The Control of Land Rent in the Fortified Farming Town

John Hartwick
Queen’s University

Department of Economics
Queen’s University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

10-2006
The Control of Land Rent in the Fortified Farming Town

John M. Hartwick\textsuperscript{1} (hartwick@econ.queensu.ca)
October 16, 2006

Abstract

We consider costly administration at the center of a farming community surrounding a fortified village. Land rent taxation is high cost mode of financing central administration in a tax incidence sense. Participatory administration by the governed is a lower cost alternative. We speculate why the low cost option has been out-competed by its higher cost alternative throughout history. We also take up constraints on predation on farmers by a landlord at the center. (lo_costdict_oct06.tex)

\begin{itemize}
\item key words: administrative structure, public goods, welfare cost
\item classification numbers: H41, H11
\end{itemize}

1 Introduction

With the aid of a basic land-use model of farmers working plots around a fortified town we investigate financing administration at "the center", costing $A$, with land-rent taxation. We contrast the funding of an administrator by land rent taxation with the "funding" of administration by rotating participation by all farmers in the community. These two distinct approaches (democratic versus "professional") have distinct tax incidence analyses which we take up and observe that participatory administration corresponds to least welfare loss from "funding". How do we then account over history of the prevalence of administration by a professional or lord? Our answer is that transactions costs make the operation of the participatory system the more costly and once a professional administrator is in office, she finds many ways of entrenching herself for the long term. Our inquiry has itself evolved from early work on an optimal or Henry George economy, comprising many farmers working plots around a fortified village, fortifications being a public good. Since a convention in this approach to model-building has each farmer’s land rent remitted to the center and then re-cycled back as an equal "dividend" per farmer, we were motivated to inquire about how the center was in fact being manned. Hence our analysis moved to the question of the dead weight loss from the extraction of funding in a Henry George economy for administration

\textsuperscript{1} I am indebted to Thor Keoppl, Frank Lewis and Dan Usher for helpful discussions and to Canada’s SSHRC for financial support.
at the center. The two distinct forms of administration which captured our attention, one "professional" and one participatory, turned out to have distinct incidence aspects when the funding question is confronted.

The tax-incidence costs of financing administration were a central topic in the well-known Olson-McGuire [1976] analysis of predatory government and our analysis can be viewed as being in the tradition of their pioneering work. The central idea in Olson-McGuire is that a predatory "administrator" will find it in her interest to set her tax or take rate at less than 100% in order to maximize her absolute "take". Such restraint in tax-rate setting, they refer to as the beneficent effect of the invisible hand of governing. Our approach is to take the funding level as given \( ab \text{ initio} \) and to compare two ways of carrying out administration. In contrast with their work, we have a complete and detailed base model with explicit land rent and an explicit public good. Our incidence issue revolves around the clearly defined requirement for a wage or support more generally of \( A \) for the administrator. We appear to be proceeding along a novel path in this literature by equating democracy with the administration of a community on a universal participatory basis. This is clearly a textbook form of democracy rather than one observed over history.

Financing a "professional" administrator with some land rent, we argue leads very naturally to entrenchment by the administrator at the center and to hereditary landlordism as the prevalent mode of government in many places over many centuries. This leads us to the familiar argument that democracy, here voting for a new administrator, is a device to forestall entrenchment by the administrator at the center. And we argue that the introduction of democracy is not easy and is not a natural low cost alternative to the hiring of a permanent administrator at the center, an administrator supported by a tax on land rent. The traditional view would be that out of some local violence a strongman emerges who entrenches himself as the local lord. This local lord proceeds to "tax" his local farmers to gain a high income and to maintain his incumbancy as the local lord. Our view does not contradict this scenario but we add the observation that "administration" at the center financed via land rent "taxation" is not initially the low incidence cost mode of local government, given the fixed bill of \( A \).

The other large issue concerning democracy is having the laws enacted by elected legislators reflect the preferences of those who participated in the election of the legislators. Our view of democracy is in a sense prior to this other. We see democracy as a mechanism for having the voters control the government in two senses: have "the government" (the administration) stick to an agenda of activities which the governed agree upon and secondly have the administration or government act in a cost-effective manner. On the latter point, the governed find it important that the administration not feather its nest with funds which should be in the hands of the governed.

The manorial system is the term used often by historians to capture the arrangement of a landlord providing protection to his velleins (peasant farmers) in return for some form of payment, generally labor services. Significant in this system was almost no trade in commodities between communities. The post manorial system was characterized by trade in commodities between communities, relatively free movement of labor and fairly well understood rights of ownership in land. See for example North and Thomas [1971] and Allen [1992]. We are dealing with a fairly stylized manorial system. We do not have inter-community trade but our economy is making use of a price system and our farmers are independent decision-makers. Early on our farmers are owners of their plots and can be viewed as hiring an administrator for "the center". 
needed for current administration. The trend over history against local administration by co-operative participation must be accounted for by something more than relative simple dead weight loss measure comparisons. Hence we turn to a consideration of the transactions costs of operating a participatory scheme as well as a consideration of the advantages of having a longer-term "professional" as administrator.

We also take up an intermediate case, a permanent and predatory landlord who finds it in his interest to tax or take at a rate less than 100% of his rightful rent in order to maximize the value of his taking. In our case, farmers have the option of migrating to the next community when their per capita welfare falls below that achievable down the road. This is the constraint which makes our landlord temper her rate of "taxation".

Our model is a rent-tax variant of one familiar to land-use theorists, the fixed coefficient Henry George model.4 In this model, each citizen is a farmer consuming some portion $c$ of the private good and a public good, $G$ (here defense services). $n$ farmers live in a fortified village surrounded by small farms of equal size. Each farmer commutes out to her farm each day, labors, and returns to the village at night. Maximin is the social objective and this allows us to define the "optimal size" of the community. The optimal community framework turns out to be a convenient setting for analyzing the tax incidence issue. For our later variant, with land owned by a predatory lord at the center, community size gets defined by the lord maximizing her tax-take from land rent subject to each tenant-farmer facing the option of migrating to the next community.

2 The Model

We first set out a spatial, optimal size economy ("optimal population", maximin, "Henry George") with a given tax on land rent. We calculate the corresponding optimal economy for this tax rate, revenues funding the administrator at "the center", and then obtain the tax rate for $A$ raised to support the administrator. (Later we will redo these steps for a "tax" supporting universal participation by citizens in the administrative task.) The economy has a spaceless "town" in the center, surrounded by a wall for protection, which is in turn surrounded by farms of unit size for each citizen. Each farmer "commutes" out to his farm and back each day.

Each farming family is jointly consuming the protection afforded by the village walls. $G$ is a public good (defense), produced centrally, and financed by a levy, $\tau$ on each household. With $n$ household-farmers, we have $n\tau = G$ in units of the numeraire, say wheat grown on each farmer’s ONE hectare plot around the town. This tax and expenditure is different from

4 Our model with fixed "lot" sizes and linear transportation costs is an easily recognized variant of the one in Arnott and Stiglitz [1979]. More recent users of the simple model include Helsley and Strange [1990] and Black and Henderson [1999]. In the appendix, we investigate the case of endogenous farm sizes. The resulting model is difficult to work with computationally.
that below for funding administration at "the center". Let \( A \) be each farmer’s production per period on his one hectare plot, and \( r(x) \) be a rent payment for use of the hectare and \( tx \) be roundtrip commuting cost per period for accessing the farm from the town. In addition, we assume that each household shares equally in the net total land rent \((1 - Y)RT\) as part of the gross household income. \( Y \) defines the fraction of total land rent that goes to a distinct "outside" agent. \( RT \) is total land rent from the farms around the town and we treat the town as a point in the center of a circular disc. In addition, each farmer has a house in the center but we gloss over the maintenance costs of this dwelling. (We can appeal to the "convention" that each farmer lives in a tent in the center, each tent occupying no space!)

A household’s utility is then \( U(c; G) \) for \( c = A - r(x) - tx + [(1 - Y)RT/n] - \tau \). \( c \) is consumption of private goods, say wheat, by the farmer. Since \( \tau = G/n \), we have \( U(A - r(x) - tx + \frac{1 - Y}{n}RT - \frac{G}{n}, G) \) as the household utility. A farmer treats \( RT, G \) and \( Y \) as parameters.

(a) For the same over each farmer, we require that land rent satisfy \( \frac{d[r(x) + tx]}{dx} = 0 \) or \( r(x) = \bar{r} + tx \). \( \bar{r} \) is land rent at the edge, \( x \). Given this land rent function, the household budget constraint reduces to \( c = A - \bar{r} - tx + \frac{1 - Y}{n}RT - \frac{G}{n} \).

Total rent is

\[
RT = \int_0^\pi r(x)2\pi x dx = \int_0^\pi [\bar{r} + tx]2\pi x dx - \int_0^\pi tx2\pi x dx
\]

\[
= \frac{1}{3}\pi tx^3 + \bar{r}\pi x^2.
\]

\( \frac{1}{3}\pi tx^3 \) is referred to as "differential land rent", leaving \( \bar{r}\pi x^2 \) as base land rent. Since each farmer farms a unit of land, the number of farmers is \( n(x) = \pi x^2 \).

For \( G, Y \), and edge \( x \) given for the moment, we have the economy account in

\[
nc = nA - nr(\bar{x}) - nt\bar{x} + [(1 - Y)RT - G].
\]

Hence

\[
nc = nA - nr(\bar{x}) - nt\bar{x} + [(1 - Y)RT - G]
\]

\[
= nA - \pi tx^3 + \frac{1}{3}\pi tx^3 - YRT - G \text{ since } RT = \frac{1}{3}\pi tx^3 + nr(\bar{x})
\]

\[
= nA - \pi tx^3 - \pi tx^3 - G \text{ since } Z = \frac{2}{3}\pi tx^3 \text{ (total commuting costs)}.
\]

This yields the basic final product \( nc + G \) equal to the net value of input, namely \( nA - Z - YRT \). Here total commuting costs, \( Z \) and administrative costs, \( YRT \) are intermediate consumption goods, frictions in a sense needed to obtain final demand but

---

5 Consider \( A = nN^L L^{1-s} \) with \( N = 1 \) unit of labor and \( L = 1 \) unit of land.

6 \( Y = 1 \) might be thought of as private land ownership with all rent going to the outside agent while \( Y = 0 \) defines pure collective ownership of the farmland. The familiar Henry George case has \( Y = 0 \).

7 Our analysis could be carried out completely for a community on a line with the fortified town in the center.
yielding no direct utility in themselves. These "frictions", $Z$ and $Y R^T$ are derived from primary inputs and are thus intermediate. In the classic Henry George case, $Y = 0$, and land rent fails to appear in the accounts for the economy. Above land rent only appears as an accounting entry in the definition of costs of administration.

(b) To obtain values for $c$ and $G$, we observe that maximization of utility by choice of $G$ (by a planner, say) yields the well-known Samuelson condition

\[ U_c = n U_G \]

for $U_z$ a partial derivative with respect to $z$. Note that $G$ is a flow of services per period, relative to the fixed durable fortification, say a wall around the village. Payment for the services is also a flow, as in maintenance costs per period. Here $G$, protection, is a public good since it enters "equally" in each household’s utility function. The Samuelson condition here indicates the appropriate level of output of $G$ relative to the level of output of the private good, $c$. This first order condition, combined with $\tau = G/n$, implies that $\tau = \frac{GU_c}{U_G}$; i.e. that $\tau$ is the value of $G$ to a household, given price $\frac{U_c}{U_G}$ defined in terms of consumption good, $c$.

(c) To obtain a value for edge, $\overline{r}$, we maximize the utility of a representative farmer given $Y$ fixed at some positive value between 0 and unity. We examine \( \frac{\partial}{\partial \overline{r}} U(\overline{A} - \tau \overline{r} - r(\overline{r}) - G(\overline{r}) + \frac{[1-Y]R^T(\overline{r})}{n(\overline{r})}, G(\overline{r})) = 0 \). First note that $-U_c \frac{G(\overline{r})}{n(\overline{r})} + U_G \frac{G(\overline{r})}{n(\overline{r})} \frac{dn(\overline{r})}{d\overline{r}} = 0$ by satisfaction of the Samuelson condition for each farmer. This leaves us with \( \frac{d}{d\overline{r}} \left[ -\overline{r} + \frac{[1-Y]R^T(\overline{r})}{n(\overline{r})} + \frac{G(\overline{r})}{n(\overline{r})} \frac{dn(\overline{r})}{d\overline{r}} \right] = 0 \) to consider. This expression reduces to

\[ -n(\overline{r}) t + [1-Y] \frac{dR^T(\overline{r})}{d\overline{r}} + \left\{ \frac{G(\overline{r})}{n(\overline{r})} - \frac{[1-Y]R^T(\overline{r})}{n(\overline{r})} \right\} \frac{dn(\overline{r})}{d\overline{r}} = 0. \]

Now $\frac{dR^T(\overline{r})}{d\overline{r}} = \int_0^{\overline{r}} \frac{dr(x)}{d\overline{r}} 2\pi x dx$ and $-n(\overline{r}) t + \int_0^{\overline{r}} \frac{dr(x)}{d\overline{r}} 2\pi x dx = 0$. Hence the central result:

\[ \left\{ \frac{[1-Y]R^T(\overline{r})}{n(\overline{r})} - \frac{G(\overline{r})}{n(\overline{r})} \right\} = [1-Y] \frac{[r(\overline{r})] 2\pi \overline{r} \frac{dr(x)}{d\overline{r}}}{dn(\overline{r})} - Y \int_0^{\overline{r}} \frac{dr(x)}{d\overline{r}} 2\pi x dx \frac{d\overline{r}}{dn(\overline{r})} \]

\[ = \left\{ [1-Y]r(\overline{r}) - Y(\overline{r}/2) \right\}. \]

On the left is the average cost of one more farmer in the economy and on the right is the marginal cost of one more person. This is one way of describing the Henry George Rule for optimal population (Arnott and Stiglitz [1979]). Given a specification for $U(c, G)$ we can solve for $\overline{r}, c, G$ and level $U$ as functions of $Y$.

Our economics proceeds as follows. The tax at rate $Y$ on total rent must extract $A$ dollars as a salary for the administrator. We will see below why she is paid this amount. Hence we

---

8. Our center is organizing the production of the public good and collecting the "fees". We are assuming that preferences of farmers have been revealed. This is public provision of the public good in contrast with Olszewski and Rosenthal [2006] who have private provision of the public good. In the latter’s interesting voting model, farmers vote for their richest compatriot to be the leader because he will create the largest public good, given his relatively large income.
can substitute for $Y$ with $A/R^T(\pi)$ everywhere above. In particular, (2) becomes

$$R^T(\pi) - G = -\frac{A}{2} + \left[ 1 - \frac{A}{R^T(\pi)} \right] r(\pi)n(\pi)$$  \hspace{1cm} (3)

We will then have two equations, (1) and (3), in $G$ and $\pi$ to solve for an equilibrium for the case of the "land tax" administrator.

3 Shared Administration by Farmers

Instead of hiring a "professional" administrator from "outside" at wage $A$ per period, we have each citizen do a small fraction of the administration per period in rotation. We proceed to obtain a maximin outcome under this institutional arrangement. Each farmer gives up some farming time, per period, to act as the administrator. Each farmer rotates through the job, once per period, at a total cost of $A$ to the economy per period. In this case, each period each farmer serves as the administrator for a spell of $1/n(\pi)$ of the the length of the interval. This is a participatory or a highly democratic approach to getting administration done. Hence each farmer's net output becomes $(n(\pi) - 1)A$. Summing over all $n(\pi)$ farmers leaves $A$ as total lost time for administration per period.\(^9\) The new allocation problem is to maximize per capita utility with this new definition of per capita consumption, namely $c = A - r(x) - tx + \frac{R^T - A}{n} - \frac{G}{n}$. That is, per capita utility is now $U(A - r(x) - tx + \frac{R^T - A}{n} - \frac{G}{n}, G)$. Again we assume the planner has the solution satisfy the Samuelson condition in (1) plus $\frac{dU}{dx} = 0$. This latter reduces to

$$R^T(\pi) - G = A + r(\pi)n(\pi).$$  \hspace{1cm} (4)

The equilibrium with shared administration is the pair $(\pi, G)$ satisfying (1), with the new definition of consumption, $c$ and (4).\(^10\) Formally then the type of administration reduces to the solution for a particular equilibrium, one with $A$ drawn from the economy by a tax on land rent and the other with $A$ drawn from the economy by a "take" of amount $A$, which can be thought of as from land rent, also. One thinks of explicit land rent taxation as distortionary, while the alternate form of "taxation" resembles a lump-sum taking. This intuition "goes through" and we do indeed see that to raise $A$, explicit land rent taxation has a higher dead weight loss. (This is our central result for the first half of this paper.) Hence the seeming evolutionary superiority of doing administration at the center by co-operative participation of citizenry. We invoke a Cobb-Douglas utility function, $U = e^x G^{1-\alpha}$ with

\(^9\) In fact there is the prospective administrator "loose" in the economy and we assume that this person fills in for each farmer while the latter is administering. The filler-in acts in every way like the farmer he is filling in for, including sending land rent to the center. This assumption about the use of the "extra" person guarantees that total land rent in the economy is not reduced at each instant by the shift of a farmer from farming to administering.

\(^10\) Once the tax rate is substituted for, each problem is left with equation (1) the same to be used for solving each model. Hence solving the two models involves equations (1) and (3) in one case and equations (1) and (4) in the other.
We set \( t = 0.1, A = 3.0 \) and \( r(x) = 0.00001 \). The participatory administration case solves with \((c, G, \pi, U, R^T, n)\) as \((1.308, 504.3, 16.919, 7.80588, 507.18, 899.29)\) and the "professional" administrator case with the same vector has values \((1.3124, 500.3, 16.82, 7.805749, 498.33, 888.798)\). Our conjecture is that these results are independent of the form of the utility function, as long as it is homogeneous of degree unity in the arguments. Note that we are raising a very small sum to hire the administrator and hence the small difference in utility levels. The tax rate for raising funds for the "professional" administrator case is \( A/R^T = 0.602\% \). Our tax incidence "wedge" for the two contrasting institutional arrangements is also small \((U = 7.80588 \text{ versus } 7.805749)\) because our economy is raising a small amount of revenue relative to its size. Striking is the relatively large difference in community size (participatory administration larger, with a larger public good) even with the seemingly small distortion.

We perturbed these solutions by shifting edge value \( r(\pi) \) and \( A \) and found no deviation from the low dead weight loss associated with participatory administration. The case of \( r(\pi) = 0 \) was a focus of our attention. It is not difficult to generate an alternate solution in a pair of solutions with a very small community size for the case of explicit land rent taxation. Such alternate solutions exhibit low utility levels and we do not spend more time on them.

4 Comments

Our incidence result suggests to us the evolutionary fitness of administration by co-operative participation at the center rather than administration by an "outside professional". We have introduced no "transactions" costs for operating the co-operative participation scheme. Nor have we considered the matter of the level of performance which each of the alternate modes exhibits. Clearly hiring an outside professional circumvents the difficult task of rotating many farmers through the job each period. And a "permanent professional" can be chosen by merit and can learn on the job and become skilled at doing the required administration. A hired agent can specialize and build up experience useful to those governed. To paraphrase Adam Smith, useful learning is limited by the time in the job. The flip-side is of course that learning to exploit the governed is also limited by the time in the job. Regular rotation of the governor can mitigate against the latter tendency. Hence the attractive "middle" case in which an administrator is say elected by universal franchise for a fixed term with say a maximum of two terms.

We are reflecting on three stylized modes of having administration at the center get done. Our co-operative, participatory mode (a kibbutz of sorts) is the most "democratic". Each of the governed is rotated through the job at the center on a regular basis. Competence or fitness for the task is sacrificed as well as the development of experience useful to the governed. The payoff is presumably in avoiding entrenchment by a potentially exploitative
administrator at the center. Our other polar case, is an administrator selected for skill by the governed and placed in office until a lack of competence is manifested. The payoff here is in having (a) a skilled person in the post and (b) allowing for the development of experience useful to the governed. The downside of this mode is in having the incumbent entrench him or herself and emerge as an autocrat. Learning to exploit the governed is to a large extent limited by the time in the position. There is a clear trade-off here. A lengthy period of incumbancy by the executive allows for experience and specialization to be exploited in the interest of the governed but longer incumbancy opens the door to possible dislodgeability and dictatorship. Our third mode is a “convex combination” of the above two polar cases. The post of administrator is filled for a fixed term and the incumbent is then rotated out in an election for a replacement. The fixed term is introduced in an attempt to prevent the incumbent from turning to practices to exploit the governed. Filling by a vote is designed to bestow legitimacy on the appointee. We see a significant payoff to having a very large fraction of the governed actually voting for the new administrator. One wants to avoid factionalism as in having a coalition of the governed arrange to have their favorite candidate become the new administrator. Farmers, the governed must give up some farming time to learn about the candidates and to actually cast their ballots. Civic participation is thus costly. It would seem appropriate to fine citizens who fail to vote.11 Not being able to construct a mechanism for rotation of the administrator leads to local autocracy and exploitation of the governed by the center. There appears to be a plausible evolutionary fitness of local exploitative landlordism as the dominant mode of government over much of history and over many places.

A specialized administrator can entrench himself, given inside information and perhaps links with administrators in neighboring communities, and become predatory on the whole flow of land rent moving through the center. Part of entrenchment is related to threats to the security of the community from attack from the outside. An administrator who can display prowess in battle and in defense of the community will win the favor of the farmers and will be difficult to rotate out of his post.12 In the broad sweep of history, autocrats who came

---

11 The standard argument is that person i has little incentive to take time to vote because the probability of her vote being decisive in the question of whether candidate A or candidate B wins is very small. Hence a very small payoff to voting weighed against a demonstrable cost of turning out and casting a ballot. We are actually interested in a slightly different phenomenon. An election for a new administrator in our set-up is a legitimization process. We assume that the candidates for administrator will be facing the same agenda and performing very similarly. Candidates do not have legislative agendas in our simple set-up. A poor turn-out in the election of a new administrator means a lower quality legitimization process. Hence if person i fails to vote because the process is personally costly, she is cutting into the quality of the legitimization process and this is our concern. Large turn-outs signal high quality legitimization of the person as administrator. Low quality legitimization is connected to the possibility of the administrator being unable to carry out his duties effectively, particularly in leading the community in say a defensive campaign against attack by a neighboring community.

12 Green [1993] explores the idea that the center ramps up its predation on the farmers (barons and their tenants) when a threat of violence from outside is imminent. Farmers are willing to be “exploited” in such circumstances because they want protection from outside violence. Green’s twist is that the center
to power "from violence" will have the glow of victory about them and will be difficult to rotate out of office. The soldier-administrator will generally have a palace guard and require a standing army. Incumbancy leads generally to entrenchment and an aggrandizement of power. This is part of entrenchment of the autocrat at the center. It also seems true that he who can impress with his splendor aids in his entrenchment in his position. Hence the need for display and ostentation by the chief administrator. This is of course a two edged sword. Ostentatious display is costly and can breed ill-will in the minds of taxpayers. Louis XIV had his sprawling palace built at Versailles to impress his peers and subjects with his "divine right" of rule.

It is worth emphasizing that democracy raises two problems from the perspective of this paper. First the transition from a landlord-autocracy to some form of popularly rotating administrator (rudimentary democracy) is not natural. One might say colloquially that it is an uphill struggle. One has to get the autocrat out of power and then has to institute a system of rotating and monitored administration. Large co-ordination problems are posed by this prospective transition. For example, the setting up of the United States after its war for independence was by no means a smooth, "natural" process. Basically all participants expected the new executive to entrench itself in some form of autocracy or oligarchy as in the northeast crowd dominating the new government or some analogous alternative.

Secondly, once a system is in place for rotating the administrator, it is not a trivial problem to make the system function effectively. There is the large problem of getting incumbant administrators to leave tranquilly after there prescribed term is over. Then there is the difficulty of finding a replacement that is competent, has the support of a large majority of "the governed" and is willing to leave tranquilly when his or her term is up. There is the perennial problem of a replacement representing the interests of a faction at the expense of those not part of the faction in question. Once in position, there remains the basic problem of having a mechanism for effective oversight by "the governed". There is first monitoring the actions of the administrator to see that he or she works only on the agenda that he or she has been assigned to work on by "the governed". "The governed" must guard against the administrator taking actions to entrench him or herself in a way that indicates that he or she will be difficult to rotate out of office when the term expires. Then there is the "monitoring" activity required for a legitimate turnover of administrators. "The governed" must participate in the process of selection and legitimization of the new administrator. Usually this means taking an interest in the candidates for the new incumbancy and participating in the selection (i.e. voting, if election is central to the process, which we think it must be). These problems has better information about the credibility of the imminent threat and will occasionally ramp up predation purely for its personal gain when no threat is on the horizon. Green shows in a stylized model that free information exchange can make both the center and the farmers better off on average over the long run; a parliament is characterized as the mechanism for information pooling. Hence the emergence of a parliament in thirteenth century England when threats of invasion from France were of great concern.
are in a sense prior to some of the standard problems with democracy. Some classic issues
are (a) safe-guarding against the majority of low-income legislators depriving asset owners
of their private property and (b) getting the representatives of "the governed" to legislate
in a way that reflects the preferences of "the governed". Our sketch above of problems of
democracy are as we noted in a sense prior to these classical difficulties.

Kiser and Barzel [1991] argue that England was able to get protodemocratic modes of
government started after 1066 because it was somewhat insulated from invasion. They note
also in their last footnote the fact "that Swiss mountain cantons had the most democratic
form of government in medieval Europe also supports the notion that isolation may facilitate
the development of democracy". They cite Blockmans [1978] to support the contention that
Swiss cantons were indeed highly democratic. We find this view to be rather fundamental.
Credible threats to a community from the outside allow the current administrator to cast
him or herself in the role of community-protector or to essentially surround him or herself
with significant military paraphenalia including a personal contingent of guards, etc. This
military apparatus can then be used against intervenors from inside the community as well
as against outside intervenors. Simply put, credible threats from the outside allow the
administrator to become his own military force and of course to become difficult to dislodge
from office by members of "his" community. Such military leaders acquire experience in
action and can further justify their continued incumbancy by invoking the fact that they have
special knowledge for defending the community. The leader argues persuasively that he or
she is indispensible to the community. Once in power, they then have a variety of ways to
entrench themselves as the permanent leader. The practical implication of a credible outside
threat is that it allows the leader to fortify his personal dwelling greatly and to develop a
standing army. In the absence of the outside threat, a police force will suffice to maintain
internal security.

The agenda of political discourse is very different for a community, like a valley canton
in Switzerland, free of violence from the outside. The governed can debate what the
administrator should be doing and can contemplate rotating the administrator after a period
in office. The administrator in turn could not justify militarizing his office. There is then
the large question of whether democracy can co-exist in a nation with incipient or actual
warfare on its doorstep. Churchill seems to have functioned as a wartime leader pretty well
within the democratic institutions of Britain between 1940-45. Lincoln fought an election
for President, within the US democratic framework, while the civil war was taking place in
the United States. The Isrealis have preserved their democracy while being threatened by
their neighbors and while striking out at their neighbors in small wars. These seem to be
exceptions. More typical would be the shelving of democratic practices while an outside
threat is addressed by the nation. We might generalize to say that adulterated democracy can
5 Constraining Predation of the Lord-landowner

McGuire and Olson [1997] developed the idea that the lord at the center would avoid "taxing" farmers too heavily in order to maintain a profitably large tax base. Very high tax rates could shrink the economy to an extent that the lord actually reduces his potential income. In particular, McGuire and Olson argue that the lord would provide public goods (eg. law and order) which enhance the productivity of farmers and thus contribute to enlarging the tax base for the lord. Over time a dynasty would be interested in having the economy and the "tax base" grow and McGuire and Olson contended that "an invisible hand" led to the interests of lord-administrator moving in parallel with those of the governed. This might be labelled the Laffer Curve theory of lordship restraint. At some rate $Y$ less unity, the lord administrator would find his personal income a maximum, apparently at an instant of time as well as over time as the economy was expanding. Here we take up a "market" constraint on the landlord’s predation on her tenant farmers. We introduce an outside option for a tenant farmer. She can move to another community if $Y$ becomes too high locally so that she is being immiserized by her local lord. The other community must be "offering" a higher utility level.

Consider now the center as landowner maximizing his income, $Y R_T(\pi)$ per period, but now facing the possibility of farmers migrating down the road to the next community if their utility falls below that observable in the next community, say, $\overline{U}$. A "high" take rate, $Y$ may drive local farmers away when the local equilibrium utility level is driven below that for a neighboring community.14 This exogenous utility level, $\overline{U}$ represents the outside option for a local farmer (not a serf legally tied to the lord’s domain). The lord faces this mobility constraint and selects his community size, $\pi$ and his take rate $Y$ in order to maximize his income, $Y R_T$.15

---

13 One thinks of Joseph Stalin becoming Secretary of the Central Committe of the Communist Party in 1922 in the Soviet Union and gradually consolidating all power in his office.

14 We leave open the question of how the equilibrium level of $U$ gets set in a system of many communities. One naturally thinks of a "marginal" community with the lowest productivity parameter $A$ yielding an infintesimal surplus for its lord.

15 North and Thomas [1971] contend that serfs in medieval England were not tied to the manor, like slaves. They then argue that predation on the output of his serfs by the lord of the manor would be restrained. "The lord’s power to exploit his serfs, however, was not unlimited; in the extreme, the serf could illegally steal away to seek asylum on another manor or, somewhat later, in one of a growing number of medieval towns. Nor were such fugitives likely to be returned by the lord’s neighboring rival. The abundance of land during the high Middle Ages made labor a very scarce and therefore valuable factor of production. Since the provision of public goods (in this case, protection and justice) is subject over some range to decreasing costs, some medieval lords were always in active competition with their peers to enlarge their estates.” (p. 788)
For a given value of \( Y \), the two equations

\[
U(c, G) = \mathcal{U} \quad \text{(5)}
\]

\[
U_c - nU_G = 0 \quad \text{(6)}
\]
define equilibrium values \( \pi \) and \( G \), assuming \( \mathcal{U} \) is a binding value. From these two equations we can solve for \( \frac{d\pi}{dY} \) and \( \frac{dG}{dY} \). We need \( \frac{d\pi}{dY} \) in the calculation of the rent maximizing tax rate \( Y \) for the landlord, given the outside option constraint, \( \mathcal{U} \). The landlord maximizes \( Y R^T(\pi) \) by choice of \( Y \) subject to (5) and (6). This leaves us with three equations (the first order condition, and (5) and (6)) to solve for \( \pi, Y \) and \( G \). The first order condition is

\[
R^T(\pi) = Y \frac{dR^T(\pi)}{d\pi} \frac{d\pi}{dY} \quad \text{for} \quad R^T(\pi) = \pi \pi^2 \{r(\pi) + \frac{1}{3} t \pi\}. \quad \text{(7)}
\]

For the case of utility Cobb-Douglas \( (U = e^{\alpha G^{1-\alpha}}) \), we get

\[
\frac{d\pi}{dY} = \frac{-\pi r(\pi) / 2 + t \pi^2 / 6}{(\pi - Y) r(\pi) + t \pi^2}.
\]

We are able to "eliminate" \( G \) and solve for \( \pi \) and \( Y \) with equations (5), (6) and (7) defining the landlord’s rent maximizing community, given each farmer’s outside option in \( \mathcal{U} \). For \( \alpha = 0.7, t = 0.1, \mathcal{U} = 7.0, A = 3 \), and \( r(\pi) = 0.00001 \), we obtain a community with \( \pi = 14.0346 \), slightly smaller than our earlier optimal community with \( \mathcal{U} \approx 7.8 \). And striking is the relatively high value of \( Y = 0.4054 \) for the "tax" rate on land rent.

In a world of very abundant labor, a free farmer would be doing poorly whether she stayed put or she re-located to a neighboring community. The irony here is that serfdom and slavery only make sense in a world of scarce labor (\( \mathcal{U} \) at a resolvable positive value). In world of abundant labor \( \mathcal{U} \) is very low and farmers have no attractive alternatives to staying put and working for a small reward. With \( \mathcal{U} \) at a reasonable value, a landlord must set a low take rate if she wants a large domain or she must somehow chain tenants or serfs or laborers to her domain by force. In recent history, large-scale slavery is associated with areas of significant labor scarcity, eg. the nineteenth century south in the US and the eighteenth and nineteenth century West Indies. The Black Death of 1347-48 created a labor shortage and we might say a high \( \mathcal{U} \). Peasant-landlord relations seemingly changed for good.\(^{16}\) Labor faced good prospects for some decades but as the population rose back to trend and farmers faced lower wages and presumably poorer outside options (reasonable \( \mathcal{U}'s \)). Nevertheless, in England, a return to serf-like conditions did not occur. The small capitalist farmer (yeoman) emerged in many places because his rights were defended in the courts. Elsewhere, powerful landlords enclosed open lands and drove many small farmers away while converting their large holdings to pasturage rather than crop-raising (Allen [1992; pp. 64-77]).\(^{17}\) It would not

\(^{16}\) Why did landlords turn to some form of slavery in order to obtain low cost labor? This is clearly a large and interesting question to ponder.

\(^{17}\) There are echoes of McGuire-Olson here. Roughly speaking, by subsidizing in-migration, each local farmer is willing to shoulder marginally more of the cost of the public good. Hence one might
be surprising to hear that labor mobility represented a historical break, sudden labor scarcity opened up the possibility of a market for free labor, and the beginning of a democratic organization of life but here we have filled in some detail of this argument. Allen’s principal point is that though labor abundance threatened the improved status of the yeoman farmer in the sixteenth and seventeenth centuries, the improved status was defended by an enlightened system of customs and laws. Why the courts moved in to defend the property rights of small, nascent-capitalistic farmers in such a significant way is unclear. Across the Atlantic Ocean, the American Revolution could well have settled into some form of government by aristocracy but labor scarcity meant an elevated status for workers and as matters evolved, resulted in a fairly democratic form of government, a beacon to the rest of the world.

Two other theories of "tax restraint" by the center involve the threat of being deposed, one scenario involving local farmers and another involving a neighboring lord. The first case has been investigated by Usher [1989] in an interesting model of weakening at the center, the spreading of chaos at the periphery (widespread banditry), and the overthrow of the current autocracy. The exploitative lord-administrator is always exposed to potential overthrow by an organized group of farmers who convince the masses that the new group of outsiders can do the administration at less cost than is being done by the current lord-administrator. This seems to capture the essence of the French Revolution and perhaps even the American Revolution. An articulate group of revolutionaries convinces "the governed" that they will be better off with the articulate "outside" group in power. The Russian Revolution of 1917 was certainly based on the revolutionaries promising an improvement in living standards for the governed, but the program of reform was or certainly turned out to be more extensive than simply doing public administration more efficiently. The second case has been investigated in an interesting variant of the Usher model by Konrad and Skaperdas [2004]. Konrad and Skaperdas formulate and equilibrium with local lords threatening each other. The equilibrium they develop involves each lord maintaining large defenses against

say that our lord, by voluntarily limiting his take, is indirectly offering more public good to his farmers by subsidizing entry to the community and in so doing is "optimizing" his "tax base" by making each farmer "more productive".

The emergence of democracy is a complicated matter, it seems. However, one can make the case for the British aristocracy ceding power to lower classes via the Reform Act of 1832, not out of fear of revolution, but out of hope for the economy to evolve in a way that improved their relative position. For example, the aristocracy might have envisaged transferring some of the burden of constructing new infrastructure and maintaining the military to the newly enfranchised classes; that is, more taxation with representation. This point in made by Engineer [1997] in an abstract framework. From this perspective, the Reforms could be viewed as reducing future tax liabilities on the aristocracy. Acemoglu and Robinson [2000] argue that franchise extension represented a device to assure restless lower classes that reforms, granted by the upper classes, could not be easily reneged upon in the near future. Franchise extension represented a commitment device by the upper classes to reforms favoring the lower classes. In our suggestion, the upper classes would have needed a commitment from the newly enfranchised folk that they would not in fact raise taxes on the upper classes in the future. No such device emerged and it appears that indeed the upper classes faced much higher taxes, in later decades. Franchise extension became the lesser of two evils: revolution or gradual "confiscation" by "the people" over future decades. Jack and Lagunoff [2006] develop an infinite horizon, median voter model in which incremental extension of the franchise, period by period, is an equilibrium outcome. Democracy as a form of government poses two problems to the analyst: how does it get started and how does it remain as the dominant governing mode over the long haul, given persistent threats to its viability.
neighbors and a general impoverishment of the communities in general.

The Usher model builds on a primitive state in which farmers and bandits co-exist with the same utility level. Bandits derive "income" by stealing a fraction of the output of the farmers. A farmer in turn must use up some of her current product in defensive expenditure. From this primitive equilibrium, one moves on to consider the introduction of "a center" which organizes community defense via economies of scale (by introducing defense as a public good). Our formulation leads to an interesting issue in model extension. Our farmers, in contrast to those in the Usher model, consume both private and public goods explicitly. Bandits would presumably prey on crops perhaps or somehow on the private consumption flow of a farmer in our set-up but would hardly prey on the public good (defensive fortifications). This raises the question about productive activity being channeled in a way so as to minimize theft by bandits. In earlier centuries, production could well have been directed to immobile outputs such as walls, moats, castles, and maybe very heavy artillery pieces. Light durable equipment such as looms, spinning wheels, foundary equipment, pottery-making equipment, tools, etc. would have been invested in sparingly in order to discourage roving bandits from bearing down and stealing. In modern times, banditry is endemic in the slums of large cities in the third world and many otherwise very productive people end up staying home in order to guard against theft. One thinks of things such as bicycles, pots and pans, tools, etc. being stolen. One does not need large-scale banditry then to have economic progress impeded. Small-scale banditry can skew economic progress in significant ways.19

6 Concluding Remarks

Our first and principal result is the low incidence cost of raising $A by participatory co-operation in administration relative to a land rent tax, for funding a "professional" administrator. We then reflected on why the democratic form of administration of a community did not emerge as a general mode over the sweep of history. Our thinking here was that the "professional" mode of administration has been dominant and has generally involved "confiscatory" rates of remuneration at "the center". We then reported on a standard mechanism constraining a predatory administrator from taxing her "subjects" too highly, namely the possibility of a citizen migrating away when the "tax-rate" became too high. Our analysis has been conducted in a variant of the textbook "optimal" spatial economy, a variant of the Henry George economy. This framework, we suggest, has made our analysis

19 The New York Times (October 17, 2006, p. C1) reports on a study of the impact of crime on national output in Latin America. Researchers suggest that had Brazil’s homicide rate been as low as Costa Rica’s in the early 1990’s, GDP in Brazil would have 3 to 8 per cent higher. Rampant crime discourages foreign investment. In addition the costs of guarding people and property is a form of waste since the guards could be doing more productive things if the society were less plagued by crime.
quite complete and straightforward. The Olson-McGuire framework strikes us as deficient in detail. The downside of our approach is that the optimal spatial economy framework is not easy to interpret as a real-world phenomenon, as with other optimal frameworks in economic analysis. Hence though our analysis seems straightforward, we may not have been working in the best framework for getting a grasp on real-world history. For example, our assumption that Samuelson’s public goods allocation rule is holding always is not very realistic. Thus our speculations on historical events through the lens of our model must regretably be discounted at a non-zero rate.
Appendix 1: Henry George Theory with Variable Farm Size

It is easy to come away with the impression that the Henry George property of our basic model above depends on a careful fixing of margins; i.e. fixed farm size and fixed production per farm. We can however extend the model and see things largely unchanged with farm sizes varying with land rents. Let \( h(x) \) be farm size at radial distance \( x \), with land rent \( R(x) \) per unit of land. Let farm output be \( \alpha h(x)^\beta \), \( \alpha \) and \( \beta \) positive with \( \beta \) less than unity (output is concave in the land input). Efficiency in the use of land requires that \( \frac{d(\alpha h(x)^\beta)}{dh(x)} = R(x) \).

Now residual consumption \( c \) is defined, as above, in
\[
c = \alpha h(x)^\beta - h(x)R(x) - tx + \frac{R^T}{n} - \frac{G^T}{n}.
\]
For each farmer to achieve the same utility, we require that the \( R(x) \) and \( h(x) \) functions satisfy
\[
-\frac{hc}{dh} = t.
\]
From the efficiency condition we obtain directly
\[
h(x) = \frac{(\alpha\beta)^{1/(1-\beta)}}{R(x)^{1/(1-\beta)}}.
\]
Hence the land rent function is defined by
\[
\frac{dR(x)}{dx} = -\frac{tR(x)^{1/(1-\beta)}}{(\alpha\beta)^{1/(1-\beta)}}.
\]
For \( R(\pi) \) a parameter and \( \pi \) temporarily fixed at some positive value, the general solution for rent is
\[
R(x) = \frac{R(\pi)}{1 + (\gamma - 1)kR(\pi)^{\gamma-1}[\pi - x]^{1/(\gamma-1)}}
\]
for \( \gamma = 1/(1 - \beta) \) and \( k = \frac{t}{(\alpha\beta)^{(1/(1-\beta))}} \). It is apparent that \( \frac{dR(x)}{dx} = -\frac{dR(x)}{dx} \) for this general case. Thus
\[
h(x) \frac{dR(x)}{dx} = -h(x) \frac{dR(x)}{dx} = -t.
\]

\[20\] We could have the one farmer per farm exerting a fixed amount of effort in farming but this would formally be an extra constant in the production function.

\[21\] The special case of \( \beta = 0.5 \) is less cluttered and will be useful below. For \( \beta = 0.5 \), we have \( \frac{dR(x)}{dx} + kR(x)^2 = 0 \) for \( k = \frac{t}{(\alpha\beta)^{1/(1-\beta)}} \). This differential equation solves as
\[
R(x) = \frac{1}{R(\pi) + k[\pi - x]}.
\]
This property of the model is the important link in establishing the Henry George Theorem for this model. Our preliminaries are complete.

We turn to equilibrium for each farmer-household. The representative farmer's utility is $U(\alpha h(x)^\beta - h(x)R(x) - tx + \frac{R^T}{\pi} - \frac{G}{\pi}, G)$ and maximization with respect to $G$ yields the familiar Samuelson condition

$$U_c = nU_G.$$ 

We assume that this condition is then satisfied for each farmer and consider the matter of an optimal size $\pi^*$ for the town with varying farm size. That is, we examine

$$\frac{dU(\pi^*, G(\pi^*))}{dx} = 0 \text{ or } \frac{d}{dx}U(\alpha h(x)^\beta - h(x)R(x) - tx + \frac{R^T(\pi)}{\pi} - \frac{G(\pi)}{\pi}, G(\pi)) = 0$$

for $h(x)$ and $R(x)$ defined above and $R^T(\pi) = \int_0^{\pi} R(x)2\pi xdx$ and $n(\pi) = \int_0^{\pi} \frac{2\pi x}{R(x)} dx$. We invoke the efficiency condition and the Samuelson condition and this reduces our problem to a consideration of $[-h(x)\frac{dR(x)}{dx} + \frac{d}{dx}R^T(\pi) - \left\{ \frac{R^T(\pi)}{n(\pi)} - \frac{G(\pi)}{n(\pi)^2} \right\} \frac{dn(\pi)}{dx}] = 0$. Since

$$\frac{d}{dx}R^T(\pi) = \int_0^{\pi} \frac{dR(x)}{dx}2\pi xdx + R(\pi)2\pi \pi dx,$$

we are left with

$$[-h(x)\frac{dR(x)}{dx}n(\pi) + \int_0^{\pi} \frac{dR(x)}{dx}2\pi xdx] + R(\pi)2\pi \pi dx - \left( \frac{R^T(\pi) - G(\pi)}{n(\pi)} \right) \frac{dn(\pi)}{dx} = 0$$

to consider. Using the definition of $n(\pi)$, the term in square brackets appears as

$$[-h(x)\frac{dR(x)}{dx} \int_0^{\pi} \frac{2\pi x}{h(x)} dx + \int_0^{\pi} \frac{dR(x)}{dx}2\pi xdx].$$

Above, we established the crucial property: $h(x)\frac{dR(x)}{dx} = -t$ independent of the value of $x$. Hence we are allowed to move $h(x)\frac{dR(x)}{dx}$ under the first integral sign and we obtain directly that the term in square brackets is zero. From the preceding equation it then follows that

$$\frac{R^T(\pi^*)}{n(\pi^*)} - R(\pi)2\pi \pi^* \frac{d\pi^*}{dn(\pi^*)} = \frac{G(\pi^*)}{n(\pi^*)}.$$ 

This is the Henry George property, here for an economy with a positive edge-rent, $R(\pi)$. $\frac{R^T(\pi^*)}{n(\pi^*)} - R(\pi)2\pi \pi^* \frac{d\pi^*}{dn(\pi^*)}$ is then per capita differential land rent. We have then established the Henry George result for an economy with varying farm size and an associated non-linear land rent function. The interpretation of this "optimal size" condition is: the marginal cost of an entrant, namely $R(\pi)2\pi \pi^* \frac{d\pi^*}{dn(\pi^*)}$, given $\pi^*$ equals the average cost of an entrant, namely $\frac{R^T(\pi^*)}{n(\pi^*)} - \frac{G(\pi^*)}{n(\pi^*)}$. Or "the center" might say to the marginal entrant: if you enter our town we will pay you, $\frac{R^T(\pi^*)}{n(\pi^*)} - R(\pi)2\pi \pi^* \frac{d\pi^*}{dn(\pi^*)}$, and you will have to pay the entry fee, $\frac{G(\pi^*)}{n(\pi^*)}$. At the margin, $\pi^*$ the marginal entrant is indifferent between entering and staying out.

This analysis immediately above establishes that the Henry George Theorem holds for quite general "monocentric cities", not just those with fixed lot sizes and linear rent.

---

Since $h(x) = \frac{t}{\pi R(x)/\pi x} = -\frac{t}{k h(x)}$, our solution for $R(x)$ leads directly to the solution for $h(x)$, namely:

$$h(x) = \frac{t}{k}\left[ \frac{1}{R(\pi)} + k[x - \pi]^2 \right].$$
functions. 

6.1 "Failure" of the Henry George Result

What about our restriction to linear commuting costs? We can relax this linear form and see the consequences. Thus a quadratic commuting cost, $\frac{1}{2}tx^2$ and $\beta = 0.5$ in the production function, yields rent function

$$R(x) = \frac{2}{h(x)} + k[x^2 - \frac{x^4}{4}]$$

This function exhibits the crucial property, $\frac{dR(x)}{dx} = -\frac{dR(x)}{d\bar{x}}$. However, in this case we have $h(x) \frac{dR(x)}{dx} = -h(x) \frac{dR(x)}{d\bar{x}} = -tx$, this latter NOT a constant. This means we cannot move $h(x) \frac{dR(x)}{d\bar{x}}$ under the integral sign in $\int_0^\infty \frac{2\pi x}{h(x)} dx$ to obtain the pure Henry George Result. We are left with a cluttered or approximate Henry George result, namely:

$$\left(\frac{R(x)}{n(\pi)} - \frac{G(\pi)}{n(\pi)}\right) = R(\pi) \frac{2\pi x}{m(\pi)} + \left[ -h(x) \frac{dR(x)}{d\bar{x}} n(\pi) + \int_0^\pi \frac{dR(x)}{d\bar{x}} 2\pi x dx \right] \frac{2\pi x}{m(\pi)}$$

"Pure" Henry George theory has the term in square brackets equal to zero. This term fails to equal zero for this case of non-linear commuting costs. Hence we observe that linearity in commuting costs are crucial for obtaining the standard Henry George Result for our class of land use models. We still have a Henry George result satisfying: average cost of entry of the marginal farmer equals the marginal cost, but the latter has become a quite messy expression. Our suspicion is that this linearity is necessary for a quite wide class of models.

This model turns out to solve consistently with the level of output of the public good near zero. In our view it was sufficiently ill-behaved numerically that we did not use it as the basis for additional analyses.

Arnott and Stiglitz [1979] abstract from the details of commuting costs in their analysis and represent the location of a household by a simple distance variable. Hence rent-distance functions do not play a central role in their analysis. They also frame their analysis in terms of an explicit optimal population argument, detouring around our concern with an economy with an optimal size or edge $x$. It is thus somewhat difficult to compare their arguments with ours.
References:


