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Commitment and Matching Contributions to Public Goods

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ABSTRACT

This paper studies multi-stage processes of non-cooperative voluntary provision of public goods. In the first stage, one or more players announce contributions that may be conditional on the subsequent contributions of others. In later stages, players choose their own contributions and fulfill any commitments made in the first stage. Equilibrium contributions are characterized under different assumptions about the commitment ability of players, the number of public goods and whether players commit to matching rates or to discrete quantities. We focus on contribution mechanisms that can emerge and be sustainable without a central authority, and that therefore may be particularly relevant for the provision of international public goods. Efficient levels of public goods can be achieved under some circumstances.

Keywords: Voluntary provision, matching contributions, commitment, multiple public goods

JEL classification: H41, H87

1 Introduction

The search for mechanisms for overcoming the free-rider problem is at the heart of public economics. Much of the rationale for government can be traced to addressing that issue. But despite the coercive power of government, it too faces almost insuperable difficulties in achieving optimal levels of public goods. In the absence of markets, there is no automatic way of determining the benefits that households obtain from public goods. Nonetheless, it is well known in principle how mechanisms can be designed to induce households to reveal their preferences for public goods, or even to lead them to voluntarily contribute efficient amounts of payments (subscriptions) to public goods. Mechanisms such as those proposed by Clarke (1971) and Groves and Ledyard (1977) can in principle induce truthful revelation of individual preferences for public goods, and those summarized by Varian (1994a, 1994b) can induce optimal voluntary subscriptions.

In an international context, where voluntary public goods provision is arguably most relevant, a further problem arises. Many of the mechanisms proposed require a government authority that can design and operate the mechanism, and no such authority exists in an international setting. This poses a challenging problem for some of the key emerging issues where international free-riding is important. For example, achievement of the United Nation's Millennium Development Goals and the meeting of the Kyoto Protocol targets for global emissions reduction involve voluntary actions by nations acting without the coercive power of an overriding governing authority. Mechanisms of the sort mentioned above that require central coordination are no longer available.

There have been a variety of proposals for generating additional revenues for international assistance, such as global carbon taxes, taxes on currency exchange, world lotteries and international capital taxes. However, each of these requires a central authority for their implementation. Our purpose in this paper is to explore mechanisms that can emerge to address the free-rider problem in a setting in which no central government exists to induce cooperation. We focus on the extreme case where nations behave non-cooperatively, although conceivably cooperative mechanisms could also surface, as the Kyoto Protocol illustrates. Our inspiration is drawn from a remarkable paper by Guttman (1978), who showed how extending the voluntary-contribution model to a two-stage setting can have profound consequences. In his model, agents in a first stage announce rates at which they will match the contributions of other agents. Then in the second stage, given the announced matching rates, agents choose their own contributions. He showed, using the special case of quasilinear preferences, that the sub-game perfect equilibrium in such a two-stage non-cooperative game is fully efficient. Subsequently, Danziger and Schnytzer (1991) showed that the result could be generalized to more general preferences and to any number of players. Moreover, equally remarkably, the equilibrium in such a two-stage contribution game would replicate the Lindahl equilibrium.

The Guttman mechanism, although seductively simple, does involve some strong assumptions. Two of these will be our special focus. The first is that the efficiency of the mechanism requires that all agents be able to commit to matching the contributions of others in the second stage. Such commitment cannot be taken for granted, as the evidence of unfulfilled promises in response to recent aid campaigns indicates (not to mention unkept promises under the Kyoto Protocol). Our first purpose will be to study the consequences of more limited commitment ability. In particular, we shall consider the consequences of only a subset of agents being able to commit. Indeed, in the case where only one agent can commit to a future contribution policy, some interesting new possibilities arise. First-stage announcements can then reasonably include not just matching rates, but also levels of contributions, possibly contingent on the contributions of others. It turns out to be the case that in these circumstances, even if only one country can commit, an efficient outcome can occur, albeit not the Lindahl one.

The second assumption in the Guttman mechanism is that there is only one public good. We shall consider the consequences of extending the analysis to a world with more than one public good. In a recent paper, Cornes and Itaya (2004) have studied the properties of non-cooperative contribution games when there are two public goods. We adapt their model to a two-stage setting where contribution policies are announced in the first period as above. In this context, the issue of commitment involves not only whether an agent can commit, but also whether the commitment is over one or both public goods.

The efficiency properties of the two-stage mechanisms depend on both these features of commitment, as well as on whether public goods are supplied simultaneously or sequentially.

To facilitate both the analysis and the intuitive understanding of the results, we conduct our analysis in the simplest of settings. We assume that there are two contributing agents, which we refer to as countries with the international public goods case clearly in mind. Country utility functions are taken to be loglinear (Cobb-Douglas). This yields linear reaction functions in the second stage, which enables us both to obtain explicit solutions to the two-stage contribution game and to provide simple geometric interpretations. The alternative would be to follow Guttman (1978) and Varian (1994a) and assume quasi-linear utility functions. However, these typically lead to indeterminate or corner solutions in contributions since reaction functions for the agents are parallel.

We proceed by considering first a base case in which there is only one public good and both countries can commit. This replicates the Guttman-Danziger-Schnytzer analysis, though some additional insights are obtained by the constructive solutions to which the loglinear utility function gives rise. We then turn to the case where only one of the countries can commit, and study different contribution policies that can be the basis of commitments. Next, we allow for two public goods and analyze the consequences of commitment applying to either one or both public goods. In a final section, we briefly consider generalizations and possible extensions.

2 The Base Case with One Public Good and Full Commitment

The setting consists of two countries, which we call Rome and Greece. Variables applying to Rome will be denoted using Roman letters, while those for Greece use corresponding Greek letters. There are initially two goods, a private composite commodity and an international public good. Consumption levels of the private good are x in Rome and χ in Greece. The total quantity of the public good, whose benefits accrue to both countries, is denoted G. Utility functions in the two countries are given by $U(G, x) = a \ln G + \ln x$ and $\Upsilon(G, \chi) = \alpha \ln G + \ln \chi$. Differences in the preference parameters for public goods, a and α, could be interpreted as reflecting different population levels. As Boadway and Hayashi (1999) and Lim (2003) show, different population levels give rise to different marginal benefits if national governments internalize the benefits of their own residents in choosing their contributions to public goods.

The public good is supplied privately by Rome and Greece. Both can make direct contributions of g and γ, but both can also match the contributions of the other country at the rates m and μ. Both direct contributions and matching contributions are chosen non-cooperatively by the two countries. The supply prices of the private and the public good are both fixed and the same in both countries, and are normalized to be unity. Purchases of the private good and contributions to the public good are financed in each country from exogenous endowments, w and ω. Thus, the budget constraints in Rome and Greece are $w = x + g + m\gamma$ and $\omega = \chi + \gamma + \mu g$, and the total supply of the public good is $G = g + m\gamma + \gamma + \mu g = (1 + \mu)g + (1 + m)\gamma$. Notice that countries can differ in two respects, preferences and endowments. These differences play no particular role in our analysis, but simply serve to illustrate some generality in the results.

In what follows, the timing of decisions and events will be important. When there is only one public good, timing can be divided into two or more stages. In the first stage, one or both countries may be able to commit to a contribution level in the next stages or to matching the contribution of the other country at some chosen rate. In the following stages, all base and matching contributions to the international public good are made, subject to commitments made in the first stage. There will be more than one such stage if subsequent contributions are sequential. We assume that all equilibria are subgame perfect, so we proceed by backward induction. Wherever convenient, interior solutions will be assumed. This requires that countries are not too different either in their endowments or in their preferences.

In the remainder of this opening section, we focus on the case in which both can commit to matching contributions announced in the first stage. This corresponds with the analysis of Guttman (1978) and Danziger and Schnytzer (1991), where full efficiency is achieved. It serves as a benchmark for the cases where there is not full commitment.

With full commitment, it suffices to suppose there are two stages that determine voluntary contributions. In Stage 1, both Rome and Greece announce rates at which they will match the contributions of the other. In Stage 2, the two countries make their contributions and the matching commitments are fulfilled. We begin with Stage 2, and then turn to Stage 1.

Stage 2

Given the matching rates m and μ announced in Stage 1, Rome and Greece choose their contributions g and γ. The solution will be a Nash equilibrium (NE). Assume to begin with that it is an interior one. The problem of Rome is to choose g to maximize $a\ln[(1+\mu)g+(1+m)\gamma]+\ln[w-g-m\gamma]$, given γ, m and μ. The first-order condition is:

$$\frac{a(1+\mu)}{(1+\mu)g+(1+m)\gamma}-\frac{1}{w-g-m\gamma}=0 \qquad \text{or} \qquad \frac{U_G}{U_x}=\frac{1}{1+\mu} \tag{1}$$

where U_G and U_x are the derivatives of $U(G,x)$ with respect to G and x. The solution to (1) yields Rome's reaction curve, which we write as:

$$g(\gamma,m,\mu)=\frac{aw}{1+a}-\frac{am+(1+m)/(1+\mu)}{1+a}\gamma \tag{2}$$

Similarly, Greece chooses γ to maximize $\alpha\ln[(1+\mu)g+(1+m)\gamma]+\ln[\omega-\gamma-\mu g]$, given g, m and μ. The first-order condition is:

$$\frac{\alpha(1+m)}{(1+\mu)g+(1+m)\gamma}-\frac{1}{\omega-\gamma-\mu g}=0 \qquad \text{or} \qquad \frac{\Upsilon_G}{\Upsilon_\chi}=\frac{1}{1+m} \tag{3}$$

which yields the reaction function:

$$\gamma(g,m,\mu)=\frac{\alpha\omega}{1+\alpha}-\frac{\alpha\mu+(1+\mu)/(1+m)}{1+\alpha}g \tag{4}$$

Note that both reaction curves are linear.

Figure 1 depicts the NE for the case in which both Rome and Greece are contributors and the NE is stable. In particular, the NE outcomes g^* and γ^* together solve (2) and (4):

$$\begin{aligned} g^* &= D^{-1}\left[(1+\alpha)aw-(am+(1+m)/(1+\mu))\alpha\omega\right] \\ \gamma^* &= D^{-1}\left[(1+a)\alpha\omega-(\alpha\mu+(1+\mu)/(1+m))aw\right] \end{aligned} \tag{5}$$

where

$$D = \begin{vmatrix} 1+a & am + \frac{1+m}{1+\mu} \\ \alpha\mu + \frac{1+\mu}{1+m} & 1+\alpha \end{vmatrix} = (1+\alpha)(1+a) - \left(am + \frac{1+m}{1+\mu}\right)\left(\alpha\mu + \frac{1+\mu}{1+m}\right)$$

The solution will be a stable interior one if Rome's reaction function, $g(\gamma, m, \mu)$, is flatter than Greece's reaction function, $\gamma(g, m, \mu)$, that is, if

$$\frac{1+\alpha}{\alpha\mu + (1+\mu)/(1+m)} > \frac{am + (1+m)/(1+\mu)}{1+a} \tag{6}$$

In this case, we can see that $D > 0$.

The following lemma will prove to be useful in what follows.[1]

Lemma 1: $1 - m\mu \gtreqless 0 \iff D \gtreqless 0$

Figure 1 also shows the effect of an increase in Rome's matching rate m, holding μ constant. This causes the slopes of both reaction curves (2) and (4) to become steeper, which in turn causes the NE value of g to fall to $g^{*\prime}$ and γ to rise to $\gamma^{*\prime}$. Conversely, an increase in μ would cause the reaction curves to become flatter, so g would rise and γ would fall. More formally, differentiating the reaction functions $g(\gamma, m, \mu)$ and $\gamma(g, m, \mu)$ in (2) and (4), we obtain:

$$\frac{\partial\gamma}{\partial m} = D^{-1} \begin{vmatrix} 1+a & -(a+1/(1+\mu))\gamma \\ \alpha\mu + (1+\mu)/(1+m) & (1+\mu)g/(1+m)^2 \end{vmatrix} \tag{7}$$

Therefore, in a stable interior equilibrium where $D > 0$, $\partial\gamma/\partial m > 0$ since both determinants on the right-hand side of (7) are positive. Similarly, it is straightforward to show that $\partial g/\partial m < 0$ if $D < 0$.

Note finally that, as the proof of Lemma 1 indicates, the larger is the difference between $m\mu$ and unity, the larger will be the difference in the absolute value of the slopes

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[1] The proof follows by rearranging (6), for the case of $D > 0$, to yield:

$$\frac{(1+a)(1+m)}{(1+m)\alpha\mu + 1 + \mu} > \frac{(1+\mu)am + 1 + m}{(1+\alpha)(1+\mu)}, \text{ or } \frac{1+a+m+am}{1+\mu+m\mu\alpha+\alpha\mu} > \frac{1+m\mu a+m+am}{1+\mu+\alpha+\alpha\mu}$$

By inspection, the inequality holds if $m\mu < 1$. The same argument applies for the case where the inequality in (6) goes the other way, or is an equality.

of the two reaction curves. As the sizes of the matching contributions—and therefore $m\mu$—rise, the two curves approach the same slope. When the reaction curves overlap, as shown by the dashed line in Figure 1, $m\mu = 1$ and the Stage 2 NE contributions g and γ are indeterminate (although as we shall see, total contributions by each country are determinate). Moreover, as $m\mu$ increases above unity, at least one reaction curve falls below the dashed line and the relative sizes of their slopes are reversed.

Stage 1

The NE in Stage 2 yields contribution functions $g(m, \mu)$ and $\gamma(m, \mu)$ (abusing notation) as well as indirect utility functions $u(m, \mu)$ and $\upsilon(m, \mu)$, where:

$$\begin{aligned} u(m,\mu) &= a\ln[(1+\mu)g(m,\mu) + (1+m)\gamma(m,\mu)] + \ln[w - g(m,\mu) - m\gamma(m,\mu)] \\ \upsilon(m,\mu) &= \alpha\ln[(1+\mu)g(m,\mu) + (1+m)\gamma(m,\mu)] + \ln[\omega - \gamma(m,\mu) - \mu g(m,\mu)] \end{aligned} \tag{8}$$

Both countries anticipate these outcomes in Stage 1. Begin with the case where $m\mu < 1$, so the Stage 2 NE is unique and, by assumption, interior.

Consider Rome's choice of m, given μ. Differentiating $u(m, \mu)$ in (8) with respect to m and using Rome's first-order condition (1) from Stage 2, we obtain after simplification:

$$\left.\frac{du}{dm}\right|_{\mu} = \frac{a(1-m\mu)}{G}\frac{\partial\gamma}{\partial m} - \frac{a\mu\gamma}{G} = D^{-1}\frac{a(1+a)(1-m\mu)}{(1+m)^2} \tag{9}$$

where the last equality uses $\partial\gamma/\partial m$ from (7). Given that we are evaluating this starting at an interior stable solution, $D > 0$ and therefore, $1 - m\mu > 0$ by Lemma 1. This implies that Rome's utility is increasing in m in a stable interior solution. Starting at an interior solution, Rome would want to increase m until the Stage 2 equilibrium goes to a corner. In terms of Figure 1, Rome will increase m until its own reaction curve coincides with the dashed line joining the two intercepts for the reaction curves, causing its own contribution to fall to zero, and Greece's direct contribution to become $\gamma = \alpha\omega/(1 + \alpha)$.

In fact, in this scenario, Rome will not increase m beyond the point at which this corner solution is reached. To see this, first find the value of m at which g just becomes zero, so the reaction curves just intersect along the γ axis. The value of m at which $g = 0$ can be obtained from (6) as:

$$m^0 = \frac{(1+\alpha)aw - \alpha\omega/(1+\mu)}{(a + 1/(1+\mu))\alpha\omega}$$

When $g = 0$, Greece's reaction function (4) simplifies to $\gamma = \alpha\omega/(1+\alpha)$, which is independent of m. Any further increase in m increases G through an increase in Rome's matching contribution alone. Rome's preferred choice of m when $g = 0$ can be determined by maximizing $a \ln[(1+m)\gamma] + \ln[w - m\gamma]$, giving:

$$m = \frac{(1+\alpha)aw - \alpha\omega}{(1+a)\alpha\omega} \leqslant m^0 \text{ for } \mu \geqslant 0$$

That means that Rome would not want to increase m above m^0, and would even like to reduce m *if* it could hold the value of g at zero, which we assume it cannot. Therefore, given $\mu \geqslant 0$ and starting in an interior stable contribution equilibrium, Rome will increase m until it just crowds out itself and no more, that is, until its reaction curve just coincides with the dashed line joining the two intercepts in Figure 1.

The same logic applies to Greece's behavior. Starting at an interior solution, Greece will want to increase μ until its reaction curve coincides with the dashed line. If both Rome and Greece behave this way, m and μ will increase to the point where both reaction curves coincide along the dashed line. When this occurs, $m\mu = 1$.

The intuition for these results is as follows. When $m\mu < 1$, it will be the case that $1/(1+\mu) > m/(1+m)$. Now, $1/(1+\mu)$ is the marginal cost to Rome of contributing directly to the public good when its contributions receive a matching rate of μ from Greece. And, $m/(1+m)$ is the marginal cost of contributing indirectly by subsidizing Greece's contributions at the rate m. Therefore, when $m\mu < 1$, it will be cheaper for Rome to subsidize Greece than to contribute directly itself. As m increases, γ will rise and g will fall until $g = 0$. At that point, Greece is at a corner solution and any further contribution by Rome will be equivalent to a direct contribution. The same logic applies to Greece.

Suppose now that we are at the point where both reaction curves overlap and $m\mu = 1$. Let m^* and μ^* be the matching rates that lead to the reaction curves of Rome and Greece coinciding with the dashed line in Figure 1, where $m^*\mu^* = 1$. It is straightforward to show that these will satisfy:[2]

$$m^* = \frac{(1+\alpha)aw}{(1+a)\alpha\omega} \qquad \text{and} \qquad \mu^* = \frac{(1+a)\alpha\omega}{(1+\alpha)aw} \tag{10}$$

This will be an equilibrium for Stage 1 since, as we now show, neither country would deviate from it. If, say, Rome increases m above m^*, then its reaction curve lies below the common reaction curve for (m^*, μ^*), while Greece's will be above. A stable Stage 2 equilibrium will be a corner solution with $g = 0$ and $\gamma = \alpha\omega/(1+\alpha)$. The change in Rome's utility for a change in m, from (8), is:

$$\frac{du}{dm} = \frac{\partial U}{\partial m} + \frac{\partial U}{\partial g}\frac{\partial g}{\partial m} + \frac{\partial U}{\partial \gamma}\frac{\partial \gamma}{\partial m} = \frac{\partial U}{\partial m}$$

since both g and γ are now independent of m. Given that $g = 0$ and $\gamma = \alpha\omega/(1+\alpha)$, we obtain, after some manipulation, that for $m = m^*$:

$$\frac{du}{dm} = \frac{-a\left[(1+a)\alpha\omega\right]^2}{(1+\alpha)w\left[(1+a)\alpha\omega + (1+\alpha)aw\right]} < 0$$

Therefore, Rome will be worse off by increasing its matching rate above m^*. Similarly, if it reduces m below m^*, its reaction curve will move above the common reaction curve for (m^*, μ^*) and Greece's will move below. A corner solution will result with $\gamma = 0$ and $g = aw/(1+a)$. In this case, if the Stage 2 equilibrium is $\gamma = 0$ and $g = aw/(1+a)$, then $du/dm = 0$, so Rome can be no better off by increasing m. An analogous argument applies for Greece.

The intuition developed above applies here as well. When $m^*\mu^* = 1$, $1/(1+\mu^*) = m^*/(1+m^*)$ and $1/(1+m^*) = \mu^*/(1+\mu^*)$. The first of these says that the marginal cost to Rome of contributing directly at the subsidized rate μ^* is equal to the marginal cost of contributing indirectly using a matching rate m^*. The second says the same for Greece. Thus, if Rome were to increase its matching rate, starting from an equilibrium with (m^*, μ^*), Rome would be contributing indirectly at a cost higher than the cost at which it can contribute directly. The same would apply for Greece. Therefore, starting

[2] When $m = 1/\mu$, the reaction curve (2) for Rome becomes $g(\gamma) = aw/(1+a) - m\gamma$. The slope, $-m$, equals the slope of the dashed line, $-(aw/\alpha\omega)\cdot(1+\alpha)/(1+a)$, yielding the result.

from an equilibrium where matching rates are (m^*, μ^*), neither country would want to increase their matching rate further.[3]

Some properties of the Stage 1 equilibrium with m^* and μ^* given by (10) are as follows.

1) Given that the reaction curves in Stage 2 coincide, direct contributions are indeterminate. Nonetheless, total contributions are determinate. This follows from the fact that, as noted in footnote 2, the reaction functions become in this case:

$$g(\gamma) = \frac{aw}{1+a} - m^*\gamma \qquad \text{and} \qquad \gamma(g) = \frac{\alpha\omega}{1+\alpha} - \mu^* g$$

Total contributions are determined uniquely by:

$$g + m^*\gamma = \frac{aw}{1+a} \qquad \text{and} \qquad \gamma + \mu^* g = \frac{\alpha\omega}{1+\alpha} \tag{11}$$

2) The subgame perfect equilibrium is efficient. To see this, note that $1 = m^*\mu^*$ implies that $1/(1+m^*) + 1/(1+\mu^*) = 1$. From the Stage 2 first-order conditions for Rome and Greece, (1) and (3), we have that:

$$\frac{U_G}{U_x} = \frac{1}{1+\mu^*} \qquad \text{and} \qquad \frac{\Upsilon_G}{\Upsilon_\chi} = \frac{1}{1+m^*}$$

We obtain immediately the Samuelson condition:

$$\frac{U_G}{U_x} + \frac{\Upsilon_G}{\Upsilon_\chi} = 1$$

[3] The outcome (m^*, μ^*) is not the only conceivable equilibrium. Suppose $m\mu > 1$ and the reaction curves intersect below the dashed line in Figure 1. Then, $D < 0$ by Lemma 1, and we have an unstable equilibrium. If Rome expects that an increase in m will cause the equilibrium to move to another unstable one, $u(\cdot)$ will rise with an increase in m by (9). Greece will reason the same way so will increase μ. In this case, both countries will continue to increase their subsidy rates until the reaction curves coincide with the axes of Figure 1 leading to an equilibrium at the origin with $g = \gamma = 0$. In effect, the ability of both countries to commit will lead to an outcome that is inferior to the case of Figure 1 where neither can commit! We might reasonably rule out this outcome, since it requires that each country anticipates that equilibria attained for $m\mu > 1$ will be the unstable ones.

3) The total contributions each country makes replicate their Lindahl contributions, as shown by Danziger and Schnytzer (1991). To see this, note that if $1/(1+\mu^*)$ is Rome's Lindahl price, Rome's total contribution would be the Lindahl price times G:

$$\frac{1}{1+\mu^*}G = \frac{(1+\mu^*)g + (1+m^*)\gamma}{1+\mu^*} = g + \frac{1+m^*}{1+\mu^*}\gamma = g + m^*\gamma$$

using $\mu^* = 1/m^*$. Thus, Rome's total contribution, $g + m^*\gamma$, equals its marginal rate of substitution, $1/(1+\mu^*)$, applied to G. The same applies for Greece.

4) The well-known Neutrality Theorem analyzed by Bergstrom, Blume and Varian (1986) (see also Shibata (1971) and Warr (1983)) does not apply in this two-stage matching contributions process. Using (11), total public good supply is:

$$G = \frac{aw}{1+a} + \frac{\alpha\omega}{1+\alpha}$$

For a reallocation of endowments so that $dw = -d\omega$, the change in the level of provision of the public good is:

$$dG = \frac{(a-\alpha)dw}{(1+a)(1+\alpha)}$$

Therefore, a redistribution of endowments toward the country with the higher relative valuation for the public good will increase the level of provision. Of course, this is not surprising given that the equilibrium is Lindahl, so it depends on the marginal valuations of G by both countries, which in turn depends on country wealth levels.

3 The Base Case with Limited Commitment

When both countries cannot commit, there are various options to consider depending on the policy instrument that can be used for commitment. We consider two classes of cases. The first one is when one country, say Rome, can commit to a matching contribution rate in the first stage, but not to any Stage 2 direct contributions. The second is when the commitment is to a quantity contribution in Stage 2. In this case, commitment is more complicated, and both countries may have some commitment ability as we shall see.

3.1 Rome Commits to a Matching Rate

In this case, Rome commits only to a matching rate m. Both Rome and Greece then participate in a Nash contribution game in Stage 2, since Rome cannot commit to a contribution. The analysis parallels the full commitment case above except that $\mu = 0$. As usual, we solve Stage 2 first and then move backward to Stage 1.

Stage 2

Given $\mu = 0$, we can obtain the reaction curves for Rome and Greece immediately as special cases of (2) and (4). Assuming an interior solution as before, the solution to these equations gives the NE contributions in Stage 2:

$$g(m) = \frac{(1+\alpha)aw - (1+m(1+a))\alpha\omega}{D}, \quad \gamma(m) = \frac{(1+a)\alpha\omega - aw/(1+m)}{D} \tag{12}$$

where $D = (1+\alpha)(1+a) - (1+(1+a)m)/(1+m) > 0$ so that an interior solution is always stable. As before, we can verify that $\partial g/\partial m < 0$ and $\partial\gamma/\partial m > 0$ as long as the solution is interior. Utility levels resulting from the NE in Stage 2 are then analogs of (8):

$$\begin{aligned} u(m) &= a\ln[g(m) + (1+m)\gamma(m)] + \ln[w - g(m) - m\gamma(m)] \\ \upsilon(m) &= \alpha\ln[g(m) + (1+m)\gamma(m)] + \ln[\omega - \gamma(m)] \end{aligned} \tag{13}$$

Stage 1

In Stage 1, Rome chooses m, taking $\mu = 0$ as given. Differentiating the first equation in (13) with respect to m, we obtain in an interior solution:

$$\left.\frac{du}{dm}\right|_{\mu=0} = D^{-1}\frac{a(1+a)(g/(1+m)+\gamma)}{(1+m)G} > 0$$

Therefore, Rome will increase its Stage 2 utility by increasing its matching contribution m as long as the NE in contributions remains an interior one. Since g is declining in m, Rome will in fact increase m until it crowds itself out. In terms of Figure 1, Rome will increase m until its reaction curve coincides with the dashed line.

It is straightforward to verify that Rome would not increase m beyond the point at which g just gets crowded out.[4] The equilibrium level of provision of the public good, denoted by G^m, is then:

$$G^m = (1+m)\gamma = \frac{aw}{1+a} + \frac{\alpha\omega}{1+\alpha}\left(1 - \frac{1}{1+a}\right) \tag{14}$$

which is less the level of G satisfying the Lindahl equilibrium with full commitment, denoted G^*:

$$G^* = \frac{aw}{1+a} + \frac{\alpha\omega}{1+\alpha} > G^m$$

Notice that Greece's contribution in this case is equal to the sum of its direct and matching contributions in the full-commitment case. Given that Greece's contribution is independent of m when $g = 0$, Rome could choose to replicate the full-commitment outcome by increasing its matching rate until $G^m = G^*$. Since the equilibrium induced by Rome's choice of m in Stage 1 involves $G^m < G^*$, Rome's utility must be higher in this case than in the full-commitment case, despite the fact that the outcome is inefficient. However, since Greece's total contribution is the same in both cases but $G^m < G^*$, Greece's utility is lower in this case than when both countries can commit.

Moreover, Greece is in fact worse-off in this case than in the NE with no commitment ($m = \mu = 0$). Since Rome increases its matching rate until it crowds out its direct contribution, Rome's total contribution is the product of the equilibrium matching rate given by (14) and $\gamma = \alpha\omega/(1+\alpha)$, which gives

$$g = \frac{aw}{1+a} - \frac{\alpha\omega}{(1+a)(1+\alpha)}$$

We can easily verify that this is smaller than Rome's contribution in the no-commitment case. Therefore, the introduction of the matching scheme increases γ more than it increases G, which necessarily makes Greece worse-off given that, at the no-commitment equilibrium, an equal increase in γ and G would reduce Greece's utility.

[4] From (12), the value of m for which $g = 0$ is: $m = ((1+\alpha)aw - \alpha\omega)/((1+a)\alpha\omega)$. Moreover, with $\mu = g = 0$, Greece's reaction function is simply $\gamma = \alpha\omega/(1+\alpha)$, independent of m. Any further increase in m will still increase the total supply of the public good, but it will all come from Rome. Given $g = 0$, Rome's choice of m maximizes $a\ln[(1+m)\gamma] + \ln[w - m\gamma]$. From the first-order condition, we obtain the above. Therefore, Rome's preferred choice of m just crowds out g.

3.2 Rome Commits to an Unconditional Contribution

Suppose now that Rome can commit to a quantity contribution in Stage 2. Two alternatives are of interest. The first, considered in this subsection, is that the contribution is not conditional on Greece's contribution. This is simply the case in which Rome acts as a Stackelberg leader with respect to Greece (as in Varian, 1994a). The second case is that in which Rome's contribution is contingent on Greece's contribution being at least some stated level. This case, which we refer to as a *quantity-contingent mechanism* (QCM), is taken up in the next subsection.

The problem involves two stages. In Stage 1, Rome commits unconditionally to a level of contribution g regardless of what Greece contributes in Stage 2. Naturally, in calculating g, Rome anticipates the effect of its choice on Greece's contribution γ.

Beginning again with Stage 2, Greece chooses γ to maximize $\alpha \ln[g + \gamma] + \ln[\omega - \gamma]$, taking g as given. From the first-order condition, we obtain Greece's reaction function:

$$\gamma(g) = \frac{\alpha\omega}{1+\alpha} - \frac{g}{1+\alpha}$$

In Stage 1, Rome anticipates $\gamma(g)$ and chooses g to maximize $a \ln[g + \gamma(g)] + \ln[w - g]$. The first-order condition yields, using the fact that $\partial\gamma/\partial g = -1/(1+\alpha)$:

$$g = \frac{aw - \omega}{1+a} \qquad \text{and} \qquad \gamma = \frac{(1+a)\alpha\omega - aw + \omega}{(1+a)(1+\alpha)} \tag{15}$$

The equilibrium level of provision of public good, denoted G^s, is then:

$$G^s = g + \gamma = \frac{a\alpha(w + \omega)}{(1+a)(1+\alpha)} \; < \; G^m \; < \; G^*$$

It is clear from this that a reallocation of endowments across countries will not affect the equilibrium provision of the public good. Moreover, if $dw = -d\omega$, (15) implies that $dg = dw$ and $d\gamma = d\omega$. Therefore, as noted in Varian (1994a), the analog of the Neutrality Theorem applies, as in the NE with no commitment.

We can compare Rome's utility from this Stackelberg outcome with the case where Rome can commit to a matching contribution. In the Stackelberg case, Greece's maximum contribution, obtained when $g = 0$, is $\gamma = \alpha\omega/(1+\alpha)$, which is the same as Greece's equilibrium contribution in the case where Rome commits to a matching rate. Suppose that Greece's contribution were held fixed at that level for any g. Then, if we choose g to maximize Rome's utility, taking γ to be fixed at that level, the solution is

$$g = \frac{aw}{1+a} - \frac{\alpha\omega}{(1+a)(1+\alpha)}$$

which is equal to Rome's matching (and total) contribution in the case where it commits to a matching rate. Therefore, Rome would choose to replicate the outcome of the matching-contribution case. But clearly, the fact that Greece's contribution in the Stackelberg case does not remain fixed when Rome increases g reduces Rome's utility. We therefore conclude that, when only Rome can commit, Rome is better-off if it commits to a matching rate than to an unconditional quantity contribution.

3.3 Rome Commits to a Quantity-Contingent Mechanism

A QCM is a matching plan involving discrete quantities rather than a matching rate applying to quantities continuously. Under a QCM, Rome commits to a given contribution contingent on Greece's contribution being at least some threshold amount. If Greece does not meet the threshold, Rome no longer provides its committed amount, and the outcome reverts to some fallback situation, which itself depends on the ability of the two countries to commit. Rome takes into consideration the fallback situation and designs the QCM to induce Greece to participate. The fallback situation is thus important in determining the parameters of the QCM and therefore the payoffs attained by each country.

To take account of the commitments that determine the fallback position, it is useful to characterize the QCM as a three-stage procedure. In the Stage 1, Rome announces its QCM. This specifies that if Greece's contribution γ in Stage 2 is no less than some threshold level, $\tilde{\gamma}$, Rome will contribute an amount $\tilde{g}$ in Stage 3. Otherwise, Rome will no longer be obliged to contribute $\tilde{g}$, and, instead, it will contribute some fallback level of g (lower than $\tilde{g}$). Moreover, Greece may make a further contribution in Stage 3. The fallback contributions depend upon the ability of either Rome or Greece to commit to a contribution in Stage 3 in the event that $\tilde{g}$ and $\tilde{\gamma}$ are not realized. In equilibrium, $\tilde{\gamma}$ and

$\tilde{g}$ will in fact be realized, but the values set by Rome depend upon the fallback outcomes. As we will see below, $\tilde{g}$ and $\tilde{\gamma}$ calculated by Rome and also g and γ that would emerge in the fallback situation depend on how we specify each country's commitment ability and the levels of discrete quantities to which they can commit. These differences affect the two countries' utilities in fallback situations, and thus affect the divisions of surplus between them when a QCM is actually implemented.

In what follows, we consider three different fallback situations. In the first, Rome can commit to a zero contribution in the event that Greece does not meet the contribution threshold $\tilde{\gamma}$. In the second, Greece can commit to a zero contribution in Stage 3 (after having made a contribution γ in Stage 2). Thus, Greece is effectively a Stackelberg leader in the fallback situation. In the final case, neither country can commit to a contribution in Stage 3, so the fallback outcome is effectively a NE in contributions. The QCM equilibrium outcomes turn out to be similar in one key respect: all yield an efficient supply of the public good. The only difference among them is the division of the surplus. That being so, it suffices to undertake the full analysis of equilibrium in the first case only. The other cases follow immediately once the fallback situation is specified.

3.3.1 Rome can Commit to a Zero Contribution if $\gamma < \tilde{\gamma}$

The order of events is as follows. Rome announces $\tilde{\gamma}$ and $\tilde{g}$ first. Then, Greece chooses γ in Stage 2. In Stage 3, if $\gamma \geqslant \tilde{\gamma}$, Rome supplies $\tilde{g}$; otherwise, $g = 0$. Whether Greece chooses $\gamma \geqslant \tilde{\gamma}$ depends upon the utility obtained in the fallback position, that is, Greece's *reservation utility.* Rome must ensure that the QCM gives Greece its reservation utility for the QCM to operate. We begin by specifying Greece's reservation utility. Rome will then use that to determine its choice of $\tilde{\gamma}$ and $\tilde{g}$.

Greece's Reservation Utility

In the fallback position, Rome fulfills its commitment to set $g = 0$. In Stage 2, Greece anticipates this and selects its contribution γ to solve

$$\max_{\{\gamma\}} \ \Upsilon = \alpha \ln[\gamma] + \ln[\omega - \gamma]$$

This yield Greece's optimal contribution, which is just the value of its reaction function where $g = 0$ (at its intercept with the horizontal axis in Figure 1):

$$\gamma = \frac{\alpha}{1+\alpha}\omega$$

Note for future reference that this is the same amount that Greece contributes when Rome has committed to a matching rate. Given this and $g = 0$, Greece's reservation utility is:

$$\Upsilon^{res} = \alpha \ln\left[\frac{\alpha}{1+\alpha}\omega\right] + \ln\left[\frac{1}{1+\alpha}\omega\right] \tag{16}$$

Rome's Choice of a QCM

Rome anticipates that Greece will only participate in the QCM if Greece achieves at least its reservation utility level. The problem for Rome in designing the QCM is therefore to choose the values of g and γ that maximize its own utility subject to the constraint that Greece obtains at least its reservation utility level. Naturally, the constraint will be binding: Greece will just obtain its reservation utility by participating in the QCM. The Lagrangian for Rome's problem is:

$$\mathcal{L} = a \ln[g+\gamma] + \ln[w-g] + \lambda[\alpha \ln[g+\gamma] + \ln[\omega-\gamma] - \Upsilon^{res}] \tag{17}$$

The first-order condition are:

$$\frac{a}{g+\gamma} - \frac{1}{w-g} + \lambda\frac{\alpha}{g+\gamma} = 0 \qquad \text{or} \qquad U_G - U_x + \lambda\Upsilon_G = 0 \tag{18}$$

$$\frac{a}{g+\gamma} + \lambda\left(\frac{\alpha}{g+\gamma} - \frac{1}{\omega-\gamma}\right) = 0 \qquad \text{or} \qquad U_G + \lambda(\Upsilon_G - \Upsilon_\chi) = 0 \tag{19}$$

and the binding constraint, where Υ^{res} is given by (16). The solutions to this problem then give $\tilde{g}$ and $\tilde{\gamma}$ in the QCM. Thus, Rome commits to $g = \tilde{g}$ in Stage 3 if Greece contributes $\gamma \geqslant \tilde{\gamma}$ in the Stage 2, but zero if Greece contributes any less than $\tilde{\gamma}$.

It is straightforward to show that the QCM yields an equilibrium in which Greece contributes $\tilde{\gamma}$ and Rome contributes $\tilde{g}$.[5] First, neither Rome nor Greece would want to

[5] This proof is analogous to the proof of Proposition 3 in Andreoni (1998).

contribute more than $\tilde{g}$ or $\tilde{\gamma}$. To see this, note that by the first-order conditions (18) and (19), it must be the case that $U_G - U_x < 0$ and $\Upsilon_G - \Upsilon_\chi < 0$, since $\lambda > 0$. That is, total G is too high from each individual country's point of view. Second, neither country would contribute less than the amounts $\tilde{g}$ and $\tilde{\gamma}$ specified by the QCM. In the case of Rome, it is assumed that a commitment to the QCM is binding, so g cannot be reduced below $\tilde{g}$ (although doing so would make Rome better off *ex post* since $U_G - U_x < 0$). If Greece sets γ lower than $\tilde{\gamma}$, anticipating Rome's response as specified by the announced plan, Rome acts according to the plan and each gets its reservation utility. Thus, Greece cannot be better off by lowering γ below $\tilde{\gamma}$. Thus $\tilde{g}$ and $\tilde{\gamma}$ can be sustained as Nash equilibrium in the game.

Remarkably, the QCM equilibrium is efficient. To see this, note that Rome's problem in (17) is essentially a Pareto-optimizing one. Combining the last two equations in (18) and (19), we obtain the Samuelson condition:

$$\frac{U_G}{U_x} + \frac{\Upsilon_G}{\Upsilon_\chi} = 1$$

In this efficient outcome, Greece just obtains the reservation utility level Υ^{res}, while Rome gets the remaining surplus from internalizing the free-rider problem.

It follows immediately that whatever the reservation utility level that Greece must be given in order to willingly provide $\tilde{\gamma}$, the above analysis will apply. As we have seen, the reservation level of utility is determined by the fallback outcome, which in turn depends upon commitment ability in Stage 3. The implication is that, whatever we assume about commitment in Stage 3, the QCM will yield an efficient outcome. Different outcomes will divide this surplus from internalizing the free-rider problem between Rome and Greece differently, and we briefly consider two other fallback situations below. First, some comments on this QCM equilibrium should be made.

The QCM equilibrium resembles the point provision results found in Bagnoli and Lipman (1989), Admati and Perry (1991) and Andreoni (1998). In these papers, the technology of the public good is such that it is produced or purchased if and only if total contributions exceed a certain threshold. Then, in an important subset of equilibria found in Bagnoli and Lipman (1989) and in some of the cases analyzed by Admati and Perry (1991) and Andreoni (1998), the aggregate level of private provision is exactly at the threshold. Efficiency obtains in many of the cases analyzed by these papers. The experimental outcomes of Bagnoli and McKee (1991) and Cadsby and Maynes (1999) support the theoretical results in Bagnoli and Lipman (1989). A QCM does not involve a technological threshold in the production or purchase of the public good: continuous amounts can be supplied. However, the player who commits to a QCM effectively uses a threshold to leverage a given contribution by the other player.

As mentioned, a property of the QCM outcome is that Rome, who sets the matching plan and moves in the last stage, gets all surplus from trade, while Greece, whose contribution is being matched, only gets its reservation utility. In principle, we might be able to introduce some reduced-form bargaining over surplus division between the two countries and allow Greece to get some of the surplus. Suppose, for example, that Greece can bargain with Rome and manage to get a level of utility of $k\Upsilon^{res}$ in the QCM, where k is some constant greater than unity. Then Rome needs to change $\tilde{g}$ and $\tilde{\gamma}$ accordingly to make Greece participation constraint hold with equality at $k\Upsilon^{res}$. The outcome would still be efficient but at a different point on the utility possibility frontier.

3.3.2 Greece can Commit to a Zero Contribution in Stage 3

In this case, if Greece contributes less than $\tilde{\gamma}$ in Stage 2, it, but not Rome, can commit to contributing zero in Stage 3. This effectively makes Greece a Stackelberg leader in the fallback situation. The calculation of Greece's reservation utility involves solving the Stackelberg game backward. In the last stage, Rome takes γ as given and chooses g to maximize $a\ln[g+\gamma]+\ln[w-g]$. The solution to this gives Rome's reaction function:

$$g(\gamma)=\frac{a}{1+a}w-\frac{1}{1+a}\gamma\geqslant 0$$

In Stage 2, Greece solves

$$\max_{\{\gamma\}}\ \Upsilon=\alpha\ln\left[\max\left\{\frac{a}{1+a}w-\frac{1}{1+a}\gamma,\,0\right\}+\gamma\right]+\ln[\omega-\gamma]$$

As in Varian (1994a), the solution to this depends on the comparison between the utility Greece obtains if it crowds out Rome's contribution completely and the utility Greece gets if it does not. That is, Greece compares the maximized utilities

$$\max_{\{\gamma\}} \left\{ \alpha \ln \left[\frac{a}{1+a} w - \frac{1}{1+a} \gamma + \gamma \right] + \ln[\omega - \gamma] \right\} \text{ and } \max_{\{\gamma\}} \{ \alpha \ln[\gamma] + \ln[\omega - \gamma] \}$$

Comparing this with the case above where Rome can commit to a zero contribution in Stage 3, if Greece finds it optimal to crowd out Rome's contribution, reservation utilities will be exactly the same as in (16). On the other hand, if Greece finds it optimal not to crowd out Rome's contribution, its reservation utility is higher in this case than when Rome can commit to a zero contribution in Stage 3.

Given this reservation utility level, the remainder of the analysis of the QCM equilibrium is basically the same as in the previous case. The QCM equilibrium is efficient. Greece gets its reservation utility, and Rome gets the rest of the surplus. If Greece chooses not to crowd out Rome in the Stackelberg fallback position, Greece will be better off and Rome worse off than in the case of subsection 3.3.1. Thus, the ability to commit determines the division of the surplus.

3.3.3 Neither Country can Commit in Stage 3

In this case, both countries behave like Nash competitors in Stage 3 if $\gamma < \tilde{\gamma}$. In the case of Greece, even though it has made a contribution in Stage 2, it cannot refrain from making further contributions in Stage 3. Then, the outcome will be the same as in a static Nash equilibrium. Let us label the static Nash equilibrium quantities by a superscript N and the equilibrium quantities when Greece is a Stackelberg leader (the case in 3.3.2) by a superscript S.

Theorem 2 of Varian (1994a) implies that if Rome contributes nothing in a static Nash equilibrium so $g^N = 0$, then the Nash equilibrium contributions g^N and γ^N are also Stackelberg equilibrium quantities. On the other hand, if Rome contributes some positive amount in a static Nash equilibrium ($g^N > 0$), then $\gamma^S < \gamma^N$ and $\Upsilon^S > \Upsilon^N$. That is, Greece's contribution in a Stackelberg equilibrium will be less than its contribution in a static Nash equilibrium, and its utility is higher in a Stackelberg equilibrium than in a static Nash equilibrium. Thus, if $g^N = 0$, then it is also the case that $g^S = 0$—meaning that Greece is the only contributor in both the Stackelberg and the static NE because of relatively large differences in wealth or valuations—and the reservation utilities would be identical in all of the three cases considered. If $g^N > 0$, then Υ^{res} is most likely highest in case 2, where Greece but not Rome can commit to zero contribution in the last stage, followed by case 3, which is the static Nash equilibrium, and then by case 1, where Rome can commit to zero contribution and effectively makes itself a Stackelberg leader. Equivalently, since all QCMs are efficient, Rome obtains the highest utility in case 1, followed by case 3, and then case 2.

3.4 Comparing the QCM with other Outcomes

We saw earlier that Rome is better off when it can commit to a matching rate than when it can make an unconditional quantity commitment (and be a Stackelberg leader). Here we consider whether the QCM would be preferred by Rome to a matching rate. In fact, comparisons turn out generally to be ambiguous. From Rome's point of view, the QCM is attractive because it gets all the incremental surplus from moving to a fully efficient outcome. On the other hand, the value of that depends upon the two countries' ability to commit in Stage 3, which determines the reservation utility that Greece must be given in the QCM. If the fallback utility that Greece obtains in the QCM is no greater than it obtains in one of the other remedies, then Rome will prefer the QCM.

Suppose, to take the case most favorable for Rome, that Rome can commit to a fallback position in which it makes zero contributions. As we have seen, Greece's contribution in the fallback situation is $\alpha\omega/(1+\alpha)$, which is the same amount it contributes when Rome sets a matching rate. However, while $g = 0$ in both cases, Rome makes an indirect contribution in the matching case since it subsidizes Greece. Therefore, the utility that Greece obtains in the fallback situation will be less than it receives under a matching contribution scheme. Since the matching contribution equilibrium is inefficient, less total surplus is generated. Given that Rome obtains all the incremental surplus in the QCM game, Rome must be better off in this case and would prefer to implement a QCM rather than a matching contribution. (If Rome can commit to a zero contribution in the Stage

3, it can presumably also implement the unconditional contribution scheme. But, we have already seen that such a scheme is inferior from its perspective than a matching contribution scheme.)

However, if Greece can commit to a zero contribution in Stage 3, Greece is effectively a Stackelberg leader in the fallback equilibrium. In this case, Greece's reservation utility may well be higher than what it achieves in the matching contribution equilibrium. If so, the additional incremental surplus that Rome obtains in the QCM may not be enough to compensate for the fact that Greece obtains higher utility in the QCM than in the matching case. By the same token, the QCM may or may not be preferable to an unconditional contribution where Rome assumes Stackelberg leadership. On the other hand, it is not clear that such a scheme is feasible in this case, because Rome cannot commit to a zero contribution in Stage 3.

3.5 Summary of Results

The main results from this section can be summarized as follows:

i. If only one country can commit to a matching rate, it will choose its matching rate to exactly crowd out its own contribution and the level of provision of the public good will be lower than the efficient level. The country that offers a matching contribution will be better off than in both the full commitment equilibrium and the NE with no commitment. The opposite will hold for the country that is unable to commit to a matching rate.

ii. If one country commits to an unconditional contribution, the level of provision of the public good and the level of utility of the country that commits will both be lower than in the case where commitment is over a matching rate.

iii. If one country commits to a QCM, the level of provision of the public good will be efficient, independently of the abilities of countries to commit to particular contributions in the fallback outcome. However, the countries' abilities to commit in the fallback situations determine how the surplus from implementing the QCM is divided.

If the country that sets the QCM can commit to a zero contribution in the fallback situation, it would prefer to set a QCM than committing to a matching rate. However, this may not be the case in the other fallback situations.

4 Two Public Goods

Suppose now there are two public goods denoted by subscripts 1 and 2. We explore the consequences of one country—Rome again—being able to commit to various contribution schemes. We again assume that utility functions are loglinear and take the following form:

$$U(G_1, G_2, x) = a_1 \ln G_1 + a_2 \ln G_2 + \ln x, \quad \Upsilon(G_1, G_2, \chi) = \alpha_1 \ln G_1 + \alpha_2 \ln G_2 + \ln \chi$$

This formulation follows Cornes and Itaya (2004), who concentrate on the properties of a single-stage contribution game.

The possibilities for limited commitment are more extensive now, since only one country may be able to commit, and the commitment may be over one or both public goods. We proceed by first supposing that Rome can commit to matching and QCM schemes involving both public goods, and then turn to the case where the commitment is only over one public good, say G_1. These two cases give rise to different natural assumptions about timing. When Rome can commit to both public goods, we assume that contributions by Rome and Greece to the two public goods are simultaneous, as in Cornes and Itaya. On the other hand, when Rome can only commit to one public good, contributions are sequential: those to G_1 are made first, followed by contributions to G_2. While the simultaneous contribution case yields results that are analogous to the one-public good case already considered, in the sequential case commitments turn out to be of no consequence.

4.1 Public Goods Determined Simultaneously

Consider first the case where Rome can commit to matching rates m_1 and m_2 applied to Greece's contributions γ_1 and γ_2. Subsequently, the alternative of applying a QCM to both public goods will be analyzed.

4.1.1 Rome Commits to Matching Rates

There are two stages as in the one-public good case. In Stage 1, Rome announces matching rates m_i, $i = 1, 2$. Then, in Stage 2, both countries make contributions g_i and γ_i. As usual, we solve for the subgame perfect equilibrium by backward induction.

Stage 2

At this stage, matching rates m_1 and m_2 have been announced, and both countries choose their contributions, given those of the other country. Rome's problem is:

$$\max_{\{g_1,g_2\}} a_1 \ln\left[g_1 + (1+m_1)\gamma_1\right] + a_2 \ln\left[g_2 + (1+m_2)\gamma_2\right] + \ln\left[w - g_1 - m_1\gamma_1 - g_2 - m_2\gamma_2\right]$$

If an interior solution applies, the first-order conditions are:

$$\frac{a_i}{g_i + (1+m_i)\gamma_i} - \frac{1}{w - g_1 - m_1\gamma_1 - g_2 - m_2\gamma_2} = 0 \quad \text{or} \quad \frac{a_i}{G_i} - \frac{1}{x} = 0 \qquad i = 1, 2 \quad (20)$$

Thus, in an interior solution for Rome, public good output levels must satisfy $G_1/G_2 = a_1/a_2$. If $G_1/G_2 > a_1/a_2$ in equilibrium, Rome would only contribute to public good 2, and vice versa. The analogous problem for Greece yields the first-order condition:

$$\frac{\alpha_i(1+m_i)}{g_i + (1+m_i)\gamma_i} - \frac{1}{\omega - \gamma_1 - \gamma_2} = 0 \qquad \text{or} \qquad \frac{\alpha_i(1+m_i)}{G_i} - \frac{1}{\chi} = 0 \qquad i = 1, 2 \quad (21)$$

In this case, an interior solution would require $G_1/G_2 = (\alpha_1/\alpha_2)(1+m_1)/(1+m_2)$. If this equality is not satisfied in equilibrium, Greece would only contribute to one public good.

Both countries will contribute to both public goods only in the improbable case in which $a_1/a_2 = (\alpha_1/\alpha_2)(1+m_1)/(1+m_2)$. If we ignore this knife-edge case, the implication is that there are three types of equilibria that are of interest (ruling out the corner solution where only one country is a contributor, as in our earlier analysis). In Case 1, which we shall see is the typical equilibrium outcome, Rome contributes to one public good, and Greece to both. In Case 2, each country contributes only to one of the public goods. Case 3 is where Rome contributes to both and Greece to one. Suppose $a_1/a_2 > \alpha_1/\alpha_2$, so Rome has a relative preference for G_1. The Stage 2 outcomes in these three cases are as follows.

Case 1: $g_1 > 0, g_2 = 0$ and $\gamma_1 > 0, \gamma_2 > 0$

Relatively too much of public good 2 is being supplied from Rome's perspective, so it will set $m_2 = 0$. The first-order conditions on g_1, γ_1 and γ_2 in (20) and (21) apply. Solving them jointly and using $m_2 = 0$ yields the following NE contributions in Stage 2:

$$g_1 = D^{-1}\left[a_1 w(1+m_1)(1+\alpha_1+\alpha_2) - (1+m_1+a_1 m_1)(1+m_1)\alpha_1\omega\right]$$

$$\gamma_1 = D^{-1}\left[(1+a_1)(1+m_1)\alpha_1\omega - a_1(1+\alpha_2)w\right] \tag{22}$$

$$\gamma_2 = D^{-1}\left[a_1\alpha_2\omega\right]$$

where $D = \alpha_1(1+a_1)(1+m_1) + a_1(1+\alpha_2) > 0$. Differentiating with respect to m_1 yields expressions that will be useful in what follows:

$$\frac{\partial\gamma_1}{\partial m_1} = \frac{(1+a_1)\alpha_1(\omega-\gamma_1)}{D} > 0, \qquad \frac{\partial\gamma_2}{\partial m_1} = \frac{\partial G_2}{\partial m_1} = \frac{-a_1(1+a_1)\alpha_1\alpha_2\omega}{D^2} < 0 \tag{23}$$

Using these solutions for Nash equilibrium contributions, we can write Stage 2 indirect utility functions for Rome and Greece as follows:

$$u(m_1) = a_1\ln[g_1(m_1) + (1+m_1)\gamma_1(m_1)] + a_2\ln[\gamma_2(m_1)] + \ln[w - g_1(m_1) - m_1\gamma_1(m_1)]$$

$$v(m_1) = \alpha_1\ln[g_1(m_1) + (1+m_1)\gamma_1(m_1)] + \alpha_2\ln[\gamma_2(m_1)] + \ln[\omega - \gamma_1(m_1) - \gamma_2(m_1)] \tag{24}$$

Case 2: $g_1 > 0, g_2 = 0$ and $\gamma_1 = 0, \gamma_2 > 0$

The first-order conditions for g_1 and γ_2 apply and yield the following reaction functions:

$$g_1 = \frac{a_1(w - m_2\gamma_2)}{1+a_1}, \qquad \gamma_2 = \frac{\alpha_2\omega}{1+\alpha_2} \tag{25}$$

Greece's contribution is independent of the matching rate offered by Rome, while g_1 will be decreasing in m_2. A matching rate m_1 would be ineffective in this case unless it is large enough to induce Greece to contribute, that is, to switch to Case 1.

Case 3: $g_1 > 0, g_2 > 0$ and $\gamma_1 > 0, \gamma_2 = 0$

In this case, first-order conditions for g_1, g_2 and γ_1 in (20) and (21) apply. Solving these for their Nash equilibrium values yields:

$$g_1 = D^{-1}\left[a_1(1+\alpha_1)(1+m_1)w - \alpha_1(1+m_1)(1+a_2+(1+a_1+a_2)m_1)\omega\right]$$

$$g_2 = D^{-1}\left[a_2\alpha_1(1+m_1)(w+\omega)\right] \tag{26}$$

$$\gamma_1 = D^{-1}\left[(1+a_1+a_2)\alpha_1(1+m_1)\omega - a_1 w\right]$$

where $D = a_1 + (1 + a_1 + a_2)\alpha_1(1 + m_1) > 0$. It can be shown that γ_1 and g_2 are both increasing in m_1 while the effect on g_1 is ambiguous. Stage 2 indirect utility functions analogous to (24) apply here as well.

Stage 1

In this stage, Rome chooses its matching rates to apply to Greece's contributions. Consider first Case 1. As mentioned, Rome would not use m_2 since from its point of view, G_2 is already relatively too high. To determine whether it would use m_1, differentiate $u(m_1)$ in (24) and use the first-order condition for Rome's Stage 2 problem and (23) to obtain:

$$\frac{du}{dm_1} = U_{G_1}\frac{\partial \gamma_1}{\partial m_1} + U_{G_2}\frac{\partial \gamma_2}{\partial m_1} = \frac{a_1(1 + a_1)(1 + \alpha_2)}{(1 + m_1)D} - \frac{a_2\alpha_1(1 + a_1)}{D}$$

Evaluating this expression at $m_1 = 0$ yields:[6]

$$\left.\frac{du}{dm_1}\right|_{m_1=0} > 0 \qquad \text{since} \qquad \frac{a_1}{a_2} > \frac{\alpha_1}{1 + \alpha_2}$$

Therefore, Rome will offer a matching rate on γ_1. Inspection of the expression for du/dm_1 indicates that there will be an optimal value of $m_1 \equiv m_1^* > 0$ for which $du/dm_1 = 0$.

Suppose next that, in the absence of subsidies, the outcome is in Case 2 where both countries are specializing in one of the public goods. As the reaction functions in (25) indicate, Greece's contribution γ_2 will not be affected by changes in m_2, so any increase in m_2 starting from zero will be equivalent to a direct contribution by Rome to G_2. Since Rome has already chosen not to contribute, this must be welfare-decreasing. Moreover, a small increase in m_1 starting from zero will have no effect either unless G_1/G_2 is very close to α_1/α_2 so that Greece is initially just on the margin of contributing to public good 1. Otherwise, as m_1 is increased from zero, it will have no effect until it reaches the value, say $\overline{m}_1$, such that $G_1/G_2 = \alpha_1(1 + \overline{m}_1)/\alpha_2$. At this point, Greece will be on the verge of contributing to G_1. As m_1 increases from $\overline{m}_1$, Greece will gradually increase γ_1 starting from zero. If $\overline{m}_1 < m_1^*$, Rome will increase m_1: its utility will gradually increase from that attained in Case 2 to that attained at the optimum in Case 1.

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[6] The last inequality holds because in Case 2, $a_1/a_2 > G_1/G_2 = \alpha_1/\alpha_2$.

Finally, consider Case 3. A small subsidy on good 2 will not be effective since $\gamma_2 = 0$. Suppose a subsidy on good 1 is contemplated. Differentiating Rome's utility in this case, we obtain after simplification and using the first-order conditions from Stage 2:

$$\frac{du}{dm_1} = \frac{a_1}{g_1 + (1 + m_1)\gamma_1} > 0$$

This would seem to indicate that Rome would want to match Greece's contributions at some rate. However, even a small matching rate on Greece's contribution γ_1 will cause G_1/G_2 to rise. Rome will go to a corner in which it contributes only to good 2, that is, Case 2 above. The analysis of Case 2 will then apply: provided $\overline{m}_1 < m_1^*$, Rome will want to increase m_1 up to m_1^*.

The upshot of this section is that as long as $m_1^* > \overline{m}_1$, Rome will find it useful to impose a matching rate of $m_1 = m_1^*$ regardless of which case applies in the absence of matching contributions. On the other hand, if $m_1^* < \overline{m}_1$, a subsidy will only be of value to Rome if the initial equilibrium is in Case 1.

4.1.2 Rome Commits to a QCM

The idea of a QCM in the context of two public goods is similar to that with one public good. The plan consists of announced contribution levels $(\tilde{\gamma}_1, \tilde{g}_1, \tilde{\gamma}_2, \tilde{g}_2)$ in Stage 1. If both $\gamma_1 \geqslant \tilde{\gamma}_1$ and $\gamma_2 \geqslant \tilde{\gamma}2$ in Stage 2, Rome contributes $\tilde{g}_1$ and $\tilde{g}_2$. Otherwise, Rome and Greece move to some fallback situation in Stage 3, which depends upon the ability of each party to commit in this stage. Fallback situations might be complicated to characterize for the two-public-good case, given the possibility of the different cases outlined above and the fact that a wide variety of commitment options exist for Stage 3. Fortunately for our purposes, we do not need an explicit characterization of fallback situations, as long as we know that some reservation utility for Greece would emerge. Rome then incorporates this reservation utility into its calculation of the four QCM quantities.

Suppose Greece's reservation utility is Υ^{res}. Rome's problem is then the analog of

(17) above:

$$\max_{\substack{\{g_1,g_2,\\ \gamma_1,\gamma_2\}}} U(G_1,G_2,x) = a_1 \ln[g_1+\gamma_1] + a_2 \ln[g_2+\gamma_2] + \ln[w-g_1-g_2]$$
$$\text{s.t. } \Upsilon(G_1,G_2,\chi) = \alpha_1 \ln[g_1+\gamma_1] + \alpha_2 \ln[g_2+\gamma_2] + \ln[\omega-\gamma_1-\gamma_2] \geqslant \Upsilon^{res}$$

We can write the Lagrangian in shorthand form as follows:

$$\mathcal{L} = U(G_1,G_2,x) + \lambda[\Upsilon(G_1,G_2,\chi) - \Upsilon^{res}]$$

where $G_i = g_i + \gamma_i$, $x = w - g_1 - g_2$, and $\chi = \omega - \gamma_1 - \gamma_2$. The first-order conditions on g_i and γ_i can be written:

$$U_{G_i} - U_x + \lambda \Upsilon_{G_i} = 0 \qquad i = 1,2 \tag{27}$$

$$U_{G_i} + \lambda(\Upsilon_{G_i} - \Upsilon_\chi) = 0 \qquad i = 1,2 \tag{28}$$

Proceeding as before, we can combine (27) and (28) for each public good to yield the Samuelson conditions:

$$\frac{U_{G_1}}{U_x} + \frac{\Upsilon_{G_1}}{\Upsilon_\chi} = 1, \qquad \text{and} \qquad \frac{U_{G_2}}{U_x} + \frac{\Upsilon_{G_2}}{\Upsilon_\chi} = 1$$

Thus, efficiency can be achieved if one country can commit to a QCM, regardless of the fallback position. We can show, following the same procedure as before, that this QCM outcome is actually an equilibrium. Neither Greece nor Rome would want to increase their contributions above the QCM. Greece would not want to decrease its contributions, and Rome has committed not to do so.

4.2 Public Goods Determined Sequentially

Suppose now commitments apply only to G_1. As above, we begin with the case where Rome commits to a matching rate, and then turn to the QCM case.

4.2.1 Rome Commits to a Matching Rate

Now we need to distinguish three stages. In Stage 1, Rome commits to a matching rate m_1 applied to Greece's contribution γ_1. In Stage 2, Rome and Greece simultaneously contribute g_1 and γ_1. In Stage 3, they contribute g_2 and γ_2 to the second public good.

Equilibrium outcomes are sub-game perfect, and we assume that contributions are interior in the last stage, though as we shall see not necessarily in Stage 2. Begin with Stage 3.

Stage 3

In this stage, Rome and Greece simultaneously choose g_2 and γ_2, given the values of g_1, γ_1 and m_1 chosen in the previous two stages. The problems for Rome and Greece are:

$$\max_{g_2} \; a_2 \ln[g_2 + \gamma_2] + \ln[w - g_1 - m_1\gamma_1 - g_2], \qquad \max_{\gamma_2} \; \alpha_2 \ln[g_2 + \gamma_2] + \ln[\omega - \gamma_1 - \gamma_2]$$

Assuming an interior solution, the first-order conditions yield the reaction functions:

$$g_2 = \frac{a_2(w - g_1 - m_1\gamma_1)}{1 + a_2} - \frac{\gamma_2}{1 + a_2}, \qquad \gamma_2 = \frac{\alpha_2(\omega - \gamma_1)}{1 + \alpha_2} - \frac{g_2}{1 + \alpha_2}$$

Solving these for g_2 and γ_2, we find Nash equilibrium contributions to be:

$$\begin{aligned} g_2(g_1, \gamma_1, m_1) &= D^{-1}[(1 + \alpha_2)a_2(w - g_1 - m_1\gamma_1) - \alpha_2(\omega - \gamma_1)] \\ \gamma_2(g_1, \gamma_1, m_1) &= D^{-1}[(1 + a_2)\alpha_2(\omega - \gamma_1) - a_2(w - g_1 - m_1\gamma_1)] \end{aligned} \tag{29}$$

where $D = (1 + a_2)(1 + \alpha_2) - 1 > 0$. Total contributions are given by $G_2(g_1, \gamma_1, m_1) = g_2(g_1, \gamma_1, m_1) + \gamma_2(g_1, \gamma_1, m_1)$, which using (29) can be written:

$$G_2(G_1) = D^{-1}[a_2\alpha_2(w + \omega - G_1)] \tag{30}$$

where $G_1 = g_1 + (1 + m_1)\gamma_1$. Thus, G_2 is decreasing in total contributions to G_1, but is independent of the relative contributions of each country.

Using (29), private goods' consumption in Rome and Greece can be obtained from $x = w - g_1 - m_1\gamma_1 - g_2$ and $\chi = \omega - \gamma_1 - \gamma_2$:

$$\begin{aligned} x(G_1) &= D^{-1}[\alpha_2(w + \omega - G_1] \\ \chi(G_1) &= D^{-1}[a_2(w + \omega - G_1] \end{aligned} \tag{31}$$

These results imply that whatever the value of m_1, a Neutrality Theorem applies to Stage 3 contributions, assuming there is an interior solution. That is, G_2, x, χ and the utility levels of each country are not only independent of income redistribution across countries but are also independent of the relative contributions of each country to G_1, given its level. Especially relevant is the fact that Stage 3 allocations are also independent of m_1.

Stage 2

In Stage 2, Rome and Greece choose g_1 and γ_1 simultaneously anticipating Nash equilibrium outcomes in Stage 3. The problem of Rome is the following, taking as given γ_1:

$$\max_{g_1} \ a_1 \ln[g_1 + (1+m_1)\gamma_1] + a_2 \ln[g_2(g_1,\gamma_1,m_1) + \gamma_2(g_1,\gamma_1,m_1)]$$
$$+ \ln[w - g_1 - m_1\gamma_1 - g_2(g_1,\gamma_1,m_1)]$$

The first-order conditions to this problem, using (29) and simplifying, give Rome's reaction function:

$$g_1 = \max\left\{0,\ \frac{a_1(w+\omega)}{1+a_1+a_2} - (1+m_1)\gamma_1\right\}$$

The analogous problem for Greece yields the following reaction function:

$$\gamma_1 = \max\left\{0,\ \frac{\alpha_1(w+\omega)}{(1+\alpha_1+\alpha_2)(1+m_1)} - \frac{g_1}{(1+m_1)}\right\}$$

Therefore, the two reaction curves have the same slopes, implying that there will be a corner solution with only one country contributing. The contributing country will be determined by the relative size of the intercepts. In particular, the Nash equilibrium solution is as follows (using $G_1 = g_1 + (1+m_1)\gamma_1 = \max\{g_1, (1+m_1)\gamma_1\}$):

$$G_1 = \max\left\{\frac{a_1(w+\omega)}{(1+a_1+a_2)},\ \frac{\alpha_1(w+\omega)}{(1+\alpha_1+\alpha_2)}\right\} \tag{32}$$

Figure 2 illustrates the case in which Rome is the contributing country. Rome's reaction curve is shown as a dashed line, while Greece's is a dotted line.

Stage 1

As the above analysis makes clear, the equilibrium values of G_1, G_2, x and χ are all independent of m_1. This is a consequence of the Neutrality Theorem applying in Stage 3. Any subsidy Rome imposes is neutralized by changes in Stage 3 contributions. Therefore, Rome has no incentive to offer a matching contribution, since it has no effect on the outcome.

4.2.2 Rome Commits to a QCM

We assume now that the QCM applies only to G_1, and equilibrium in G_1 occurs before contributions are made to G_2. Outcomes are determined over four stages. In Stage 1, Rome announces its QCM. Greece then contributes γ_1 in Stage 2. In Stage 3, Rome, observing γ_1, contributes g_1 and Greece may make a further contribution to good 1. Finally, in Stage 4, Rome and Greece simultaneously contribute g_2 and γ_2. In each stage, outcomes of subsequent stages are anticipated.

The QCM applying to G_1 is analogous to the one-public-good case. Rome announces and commits to a QCM which specifies that, if Greece contributes $\gamma_1 \geqslant \tilde{\gamma}_1$, Rome will contribute $g_1 = \tilde{g}_1$. Otherwise, the fallback outcome applies. For concreteness, we assume that Rome can commit to $g_1 = 0$ if $\gamma_1 < \tilde{\gamma}_1$. (Neither country can commit to a contribution to G_2 in Stage 4.) This implies that Greece's reservation utility is relatively low, and one might expect that Rome would be able extract a relatively high surplus by applying the QCM. However, as we shall see, that will not be the case for reasons that are analogous to the sequential contribution case involving matching rates just considered.

We assume, again, that both countries make positive contributions (g_2, γ_2) in Stage 4. This requires that, after the QCM outcome is determined, the distribution of remaining wealth $(w - g_1, \omega - \gamma_1)$ is not very unequal. Of course, this distribution is determined endogenously, but, roughly speaking, a relatively even initial wealth distribution is more likely to lead to a more even post-Stage 2 wealth distribution.

We begin with Stage 4 and then move back to the prior stages.

Stage 4

The outcomes in this stage follow directly from the analysis of Stage 3 in the matching rate case, except that now $m_1 = 0$. Given g_1 and γ_1, Rome and Greece choose g_2 and γ_2 to maximize their respective utilities, taking the others' contribution as given. The Nash equilibrium solution follows immediately from (29) with $m_1 = 0$:

$$\begin{aligned} g_2(g_1, \gamma_1) &= D^{-1}[(1+\alpha_2)a_2(w - g_1) - \alpha_2(\omega - \gamma_1)] \\ \gamma_2(g_1, \gamma_1) &= D^{-1}[(1+a_2)\alpha_2(\omega - \gamma_1) - a_2(w - g_1] \end{aligned} \tag{33}$$

where $D = (1+a_2)(1+\alpha_2) - 1 > 0$. Total contributions, $G_2 = g_2(g_1, \gamma_1) + \gamma_2(g_1, \gamma_1)$, are:

$$G_2(G_1) = D^{-1}[a_2\alpha_2(w + \omega - G_1)] \tag{34}$$

Using these, private goods' consumption in Rome and Greece may be written again as (31). Thus, we have that NE outcomes in Stage 4, $G_2(G_1)$, $x(G_1)$ and $\chi(G_1)$, are independent of the distribution of wealth and of the relative contributions of the two countries.

We can then write utility levels for Rome and Greece arising from the prior stages as

$$U(G_1, G_2(G_1), x(G_1)) = a_1 \ln[G_1] + a_2 \ln[G_2(G_1)] + \ln[x(G_1)]$$

$$\Upsilon(G_1, G_2(G_1), \chi(G_1)) = \alpha_1 \ln[G_1] + \alpha_2 \ln[G_2(G_1)] + \ln[\chi(G_1)]$$

where $G_2(g_1), x(G_1), \chi(G_2)$ are given by (34) and (31). The level of G_1 is determined in Stages 1 to 3. However, we do not need to characterize the three stages explicitly in this case. In fact, it is fairly straightforward to see that Rome will not have any incentive to offer a QCM. To see this, note that from the above utility functions, the level of G_1 is the only variable about which both countries are concerned in Stages 1 to 3. We can characterize the levels of G_1 that would be preferred by Rome and Greece, given their anticipation of the solution to Stage 4. These levels of G_1 solve the following problems:

$$\max_{G_1}\ a_1 \ln[G_1] + a_2 \ln[G_2(G_1)] + \ln[x(G_1)], \quad \max_{G_1}\ \alpha_1 \ln[G_1] + \alpha_2 \ln[G_2(G_1)] + \ln[\chi(G_1)]$$

Using (33), the solutions, denoted by G_1^R and G_1^G for Rome and Greece respectively, are:

$$G_1^R = \frac{a_1(w+\omega)}{1+a_1+a_2}, \qquad G_1^G = \frac{\alpha_1(w+\omega)}{1+\alpha_1+\alpha_2},$$

These are the levels of G_1 that maximize the utility of each country independently of their relative contributions to public good 1.

If $[a_1/(1+a_1+a_2)] > [\alpha_1/(1+\alpha_1+\alpha_2)]$ (the case depicted in Figure 2), then Rome prefers a higher level of G_1 than Greece. In this case, Rome has no incentive to offer a QCM since, by (32), the Nash equilibrium in contributions would yield its preferred level of G_1. Alternatively, if $[a_1/(1+a_1+a_2)] < [\alpha_1/(1+\alpha_1+\alpha_2)]$, Rome would not want to use a QCM to induce a higher level of provision of G_1 since the Nash equilibrium level of G_1, given again by (32), is already above Rome's preferred level.[7] Therefore, when the two public goods are provided sequentially, Rome cannot make itself better off than in the Nash equilibrium, and hence would not offer a QCM.

4.3 Summary of Results

The main results from this section can be summarized as follows:

i. If contributions to the two public goods occur simultaneously, and if one country can fully commit to matching rates on either public goods, a matching subsidy on one of the goods will always be used provided the optimal matching rate in Case 1 exceeds the rate required to move from Cases 2 and 3 to Case 1. Otherwise, a matching subsidy will be used only in Case 1 (where the country offering the matching rate contributes to both public goods).

ii. If contributions to the two public goods take place at the same time and one country can commit to a QCM applying on both public goods, the level of provision of both public goods will be efficient.

iii. When contributions to the public goods take place sequentially, a Neutrality Theorem applies to contributions to the second public good if these contributions are interior. In turn, that implies that neither a matching subsidy nor a QCM for the public good contributed first will be offered, as the country with the ability to commit cannot make itself better off under either of these schemes than in the Nash equilibrium in contributions.

[7] Note that in this latter case, where Rome prefers a lower level of G_1 than Greece, Rome could induce a level of G_1 equal to its preferred level by using a QCM which would specify that, if Greece contributes $\gamma_1 \leqslant \tilde{\gamma}_1$, Rome will contribute $g_1 = \tilde{g}_1$, otherwise Rome will contribute $g_1 > \hat{g}_1$. However, that requires that Rome be able to commit to a relatively high level of contribution in the fallback outcome. If this fallback contribution is large enough, it would induce a level of G_2 low enough to make Greece worse off than under the contributions specified in the QCM, despite the large contribution of Rome, and would therefore induce Greece to comply with the QCM.

5 Concluding Remarks

This paper can be viewed as exploratory in nature. We have considered in a specific setting the scope for non-cooperative multi-stage games to emerge that will improve the efficiency of public goods supply in a context with two agents or countries. If one or more of the countries can commit to a contribution policy, the free-rider problem may be mitigated. In some cases, the free-rider problem can be overcome fully. The use of loglinear utility functions for the analysis was adopted to facilitate clear and relatively simple analytical solutions, and many of the results do not depend on that.

Even in the base case with one public good, many comparisons are ambiguous. Thus, Rome may or may not prefer a QCM depending on the reservation utility that must be provided to Greece. It is clear that matters become considerably more complicated once we leave the simple base case. The main departure we considered was to allow for two public goods. The effect of this was to reduce the scope for matching schemes. When only one country can commit, it may not find matching contributions to be helpful even if they can be made to both public goods. On the other hand, the QCM continues to lead to efficiency when it can be applied by one country to both public goods. When either a matching contribution or a QCM can be applied to only the public good contributed first and when we assume interior contributions to the second, neither scheme can improve upon the fallback Nash equilibria.

The next obvious extension is to the multi-agent case. It can be shown that if one country can commit to a QCM in which its contributions are contingent on the aggregate level of contributions of all other countries, full efficiency is maintained. In effect, the QCM facing all other countries together is like a weakest-link public good in which an equilibrium is for all to contribute their efficient amounts.

A very large open question remains. What determines which countries can commit? The answer to that is not apparent.

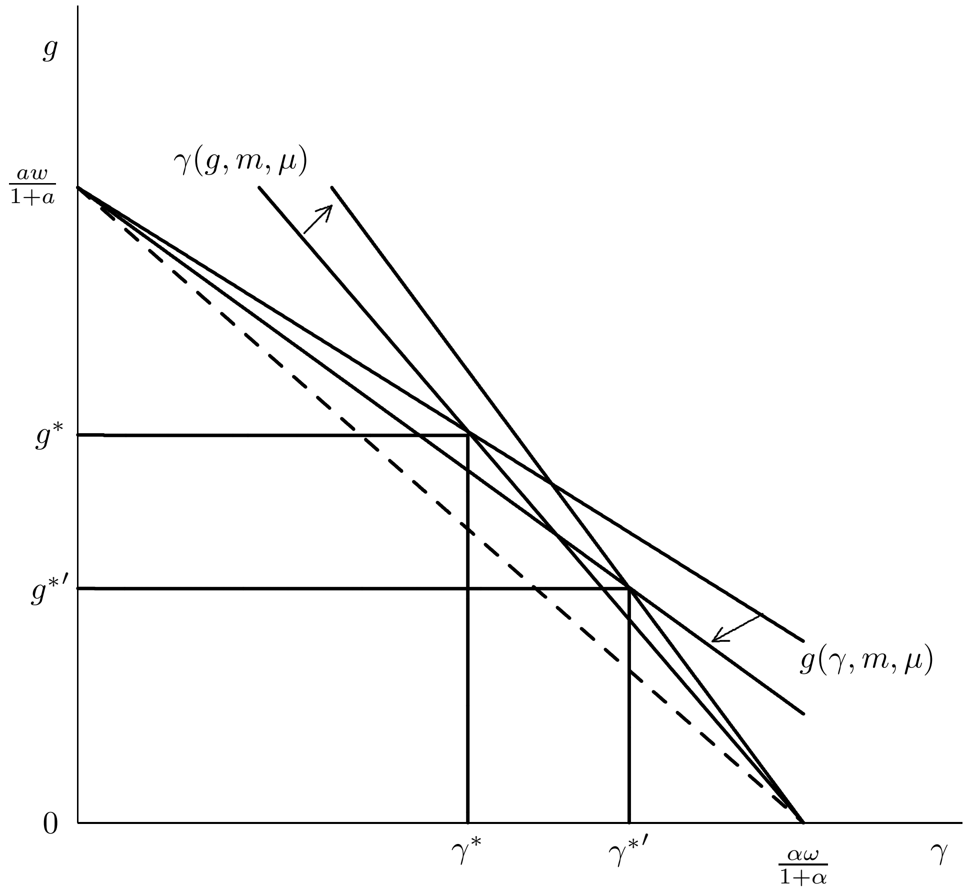

Figure 1. Stage 2 with Full Commitment

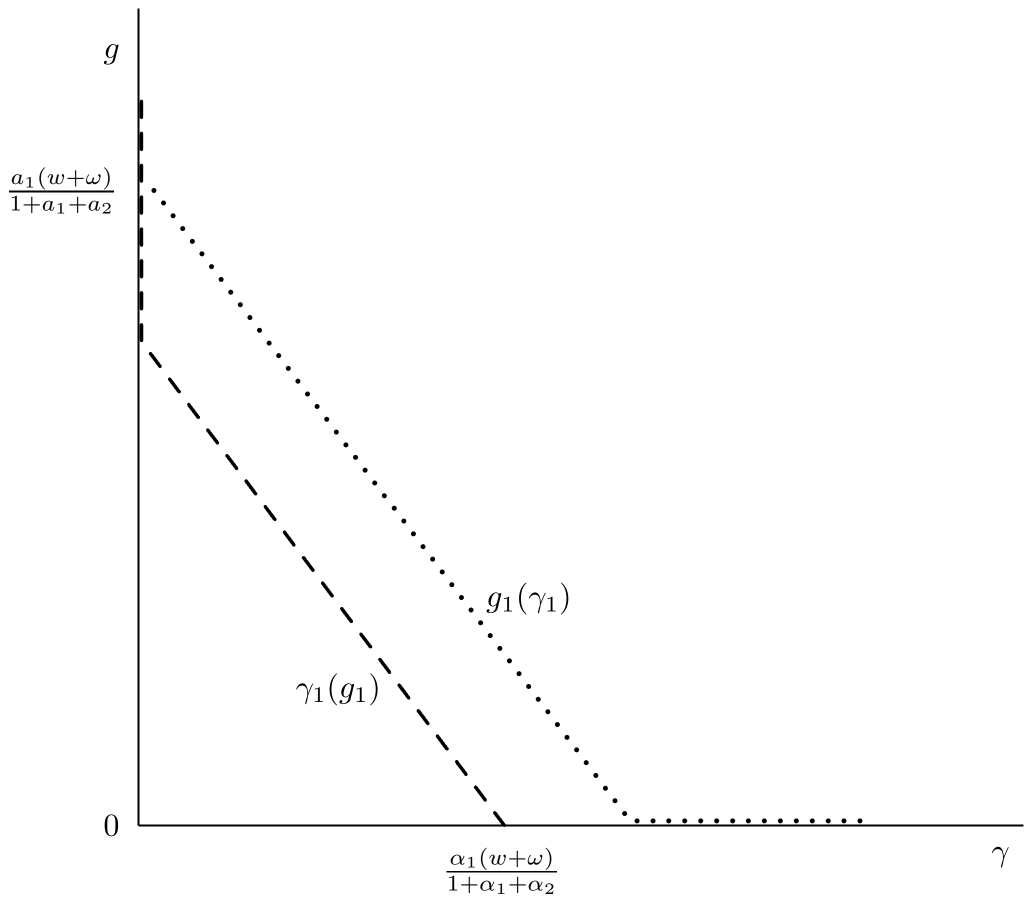

Figure 2. Stage 2, Two Public Goods, Sequential Contributions

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