boldsymbol{v} = \boldsymbol{o}$, so the Lagrangian expression under trade taxes, denoted $\mathcal{L}_t$, can be written using (8) as::

$$\mathcal{L}_t = u + (\lambda - \mu) \left(\sum_j t_j (E_{q_j} - R_{p_j}) + \theta R^F \right) + \mu \sum_j (R_{p_j} - E_{q_j}) - \lambda \overline{G} \quad (9)$$

where $\boldsymbol{p} = \boldsymbol{1} + \boldsymbol{t}$ is the vector of producer prices, which is the same in the formal and informal sectors with only trade taxes in effect. The government can differentiate all trade

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taxes, so its first-order condition on t_n can be written as follows:

$$(\lambda - \mu)\left(E_{q_n} - R_{p_n} + \theta R^F_{p_n}\right) = -\lambda \sum_j t_j (E_{q_j q_n} - R_{p_j p_n}) \qquad \forall n > 0 \qquad (10)$$

where we have used the homogeneity properties of the profit and expenditure functions, so $R^F = \sum_j (1 + t_j) R^F_{p_j}$, $\sum_j (1 + t_j) R_{p_j p_n} = 0$ and $\sum_j (1 + t_j) E_{q_j q_n} = 0$.

These optimal trade tax conditions have a straightforward and familiar interpretation. The lefthand side is the gain in tax revenue from an incremental increase in t_n weighted by the social value of a transfer of revenues from the private to the public sector, $\lambda - \mu$. The righthand side is the marginal deadweight loss from the change in compensated demands, $E_{q_j q_n}$, and from the change in producer demands, $R_{p_j p_n}$, given that tariffs distort both consumption and production choices. Notice that if $\theta = 0$, (10) gives a standard proportionate reduction rule in net imports.

These conditions along with the constraints and the first-order condition on u can be solved to yield the optimal trade tax rates in the trade tax regime (regime T) denoted $\boldsymbol{t}^T$ (with $t^T_0 = 0$ for the untaxed good), the shadow prices λ^T and μ^T, with $\lambda^T > \mu^T$, and the level of utility u^T.[8] As well, total profits in the optimum are given by:

$$R^T(\boldsymbol{p}) = R^{FT}(\boldsymbol{p}^F) + R^{ST}(\boldsymbol{p}^S) = \sum_{j \in \boldsymbol{F}} R^{jFT}(\boldsymbol{1} + \boldsymbol{t}^T) + \sum_{j \in \boldsymbol{S}} R^{jST}(\boldsymbol{1} + \boldsymbol{t}^T) \qquad (11)$$

Also, we denote the value of $\mathcal{L}_t$ when trade taxes are optimized as $\mathcal{L}^T_t$, where by (9):

$$\mathcal{L}^T_t = u^T + (\lambda^T - \mu^T)\left(\sum_j t^T_j (E^T_{q_j} - R^T_{p_j}) + \theta R^{FT}\right) + \mu^T \sum_j (R^T_{p_j} - E^T_{q_j}) - \lambda^T \overline{G} \quad (12)$$

The VAT Regime

In this case, $\boldsymbol{t} = \boldsymbol{o}$, $v_0 = 0$ and $v_k = 0$ for $k \in \boldsymbol{X}^S$. The analog of (9) derived from (8) is:

$$\mathcal{L}_v = u + (\lambda - \mu)\left(\sum_{j \notin \boldsymbol{X}^S} v_j (E_{q_j} - R^S_{p_j}) + \theta R^F\right) + \mu \sum_j (R_{p_j} - E_{q_j}) - \lambda \overline{G} \qquad (13)$$

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[8] We assume the solution to the first-order conditions is interior and unique. It is well-known that there is no guarantee that the second-order conditions will be satisfied in optimal tax problems, and this is particularly important in our context since we are investigating perturbations from an optimum.

where $\mathcal{L}_v$ denotes the value of the Lagrangian expression in the VAT regime and $R_{p_j} = \sum_{i \in \boldsymbol{F}} R_{p_j}^{iF}(\mathbf{1}) + \sum_{i \in \boldsymbol{S}} R_{p_j}^{iS}(\mathbf{1} + \boldsymbol{v})$. The first-order condition on v_n can be written as:

$$(\lambda - \mu)(E_{q_n} - R_{p_n}^S) = -\lambda \sum_{j \notin \boldsymbol{X}^S} v_j(E_{q_j q_n} - R_{p_j p_n}^S) \qquad \forall n \notin \boldsymbol{X}^S \qquad (14)$$

where we have again used the homogeneity properties of the profit and expenditure functions, which here imply $\sum_j (1 + v_j) R_{p_j p_n}^S = 0 = \sum_j (1 + v_j) E_{q_j q_n}$.

Again, the interpretation of (14) is straightforward. The lefthand side is the social value of the net gain in revenues from an increase in v_n, while the righthand side is the marginal deadweight loss created by dv_n. Relative to (10), (14) reflects the fact that the VAT does not collect any tax on rents in the formal sector, but at the same time it does not distort production there. Note that in the absence of an informal sector, the terms involving R^S disappear and (14) is just the standard Ramsey optimal tax rule.

The solution to the first-order conditions on v_n and u in the VAT regime (regime V) along with the constraints yields the optimal commodity tax rates $\{v_j^V\}_{j \notin \boldsymbol{X}^S}$ (with $v_0^V = 0$), the shadow prices μ^V and λ^V, and utility u^V. Total profits are:

$$R^V(\boldsymbol{p}) = R^{FV}(\boldsymbol{p}^F) + R^{SV}(\boldsymbol{p}^S) = \sum_{i \in \boldsymbol{F}} R^{iFV}(\mathbf{1}) + \sum_{i \in \boldsymbol{S}} R^{iSV}(\mathbf{1} + \boldsymbol{v}^V)$$

where $\boldsymbol{q}^V = \mathbf{1} + \boldsymbol{v}^V$, with $v_j^V = 0$ for $j \in \boldsymbol{X}^S$. The value of the Lagrangian (13) when the VAT is optimized is:

$$\mathcal{L}_v^V = u^V + (\lambda^V - \mu^V)\left(\sum_{j \notin \boldsymbol{X}^S} v_j^V(E_{q_j}^V - R_{p_j}^{SV}) + \theta R^{FV} \right) + \mu^V \sum_j (R_{p_j}^V - E_{q_j}^V) - \lambda^V \overline{G} \quad (15)$$

Note that in the two regimes, the optimal values of u^T and u^V depend upon the number of producers the formal sector, $\boldsymbol{F} = \boldsymbol{X}^F \cup \boldsymbol{M}^F$. The key differences between the two regimes are that i) production efficiency is maintained in the formal sector in the VAT regime but not the trade tax regime, and ii) profits in the formal sector are indirectly taxed in the trade tax regime but not in the VAT regime, since producer prices in the formal sector remain unaffected by the VAT. The first difference favors the VAT, while the second favors trade taxes. Depending on the strength of these effects, either could be preferred.[9]

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[9] Newbery (1986) shows that if rents cannot be fully taxed and commodity taxes are differentiable, it may be optimal to violate production efficiency by taxing intermediate inputs.

Before turning to a comparison between the two regimes, recall that Keen (2008) allows for a withholding tax in his VAT regime, which in his context is important for arguing in favor of the VAT. In our context, the possibility of varying the VAT rates by commodity implicitly allows for the analog of the withholding tax. To see this, consider commodities $j \in \boldsymbol{F}$ and $i \in \boldsymbol{S}$ with $q_j = 1 + v_j$ and $q_i = p_i^S = 1 + v_i$, where $v_i = 0$ for $i \in \boldsymbol{X}^S$. Suppose producers of j and i use an importable commodity k as an input, so v_k is charged as a tax. Then, v_k may interpreted as a withholding tax as in Keen. With $q_k = 1 + v_k$, v_k is borne by the informal producers of i, while the formal producers of j can claim a credit.

4 Comparing Regimes Given the Informal Sector Size

In this section, the number and composition of producers in the formal and informal sectors ($\boldsymbol{F}$ and $\boldsymbol{S}$) are taken as given and the same in both regimes, despite the fact that each regime provides firms with differing incentives to opt for sector $\boldsymbol{S}$. Later, we allow for endogenous choice of sector size, in which case both the size and composition of the informal sector will differ in the two regimes. We compare household utilities u^T and u^V generated under the two regimes, given revenue requirements $\overline{G}$, and characterize the circumstances under which one regime will be preferred to the other. We begin with a comparison in the general case in which both formal and informal sectors of given composition exist and there are no constraints on taxes. We then discuss special cases of this general result when the size of one of the sectors is restricted or when policies are limited.

Suppose then that there are both formal and informal sectors, that is, $\boldsymbol{F} \neq \emptyset$, $\boldsymbol{S} \neq \emptyset$. We want to compare utility obtained in the trade tax regime, u^T, with that in the VAT regime, u^V. Recall that in the VAT regime, since no VAT is charged on exports, informal firms in the exportable sector receive no protection from the VAT, so $v_j = 0$ for $j \in \boldsymbol{X}^S$. To facilitate comparison between the two regimes, it is useful to consider the trade tax regime when we artificially impose the comparable restriction that $t_j = 0$ for $j \in \boldsymbol{X}^S$. Let $\boldsymbol{t}^C$ be the value of optimal trade taxes in this constrained regime, which yields the utility level u^C and profits R^C. Define the difference in utilities between the unrestricted and restricted trade tax regimes as $\Delta u^T \equiv u^T - u^C \geqslant 0$, with the equality applying when

$\boldsymbol{X}^S = 0$. Finally, let $\mathcal{L}_v^C$ be the value of the Lagrange expression in the VAT regime when tax rates satisfy $\boldsymbol{v} = \boldsymbol{t}^C$ with $\mu = \mu^V$ and $\lambda = \lambda^V$. (This will involve some negative VAT rates if $t_j < 0$ for $j \in \boldsymbol{X}^F$.) Using these definitions, the following proposition is proven in the Appendix.

Proposition 1: *Given* $\boldsymbol{F}$ *and* $\boldsymbol{S}$, $u^V > u^T$ *iff*

$$\lambda^V \left(R^{FV} - \sum_j R_{p_j}^{FC} \right) + \mathcal{L}_v^V - \mathcal{L}_v^C > (\lambda^V - \mu^V)(1 - \theta)(R^{FV} - R^{FC}) + \Delta u^T \qquad (16)$$

The various terms in expression (16) reflect the relative advantages of each regime. In the first term on the lefthand side, $R^{FV} - \sum_j R_{p_j}^{FC}$ gives the gain in production efficiency in the formal sector under the VAT. That is, R^{FV} is the value of maximized profits under the VAT where producer prices are world prices ($\boldsymbol{p}^T = \boldsymbol{1}$), while $\sum_j R_{p_j}^{FC}$ is the value of profits in the trade tax regime when the world prices are used to evaluate inputs and outputs. Naturally, this term will be positive. The first term on the righthand side involves the difference in formal sector profits in the two regimes (with the restriction that $t_j = 0$ for $j \in \boldsymbol{X}^S$). The term $(1 - \theta)(R^{FV} - R^{FC})$ is the increase in after-tax rent accruing to the household from substituting a VAT for trade taxes. It reflects the opportunity cost of the government failing to extract the rent. The sign of this term is generally ambiguous without further assumption: profits can be higher in either regime in general. However, to the extent that the trade tax regime indirectly taxes rents in the formal sector, R^{FC} will be lower and that will favor trade taxes. The final term on the righthand side, Δu^T, is positive as mentioned and reflects the fact that the VAT cannot be charged on informal exportables. Finally, the term $\mathcal{L}_v^V - \mathcal{L}_v^C$ is necessarily positive.

In general, which regime is preferable depends on the relative size of these various terms. However, there are a number of special cases of interest that we can deduce from this general case. First, consider the case where there are no informal exportables, so $\boldsymbol{X}^S = \emptyset$. This case is of interest because, as we shall see, when producers are free to choose their sector of operation, producers of exportables would not choose the informal sector in the VAT regime because the price at which they sell their output is the world price, whereas they have to pay tax-inclusive prices on their inputs and cannot obtain a

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credit if they operate informally. In this case, we have $\boldsymbol{t} = \boldsymbol{t}^C$, which implies that $\Delta u^T = 0$. Then, since $\mathcal{L}_v^V > \mathcal{L}_v^C$, the following corollary is immediate.

Corollary 1: $u^V > u^T$ when $\boldsymbol{X}^S = \emptyset$ and $\theta = 1$.

A similar consequence arises if, instead of assuming there is no informal exportable sector, we assume that trade taxes only apply to importables, so $t_j = 0$ for $j \in \boldsymbol{X}$. In this case, we have again that $\boldsymbol{t} = \boldsymbol{t}^C$, so $\Delta u^T = 0$ and $R^{FV} - \sum_j R_{p_j}^{FC} > 0$. This leads to the following corollary.

Corollary 2: $u^V > u^T$ when $t_j = 0$ for $j \in \boldsymbol{X}$ and either $\theta = 1$ or $R^{FC} > R^{FV}$.

On the other hand, there are some influences tending to favor the trade tax regime. As (16) indicates, lower values of θ tend to favor the trade tax regime, given that trade taxes indirectly tax profits in the formal sector. A larger informal sector might also favor trade taxes since it reduces the value of achieving production efficiency in the formal sector.[10]

Next, consider the case where there is no informal sector, so $\boldsymbol{S} = \emptyset$ and $\boldsymbol{F} = \boldsymbol{A}$. Recall that under trade taxation, producer and consumer prices in the formal sector are $\boldsymbol{p}^F = \boldsymbol{q} = \boldsymbol{1} + \boldsymbol{t}$, while in the VAT regime, we have $\boldsymbol{p}^F = \boldsymbol{1}$ and $\boldsymbol{q} = \boldsymbol{1} + \boldsymbol{v}$. Here that applies to the entire economy. Profits under optimal taxation in the trade and VAT regimes in this case are $R^T = \sum_j (1 + t_j^T) R_{p_j} (\boldsymbol{1} + \boldsymbol{t}^T)$ and $R^V = \sum_j R_{p_j}(\boldsymbol{1})$ by the homogeneity property of the profit function, where $R_{p_j} = \sum_i R_{p_j}^i$. The next corollary follows immediately from Proposition 1 and indicates a sufficient condition for the VAT regime to be preferable to the trade tax regime:

Corollary 3: With $\boldsymbol{S} = \emptyset$, $u^V > u^T$ if

$$R^V - \sum_j R_{p_j}^T > \frac{\lambda^V - \mu^V}{\lambda^V}(1 - \theta)(R^V - R^T) \tag{17}$$

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[10] The above results can be generalized to any case in which VAT rates are more constrained than trade taxes. Thus, suppose the set of feasible VAT rates, say Ω_v, is more restricted that the set of feasible trade tax rates, Ω_t, so $\Omega_v \subset \Omega_t$. For example, the VAT may be restricted to be uniform or non-negative, or subject to limited differentiation for administrative or other reasons. Then, u^C can be interpreted as maximized utility when the restriction $\boldsymbol{t} \subset \Omega_t$ is imposed, and $\Delta u^T = u^T - u^C$ is the loss in utility arising from the restriction in tax rates. Proposition 1 applies with this reinterpretation of Δu^T.

The intuition for this result is as follows. Note first that using optimal tax conditions (14) for the VAT regime with $\boldsymbol{S} = \emptyset$, we obtain:

$$\frac{\lambda^V}{\mu^V} = \left(1 + \sum_j \frac{v_j^V}{1 + v_j^V}(1 + v_j^V)\frac{E_{q_n q_j}}{E_{q_n}}\right)^{-1} = \left(1 + \sum_j \frac{v_j^V}{1 + v_j^V}\varepsilon_{nj}\right)^{-1}$$

where ε_{nj} is the compensated elasticity of demand for good n with respect to the price of good j. The ratio λ^V/μ^V can be interpreted as the marginal cost of public funds (MCPF) in the VAT regime. It is the social cost of transferring an increment of revenue from the household to the government using optimal commodity taxes. Therefore, we can write:

$$\frac{\lambda^V - \mu^V}{\lambda^V} = \frac{\text{MCPF} - 1}{\text{MCPF}} > 0$$

which is the increase in the marginal excess burden associated with a transfer of funds from the household to the government. Thus, the righthand side of (17) is the increase in after-tax rent accruing to the household from substituting a VAT for trade taxes, $(1 - \theta)(R^V - R^T)$, weighted by $(\text{MCPF}-1)/\text{MCPF}$. The sign of $R^V - R^T$ is generally ambiguous without further assumptions, though it will be larger the greater is the extent to which trade taxes indirectly tax profits. The lefthand side is the gain from removing the production inefficiency caused by trade taxation, which is positive.

Inspection of (17) confirms that $u^V > u^T$ if $R^V \leqslant R^T$ or $\theta = 1$.[11] That is, in the absence of an informal sector, the VAT will be preferred to trade taxes if profits are fully taxed or profits are higher under trade taxes. Keen and Ligthart (2002) consider the case where $\boldsymbol{S} = \emptyset$ and $\theta = 0$, and find that the VAT regime is preferred in their basic model. This is a consequence of their base-case assumption that there are no intermediate inputs, so $R_{p_j} > 0$ for all commodities, which implies that $R^V = R(\boldsymbol{1}) < R(\boldsymbol{1} + \boldsymbol{t}^T) = R^T$. This gives $u^V > u^T$ by Corollary 1. They also assume that trade taxes are imposed only on imports, and this reduces the attractiveness of the trade tax regime. This unambiguous

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[11] If profits are taxed at 100 percent, the consumers will need some source of revenues to purchase goods. This can be accommodated by supposing that good zero is an input supplied by consumers so $E_{q_0} < 0$.

preference for the VAT vanishes in our context since with intermediate goods, some of which may be net imports, we can have $R_{p_j} < 0$ for some goods. (Keen and Ligthart subsequently extend their analysis to allow for intermediate inputs.) More generally, (17) suggests that the VAT will be preferred if θ is above some cutoff level.

Finally, consider the opposite case where there is no formal sector. In this obviously extreme case, all activities take place in the informal sector. Trade taxes $\boldsymbol{t}$ apply to all commodities, while the VAT $\boldsymbol{v}$ applies only to imported commodities, so $v_j = 0$ for $j \in \boldsymbol{X}$. Thus, the VAT is more restrictive. With $\boldsymbol{F} = \emptyset$, Proposition 1 implies:

Corollary 4: *With* $\boldsymbol{F} = \emptyset$, $u^T > u^V$.

Intuitively, the trade tax regime can replicate the VAT regime, which applies only to imports, since domestic firms are all untaxed. Starting with the case where trade taxes replicate the VAT, trade taxes can be further optimized since exports can now be taxed.

5 Changes in the Size of the Informal Sector

The above analysis assumed that the allocation of firms to the formal and informal sectors was exogenous. Here, we consider the effects of changes in $\boldsymbol{S}$ and $\boldsymbol{F}$. We begin by considering the effects of moving one producer from the informal to the formal sector in each tax regime. This will establish some preliminary results that will be useful in the following section where we allow producer to choose sectors in each regime.

The Trade Tax Regime

Given $\boldsymbol{F}$ and $\boldsymbol{S}$, we can write the maximized utility of the representative consumer in the trade tax regime as simply the maximized value of the Lagrangian in (12), or $u^T = \mathcal{L}_t^T$. Suppose now that the representative producer of commodity n moves from sector $\boldsymbol{S}$ to sector $\boldsymbol{F}$. Given $\boldsymbol{F}$, $\boldsymbol{t}^T$ is optimal and the envelope theorem applies, so using (12) we have:

$$\Delta u_n^T = (\lambda^T - \mu^T)\left(\theta \Delta R^{FT} - \sum_j t_j^T \Delta R_{p_j}^T\right) + \mu^T \sum_j \Delta R_{p_j}^T$$

$$= (\alpha_n - 1)\mu^T r^{nT} - (\alpha_n - 1)\lambda^T \sum_j t_j^T r_{p_j}^{nT} + (\lambda^T - \mu^T)\theta \Delta R^{FT} \tag{18}$$

where we have used $\Delta R_{p_j}^T = (\alpha_n - 1)r_{p_j}^{nT}$, and $r^{nT} = \sum_j (1 + t_j^T)r_{p_j}^{nT}$ by homogeneity.

To interpret this, note that producers face the same prices $\boldsymbol{p}^T = \boldsymbol{1} + \boldsymbol{t}^T$ in both the formal and informal sectors. The move of the representative producer of n will therefore simply expand inputs and outputs by $(\alpha_n - 1)$ without improving production efficiency. The first term in (18) is the increase in rent accruing to the representative household due to the expansion of production of commodity n by $\alpha_n - 1$. The second term involves the change in tariff revenue, which can be of either sign. If commodity n is importable ($n \in \boldsymbol{M}$), an increase in the domestic output of n by $(\alpha_n - 1)r_{p_n}^{nT}$ lowers imports and decreases tariff revenue, whereas increasing inputs to produce n, $-(\alpha_n - 1)r_{p_j}^{nT} > 0$ ($j \neq n$), enhances government revenue when $t_j^T > 0$. In the case where $n \in \boldsymbol{X}$, both the increase in domestic output and the increased need for inputs tend to increase trade tax revenue. The third term is the net social value of transferring profits from the household to the government since the profits of commodity n are now taxed. In general $\varDelta u_n^T \gtrless 0$, given $\alpha_n > 1$, but for $\alpha_n = 1$ the following lemma is immediate and will be useful in what follows.

Lemma 1: *If $\alpha_n = 1$, $\varDelta u_n^T \gtrless 0$ as $\theta \gtrless 0$.*

The intuition for this is clear. Given that $\alpha_n = 1$, there is no change in output when a producer moves from the informal to the formal sector. However, the profits of the producer now become subject to taxation and these have a net social value of $\lambda^T - \mu^T$ per unit of tax revenue.

Lemma 1 has the following implication. Let $|\boldsymbol{F}|$ denote the number of commodities produced in the formal sector. Figure 1(a) depicts how per capita utility in the trade tax regime varies with the number of commodities in the formal sector in the range of producers for which $\alpha_j = 1$, assuming that at each point on the curve the government is optimizing trade taxes. The presumption in this figure is that firms that are more productive in the formal sector have already moved, a presumption that will apply in the next section. When $\theta = 0$, u^T remains unchanged as more producers with $\alpha_j = 1$ move to sector F.

The VAT Regime

In this case, given $\boldsymbol{F}$ and $\boldsymbol{S}$, the maximized utility of the representative consumer is the value of the maximized Lagrangian in (15), or $u^V = \mathcal{L}_t^V$. Consider again a move of

producers of commodity n from $\boldsymbol{S}$ to $\boldsymbol{F}$. This case is more complicated than the trade tax case because, as we have seen, the relevant producer may face a different tax rate in the two sectors. In particular, if the commodity is exportable, producers will bear no tax in the informal sector and this would have to be be taken into account in considering the consequences of a producer moving from one sector to another. In fact, our interest in the following section is limited to the case where only importables are produced in the informal sector in the VAT regime since, when producers are free to choose, those producing exportables will always opt for the formal sector (in order to take advantage of input tax crediting). We therefore focus here on the case where commodity n is an importable. Applying the envelope theorem to (15), we obtain the following:

$$\Delta u_n^V = (\lambda^V - \mu^V)\Big(v_n^V(E_{q_n}^V - R_{p_n}^{SV}) - \sum_j v_j^V \Delta R_{p_j}^{SV} + \theta \Delta R^F\Big) + \mu^V \sum_j (R_{p_j}^{nFV} - R_{p_j}^{nSV}) \quad (19)$$

for $n \in \boldsymbol{M}$, where

$$\sum_j (R_{p_j}^{nFV} - R_{p_j}^{nSV}) = (\alpha_n - 1)\sum_j r_{p_j}^n(\boldsymbol{1}) + \sum_j r_{p_j}^n(\boldsymbol{1}) - \sum_j r_{p_j}^n(\boldsymbol{1} + \boldsymbol{v}^V) > 0 \quad (20)$$

The first term on the righthand side of (20) is just $(\alpha_n - 1)r^n(\boldsymbol{1})$ by the homogeneity of the profit function. The latter two terms represent the change in the value of aggregate net profits by producers of commodity n evaluated at world prices. Thus, (20) indicates that the reallocation of the producer of commodity n from the informal to the formal sector improves production efficiency and, if $\alpha_n > 1$, expands output.

Since the sign of the last term in (19) is positive, the sign of Δu_n^V will depend on the sign of $-\sum_j v_j^V \Delta R_{p_j}^{SV}$. This is the additional net revenue that the VAT can raise from producers of commodity n, given that n is importable. Since $\Delta R_{p_j}^{SV} = -R_{p_j}^{nSV}$, when the producer of commodity n leaves the informal sector, we obtain, using the homogeneity of the profit function:

$$-\sum_j v_j^V \Delta R_{p_j}^{SV} = \sum_j v_j^V R_{p_j}^{nSV} = \sum_j (1 + v_j^V)R_{p_j}^{nSV} - \sum_j R_{p_j}^{nSV} = r^{nSV} - \sum_j r_{p_j}^{nSV}$$

$$= r^n(\boldsymbol{1} + \boldsymbol{v}^V) - r^n(\boldsymbol{1}) + r^{nFV} - \sum_j r_{p_j}^{nSV} > 0, \qquad \text{if} \qquad r^n(\boldsymbol{1} + \boldsymbol{v}^V) \geqslant r^n(\boldsymbol{1}) \quad (21)$$

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Thus, we have:

Lemma 2: *If producer $n \in \boldsymbol{M}$ moves from $\boldsymbol{S}$ to $\boldsymbol{F}$, $\Delta u_n^V \geqslant 0$ if $r^n(\boldsymbol{1} + \boldsymbol{v}^V) \geqslant r^n(\boldsymbol{1})$.*

As we shall see below, when producers are free to choose their sector, the condition $r^n(\boldsymbol{1} + \boldsymbol{v}^V) \geqslant r^n(\boldsymbol{1})$ will necessarily apply for firms moving since it will apply for all firms who choose to operate in the informal sector. Figure 1(b) depicts the effect of moving producers of importables from the informal to the formal sector, assuming this condition applies, where again the horizontal axis indicates simply the number of commodities produced in the formal sector. Note that Lemma 1 applies for any value of the profit tax rate θ. However, a higher value of θ implies a larger value of Δu_n^V, and therefore a steeper curve in Figure 1(b).

Lemmas 1 and 2, and their representation in Figures 1(a) and 1(b), will be useful in the following section when we allow firms to choose whether to produce in the formal or informal sector. Note that, while the horizontal axes in these figures measure the number of commodities produced in the formal sector in the trade tax and VAT regimes, the identity of the commodities will generally not be the same in the two regimes, as we shall see.

6   Endogenous Choice of Sector

In comparing the two regimes in Proposition 1 and its corollaries, we assumed that the same producers are in the informal sector in both regimes. Suppose, following de Paula and Scheinkman (2007), that producers of each commodity can choose between the formal and the informal sector in each tax regime. Both the size of the informal sector and its composition can now differ between the two regimes since each offers different advantages to different producers, depending among other things on the size of their productivity parameter α_j, the rate of profit tax in the formal sector θ, and how taxes affect their profits. To simplify the exposition, we assume initially in this section that profits in the formal sector are not taxed ($\theta = 0$). This is not an innocuous assumption since, although the taxation of profits is the same in both regimes, the size of the informal sector in the two regimes will differ so different amounts of profit tax revenues will be generated in each. We return to the consequences of θ being positive below.

The representative producer of commodity j will choose between the formal and in-

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formal sectors by comparing profits in each regime.[12] Producer j will prefer the formal sector iff $R^{jF} > R^{jS}$. If $R^{jF} = R^{jS}$, we arbitrarily assume that the producer stays in the informal sector. This is equivalent to assuming that the default option for producers is to be in the informal sector. The rationale is that there may be additional costs of participating in the formal sector that are not captured in $r^j(\cdot)$, such as auditing, bookkeeping, regulations, and so on that we have not formulated explicitly. For example, we could have allowed there to be a small fixed cost associated with joining the formal sector. Consider the two regimes in turn.

The Trade Tax Regime

In this case, producers face the same producer prices in both sectors, which in the optimum are given by $\boldsymbol{p}^T = \boldsymbol{1} + \boldsymbol{t}^T$. The condition for producer j to join the formal sector, $R^{jFT} > R^{jST}$, becomes $\alpha_j r^j(\boldsymbol{1} + \boldsymbol{t}^T) > r^j(\boldsymbol{1} + \boldsymbol{t}^T)$, or $\alpha_j > 1$. Intuitively, since producers face the same taxes in the formal and the informal sector under trade taxes, they will join the formal sector only if they are more productive there.

More formally, define the set of producers who opt for the formal sector as $\boldsymbol{F}_t^0 \equiv \{j \in \boldsymbol{A} | \alpha_j > 1\}$. Then, all $j \in \boldsymbol{F}_t^0$ voluntarily participate in the formal sector, so that the set of firms in the formal sector under trade taxes is $\boldsymbol{F} = \boldsymbol{F}_t^0$. Analogous to above, let $|\boldsymbol{F}_t^0|$ be the size of the formal sector (i.e., number of commodities produced there).

The VAT Regime

The choice of sector by firms is more complicated in the VAT case because, besides firms possibly having different productivities in the two sectors, they may also face different tax rates for both their inputs and outputs. Suppose that in the optimum, all tax rates are non-negative, so $\boldsymbol{v}^V \geqslant 0$.

Consider first producers of exportables. For $j \in \boldsymbol{X}^S$, $v_j^V = 0$ as we have seen. A producer $j \in \boldsymbol{X}$ who opts for the informal sector will pay a tax-inclusive price on inputs purchased from those in sectors other that X^S. This implies, as mentioned earlier, that

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[12] Recall that all producers of a given commodity are identical so they will all choose the same sector. This is obviously a simplification used for tractability. In the real world, a given commodity may be produced in both sectors. Moreover, a given producer may be only partly in the informal sector, for example, if some but not all taxes are evaded.

for this producer $R^{jSV} = r^j(\mathbf{1} + \mathbf{v}^V) < r^j(\mathbf{1}) \leqslant R^{jFV}$. Producer j will choose the formal sector to obtain an input tax credit. Thus, if we assume that all producers in $\mathbf{X}$ will use some inputs that have been taxed, $\mathbf{X}^S = \emptyset$ under the VAT regime. In what follows, we assume that to be the case.

Producer j of an importable good will operate in the formal sector if and only if $R^{jFV} = \alpha_j r^j(\mathbf{1}) > r^j(\mathbf{1} + \mathbf{v}^V) = R^{jSV}$. Since producers of importables operating in the informal sector obtain consumer prices $q_j^V = 1 + v_j^V$ on their sales, it may be more profitable to be in the informal sector than in the formal sector even if $\alpha_j > 1$. The set of producers who prefer the formal sector can be defined as $\mathbf{F}_v^0 \equiv \{j \in \mathbf{A} | \alpha_j r^{jF}(\mathbf{1}) > r^{jS}(\mathbf{1} + \mathbf{v}^V)\}$, and $|\mathbf{F}_v^0|$ will be the size of the formal sector in the VAT regime.[13]

The following lemma summarizes the size of the formal sector in each regime.

Lemma 3:

 i. *In the trade tax regime, $\mathbf{F} = \mathbf{F}_t^0 \equiv \{j \in \mathbf{A} | \alpha_j > 1\}$.*

 ii. *In the VAT regime, $\mathbf{X} \subseteq \mathbf{F}_v^0 = \mathbf{F}$, where $\mathbf{F}_v^0 \equiv \{j \in \mathbf{A} | \alpha_j r^{jF}(\mathbf{1}) > r^{jS}(\mathbf{1} + \mathbf{v}^V)\}$.*

The sets of firms that join the formal and informal sectors in the two regimes will generally be overlapping. In the trade tax regime, all firms with $\alpha_j > 1$ will join the formal sector, while those with $\alpha_j = 1$ will not. With the VAT, some firms with $\alpha_j = 1$ will join the formal sector, while some with $\alpha_j > 1$ will not. Moreover, the relative size of the two regimes is ambiguous: we can have either $|\mathbf{F}_v^0| > |\mathbf{F}_t^0|$ or $|\mathbf{F}_t^0| > |\mathbf{F}_v^0|$. Below, we consider each case in turn.

To facilitate welfare comparisons in the two regimes, consider the thought experiment of starting with the formal sector existing in each regime, $\mathbf{F}_t^0$ and $\mathbf{F}_v^0$, and moving producers from the informal sector to the formal sector. The following proposition applies.

Proposition 2: *Assuming $\theta = 0$,*

 i. *In the trade tax regime, u^T is constant when $\mathbf{F}$ is expanded beyond $|\mathbf{F}_t^0|$.*

 ii. *In the VAT regime, u^V will rise when $\mathbf{F}$ is expanded beyond $|\mathbf{F}_v^0|$.*

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[13] In the formal sector in equilibrium $\mathbf{F}_v^0$ depends on $\mathbf{v}^V$, but at the same time, the choice of $\mathbf{v}^V$ will based on $\mathbf{F}_v^0$. In principle, optimal tax rates $\mathbf{v}^V$ should be chosen taking into account how $\mathbf{F}_v^0$ is affected by $\mathbf{v}$. For simplicity, we do not take this interdependency into account in deriving our optimal tax rates and consequent results since our qualitative results are not affected.

The first part follows from Lemma 1, given that $\theta = 0$ and $\alpha_j = 1$ for producers not already in $\boldsymbol{F}_\tau^0$ (see Figure 1(a).). The second part follows from the fact that firms that will move will be those in $\boldsymbol{M}$ for whom $r^j(\boldsymbol{1} + \boldsymbol{v}^V) \geqslant \alpha_j r^j(\boldsymbol{1}) \geqslant r^j(\boldsymbol{1})$.[14] Then, Lemma 2 ensures that $\Delta u_n^V > 0$. Intuitively, a move of an additional firm from $\boldsymbol{M}^S$ to $\boldsymbol{M}^F$ increases tax revenue and improves production efficiency, which are social benefits not fully taken into account by private producers.

An implication of this proposition will be exploited in the following section. If the government can influence the size of the informal sector, it would have no incentive to do so in the trade tax regime if $\theta = 0$. On the other hand, it might want to take measures to increase the size of the formal sector in the VAT regime if they are not too costly. Given that, it might be able to make u^V higher than u^T even though the trade tax regime would be preferred in the absence of such measures if producers were free to choose their own sector.

A further implication of this proposition is as follows. Suppose that condition (17) in Corollary 3 is not satisfied when $\theta = 0$, so when there is no informal sector, $u^T > u^V$. This would happen, for example, if trade taxes are very effective at indirectly taxing profits in the formal sector. In these circumstances, Proposition 1 would imply that regardless of the size of the informal sector, the trade tax regime would be preferred to the VAT regime. Alternatively, suppose that (17) is satisfied, which implies that $u^V > u^T$ when $\boldsymbol{F} = \boldsymbol{A}$. The consequences of that can be seen if we consider the two cases, one where the size of the formal sector is larger in the trade tax regime than in the VAT regime when producers freely choose their sector, and the other where the opposite is the case. We continue to assume that $\theta = 0$ for the time being.

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[14] Formally, the proof that these inequalities apply assumes that the equilibrium $\boldsymbol{F}_v$ is stable. The equilibrium value of $\boldsymbol{F}_v$ depends on $\boldsymbol{v}$, while at the same time the optimal choice of $\boldsymbol{v}$ depends on $\boldsymbol{F}_v$. (Recall that we assume optimal taxes are calculated given the size of $\boldsymbol{F}$.) With some abuse of notation, let $\boldsymbol{F}_v(\boldsymbol{v}) \equiv \{j \in \boldsymbol{A} | \alpha_j r^j(\boldsymbol{1}) > r^j(\boldsymbol{1} + \boldsymbol{v})\}$ be the size of the formal sector given $\boldsymbol{v}$, and let $\boldsymbol{v}(\boldsymbol{F}_v)$ be optimal tax rates given the size of the formal sector $\boldsymbol{F}_v$. Consider some initial value of $\boldsymbol{F}_v' \supset \boldsymbol{F}_v^0$. Given $\boldsymbol{F}_v'$, optimal tax rates will be $\boldsymbol{v}(\boldsymbol{F}_v')$, which in turn will lead to a size of the formal sector $\boldsymbol{F}_v'' = \boldsymbol{F}_v(\boldsymbol{v}(\boldsymbol{F}_v'))$. Stability at $\boldsymbol{F}_v^0$ requires that $\boldsymbol{F}_v'' \subset \boldsymbol{F}_v'$. Then, $\boldsymbol{S}' \subset \boldsymbol{S}''$, so producers in $\boldsymbol{S}'$ do not have an incentive to participate in the formal sector. That is, $r^j(\boldsymbol{1} + \boldsymbol{v}^V(\boldsymbol{F}')) \geqslant \alpha_j r^j(\boldsymbol{1}) \geqslant r^j(\boldsymbol{1})$.

Case 1: $|\boldsymbol{F}_t^0| > |\boldsymbol{F}_v^0|$

This case would apply if the number of firms for which $\alpha_j > 1$ is relatively large, the extreme case being that $\alpha_j > 1$ for all firms. As we saw in Proposition 1, for a given size of the informal sector, either regime can dominate. Naturally, the same ambiguity will generally apply when the sizes of the sectors are endogenous. Nonetheless, endogeneity of sector size may influence which sector is preferable. To illustrate the possibilities, suppose we start at with the trade tax regime, where the size of the formal sector is $|\boldsymbol{F}_t^0|$. If we switch to the VAT regime, both the number and the composition of producers in the formal sector will change if producers are free to choose their sector. Suppose we conceptually decompose the change into two steps, the first one a change in the composition of the formal sector, holding the total number of producers fixed, and the second a change in the number of producers. The change in composition holding the number of producers in the formal sector fixed could cause utility to rise or fall. Then, given that $|\boldsymbol{F}_t^0| > |\boldsymbol{F}_v^0|$, producers will reallocate from $\boldsymbol{F}$ to $\boldsymbol{S}$ and utility will fall by Proposition 2ii. The end result may be higher or lower utility in the VAT regime.

Figure 2 illustrates the case where utility is ultimately lower in the VAT regime. Start in the trade tax regime where the number of commodities produced in the formal sector is given by $|\boldsymbol{F}_t^0|$ and household utility is u^T. When the VAT is substituted for the trade tax and the size of the informal sector is held constant at $|\boldsymbol{F}| = |\boldsymbol{F}_t^0|$, utility could rise as in the figure to $u^{V'}$. However, once the size of the formal sector is allowed to adjust, utility under the VAT will fall as the size of the formal sector falls to $\boldsymbol{F}_v^0$, where $u^{V^o} < u^T$. Of course, this is a purely heuristic argument since the composition of firms in the two sectors will generally differ between the two regimes, for any given size of the formal sector. But it does illustrate that both the size and the composition of the informal sector influence the ranking of the two regimes.

Case 2: $|\boldsymbol{F}_v^0| > |\boldsymbol{F}_t^0|$

Contrary to the previous case, here the number of firms for which $\alpha_j > 1$ is small, possibly no firms. In this case, virtually anything can happen in moving from the trade tax to the VAT regime. Although Proposition 2 indicates that u^T will remain constant, u^V is only

assured to increase monotonically once $|\boldsymbol{F}_v^0|$ is reached. Whether trade taxes are preferred to the VAT depends both on the relative size of u^T and u^V at $|\boldsymbol{F}_t^0|$, but also on how the latter changes when $\boldsymbol{F}$ expands from $|\boldsymbol{F}_t^0|$ to $|\boldsymbol{F}_v^0|$.

Figure 3 illustrates one possibility. In this figure, utility under the trade tax at $|\boldsymbol{F}_t^0|$, u^T, is higher than utility under the VAT for the same number of producers in the two sectors (though a different composition), denoted $u^{V'}$. Suppose that u^V rises as producers move to the formal sector because of the regime change. It is possible that, as in Figure 3, utility can end up being higher in the VAT regime than in the trade tax regime ($u^{V^o} > u^T$). Of course, other outcomes might occur. The utility gain from the reallocation may not be sufficient to cause the VAT regime to be preferred in this case.

Reallocations induced by a change in regime could alternatively reinforce the outcome of a switch from the trade tax to the VAT regime. Thus, if $u^T > u^{V'}$ in Case 2 where $|\boldsymbol{F}_t^0| > |\boldsymbol{F}_v^0|$, the reduction in welfare from switching to the VAT will be exacerbated by the induced reduction in the size of the formal sector. Similarly, in Case 1 where $|\boldsymbol{F}_v^0| > |\boldsymbol{F}_t^0|$, if $u^{V'} > u^T$ given the set of formal producers under the trade tax, $\boldsymbol{F}_t^0$, the increase in the size of the formal sector when the VAT is introduced improves the situation further.

The above discussion assumed that $\theta = 0$. Matters become slightly more complicated when $\theta > 0$, but similar outcomes can occur. Increasing θ above zero has the following effects. Since profits are only taxed in the formal sector, the criterion for choosing the formal sector becomes $(1 - \theta)R^{jF} > R^{jS}$. The sizes of the formal sector in both regimes, $|\boldsymbol{F}_t^0|$ and $|\boldsymbol{F}_v^0|$, will thus fall, though their relative size remains ambiguous. In the trade tax regime, by Lemma 1, u^T will now rise if, starting at $\boldsymbol{F}_t$, producers were to move from sector S to F, as Figure 1(a) indicates. At the same time, as (19) shows, u^V will increase more in the VAT regime as more commodities are produced in sector F, so the curve in Figure 1(b) becomes steeper. As well, as $\boldsymbol{S}$ approaches zero, u^V would rise relative to u^T. As Corollary 3 indicates, there will be some value of $\theta \geqslant 0$ such that $u^V > u^T$ when $\boldsymbol{S} = \emptyset$. Despite these changes, the qualitative comparisons made with $\theta = 0$ still apply when $\theta > 0$.

The upshot is that the comparison between VAT and trade tax regimes must take account of changes in the size of the informal sector induced by the reform itself. In

general, both the size and the composition of the two sectors will differ between the two regimes, and that makes the preferred outcome case-specific.

7 Tax Administration

In the previous section, we showed that the choice between the two tax regimes can be influenced by the endogeneity of the size of the formal sector. Thus, for example, when $\theta = 0$, even though trade taxes may be preferred to the VAT for a given size of formal sector, the ranking can be reversed once account is taken of the fact that producers can choose their sectors endogenously. This can be the case if the chosen size of the formal sector is higher under the VAT than under trade taxes. The reason is that per capita utility is increasing in the size of the formal sector under the VAT but not under trade taxes. (Alternatively, allowing producers to leave the formal sector when a VAT is instituted may be the cause of the trade tax being preferred, as in the example of Figure 3.) Relying on producers to move endogenously to the formal sector when a VAT is substituted for trade taxes may not be sufficient to make the VAT regime preferable. Suppose that instead of the size of the informal sector being determined solely by producers, the government can also influence it by incurring a cost.[15] This might serve as a mechanism for exploiting the fact that utility under the VAT is increasing with the size of the formal sector. In this section, we discuss some possible consequences of allowing the size of the formal sector to be influenced by tax enforcement rather than relying solely on producer discretion.

Suppose for concreteness that profits R^j are not the sole influence on the choice of sector. In sector S, producers may face some expected cost associated with the chances that they will be audited or investigated and prosecuted for tax evasion. Let the monetary value of such costs for producer j by c_j^S, where c_j^S is increasing in the intensity of tax enforcement by the government. At the same time, producers in the formal sector may encounter corrupt tax officials who demand bribes or extortionary payments in return for under-

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[15] This is analogous to the case studied by Slemrod and Kopczuk (2002), where the government has an additional policy instrument that affects the number of commodities entering the tax base. The optimal use of the instrument trades off its cost with the reduction in distortions from having a broader tax base to achieve what they refer to as the optimal elasticity of taxable income with respect to the tax rate.

reporting or not over-reporting tax liabilities. Let c_j^F be the monetary value of payments to corrupt officials, where c_j^F is decreasing in administrative effort to reduce corruption. The participation rate in the formal sector may then be written $R^{Fj} - c_j^F > R^{Sj} - c_j^S$, so more producers will join the formal sector the greater is administrative effort. Increases in administrative effort will be costly and will therefore affect the government budget constraint, but we can assume for simplicity that neither c_j^F nor c_j^S directly affect the representative household's budget. In the case of c_j^F, the cost of bribery or extortion simply redistributes money from the taxpayer to the official, both of whom are included in the representative household. The expected costs of prosecution for tax evasion, c_j^S, can be considered non-pecuniary. Alternatively, it can be assumed that if they are monetary sanctions, the proceed are returned as a lump sum to the household (to avoid giving the government a purely revenue-raising incentive to improve enforcement).

With this kind of reasoning in mind, we can capture the effect of improved tax administration in a very rough-and-ready way. Assume that the government can change the size of the informal sector by incurring administrative costs that are convex in the change in size of formal sector starting from $\boldsymbol{F}_v^0$ or $\boldsymbol{F}_t^0$ depending on the regime. Our approach is to consider the value of changing the size of the formal sector in the two regimes when taxes have been set optimally and producers are choosing their preferred sector. For simplicity, we continue to assume that $\theta = 0$ as in the previous section.

In the case of the trade tax regime, the choice of enforcement is straightforward. By (18), or equivalently Lemma 1, starting with optimal trade taxes in place and producers freely choosing their sector, $\Delta u_n^T = 0$ for $n \notin \boldsymbol{F}_t^0$ since $\alpha_n = 1$. Thus, the government has no motive to expand $\boldsymbol{F}$ beyond $\boldsymbol{F}_t^0$. To do so would simply waste resources without improving consumer welfare. In the VAT regime, $\boldsymbol{F}$ should be increased if the benefit from inducing a producer to move from sector S to F exceeds the costs. The benefit is given by Δu_n^V in (19). The cost is given by the administrative cost incurred to increase the size of the informal sector by increasing c_j^S and decreasing c_j^F.

Figure 4 illustrates how the option of varying the size of the formal sector by costly enforcement may influence the choice of tax regime. The figure indicates a case where, corresponding with that in Figure 2, $u^T > u^{V^o}$ when producers freely choose their sector

and the government exerts no special administrative effort, while $u^V > u^T$ as $\boldsymbol{F}$ approaches $\boldsymbol{A}$. Enforcement induces the size of the formal sector to increase, which in the VAT regime causes u^V net of administrative costs, denoted $\overline{u}^V$, to increase as long as enforcement costs are not too large. The figure shows an example where $\overline{u}^V$ rises above u^T, implying that it may be optimal to adopt the VAT regime when administrative costs are not too high, even though the trade tax regime is better in the absence of enforcement effort.

This finding assumes that $u^V > u^T$ as $\boldsymbol{S}$ approaches zero, which may not be the case when $\theta = 0$. This might suggest that accompanying administrative reform with more effective taxation of income in the formal sector may be needed to ensure that a move to a VAT will be beneficial. However, as we have seen, increasing θ induces more producers to stay in the informal sector, so that higher administrative costs are required to increase the size of the formal sector sufficiently to make $u^V > u^T$.

This result assumes that administrative costs are not too high. One implication might be that lowering such costs through the modernization of tax administration should be a part of tax reform. Indeed, it might be a necessary requirement to justify adopting a VAT. As Ebrill *et al* (2001) point out, the implementation of a VAT is typically accompanied by an improvement in tax administration, for example, by modernizing the administrative organization and instituting better information auditing and reporting. In our model, the VAT regime gives the government an incentive to modernize its tax administration to lower the cost of tax collection. In the trade tax regime, the costs of administration may remain high since there is no incentive for improving tax administration.

8 Concluding Remarks

The purpose of this paper has been to use optimal commodity tax analysis to compare a VAT-type commodity tax system with a system of trade taxes as means of raising revenues in an economy with a non-negligible informal sector. The choice of an optimal commodity tax approach has its advantages and disadvantages. It is somewhat restrictive in limiting government policy instruments to indirect rather than direct taxes, but this may not be too unrealistic in the context of developing countries for which the analysis is mainly relevant. It is also somewhat restrictive in its focus on efficiency in the tax system to the exclusion

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of redistributive concerns. However, given that the emphasis in the literature on the choice of a tax system for developing countries has been on efficiency, this is perhaps the most suitable approach for sorting out the merits of VAT versus trade tax systems. The optimal tax approach is flexible enough to be able to consider commodity taxes and trade taxes in their most efficient forms, since we have allowed the government to differentiate tax rates across commodities for both systems.

Each tax system has its advantages, and depending on the circumstances, one or the other may be preferred on efficiency grounds. As we have emphasized, the main advantage of the VAT regime is that it leads to production efficiency in the formal sector. As well, it succeeds in taxing informal producers to the extent that they purchase inputs from the formal sector. On the other hand, it does not tax sales by the informal sector or the pure profits of formal firms. The trade tax regime, while it distorts production efficiency, does tax sales by both sectors (by assumption) and indirectly taxes the profits of both sectors as well, which is advantageous as long as there are limits to the direct taxation of profits. More generally, we have allowed the size of the informal sector to respond endogenously to the tax system. As it turns out, both tax systems have an influence on the size of the informal sector, although the relative magnitude and efficiency of that effect is ambiguous. With trade taxes, firms will choose between sectors based on their relative productivity in each sector (assuming profits taxes are not important). Under the VAT, some firms might come to the formal sector even though they have no productive advantage from doing so, and in the end endogeneity of the choice of sector by producers might itself influence which tax system is preferred.

In developing countries, weak tax administration and enforcement are often cited as major obstacles to adopting a VAT, given that the latter requires more information and special skills to administer effectively. This is reflected in the fact that previous analyses of the choice of tax regime have typically taken as given the size of the informal sector. An important aspect of our analysis is not only to let the size of the informal sector differ between tax regimes, but also to consider the effect of allowing the government to devote resources to tax administration. In our model, the government may be more motivated to enhance tax enforcement under a VAT regime than with trade taxes. Moreover, the ability

to affect the size of the informal sector may be a determinant of the preferred regime. This serves to emphasize the fact that it is necessary to consider reform of the tax system and reform of the tax administration as going hand in hand.

Some of the other assumptions we have made are also quite strong. Most importantly, we have modeled evasion in a very simple way, following the related underlying literature. We have assumed, for example, that there can be no evasion of trade taxes by informal firms, although they can evade the VAT on their sales. This facilitates our optimal policy analysis. Perhaps more important, we have ignored the possibility that firms in the formal sector might also evade some taxes by, for example, failing to report all their sales to the tax authority while claiming input tax credits. A more complete analysis would include a detailed specification of tax evasion as a risky decision with firms subject to the possibility of detection and penalty, and perhaps even facing corruption or extortion by tax administrators. We have also ignored any differences in the costs of administering the two systems, including costs borne by taxpaying firms themselves. It is not clear how a more complicated tax administration system would influence the choice between trade taxes and the VAT.

We have assumed that all commodities are taxable, which simplifies the analysis considerably. Moreover, we have assumed that informal producers are like formal producers, except perhaps less productive. In practice, informal producers tend to be small, and typically operate at the retail level selling mainly to final consumers rather than firms. Indeed, may firms in the formal sector may well fall below the threshold value of sales to qualify as VAT-registered firms. The distinction between informal firms and those legitimately below the threshold size is moot since our analysis applies to both.

We have also taken the rate of tax on profits to be given and less than 100 percent. This is important because as Proposition 3 indicates, higher profit tax rates tend to favor the VAT regime. A more general analysis would include the reform of both indirect taxes (VAT and trade taxes) and direct taxes (profit and income taxes). Such a coordinated approach would have implications not just for the choice of tax regime, but also for the optimal allocation of resources devoted to improving tax administration.

Finally, our approach has been to analyze optimal policies separately for trade taxes

and the VAT, and to do a global comparison between them. This approach is suitable as a means of determining the main factors that work in favor of one regime or the other. It would be useful to allow the government to select a mix of commodity and trade taxes, since presumably on optimal policy grounds that would improve efficiency. Each tax could then be directed at transactions for which it was best suited. (See Munk (2005) for an analysis of the optimal mix of commodity taxes and tariffs with administrative costs.) Presumably a disadvantage of that approach in practice is that two tax systems operating at the same time would involve some duplication of administrative costs.
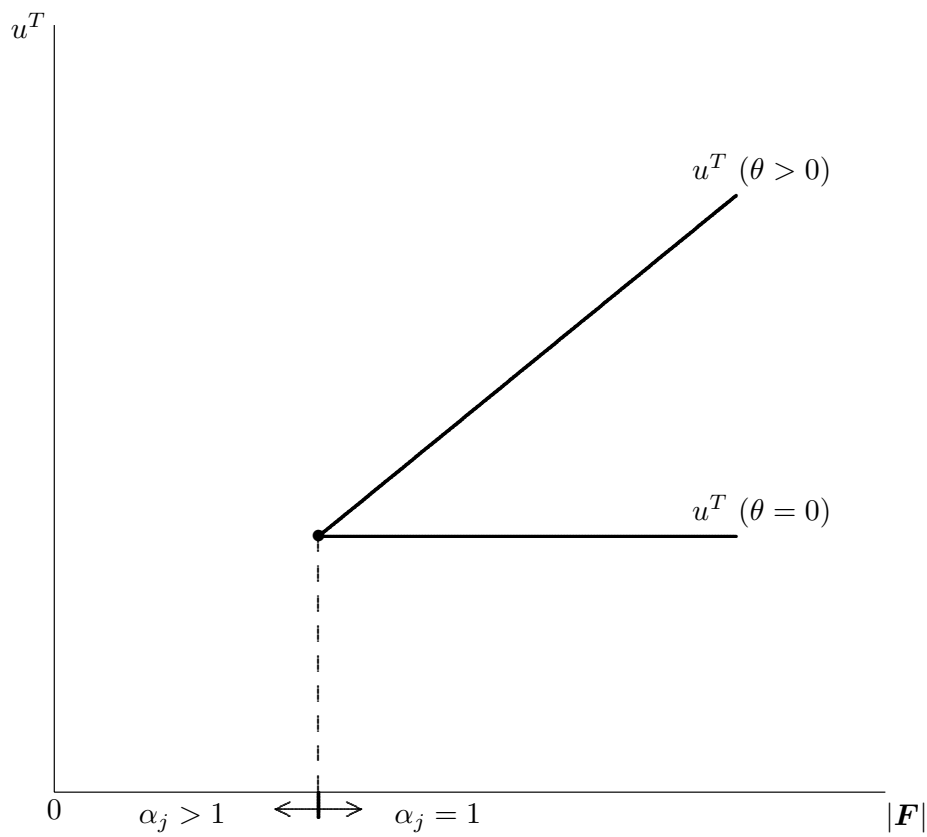
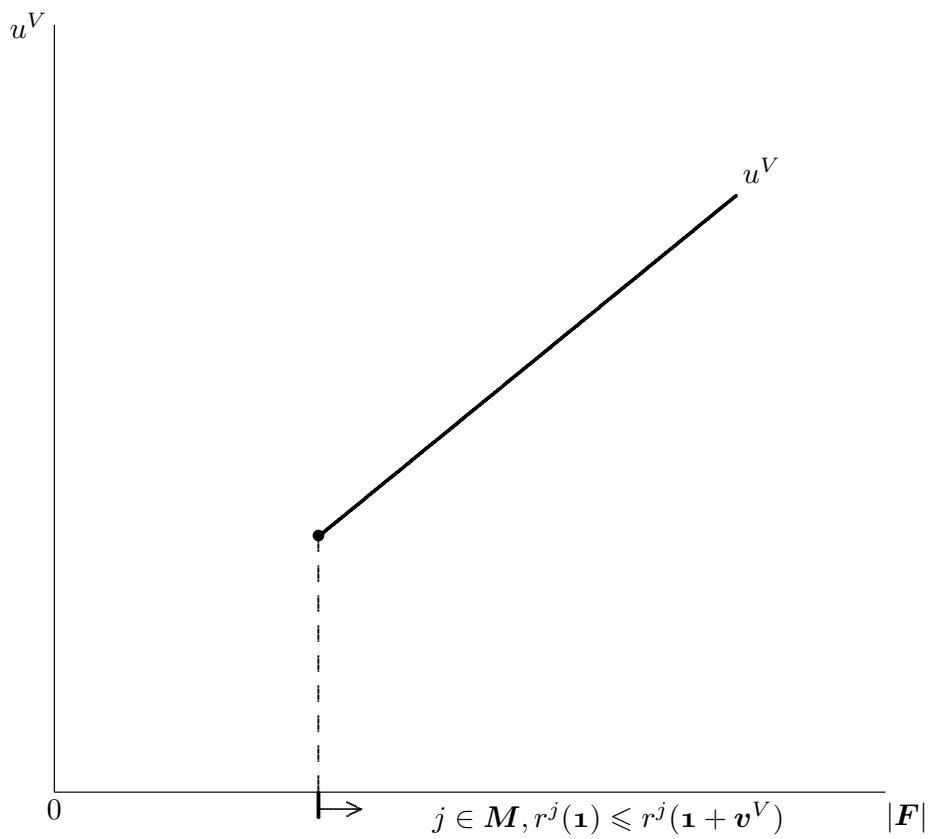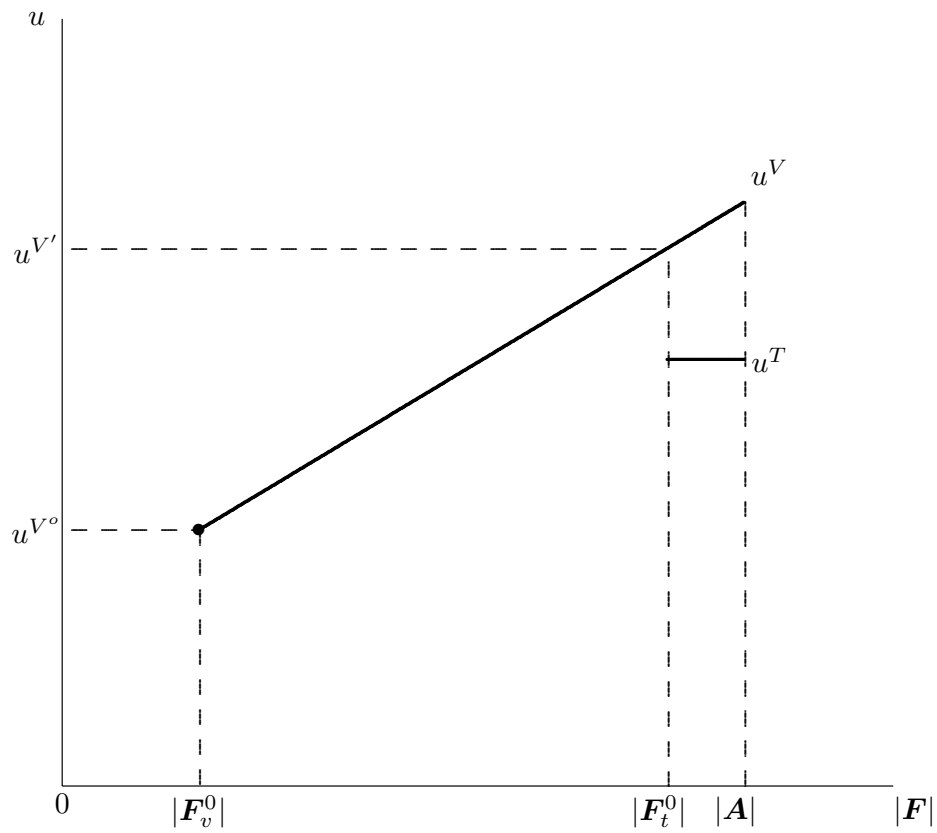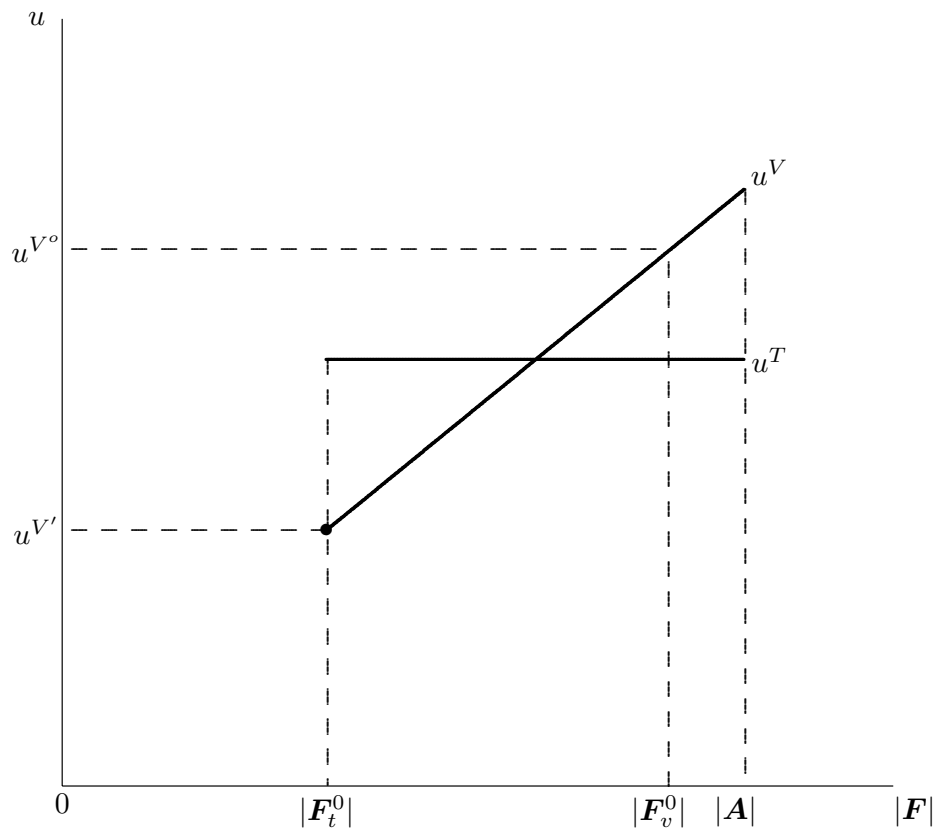**Figure 1(a). Utility and the Size of the Formal Sector with Trade Taxes**

Figure 1(b). Utility and the Size of the Formal Sector with a VAT

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Figure 2. VAT vs. Trade Taxes when $|\boldsymbol{F}_t^0| > |\boldsymbol{F}_v^0|$

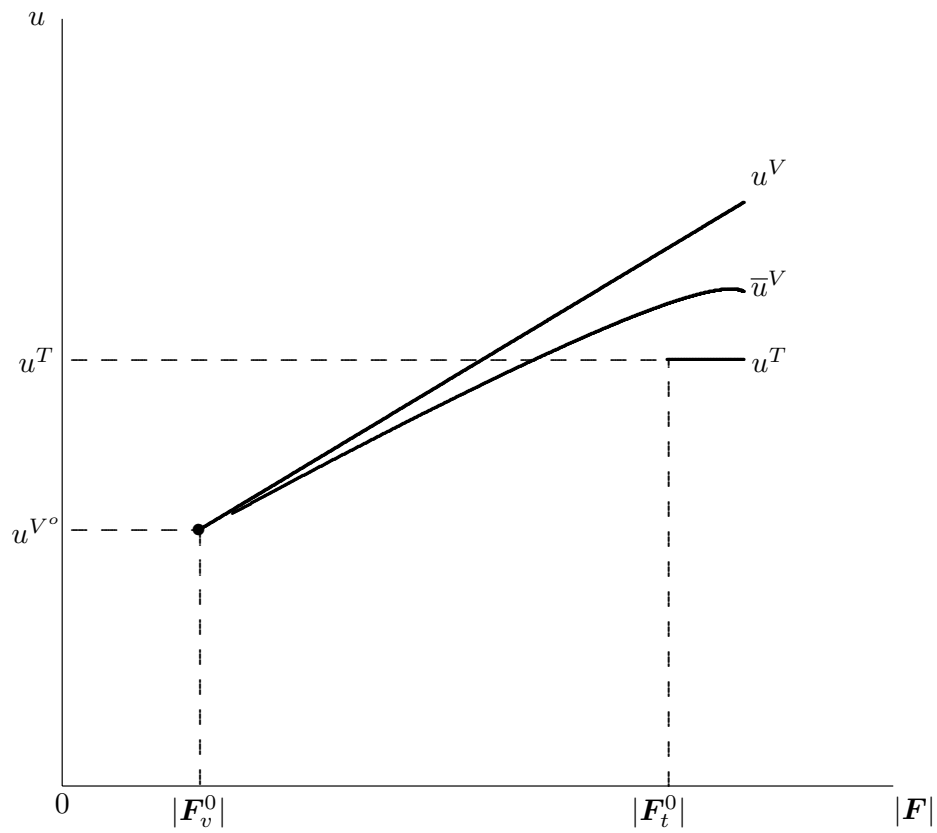**Figure 3. VAT vs. Trade Taxes when $|\boldsymbol{F}_v^0| > |\boldsymbol{F}_t^0|$**

Figure 4. Effect of Tax Administration Effort

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Appendix

Proof of Proposition 1

Note first that the government budget constraint (7) can be written, using the homogeneity property $R^F = \sum_j (1 + t_j) R_{p_j}^F$:

$$\sum_j t_j (E_{q_j} - R_{p_j}^S) + \sum_{j \notin \boldsymbol{X}^S} v_j (E_{q_j} - R_{p_j}^S) + \sum_j R_{p_j}^F - (1 - \theta) R^F = \overline{G} \qquad (A.1)$$

Substituting (A.1) into (8) yields:

$$\mathcal{L} = u + (\lambda - \mu) \left(\sum_j t_j (E_{q_j} - R_{p_j}^S) + \sum_{j \notin \boldsymbol{X}^S} v_j (E_{q_j} - R_{p_j}^S) + \sum_j R_{p_j}^F - (1 - \theta) R^F \right)$$

$$+ \mu \sum_j (R_{p_j} - E_{q_j}) - \lambda \overline{G} \qquad (A.2)$$

Denote by superscript C outcomes in the constrained trade tax regime where $t_j = 0$ for $j \in \boldsymbol{X}^S$. The optimized value of Lagrangian for this constrained case is $\mathcal{L}_t^C$ and may be written as follows:

$$\mathcal{L}_t^C = u^C + (\lambda^C - \mu^C) \left(\sum_{j \notin \boldsymbol{X}^S} t_j^C (E_{q_j}^C - R_{p_j}^{SC}) + \sum_j R_{p_j}^{FC} - (1 - \theta) R^{FC} \right)$$

$$+ \mu^C \sum_j (R_{p_j}^C - E_{q_j}^C) - \lambda^C \overline{G} \qquad (A.3)$$

where superscript C indicates that all variables and Lagrange multipliers are being valued at the constrained trade tax optimum. Since the constraints in (A.3) are binding at the optimum, we have that $\mathcal{L}_t^C = u^C$. Then, without changing the value of the Lagrangian, we can replace the Lagrange multipliers by those evaluated at the optimum of the VAT regime, λ^V and μ^V, so that:

$$\mathcal{L}_t^C = u^C = u^C + (\lambda^V - \mu^V) \left(\sum_{j \notin \boldsymbol{X}^S} t_j^C (E_{q_j}^C - R_{p_j}^{SC}) + \sum_j R_{p_j}^{FC} - (1 - \theta) R^{FC} \right)$$

$$+ \mu^V \sum_j (R_{p_j}^C - E_{q_j}^C) - \lambda^V \overline{G}$$

Letting $\Delta u^T = u^T - u^C$, we can make use of this expression to write u^T as:

$$u^T = u^C + \Delta u^T = u^C + (\lambda^V - \mu^V) \left(\sum_{j \notin \boldsymbol{X}^S} t_j^C (E_{q_j}^C - R_{p_j}^{SC}) + \sum_j R_{p_j}^{FC} - (1 - \theta) R^{FC} \right)$$

$$+\mu^V \sum_j (R^C_{p_j} - E^C_{q_j}) - \lambda^V \overline{G} + \Delta u^T$$

Suppose now that the VAT rates v are set equal to the restricted trade tax rates t^C. Switching to the VAT regime changes the rent and net output in the formal sector by $R^{FV} - R^{FC}$ and $R^{FV}_{p_j} - R^{FC}_{p_j}$, respectively. Recall that R^{FV} and $R^{FV}_{p_j}$ are evaluated at world prices in the VAT regime. Then, denoting by $\mathcal{L}^C_v$ the value of the Lagrange function in the VAT regime with $v = t^C$, $\lambda = \lambda^V$ and $\mu = \mu^V$,

$$u^T = \mathcal{L}^C_v + (\lambda^V - \mu^V)\left(\sum_j R^{FC}_{p_j} - (1-\theta)R^{FC} - \theta R^{FV}\right) + \mu^V \sum_j \left(R^{FC}_{p_j} - R^{FV}_{p_j}\right) + \Delta u^T$$

$$= \mathcal{L}^C_v + (\lambda^V - \mu^V)\left(\sum_j R^{FC}_{p_j} + (1-\theta)(R^{FV} - R^{FC}) - R^{FV}\right) + \mu^V \left(\sum_j R^{FC}_{p_j} - R^{FV}\right) + \Delta u^T$$

$$= \mathcal{L}^C_v - \lambda^V \left(R^{FV} - \sum_j R^{FC}_{p_j}\right) + (\lambda^V - \mu^V)(1-\theta)(R^{FV} - R^{FC}) + \Delta u^T \gtreqless u^V = \mathcal{L}^V_v$$

where in the second equality, we use $R^{FV} = \sum_j R^{FV}_{p_j}$. Proposition 1 follows immediately.

In the case of $\boldsymbol{F} = \emptyset$, the rent and net output in the formal sector vanish, and $\mathcal{L}^C_v = \mathcal{L}^V_v$ since $v = t^C$ in the optimum, i.e., the VAT is equivalent to import taxes. Then, Corollary 4 follows.