Queen's Economics Department Working Paper No. 1111

Taxation and Transaction Costs in a General Equilibrium Asset Economy

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10-2006

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Summary. Most financial asset pricing models assume frictionless, competitive markets that imply the absence of arbitrage opportunities. Given the absence of arbitrage opportunities and complete asset markets, there exists a unique martingale measure that implies martingale pricing formulae and replicating asset portfolios. In incomplete markets, or markets with transaction costs, these results must be modified to admit non-unique measures and the possibility of imperfectly replicating portfolios. Similar difficulties arise in markets with taxation. Some theoretical research has argued that some taxation functions will imply arbitrage opportunities and the non-existence of a competitive asset economy. In this paper we construct a multi-period, discrete time/state general equilibrium model of asset markets with transaction costs and taxes. The transaction cost technology and the tax system are quite general, so that we can include most discrete time/state models with transaction costs and taxation. We show that a competitive equilibrium exists. Our results require careful modeling of the government budget constraints to rule out tax arbitrage possibilities.

Key words: Taxation, Transaction Costs, General Equilibrium, Asset Economy

1 Introduction

There is an extensive literature addressing the role of taxation and transaction costs in competitive financial markets. In an earlier paper, Jin and Milne [15], we proposed a general competitive asset economy with very general transaction technologies and provided sufficient conditions for the existence of a competitive equilibrium . This model allowed us to consider economies with brokers, dealers and a wide range of market constraints on trade (e.g., short sale constraints). We considered economies with convex and non-convex transaction technologies. In a complimentary paper, Milne and Neave [18] provide characterizations of agent and competitive equilibria in such an economy and demonstrate that most known discrete time/state models can be accommodated as special cases.

In this paper we address another major imperfection in asset markets: taxation. As pointed out by Schaefer [20] and Dammon and Green [6], there are difficulties in dealing with taxes in a general equilibrium one-period setting. To clear markets, when there are no restrictions on asset trading, relative prices must reflect the after tax marginal rates of substitution of all agents simultaneously. When tax rates differ across investors, however, this condition can be impossible to achieve. Schaefer [20] and Dammon and Green [6] give examples showing that when tax rates between investors are sufficiently different, there may exist arbitrage opportunities for at least one investor: this is inconsistent with the existence of general competitive equilibrium. The main reason that investors can exploit tax arbitrage opportunities is that they can infinitely short sell assets, and exploit unlimited tax rebates on capital losses. An alternative and more realistic view, which can accommodate asset short selling and individuals in different tax brackets, is where potential tax arbitrage opportunities exist, but there are realistic restrictions on their exploitation.

Unlike Schaefer [20] and Dammon and Green [6], we will investigate a closed model where the government is modeled explicitly, and tax rebates are limited (ultimately) by the government budget constraint. Long before government revenues and wealth are exhausted, sections of the tax code limit the ability of private agents or organizations to exploit unbounded loopholes in the tax law. For example, our modeling captures the essential features of the current US tax code. The Tax Reform Act of 1986 has eliminated the net capital gain deduction. Prior to this date, individuals could deduct 60 percent of net long-term capital gains from income. A $3000 loss limitation was applied to net long-term capital losses. Under the new rules, the capital gain deduction is eliminated, but the $3000 loss limitation is retained. More generally, the government can always make contingent provisions in the application of tax laws to avoid large revenue losses from implausible (unbounded) claims. More realistically, its revenues from wages and salaries dwarf any revenue or drains from financial taxes: in other words, the government cannot, and will not, promise infeasible tax rebates to consumers and firms. We call this a No Ponzi Game(NPG) condition. We stress that this constraint is a weak bound and that more restrictive conditions could be introduced by appealing to detailed tax laws or financial regulations. These tighter constraints would not destroy our equilibrium existence result.

Conceptually, the forms of tax on capital gains and losses are very complicated to analyze because the investment strategies depend on the whole history of the investment (For example see Dammon and Spatt [7] and Dammon, Spatt and Zhang [8]). Instead of giving concrete examples of capital gains and tax rebate rules, we will impose an upper bound on rebates for each investor. The bound can be regarded as being exogenously determined by government and legal considerations. Realistically it will be far less than the bound that would exhaust government resources.

Rather than considering bounds on short positions, the upper bound on rebates is the first friction in the present paper and is an asymmetric treatment of taxes on long and short positions. If the asymmetry is sufficiently pronounced to eliminate arbitrage opportunities, the only motivation for individuals to take short positions is to construct a profile of portfolio cash flows which is not feasible with entirely nonnegative portfolio weights. However, when there is a large number of securities, and tax asymmetries have eliminated arbitrage opportunities, individuals may have little motivation to take short positions. Consequently prices may be similar to those which would result if short sales were explicitly disallowed.

The second friction considered in this paper is transaction costs. Effectively we have extended our earlier paper (Jin and Milne [15]) on transaction costs to include taxation. This means short-sales have additional costs over and above taxation asymmetries, and is consistent with the discussion of Allen and Gale [1]. In this paper, we do not impose any constraints on short-selling, and short-sales are determined endogenously. Limited tax rebates on capital losses and transaction costs are suggested as possible explanations for the lack of apparent arbitrage and why a general competitive equilibrium can exist.

Another difficult issue is to construct a model which is sufficiently general to deal with the complexities of the tax law, and yet remain tractable. There are many papers that consider the implications for asset prices or asset allocations in specialized models (for a small sample see: Constantinides [4], Dammon and Green [6], Dammon and Spatt [7], Dammon, Spatt and Zhang [8], Dybvig and Ross [12], Ross [19], Green [14], Zechner [22]). Before discussing the properties of an equilibrium, it is important that there are sufficient restrictions imposed on the economy to imply the existence of an equilibrium. As we indicated above, if agents face agent specific tax functions that allow tax arbitrage possibilities, then an equilibrium may not exist. This issue has been addressed in a two period exchange economy by Dammon and Green [6] and Jones and Milne [16]. Dammon and Green [6] consider restrictions on tax functions to eliminate arbitrage and define a set of arbitrage-free asset prices. Their paper considered tax functions of considerable generality and exploited the theory of recession (asymptotic) cones to determine arbitrage free prices. Jones and Milne [16] argued that Dammon and Green [6] did not include the government sector explicitly, ignoring the feasibility constraints implicit in the government budget constraint. Once these constraints were introduced and recognized by the agents, there were natural bounds on the possible asset trades consistent with tax arbitrage. These restrictions allowed more general and complex tax functions to be consistent with equilibrium. Of course the differences in the two models could be resolved by assuming that Dammon and Green's tax functions included the implicit tax rules that come into play as soon as large tax arbitrages were claimed from the government by consumers. In that sense the two models could be made consistent.

The models in Dammon and Green [6] and Jones and Milne [16] were two-period exchange economies. In this paper we extend the two-period model to a more general multi-period economy with an explicit government sector, productive firms, brokers/dealers who operate the costly transaction technology, and spot commodity markets. The aim of the paper is to show how the ideas of earlier, simpler models can be extended in a number of realistic directions. In particular the introduction of firms allows us to accommodate models (see Zechner [22], Swoboda and Zechner [21], Graham [13] for a sample) that discuss the interaction of personal and corporate taxation, the trade-off between corporate equity and debt taxation, and their impact on corporate financial structure. The introduction of government is more general than the modeling in Jones and Milne [16] in that we allow the government to choose commodity trades optimally given its net tax revenues. We show that the introduction of spot commodity markets is easily accommodated, although we exclude commodity and wage income taxation for simplicity. We allow general tax functions on asset capital gains and dividends that can accommodate most properties of tax codes on financial assets. For example: the tax functions can be non-linear, convex or piece-wise linear; they can depend upon dynamic asset strategies so that capital gains or other complex tax systems can be incorporated; the tax functions can include financial subsidies as well as taxes; and the state contingent taxation functions can be interpreted to include (random) legal interpretations of the tax code where a dynamic asset position is deemed to violate the code and subsequent income or capital gains are taxed at a higher rate with possible penalties. In the body of the paper we rule out non-convex tax functions due to subsidy/tax thresholds- later in the paper, we discuss how this (and other extensions) could be incorporated in a more extensive model.

The model can be used as a basic structure to discuss incomplete asset markets and government policies to use the tax code and subsidies to complete asset markets, or at least improve the welfare of some agents.

The remainder of the paper is organized as follows. In Section 2 we set out the model and present fundamental assumptions; Section 3 is devoted to the proof of the main theorem; in Section 4 we discuss extensions or variations of the model and how they could be incorporated in an extended or modified version of the model. Also we discuss equilibrium efficiency properties and the complexity of general characterizations of an equilibrium.

2 The Economic Setting

Consider an economy with uncertainty characterized by a event tree such as that depicted in Fig.1 of Duffie [11]. This tree consists of a finite set of nodes $\mathbf{E}$ and directed arcs $\mathbf{A} \subset \mathbf{E} \times \mathbf{E}$, such that $(\mathbf{E}, \mathbf{A})$ forms a tree with a distinguished root e_0. The number of immediate successor nodes of any $e \in \mathbf{E}$ is denoted $\#e$. A node $e \in \mathbf{E}$ is terminal if it has no successor node. Let $\mathbf{T}$ denote the set of all terminal nodes. The immediate successor nodes of any non-terminal node $e \in \mathbf{E}$ are labeled $e^{+1}, ..., e^{+K}$, where $K = \#e$. The subtree with root e is denoted $\mathbf{E}(e)$. Particularly, $\mathbf{E} = \mathbf{E}(e_0)$. Suppose there are $|\mathbf{E}|$ nodes(including e_0)in the tree, N securities and M commodities at any node $e \in \mathbf{E}' = \mathbf{E} - \mathbf{T}$.This assumption can be relaxed by assuming that the numbers of securities can vary across nodes because we will not require every asset to be held by agents at node e_0. This relaxation covers the case where some securities are issued and some mature in interim periods. For the sake of simplicity, we assume that there is only one commodity at each terminal node (this is largely for expositional convenience and avoids some minor technical issues). At each node, all securities first distribute dividends, and then are available for trading: that is, all security prices are ex-dividend.

Let $p^C(e) = (p_1^C(e), ..., p_M^C(e))$ denote the spot price of commodities at node $e \in \mathbf{E}'$. At each node $e \in \mathbf{E}'$, asset n$(n = 1, \cdots, N)$ has a buying price $p_n^B(e)$ and a selling price $p_n^S(e)$ and a dividend $p_1^C(e)D_n(e)$, where we take the first commodity as numeraire. Moreover, let $D_n(e)$ be the dividend of asset n at the terminal node $e' \in \mathbf{T}$. We denote $p^B(e) = (p_1^B(e), \cdots, p_N^B(e))$, $p^S(e) = (p_1^S(e), \cdots, p_N^S(e))$ and $p(e) = (p^B(e), p^S(e))$ and $D(e) = (D_1(e), \cdots, D_N(e))$. And it is assumed that dividends are always non-negative.

2.1 Intermediaries

Suppose that there are H brokers or intermediaries (indexed by h) with an objective (utility) function $U_h^B(\cdot)$ defined over the non-negative orthant, and commodity endowment vector $\omega_h^B(e)$ at each node e. They are intermediaries specializing in the transaction technology that transforms bought and sold assets. Let $\phi_{h,n}^B(e)(\phi_{h,n}^S(e))$ be the number of bought(sold) asset n supplied by intermediary h at node $e \in \mathbf{E}'$ (denote $\phi_h^B(e) = (\phi_{h,1}^B(e), ..., \phi_{h,N}^B(e))^T$ and $\phi_h^S(e) = (\phi_{h,1}^S(e), ..., \phi_{h,N}^S(e))^T$, where T is the transpose transformation) and $z_h(e) = (z_{h,1}(e), \cdots, z_{h,M}(e))^T$ be the vector of contingent commodities used up in the activity of intermediation at node $e \in \mathbf{E}'$. Then the broker pays tax on capital gains via a general tax function $T_h^{BC}(e) = T_h^{BC}(e)((p^B(e'), p^S(e'), \phi_h(e'))_{e' \in \mathbf{PA}(e)})$, where $\mathbf{PA}(e)$ is a path from e_0 to e; and pays tax on dividends via a general tax function

$$T_h^{BD}(e) = T_h^{BD}(e)(D(e)[\sum_{e' \in \mathbf{PA}(e) - \{e\}} (\phi_h^B(e') - \phi_h^S(e'))])$$

At any terminal node $e \in \mathbf{T}$, the broker receives dividends

$$D(e)[\sum_{e' \in \mathbf{PA}(e) - \{e\}} (\phi_h^B(e') - \phi_h^S(e'))]$$

and pays tax via a general tax function

$$T_h^{BD}(e) = T_h^{BD}(e)(D(e)[\sum_{e' \in \mathbf{PA}(e)-\{e\}} (\phi_h^B(e') - \phi_h^S(e'))]).$$

Denote $z_h = (..., z_h(e), ...)_{e \in \mathbf{E}'} \in R_+^{|\mathbf{E}'| \times M}$ and the portfolio plan by

$$\phi_h = (\phi_h(e))_{e \in \mathbf{E}'} = (\phi_h^B(e), \phi_h^S(e))_{e \in \mathbf{E}'} \in R_+^{2(|\mathbf{E}'| \times N)}$$

And set

$$\begin{aligned}\gamma_h = (&p^B(e_0)\phi_h^S(e_0) - p^S(e_0)\phi_h^B(e_0) - T_h^{BC}(e_0)\\ &+p(e_0)(\omega_h^B(e_0) - z_h(e_0)),\\ &(p^B(e)\phi_h^S(e) - p^S(e)\phi_h^B(e)\\ &+p_1^C(e)D(e)[\sum_{e' \in \mathbf{PA}(e)-\{e\}} (\phi_h^B(e') - \phi_h^S(e'))]\\ &-T_h^{BC}(e) - T_h^{BD}(e) + p(e)(\omega_h^B(e) - z_h(e)))_{e \in \mathbf{E}'-\{e_0\}},\\ &(D(e)[\sum_{e' \in \mathbf{PA}(e)-\{e\}} (\phi_h^B(e') - \phi_h^S(e'))] - T_h^{BD}(e))_{e \in \mathbf{T}})\end{aligned}$$

For intermediary h, let $\mathbf{T}(h,e) \subseteq R_+^N \times R_+^N \times R^M$ denote his/her technology at node e.

The maximization problem of broker h can be stated as:

$$\sup_{(\phi_h, z_h) \in \Gamma_h^B(\tilde{p})} U_h^B(\gamma_h)$$

where $\Gamma_h^B(\tilde{p})$ is the space of feasible trade-production plans $(\phi_h, z_h) = (\phi_h^B, \phi_h^S, z_h)$ given $\tilde{p} = (p^B, p^S, p^C)$, which satisfies:

(2.1)$(\phi_h^B(e), \phi_h^S(e), z_h(e)) \in \mathbf{T}(h,e)$ and $z_h(e) \geq 0$;

(2.2)$p^B(e_0)\phi_h^S(e_0) - p^S(e_0)\phi_h^B(e_0) + p(e_0)(\omega_h^B(e_0) - z_h(e_0)) - T_h^{BC}(e_0) \geq 0$, and

$p^B\phi_h^S(e) - p^S(e)\phi_h^B(e) + p^C(e)(\omega_h^B(e) - z_h(e)))$
$+p_1^C(e)D(e)[\sum_{e' \in \mathbf{PA}(e)-\{e\}}(\phi_h^B(e') - \phi_h^S(e')] - T_h^{BC}(e) - T_h^{BD}(e) \geq 0, \forall e \in \mathbf{E}' - \{e_0\}$;

(2.3)$D(e)[\sum_{e' \in \mathbf{PA}(e)-\{e\}}(\phi_h^B(e') - \phi_h^S(e')] - T_h^{BD}(e) \geq 0, \forall e \in \mathbf{T}$.

(2.4)$\gamma_h \geq 0$.

Comments: The intermediary formulation allows the agent to trade on his/her own account; or by interpreting the transaction technology more narrowly, it can be restricted to a pure broker with direct pass through of assets bought and sold(see Milne and Neave [18] for further discussion). Note: our tax functions are sufficiently general in formulation that our capital gains tax function could incorporate dividend taxes as well (apart from the final date).

2.2 Firms

Suppose there are J firms. At node e_0, the firm $j \in \mathbf{J} = \{1, ..., J\}$ has an initial endowment $\omega_j^F(e_0)$, which gives the firm a positive cash flow. The firm chooses an input plan $y_j^-(e_0) \in R_+^M$ and a trading strategy $\beta_j(e_0) = (\beta_j^B(e_0), \beta_j^S(e_0))^T = (\beta_{j,1}^B(e_0), ..., \beta_{j,N}^B(e_0), \beta_{j,1}^S(e_0), ..., \beta_{j,N}^S(e_0))^T \in R_+^{2N}$, where $\beta_{j,n}^B(e_0)(\beta_{j,n}^S(e_0))$ represents the purchase(sale) of asset n by firm j at node e_0. Then, the firm pays tax according to the tax function $T_j^F(e_0)$. At every node $e(\in \mathbf{E}')$ other than node e_0, the firm produce an output $y_j^+(e) \in R_+^M$ and receives a net dividend

$$p_1^C(e)D(e)[\sum_{e' \in \mathbf{PA}(e)-\{e\}} (\beta_j^B(e') - \beta_j^S(e'))],$$

then, chooses an input plan $y_j^-(e) \in R_+^M$ and a trading strategy $\beta_j(e) = (\beta_j^B(e), \beta_j^S(e))^T = (\beta_{j,1}^B(e), ..., \beta_{j,N}^B(e), \beta_{j,1}^S(e), ..., \beta_{j,N}^S(e))^T \in R_+^{2N}$ and pays tax $T_j^{FC}(e)$ and $T_j^{FD}(e)$ on capital gains and ordinary income. It is assumed $(y_j^+(e) - y_j^-(e)) \in \mathbf{Y}_j(e) \subseteq R^M$,where $\mathbf{Y}_j(e)$ is a production set. At each terminal node e, the firm j produces $y_j^+(e) \in R_+^M$, get its dividend and then pays tax $T_j^{FD}(e)$. Set

$$\delta(e_0) = p^C(e_0)(\omega_j^F(e_0) - y_j^-(e_0)) + p^S(e_0)\beta_j^S(e_0) - p^B(e_0)\beta_j^B(e_0) - T_j^{FC}(e_0),$$

$$\begin{aligned}\delta(e) = &p^C(e)(y_j^+(e) - y_j^-(e)) + p^S(e)\beta_j^S(e) - p^B(e)\beta_j^B(e) \\ &+p_1^C(e)D(e)[\sum_{e' \in \mathbf{PA}(e)-\{e\}} (\beta_i^B(e') - \beta_i^S(e'))] - T_j^{FD}(e) - T_j^{FC}(e), \\ &e \in \mathbf{E}' - \{e_0\},\end{aligned}$$

and

$$\begin{aligned}\delta(e) = &p^C(e)y_j^+(e) + p_1^C(e)D(e)[\sum_{e' \in \mathbf{PA}(e)-\{e\}} (\beta_i^B(e') - \beta_i^S(e'))] - T_j^{FD}(e), \\ &e \in \mathbf{T}.\end{aligned}$$

Therefore,the problem of the firm j can be stated as:

$$\max_{(y_j, \beta_j) \in \Gamma_j^F(\tilde{p})} U_j((\delta_j(e))_{e \in \mathbf{E}}),$$

where $\Gamma_j^F(\tilde{p})$ is the feasible set of production-portfolio plan of firm j given $\tilde{p}$, which satisfies:

$$\delta_j(e) \geq 0, e \in \mathbf{E}.$$

Remark:The modeling of the objective of the firm avoids the failure of profit to be well-defined, and the non-applicability of the Fisher Separation Theorem. We can think of the firm being either a sole proprietorship or that the utility function is derived from some group preference arrangement (for a more detailed discussion of these issues see Kelsey and Milne [17]). The utility function can incorporate a number of interpretations: for example, it could include job perquisite consumption by a manager, where her optimal perquisite consumption is endogenously determined in equilibrium.

2.3 Consumers

In addition to broker/intermediaries and firms, there are I consumers, indexed by $i \in \mathbf{I} = \{1, ..., I\}$. The consumer i has endowment $\omega_i^C(e)$ of commodity at any node $e \in \mathbf{E}$. At any node $e \in \mathbf{E}'$, the consumer i has a consumption $x_i(e) = (x_{i,1}(e), ..., x_{i,M}(e))^T \in R_+^M$ and chooses an asset portfolio $\alpha_i(e) = (\alpha_i^B(e), \alpha_i^S(e))^T = (\alpha_{i,1}^B(e), ..., \alpha_{i,N}^B(e), \alpha_{i,1}^S(e), ..., \alpha_{i,N}^S(e))^T \in R_+^{2N}$, where $\alpha_{i,n}^B(e)(\alpha_{i,n}^S(e))$ represents the purchase(sale) of asset n by consumer i at node e. The consumer pays capital gains tax via a tax function

$$T_i^{CC}(e)((p^B(e'), p^S(e'), \alpha_i(e'))_{e' \in \mathbf{PA}(e)})$$

and dividend taxes via a function defined similarly to the broker tax function, $T_i^{CD}(e)$. At any terminal node $e \in \mathbf{T}$, the consumer receives dividends

$$D(e)[\sum_{e' \in \mathbf{PA}(e) - \{e\}} (\alpha_i^B(e') - \alpha_i^S(e'))]$$

and pays tax via a general tax function $T_i^{CD}(e)$, and then consumes. We denote the consumption plan by $x_i = (..., x_i(e), ...)_{e \in \mathbf{E}'} \in R_+^{|\mathbf{E}'| \times M}$ (where $|\mathbf{E}'|$ denote the number of nodes in the set $\mathbf{E}'$) and portfolio plan by $\alpha_i = (\alpha_i(e))_{e \in \mathbf{E}'} = (\alpha_i^B(e), \alpha_i^S(e))_{e \in \mathbf{E}'} \in R_+^{2(|\mathbf{E}'| \times N)}$.

The consumer i has a consumption set $X_i \subseteq R_+^{\mathbf{E}' \times M}$ and a utility function $U_i^C(\cdot)$ over the consumption plan x_i and the consumption

$$D(e)[\sum_{e' \in \mathbf{PA}(e) - \{e\}} (\alpha_i^B(e') - \alpha_i^S(e'))] - T_i^{CD}(e)$$

at terminal nodes. Denote

$$\begin{aligned} \tilde{x}_i = (x_i, (D(e)[\sum_{e' \in \mathbf{PA}(e)} (\alpha_i^B(e') - \alpha_i^S(e'))] \\ -T_i^{CD}(e))_{e \in \mathbf{T}}). \end{aligned}$$

So the problem of consumer i can be expressed as:

$$\max_{(x_i,\alpha_i)\in \Gamma_i^C(\tilde{p},\gamma,\delta)} U_i^C(\tilde{x}_i),$$

$\Gamma_i^C(\tilde{p},\gamma,\delta)$ is the feasible set of portfolio-consumption plan of consumer i given $(\tilde{p},\gamma,\delta)$, which satisfies:

$$\begin{aligned} &p^C(e)x_i(e)+p^B(e)\alpha_i^B(e)-p^S(e)\alpha_i^S(e)+T_i^{CC}(e)+T_i^{CD}(e)\\ &\leq p^C(e)\omega_i(e)+p_1^C(e)D(e)[\sum_{e'\in \mathbf{PA}(e)-\{e\}}(\alpha_i^B(e')-\alpha_i^S(e'))]\\ &+\sum_j \eta_{i,j}\delta_j(e)+\sum_h \theta_{i,h}\gamma_h(e) \end{aligned}$$

$\forall e\in \mathbf{E}'$ and

$$D(e)[\sum_{e'\in \mathbf{pa}(e)}(\alpha_i^B(e')-\alpha_i^S(e'))]-T_i^{CD}(e)\geq 0.\forall e\in \mathbf{T}$$

where $\eta_{i,j}$ is the share of the profit of jth producer owned by the ith consumer. The numbers $\eta_{i,j}$ are positive or zero, and for every j, $\sum_{i=1}^{I}\eta_{i,j}=1$. The numbers $\theta_{i,h}$ have the same explanation.

2.4 Government

The government is also included as part of the economy. Rather than providing a detailed analysis of the operations of the government, we will place weak restrictions on government activity. For simplicity assume that the government has net resources $\omega_G(e)\in R_+^M$ at node e, sets tax rates ex ante, and then consumes $x_G(e)$ at node e. The government can always propose precommitted tax laws that avoid government bankruptcy due to errors or subtle legal interpretations; and its revenues from taxes on wages and salaries dwarfs any revenue or drains from financial taxes. In other words, the government cannot promise infeasible tax rebates to consumers and firms. We call this a No Ponzi Game(NPG) condition[3]. We stress that this constraint is a weak bound and that more restrictive conditions could be introduced by appealing to detailed tax laws or financial regulations.

Suppose that the government has preferences over its consumption set $X_G=R_+^{|\mathbf{E}'|\times M}$, represented by a utility function U_G (this is simplistic but avoids more complex issues of government decision-making). At node $e\in \mathbf{E}'$, it spends its income, which comes from its endowment $\omega_G(e)$ and tax revenue

[3]The No Ponzi Game condition has been introduced in macroeconomic models to eliminate unbounded borrowing positions by consumers and/or governments(see Blanchard and Fischer [3].

$$T(e) = \sum_h (T_h^{BC}(e) + T_h^{BD}(e)) + \sum_j (T_j^{FC}(e) + T_j^{FD}(e)) + \sum_i (T_i^{CC}(e) + T_i^{CD}(e)).$$

The government's problem can be stated as:

$$\max_{x_G \in \Gamma_G} U^G(x_G),$$

where Γ_G denote the set of government's consumption x_G which satisfies

$$p^C(e)x_G(e) \le p^C(e)\omega_G(e) + T(e)$$

2.5 Competitive Equilibrium and Assumptions

To conclude the section, we give the definition of a competitive equilibrium and assumptions made throughout the remainder of this paper.

Definition 2.1. A competitive equilibrium with asset taxation and transaction costs is a non-negative vector of prices $\tilde{p}^* = (p^{*C}, p^{*B}, p^{*S})$ and allocations $\{(x_i^*, \alpha_i^*)$ for all $i \in \mathbf{I}; (z_h^*, \phi_h^*)$ for all $h \in \mathbf{H}; (y_j^*, \beta_j^*)$,for all $j \in \mathbf{J}; x_G^*$ such that:

(i)(z_h^*, ϕ_h^*) solves the broker problem for each $h \in \mathbf{H}$;

(ii) (y_j^*, β_j^*) solves the firm's problem for each $j \in \mathbf{J}$;

(iii)(x_i^*, α_i^*) the consumer problem for each $i \in \mathbf{J}$;

(iv)x_G^* solves government's problem;

(v)$\sum_i x_i^* + \sum_h z_h^* + \sum_j y_j^{*-} + x_G^* = \omega_G + \sum_j y_j^{*+} + \sum_i \omega_i^C + \sum_j \omega_j^F + \sum_h \omega_h^B$;

(vi)$\sum_i \alpha_{i,n}^{*B} + \sum_j \beta_{j,n}^{*B} = \sum_h \phi_{h,n}^{*S}$ and $\sum_i \alpha_{i,n}^{*S} + \sum_j \beta_{j,n}^{*S} = \sum_h \phi_{h,n}^{*B}$ if $p_n^{*B} > p_n^{*S}$; $\sum_i \alpha_{i,n}^{*B} + \sum_j \beta_{j,n}^{*B} + \sum_h \phi_{h,n}^{*B} = \sum_h \phi_{h,n}^{*S} + \sum_i \alpha_{i,n}^{*S} + \sum_j \beta_{j,n}^{*S}$ if $p_n^{*B} = p_n^{*S}$.

Notice that condition (vi) allows for the extreme case where the transaction technology is costless.

The following assumptions are made in the remainder of this paper.

For consumer i:

(**A1**) $X_i = R_+^{|\mathbf{E}'| \times M}$;

(**A2**) U_i^C is a continuous, concave and strictly increasing function;

(**A3**) T_i^{CC} and T_i^{CD} are continuous, convex functions and there exist positive constants $c_i^{CC}(e) < 1$, $c_i^{CD}(e) < 1$ such that $T_i^{CC}(e)[x] \le c_i^{CC}(e)x, T_i^{CD}(e)[x] \le c_i^{CD}(e)x$,for any $x \ge 0$, that is, the taxes can never be larger than revenues;

(**A4**)We assume that the tax rebates on capital losses and ordinary income on long and short positions have lower bounds: that is, $T_i^{CC}(e) + T_i^{CD}(e) > \alpha_i^C(e)$, where α_i^C is determined by the government;

(**A5**) $\omega_i^C(e) > 0$, [4]$\forall e \in \mathbf{E}'$;

For broker h :

(**A6**)For each e, $\mathbf{T}(h,e)$ is a closed and convex set with $0 \in \mathbf{T}(h,e)$;

(**A7**)For any e and given $x = (x_1, \cdots, x_{2N}, z_1, \cdots, z_M) \in \mathbf{T}(h,e)$, if $y = \sum_{n=1}^{2N} x_n \longrightarrow \infty$, then $|(z_1, \cdots, z_M)| = \sum_{m=1}^{M} z_m \longrightarrow \infty$;

(**A8**)For each e, if $(\psi, z) \in \mathbf{T}(h,e)$ and $z' \geq z$, then $(\psi, z') \in \mathbf{T}(h,e)$(free disposal).

(**A9**)$U_h^B(\cdot)$ is a continuous, concave and strictly increasing function;

(**A10**) $\omega_h^B(e) > 0, \forall e \in \mathbf{E}'$.

(**A11**) T_h^{BC} and T_h^{BD} satisfy assumption (**A3**) and (**A4**) for some $c_h^{BC}(e)$, $c_h^{BD}(e)$ and $\alpha_h^B(e)$.

For firm j:

(**A12**)For each $e \in \mathbf{E}'$, $\mathbf{Y}_j(e)$ is a closed and convex set ;

(**A13**)For each $e \in \mathbf{E}'$, $\mathbf{Y}_j(e) \cap \mathbf{R}\ _+^{2M} = \{0\}$;

(**A14**)For each $e \in \mathbf{E}'$, $(\sum_j \mathbf{Y}_j(e)) \cap (-\sum_j \mathbf{Y}_j(e)) = \{0\}$;

(**A15**)$U_j^F(\cdot)$ is a continuous, concave and strictly increasing function;

(**A16**) $\omega_j^F(e_0) > 0$;

(**A17**) T_j^{FC} and T_j^{FD} satisfies assumption (**A3**) and (**A4**) for some $c_j^{FC}(e), c_j^{FD}(e)$ and $\alpha_j^F(e)$.

For government;

(**A18**) U^G is a continuous, concave and strictly increasing function;

(**A19**) $\omega_G(e) > 0$;

(**A20**) For each security and any $e \in \mathbf{E}'$ there exists a terminal node $e' \in \mathbf{E}(e)$ such that the dividend of this security is positive at this node;

(**A21**)$\sum_i \alpha_i^C(e) + \sum_j \alpha_j^F(e) + \sum_h \alpha_h^B(e) \geq 0, \forall e \in \mathbf{E}'$

.

All assumptions except (**A7**), (**A15**), (**A4**), (**A11**), (**A17**) and (**A21**) are standard. (**A7**) says that transactions must consume resources. Assumptions (**A4**), (**A11**), (**A17**) and (**A21**) say that the government has designed into the tax code, and in the application of tax laws, measures that avoid large revenue losses from implausible (unbounded) claims. In other words, the government cannot/will not promise infeasible tax rebates to consumers and firms, preventing consumers from exploiting unlimited arbitrage opportunities through tax rebates on capital losses. According to the Federal Tax Code, the government often recognizes realized capital gains and does not recognize realized capital losses. Observe that the tax functions and trading restrictions are sufficiently general to allow the government to Levy taxes on capital gains, interest and dividends, and delay paying tax rebates on capital losses. Also the tax functions are state contingent allowing for random audits or (random

[4]For $x = (x_1, ..., x_n) \in R^n, x \gg 0$ means $x_i > 0, i = 1, .., n$; $x > 0$ means $x_i \geq 0, i = 1, .., n$ but $x_{i_0} > 0$ for at least one i_0.

from the point of view of agents and the government) legal interpretations. Strictly speaking, a fully rational system would model the intricacies of the legal tax system - here we simply assume that is an exogenous mechanism that is precommitted by the government.

3 The Competitive Equilibrium Existence Theorem

The main theorem of this paper is as follows:

Theorem 3.1: Suppose that the Assumptions 1-21 hold. Then there exists an equilibrium $(\xi^*, \widetilde{p}^*)$: that is, $(\xi^*, \widetilde{p}^*)$ satisfies $(i) - (vi)$ of Definition 2.1, where $\xi^* = ((x_i^*, \alpha_i^*)_{i\in\mathbf{I}}\ , (z_h^*, \phi_h^*)_{h\in\mathbf{H}}, (y_j^*, \beta_j^*)_{j\in\mathbf{J}}, x_G^*)$.

3.1 The Modified Economy

To show the existence of equilibrium, as in Arrow and Debreu [2], or Debreu [9], we will construct a bounded commodity spot market. Unfortunately this method does not apply directly because of the possible emptiness of the budget correspondence of agents in a multi-period assets market. To overcome this problem we will approximate the original economy with a sequence of new economies with positive commodity prices which has the property that the limit of the equilibria of the new economies is an equilibrium of the original economy. In the following, we will first truncate the commodity spot market as in Arrow and Debreu [2] and then truncate the asset space.

First of all, we truncate the commodity space. For broker h, define

$\mathbf{Z}_h = \{z_h : \textit{there exists}\, (\phi_h^B, \phi_h^S) \gg 0\, \textit{such that}\, (\phi_h^B(e), \phi_h^S(e), z_h(e)) \in \mathbf{T}(h, e), \quad \forall e \in \mathbf{E}'\}$

$\widehat{\mathbf{Z}}_h = \{z_h \in \mathbf{Z}_h : \textit{there exist}\ x_i \in \mathbf{X}_i, i = 1, ..., I, y_j \in \mathbf{Y}_j, j = 1, ..., J, z_{h'} \in \mathbf{Z}_{h'}, h' \neq h$ and $x_G \in \mathbf{X}_G$ such that $\sum_i x_i(e) + \sum_j y_j^-(e) + \sum_h z_h(e) + x_G(e) - \omega_G(e) - \sum_j y_j^+(e) - \sum_i \omega_i^C(e) - \sum_j \omega_j^F(e) - \sum_h \omega_h^B(e) \ll 0, \forall e \in \mathbf{E}'\}$.

For consumer i, define

$\widehat{\mathbf{X}}_i = \{x_i \in \mathbf{X}_i : \textit{there exist}\ x_{i'} \in \mathbf{X}_{i'}, i' \neq i, y_j \in \mathbf{Y}_j, j = 1, ..., J, z_h \in \mathbf{Z}_h, h = 1, .., H$ and $x_G \in \mathbf{X}_G$ such that $\sum_i x_i(e) + \sum_j y_j^-(e) + \sum_h z_h(e) + x_G(e) - \omega_G(e) - \sum_j y_j^+(e) - \sum_i \omega_i^C(e) - \sum_j \omega_j^F(e) - \sum_h \omega_h^B(e) \ll 0, \forall e \in \mathbf{E}'\}$.

For firm j, set

$\widehat{\mathbf{Y}}_j = \{y_j \in \mathbf{Y}_j : \textit{there exist}\ x_i \in \mathbf{X}_i, i = 1, ..., I, y_{j'} \in \mathbf{Y}_{j'}, j' \neq j, z_h \in \mathbf{Z}_h, h = 1, ..., H$ and $x_G \in \mathbf{X}_G$ such that $\sum_i x_i(e) + \sum_j y_j^-(e) + \sum_h z_h(e) + x_G(e) - \omega_G(e) - \sum_j y_j^+(e) - \sum_i \omega_i^C(e) - \sum_j \omega_j^F(e) - \sum_h \omega_h^B(e) \ll 0, \forall e \in \mathbf{E}'\}$.

For the government, set

$\hat{\mathbf{X}}_G = \{x_G \in \mathbf{X}_G : \textit{there exist}\ x_i \in \mathbf{X}_i, i = 1, ..., I, y_j \in \mathbf{Y}_j, j = 1, ..., J, z_h \in \mathbf{Z}_h, h = 1, ..., H$ such that $\sum_i x_i(e) + \sum_j y_j^-(e) + \sum_h z_h(e) + x_G(e) - \omega_G(e) - \sum_j y_j^+(e) - \sum_i \omega_i^C(e) - \sum_j \omega_j^F(e) - \sum_h \omega_h^B(e) \ll 0, \forall e \in \mathbf{E}'\}$.

As in Arrow and Debreu [2], we have the boundedness of sets $\widehat{\mathbf{X}}_i$, $\widehat{\mathbf{Y}}_j$,$\widehat{\mathbf{Z}}_h$ and $\hat{\mathbf{X}}_G$, which is stated in the following lemma. The proof is omitted because it is standard.

Lemma 3.1. The sets $\widehat{\mathbf{X}}_i$, $\widehat{\mathbf{Y}}_j$,$\widehat{\mathbf{Z}}_h$ and $\hat{\mathbf{X}}_G$ are all convex and compact.

Now we turn to truncating trading sets of asset. For broker h, define

$\mathbf{\Phi}_h = \{\phi_h = (\phi_h^B, \phi_h^S)$: there exists $z_h \in \widehat{\mathbf{Z}}_h$ such that $(\phi_h(e), z_h(e)) \in \mathbf{T}(h,e), \forall e \in \mathbf{E}'\}$.

In the following lemma, $\mathbf{\Phi}_h$ will be shown to be bounded, which will play an important role in proving the existence of the general equilibrium.

Lemma 3.2. The set $\mathbf{\Phi}_h$ is a compact and convex subset of $R^{|\mathbf{E}'|\times N}$, $h = 1, ..., H$.

Proof: If the assumption (**A6**) holds, then the set $\mathbf{\Phi}_h$ is a closed convex set. The convexity of $\mathbf{\Phi}_h$ is obvious. It remains to show its closedness. To this end, suppose that for each k, there exists $\phi_h^{(k)} \in \mathbf{\Phi}_h$ and $z_h^k \in \widehat{\mathbf{Z}}_h$ such that $(\phi_h^{(k)}, z_h^k) \in \mathbf{T}(h,e), \forall e \in \mathbf{E}'$, and, In particular, by (**A8**),$(\phi_h^{(k)}, \overline{z}_h^k) \in \mathbf{T}(h,e), \forall e \in \mathbf{E}'$, where $\overline{z}_h^k = (\max_k z_{h,1}^k, \cdots, \max_k z_{h,M}^k)$. If $\{\phi_h^{(k)}\}$ is unbounded, we may suppose $(\phi^{(k)})_{h,1}^B \longrightarrow \infty$ without loss of generality. .
But, by assumption (**A7**),

$$\lim_{k\longrightarrow\infty} |\overline{z}|_h^k = \infty$$

which provides a contradiction since $\widehat{\mathbf{Z}}_h$ is bounded by Lemma 3.1 and proves the boundedness of $\{\phi_h^{(k)}\}$. Hence, we can choose a subsequence $\{\phi_h^{(k_n)}\}$ from $\{\phi_h^{(k)}\}$ such that

$$\lim_{n\longrightarrow\infty} \phi_h^{(k_n)} = \phi_h,$$

this implies, by closedness of $\mathbf{T}(h,e)$, that $\mathbf{\Phi}_h$ is compact. □

By Lemma 3.2, there exists a positive constant M_0 such that for any $(\phi_h^B, \phi_h^S) \in \mathbf{\Phi}_h$, $\phi_{h,n}^B(e) \leq M_0$ and $\phi_{h,n}^S(e) \leq M_0, n = 1, .., N, h = 1, ..., H$, $\forall e \in \mathbf{E}'$.

For consumer i, define

$$\widehat{\mathbf{\Psi}}_i = \{\alpha_i = (\alpha_i^B, \alpha_i^S) : \alpha_{i,n}^B(e) \leq HM_0, n = 1, .., N, \forall e \in \mathbf{E}'\}.$$

For firm j, define

$$\widehat{\mathbf{\Theta}}_j = \{\beta_j = (\beta_j^B, \beta_j^S) : \beta_{j,n}^B(e) \leq HM_0, n = 1, .., N, \forall e \in \mathbf{E}'\}.$$

Define

$$C^1 = \{(c_1, ..., c_{2(|\mathbf{E}'|\times N)}) : |c_i| \leq M_1, i = 1, ..., 2(|\mathbf{E}'| \times N)\}(\subseteq R_+^{2(|\mathbf{E}'|\times N)})$$

where $M_1 > HM_0$.

Set $\widehat{\mathbf{X}}_i'$ be the set of $x_i \in \mathbf{X}_i$ with property that

$$\sum_i x_i(e) + \sum_j y_j^-(e) + \sum_h z_h(e) + x_G(e)$$
$$-\omega_G(e) - \sum_j y_j^+(e) - \sum_i \omega_i^C(e) - \sum_j \omega_j^F(e) - \sum_h \omega_h^B(e)$$
$$\ll D(e)[\sum_{e' \in \mathbf{PA}(e)-\{e\}} M_1(e')], \forall e \in \mathbf{E}'$$

for some $x_{i'} \in \mathbf{X}_{i'}, i' \neq i, y_j \in \mathbf{Y}_j, j = 1, ..., J, z_h \in \mathbf{Z}_h, h = 1, .., H$ and $x_G \in \mathbf{X}_G$. Here

$$M_1(e') = (3M_1, ..., 3M_1).$$

Likewise, we can define $\widehat{\mathbf{Y}}_j'$,$\widehat{\mathbf{Z}}_h'$ and $\hat{\mathbf{X}}_G'$. Like Lemma 3.1, it can be shown that $\widehat{\mathbf{X}}_i'$, $\widehat{\mathbf{Y}}_j'$,$\widehat{\mathbf{Z}}_h'$ and $\hat{\mathbf{X}}_G'$ are compact and convex and therefore, there exists a cube $C^2 = \{(c_1, ..., c_{|\mathbf{E}'| \times M}) : |c_i| \leq M_2, i = 1, ..., |\mathbf{E}'| \times M\} (\subseteq R^{|\mathbf{E}'| \times M})$ such that C^2 contains in its interior all sets $\widehat{\mathbf{X}}_i'$, $\widehat{\mathbf{Y}}_j'$,$\widehat{\mathbf{Z}}_h'$ and $\hat{\mathbf{X}}_G'$. Define $\widetilde{\mathbf{X}}_i = C^2 \cap X_i$,$\widetilde{\mathbf{X}}_G = C^2 \cap X_G, \widetilde{\mathbf{Y}}_j = C^2 \cap Y_j, \widetilde{\mathbf{Z}}_h = C^2 \cap Z_h$ and $\widetilde{\mathbf{\Phi}}_h = \widetilde{\mathbf{\Psi}}_i = \widetilde{\mathbf{\Theta}}_j = C^1$. And let $\tilde{\Gamma}_i^C(\tilde{p}, \gamma, \delta)$, $\tilde{\Gamma}_j^F(\tilde{p})$, $\tilde{\Gamma}_h^B(\tilde{p})$ and $\tilde{\Gamma}_G$ be the resultant modification of $\Gamma_i^C(\tilde{p}, \gamma, \delta)$, $\Gamma_j^F(\tilde{p}), \Gamma_h^B(\tilde{p})$ and Γ_G respectively. Define [5]

$$\triangle = \{\tilde{p} = (p^C, p^B, p^S) : \sum_{n=1}^{N}(p_n^B(e) + p_n^S(e)) + \sum_{m=1}^{M} p_m^C(e) = 1,$$
$$p_n^B(e) \geq p_n^S(e) \geq 0, p_m^C(e) \geq 0, m = 1, ..., M, n = 1, .., N, \forall e \in \mathbf{E}'\}$$

and

$$\triangle_k = \{\tilde{p} = (p^C, p^B, p^S) \in \triangle : p_m^C(e) \geq 1/k,$$
$$m = 1, \cdots, M, \forall e \in \mathbf{E}'\},$$

where $k \geq M$.

In the next section, we will prove the existence of the modified economy and then prove the existence of equilibrium of the original economy.

3.2 The Proof of Existence of General Equilibrium

We will adopt the Arrow-Debreu technique to prove existence in the modified economy.

---

[5] In this model excess demand is not necessarily homogeneous of degree zero in prices: thus the equilibrium prices may depend on the normalization chosen. For example, this will be the case for specific taxes.

Given $\tilde{p} \in \triangle_k$, let

$$\mu_i^C = \mu_i^C(\tilde{p}, \gamma, \delta) = \{(\psi_i, x_i) : U_i^C(x_i) = \sup_{(\bar{\psi}_i, \bar{x}_i) \in \tilde{\Gamma}_i^C(\tilde{p}, \gamma, \delta)} U_i^C(\bar{x}_i)\};$$

$$\mu_j^F = \mu_j^F(\tilde{p}) = \{(\beta_j, y_j) : U_j^F(\delta_j) = \sup_{(\beta'_j, y'_j) \in \tilde{\Gamma}_i^F(\tau)} U_j^F(\delta'_j)\};$$

$$\mu_h^B = \mu_h^B(\tilde{p}) = \{(\phi_h, z_h) : U_h^B(\gamma_h) = \sup_{(\bar{\phi}_h, \bar{z}_h) \in \tilde{\Gamma}_h^B(\gamma_h)} U_h^B(\bar{\gamma}_h)\}$$

$$\upsilon_G = \upsilon_G(\tilde{p}) = \{x_G : U^G(x_G) = \sup_{\bar{x}_G \in \tilde{\Gamma}_G} U^G(\bar{x}_G)\};$$

$$\begin{aligned}
\mu^M(e) = \{p \in \triangle_k | & p^B(e)(\sum_i \alpha_i^B(e) + \sum_j \beta_j^B(e) - \sum_h \phi_h^S(e)) \\
& - p^S(e)(\sum_i \alpha_i^S(e) + \sum_j \beta_j^S(e) - \sum_h \phi_h^B(e)) \\
& + \sum_{m=2}^{M} p_m^C(e)[\sum_i x_{i,m}(e) + \sum_j y_{j,m}^-(e) + \sum_h z_{h,m}(e) + x_{G,m}(e) \\
& - (\omega_{G,m}(e) + \sum_j y_{j,m}^+(e) + \sum_i \omega_{i,m}^C(e) + \sum_j \omega_{j,m}^F(e) + \sum_h \omega_{h,m}^B(e))] \\
& + p_1^C(e)\{D(e)[\sum_{e' \in \mathbf{PA}(e) - \{e\}} (\alpha_i^B(e') - \alpha_i^S(e'))] \\
& + D(e)[\sum_{e' \in \mathbf{PA}(e) - \{e\}} (\beta_i^B(e') - \beta_i^S(e'))] \\
& + D(e)[\sum_{e' \in \mathbf{PA}(e) - \{e\}} (\phi_h^B(e') - \phi_h^S(e')])]\} \text{ is maximum}\}
\end{aligned}$$

Now we make the last assumption, the Boundary Condition for government.

(**A22**) If ,for $\tilde{p}(k) = (p^C(k), p^B(k), p^S(k))$, $p_m^C(k) \to 0$ as k goes to infinity, then $x_{G,m}(\tilde{p}(k)) \to \infty$, where $x_{G,m}(\tilde{p}(k))$ is the government's optimal consumption of commodity m corresponding to price $\tilde{p}(k)$.

This assumption makes the plausible condition that the government can increase its consumption of a commodity to infinity when the price of that commodity declines to zero. A sufficient condition to imply this, would be to assume that the government utility function has the property that the marginal utility of its consumption goes to infinity as consumption goes to zero. Here we merely assume the condition directly.

Now we turn to the proof of the lower hemi-continuity of $\widetilde{\Gamma}_i^C(p,\gamma,\delta)$, $\tilde{\Gamma}_j^F(p)$, $\tilde{\Gamma}_h^B(\gamma_h)$ and $\tilde{\Gamma}^G$.

Lemma 3.3. $\widetilde{\Gamma}_i^C(p,\gamma,\delta), \tilde{\Gamma}_j^F(p), \tilde{\Gamma}_h^B(\gamma_h)$ and $\tilde{\Gamma}^G$ are lower hemi-continuous on $\triangle_k$ for each k and therefore, $\mu_i^C(p,\gamma,\delta), \mu_j^F(\tilde{p}), \mu_h^B(\tilde{p})$ and $\upsilon_G(\tilde{p})$ are upper hemi-continuous on $\triangle_k$ for each k.

Proof. It suffices to show that the correspondences $\widetilde{\Gamma}_i^C(p,\gamma,\delta), \tilde{\Gamma}_j^F(p)$, $\tilde{\Gamma}_h^B(\gamma_h)$ and $\tilde{\Gamma}^G(\tau)$ all have interior points for the given price $\tilde{p}\in\triangle_k$. From (**A5**) $\widetilde{\Gamma}_i^C(p,\gamma,\delta)$ has an interior point; From (**A10**), $\widetilde{\Gamma}_i^B(p)$ has an interior point; from (**A16),** $\tilde{\Gamma}_j^F(p)$ has an interior point since the firm can buy some security at the initial node and sell it gradually afterwards; and from (**A19)** and (**A20),** $\widetilde{\Gamma}^G$ has an interior point. By Berge's Maximum Theorem and standard methods, we can prove that the correspondences μ_i, μ_j, μ_h and υ_G are upper hemi-continuous and convex valued. □

Define

$$\Psi(\xi,\tilde{p}) = \prod_{i=1}^{I}\mu_i^C \bigotimes \prod_{j=1}^{J}\mu_j^F \bigotimes \prod_{h=1}^{H}\mu_h^B \bigotimes \upsilon_G \bigotimes \prod_{e\in\mathbf{E}'}\mu^M$$

Ψ is also upper hemi-continuous and convex valued . Under these conditions, Ψ satisfies all the conditions of the Kakutani fixed point theorem. Thus there exists$(x_i^*(k),\alpha_i^*(k))$ for all $i\in\mathbf{I}; (z_h^*(k),\phi_h^*(k))$ for all $h\in\mathbf{H}; (y_j^*(k),\beta_j^*(k))$, for all $j\in\mathbf{J}; x_G^*(k)$ and $\tilde{p}^*(k)$ such that $(\xi^*(k),\tilde{p}^*(k))$ $\in\Psi(\xi^*(k),\tilde{p}^*(k))$, where $\xi^*(k) = ((x_i^*(k),\alpha_i^*(k))_{i\in\mathbf{I}}$, $(z_h^*(k),\phi_h^*(k))_{h\in\mathbf{H}}$, $(y_j^*(k),\beta_j^*(k))_{j\in\mathbf{J}}$, $x_G^*(k))$. Particularly, for all $\tilde{p}\in\triangle_k$

$$\begin{aligned}
& p^B(e)(\sum_i \alpha_i^{B*}(k,e) + \sum_j \beta_j^{B*}(k,e) - \sum_h \phi_h^{S*}(k,e)) \\
& -p^S(e)(\sum_i \alpha_i^{S*}(k,e) + \sum_j \beta_j^{S*}(k,e) - \sum_h \phi_h^{B*}(k,e)) \\
& +\sum_{m=2}^{M} p_m^C(e)[\sum_i x_{i,m}^*(k,e) + \sum_j y_{j,m}^{-*}(k,e) + \sum_h z_{h,m}^*(k,e) + x_{G,m}^*(k,e) \\
& -\omega_{G,m}(e) + \sum_j y_{j,m}^{+*}(k,e) + \sum_i \omega_{i,m}^C(e) + \sum_j \omega_{j,m}^F(e) + \sum_h \omega_{h,m}^B(e)] \\
& +p_1^C(e)\{D(e)[\sum_{e'\in\mathbf{PA}(e)-\{e\}} (\alpha_i^{B*}(k,e') - \alpha_i^{S*}(k,e'))] \\
& +D(e)[\sum_{e'\in\mathbf{PA}(e)-\{e\}} (\beta_i^{B*}(k,e') - \beta_i^{S*}(k,e'))]
\end{aligned}$$

$$+D(e)[\sum_{e'\in\mathbf{PA}(e)-\{e\}}(\phi_h^{B*}(k,e')-\phi_h^{S*}(k,e')])]\}$$
$$\leq 0, e\in\mathbf{E}' \quad (1)$$

where $(\alpha_i^{B*}(k,e))_{e\in\mathbf{E}'}=\alpha_i^{B*}(k)$ and the other variables are defined in exactly the same manner.

Since $(\xi^*(k),\tilde{p}^*(k))$ are bounded, we may assume that the sequence $(\xi^*(k),\tilde{p}^*(k))$ converges, say to $(\xi^*,\tilde{p}^*)$. By Lemma 3.3, in order to prove that $(\xi^*,\tilde{p}^*)$ is an equilibrium of the modified economy, we should show that $p_m^{C*}(e)>0, m=1,...,M, e\in\mathbf{E}'$ and ξ^* satisfies market clearance.

Lemma 3.4. $p^{C*}(e)\gg 0, p^{B*}(e)\gg 0, e\in\mathbf{E}'$.

Proof: First of all, we prove that $p^{C*}\gg 0$. It is obvious that for all $\tilde{p}\in\triangle$, ξ^* satisfies (1) and particularly,

$$\sum_i\alpha_i^{B*}(e)+\sum_j\beta_j^{B*}(e)-\sum_h\phi_h^{S*}(e)$$
$$-(\sum_i\alpha_i^{S*}(e)+\sum_j\beta_j^{S*}(e)-\sum_h\phi_h^{B*}(e))\ll 0; \quad (2)$$

$$\sum_i\alpha_i^{B*}(e)+\sum_j\beta_j^{B*}(e)-\sum_h\phi_h^{S*}(e)\ll 0; \quad (3)$$

$$\sum_i x_{i,m}^*(e)+\sum_j y_{j,m}^{-*}(e)+\sum_h z_{h,m}^*(e)+x_{G,m}^*(e)$$
$$-(\omega_{G,m}(e)+\sum_j y_{j,m}^{+*}(e)+\sum_i\omega_{i,m}^C(e)+\sum_j\omega_{j,m}^F(e)+\sum_h\omega_{h,m}^B(e))$$
$$\leq 0, \quad m=2,...,M, \quad (4)$$

and

$$\sum_i x_{i,1}^*(e)+\sum_j y_{j,1}^{-*}(e)+\sum_h z_{h,1}^*(e)+x_{G,1}^*(e)$$
$$-(\omega_{G,1}(e)+\sum_j y_{j,1}^{+*}(e)+\sum_i\omega_{i,1}^C(e)+\sum_j\omega_{j,1}^F(e)+\sum_h\omega_{h,1}^B(e))$$
$$\leq D(e)[\sum_{e'\in\mathbf{PA}(e)-\{e\}}(\alpha_i^{B*}(e')-\alpha_i^{S*}(e'))]$$
$$+D(e)[\sum_{e'\in\mathbf{PA}(e)-\{e\}}(\beta_i^{B*}(e')-\beta_i^{S*}(e'))]$$
$$+D(e)[\sum_{e'\in\mathbf{PA}(e)-\{e\}}(\phi_h^{B*}(e')-\phi_h^{S*}(e')])]$$
$$\leq 0. \quad (5)$$

The last inequality follows from (2). This means that $x_i^* \in \widehat{\mathbf{X}}_i$, $y_j^* \in \widehat{\mathbf{Y}}_j$,$z_h^* \in \widehat{\mathbf{Z}}_h$ and $x_G^* \in \hat{\mathbf{X}}_G$ and therefore, $(\phi_h^B, \phi_h^S) \in \mathbf{\Phi}_h,\ h = 1, ..., H, (\alpha_i^{B*}, \alpha_i^{S*}) \in \widehat{\mathbf{\Psi}}_i$ and $(\beta_i^{B*}, \beta_i^{S*}) \in \widehat{\mathbf{\Theta}}_j$. Thus, by assumption (**A22**), $p_m^{C*}(e) > 0, m = 1, .., M, \forall e \in \mathbf{E}'$, otherwise, $x_G^* \notin \hat{\mathbf{X}}_G$.

Now we turn to proving $p^{B*}(e) \gg 0, \forall e \in \mathbf{E}'$. To the contrary, suppose $p_n^{B*}(e_0) = 0$ for some n and some $\tilde{e} \in \mathbf{E}'$. By considering the consumer i and noticing that $p^{C*}(e) \gg 0, \forall e \in \mathbf{E}'$, $(\alpha_i^{B*}, \alpha_i^{S*}) \neq \widehat{\mathbf{\Psi}}_i$, since, by assumption (**A20**), the consumer can unlimitedly increase her wealth at one of terminal node by buying asset n at the node $\tilde{e}$. Consequently, $p^{B*}(e) \gg 0, \forall e \in \mathbf{E}'$. □

Lemma 3.5. ξ^* satisfies market clearance.

Proof: Note that from the proof of Lemma 3.4, $x_i^* \in \widehat{\mathbf{X}}_i$, $y_j^* \in \widehat{\mathbf{Y}}_j$,$z_h^* \in \widehat{\mathbf{Z}}_h$, $x_G^* \in \hat{\mathbf{X}}_G$. Hence,

$$
\begin{aligned}
& p^{B*}(e)(\sum_i \alpha_i^{B*}(e) + \sum_j \beta_j^{B*}(e) - \sum_h \phi_h^{S*}(e)) \\
& -p^{S*}(e)(\sum_i \alpha_i^{S*}(e) + \sum_j \beta_j^{S*}(e) - \sum_h \phi_h^{B*}(e)) \\
& + \sum_{m=2}^{M} p_m^{C*}(e)[\sum_i x_{i,m}^*(e) + \sum_j y_{j,m}^{-*}(e) + \sum_h z_{h,m}^*(e) + x_{G,m}^*(e) \\
& -(\omega_{G,m}(e) + \sum_j y_{j,m}^{+*}(e) + \sum_i \omega_{i,m}^C(e) + \sum_j \omega_{j,m}^F(e) + \sum_h \omega_{h,m}^B(e)] \\
& +p_1^{C*}(e)\{D(e)[\sum_{e' \in \mathbf{PA}(e)-\{e\}} (\alpha_i^{B*}(e') - \alpha_i^{S*}(e'))] \\
& +D(e)[\sum_{e' \in \mathbf{PA}(e)-\{e\}} (\beta_i^{B*}(e') - \beta_i^{S*}(e'))] \\
& +D(e)[\sum_{e' \in \mathbf{PA}(e)-\{e\}} (\phi_h^{B*}(e') - \phi_h^{S*}(e'))]\} \\
= {} & 0, e \in \mathbf{E}'
\end{aligned}
$$

Suppose $p_n^{B*}(e) > p_n^{S*}(e)$. Define $p_n^S(e) = p_n^{S*}(e) + \varepsilon, p_M^C(e) = p_M^{C*}(e) - \varepsilon$ for ε sufficiently small, $p_l^S(e) = p_l^{S*}(e), l \neq n, p_l^B(e) = p_l^{B*}(e), l = 1, .., N, p_m^C(e) = p_m^{C*}(e)$, $m = 1, ..., M-1$. Plugging this price in to (1) and by above equality and (5),

$$
\begin{aligned}
& \varepsilon(\sum_h \phi_{h,n}^{B*}(e) - \sum_i \alpha_{i,n}^{S*}(e) - \sum_j \beta_{j,n}^{S*}(e)) \\
\leq {} & \varepsilon[\sum_i x_{i,M}^*(e) + \sum_j y_{j,M}^{-*}(e) + \sum_h z_{h,M}^*(e) + x_{G,M}^*(e)
\end{aligned}
$$

$$-(\omega_{G,M}(e) + \sum_j y^{+*}_{j,M}(e) + \sum_i \omega^C_{i,M}(e) + \sum_j \omega^F_{j,M}(e) + \sum_h \omega^B_{h,M}(e)]$$
$$\leq 0, e \in \mathbf{E}'$$

implying
$$\sum_h \phi^{B*}_{h,n}(e) - \sum_i \alpha^{S*}_{i,n}(e) - \sum_j \beta^{S*}_{j,n}(e) \leq 0, e \in \mathbf{E}'. \tag{6}$$

On the other hand, as in the proof of $p^{B*}(e) \gg 0$, we can show that if $p^{S*}_m(e^0) = 0$ for some m and some $e^0 \in \mathbf{E}'$, then, by assumption (**A20**), $\alpha^{S*}_{i,n}(e^0) = \beta^{S*}_{j,n}(e^0) = 0$, $\forall i, j$, and therefore, by (6), $\phi^{B*}_{h,n}(e^0) = 0$. Consequently, by noticing that $p^{C*}(e) \gg 0, p^{B*}(e) \gg 0, e \in \mathbf{E}'$, and combining (2), (3), (4), (5) and (6),

$$\sum_i x^*_i(e) + \sum_j y^{-*}_j(e) + \sum_h z^*_h(e) + x^*_G(e)$$
$$-(\omega_G(e) + \sum_j y^{+*}_j(e) + \sum_i \omega^C_i(e) + \sum_j \omega^F_j(e) + \sum_h \omega^B_h(e)) = 0;$$

$$\sum_i \alpha^{B*}_{i,n}(e) + \sum_j \beta^{B*}_{j,n}(e) - \sum_h \phi^{S*}_{h,n}(e) - (\sum_i \alpha^{S*}_{i,n}(e) + \sum_j \beta^{S*}_{j,n}(e) - \sum_h \phi^{B*}_{h,n}(e)) = 0$$

if $p^{B*}_n(e) = p^{S*}_n(e)$; if $p^{B*}_n(e) > p^{S*}_n(e)$,

$$\sum_i \alpha^{B*}_{i,n}(e) + \sum_j \beta^{B*}_{j,n}(e) - \sum_h \phi^{S*}_{h,n}(e) = 0$$

and
$$\sum_i \alpha^{S*}_{i,n}(e) + \sum_j \beta^{S*}_{j,n}(e) - \sum_h \phi^{B*}_{h,n}(e) = 0$$

finishing the proof of Lemma 3.5.

□

Combining the above three lemmas, we have proved the existence of general equilibrium in the modified economy. Consequently, in exactly the same manner as in Arrow and Debreu [2], it can be shown that $(\xi^*, \tilde{p}^*)$ is an equilibrium of the original economy, completing the proof of Theorem 3.1.

4 Discussion

We have provided sufficient conditions for the existence of a competitive equilibrium for a multiperiod asset economy with taxes and transaction costs. The point of the paper is to show how common assumptions on taxes and transaction costs can be incorporated into a competitive asset economy in a consistent manner. For expositional reasons we have introduced a number of assumptions to simplify the analysis and proofs. Below we will sketch how one could relax these assumptions in a more general model and how one could modify the strategy of proof to deal with this added complexity.

Having proved the existence of a competitive equilibrium in this model, we should be interested in the efficiency properties of the equilibrium, uniqueness of the equilibria (if any) and characterizations of the equilibrium. We argue that efficiency and uniqueness are problematic except under very strong conditions. The characterization of the asset prices and portfolios can be very complex: simple arbitrage pricing relations will occur under certain restrictive conditions, but one must be careful to check that the taxation functions and transaction costs are consistent with the standard results. Dynamic portfolios with transaction costs, trading constraints and taxes will be highly sensitive to the specification of those functions. There are numerous simple models in the literature, but most rely on strong assumptions to obtain analytic results, or require numerical simulation.

4.1 Extensions and Generalizations of the Model

First, one could introduce multiple physical commodities in the final period. We assumed a single commodity in the last period to simplify the technical analysis. Apart from some additional technical restrictions, the addition of multiple commodities in the last period is a straightforward generalization.

Second, the brokers and firms have utility functions. We assumed this type of objective because profit maximization is no longer the unanimous objective for share holders with incomplete markets or transaction costs. Similar problems arise with differing taxation across firms and shareholders in multiperiod economies. Rather than attempt to address this difficult issue here, we have avoided it by assuming utility functions for brokers and firms. There have been attempts to discuss more abstract firm objectives - see Kelsey and Milne [17] and their references, but we will not pursue that line of argument here. Similar arguments can be applied to the government utility function. Clearly, assuming a utility function is a gross simplification of government decision making, but it suffices to provide sufficiently regular responses in the government demands for goods and services for our existence proof, and it closes our economy. It would be possible to introduce less regular preferences for government - see the techniques on voting outcomes for firm decision-making in Kelsey and Milne [17] for some possible ideas that could applied to firms and governments. If the economy has a single physical commodity and the government cannot trade securities then government consumption is specified by the budget constraint and we can dispense with the government utility function.

Third,we have assumed that the transaction technology is convex. It is well known that transactions involve set-up costs, or non-convex technologies. In Jin and Milne [15] we address that issue by considering approximate equilibria in economies with non-convex transaction technologies (for details see Jin and Milne [15]). Clearly these ideas could be introduced here for our transaction technologies. Similar arguments could be applied to non-convex tax functions that arise from tax/subsidy thresholds that induce sharp non-convexities in tax schedules. As long as the non-convexities in these functions are not large in comparison to the overall economy, an approximate equilibrium could be obtained.

Fourth, we assumed that taxation over dividends and capital gains are additively separable. This is assumed for expositional convenience. We could have assumed that the capital gains tax function included dividend taxation and suppressed the dividend tax function. Notice that agent's asset demands and supplies will depend upon buying and selling prices, their utility functions, endowments, technology and their tax functions. This allows considerable generality in taxation law. For example our model allows for tax timing in buying or selling assets (for more discussion and examples of this type of behavior see Dammon, Spatt and Zhang [8]).

Fifth, although we consider the simple case of broker/intermediaries, the model can be adapted quite easily to accommodate restrictions on consumer and/or firm asset positions that are imposed by regulation (see Milne and Neave [18] for either a reinterpretation of our model, or a more straightforward modification of the consumer model). Thus our model could be adapted to include discrete versions of Detemple and Murthy [10] and Cuoco and Liu [5] on trading restrictions.

Sixth one can consider the government precommitting to a general taxation system of laws that specify contingent rates and rules depending upon the behavior of the agents. Given that the government could compute the equilibrium given the competitive actions of all other agents, then the government could simulate different tax codes and choose among the equilibria. The equilibria we have discussed here is merely one of that set.

Seventh, we have omitted taxation on commodities and services at each event. It is not difficult to add those taxation functions to accommodate the interaction of financial and income taxation and subsidies. This modification would be necessary to include standard economic discussions of the interaction of the taxation and social security systems.

4.2 Efficiency and Uniqueness

It is well-known in the economics and finance literature, that an equilibrium in incomplete asset markets is generically inefficient. This result carries over to the economy with transaction costs, because the incomplete market model is simply an economy with zero or infinite transaction costs on disjoint sets of assets. There are special cases where the economy is efficient: the representative agent model is a standard example. Another case is where the agents have identical HARA v.Neumann-Morgenstern preferences. Related to the efficiency property, is the derivation of an objective function for the firm.

With transaction costs and taxation there are many cases where the objective function cannot be obtained by the Fisher Separation theorem. We have assumed the existence of a firm utility function, but a more serious construction would introduce a welfare function that weighted the utility of the controlling agents. Or more abstractly, we could allow for an efficient outcome where a social-welfare function does not exist (see Kelsey and Milne [17]).

One strand of literature in Public Economics discusses incomplete markets and taxation with respect to the social security system. It suggests that the taxation system can help overcome asset market incompleteness. As the taxation and asset return functions enter in the budget constraints, it is possible to construct tax functions that would mimic the missing asset returns and prices. It is clear that altering the taxation system will also alter the actions of agents in equilibrium and the relative prices of assets, so that the design of the taxation system would require very detailed information about economic agents.

In general the equilibria for the economy will not be unique. This is well -known in the general equilibrium literature. Often in simple models with transaction costs and taxes, the modeler assumes HARA utilities (or identical risk neutral preferences) to avoid the difficulties of non-uniqueness of equilibria. These restrictive assumptions greatly simplify the model, and in extreme cases imply a representative agent.

4.3 Characterizations of Equilibria

Our model includes the well-studied case where there are no transaction costs and taxes. This model implies martingale pricing results and the generalized Modigliani-Miller theorems for derivative or redundant assets. With complete markets, any equilibrium is efficient.

With transaction costs,or incomplete markets, it is well-known that one can still derive martingale pricing results, but the martingale measure may not be unique. This type of model is consistent with a sub-case of our model. Similar characterizations with bounds on martingale measures can be derived with the version of the model with different buying and selling prices for assets. A number of papers use this characterization to bound derivative prices in an equilibrium (see Milne and Neave [18] for a synthesis and discussion of the literature).

A more difficult issue arises with the introduction of a new derivative security, which cannot be priced by arbitrage, and the derivation of pricing bounds. In a two period setting, this model is identical to an Industrial Organization model with the introduction of a new commodity. The pricing of the commodity, and related commodities, will be very sensitive to the structure of the model and assumptions on strategic behavior by the agents. Clearly this type of model is not nested in our model as we are restricted to an equilibrium model and do not consider comparative equilibria or strategic behavior.

Turning to the economy with taxation, there are a number of results that can be nested in our model. First, the introduction of taxation introduces distortions that destroy efficiency. But the purpose of government taxation is to raise revenue to provide public goods and redistribute wealth (we could have modified our model to allow for this extension). Second, the existence of transaction costs,or incomplete asset markets, raises the possibility of government taxation acting as a proxy for the missing or inactive asset markets. This type of argument is often used to justify social security systems to redistribute wealth and to act as an implied insurance market for the poor. Third, restricting our attention to asset pricing, it is easy to show that if taxation does not discriminate across assets and falls only on net income after asset trading, then the standard arbitrage pricing results continue to hold. As these results rely on zero arbitrage profits, and there is no profit in trading dynamic portfolios, the martingale pricing results follow. Fourth, when taxation discriminates across assets or dynamic asset portfolios, then arbitrage asset pricing must include taxes in the marginal conditions. These results are well-known in corporate finance when dealing with corporate leverage, or dividend policy. In the case of capital gains taxation, characterizations will involve optimal stopping rules that will be very sensitive to the specification of the tax rules, asset price movements and the preferences and income of the agent (for an example, see Dammon, Spatt and Zhang [8]). Fifth, although we have only one government, it is easy to introduce other governments and their taxation systems. This modification would allow the model to deal with international taxation and tax arbitrage through financial markets.

5 Conclusion

We have constructed a model that is sufficiently general to include most known models of competitive asset economies with "frictions" in asset markets. We have omitted two important classes of frictions: the first is price-making behavior where agents' asset trades impact on prices. Clearly this would violate our competitive assumption. The second friction involves asymmetric information between agents. Nearly all of the latter literature is restricted to partial equilibrium frameworks with strategic behavior, and is not consistent with our competitive assumptions.

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