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Industry Dynamics with Stochastic Demand

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Abstract

We study the dynamics of an industry subject to aggregate demand shocks where the productivity of a firm's technology evolves stochastically over time. Each period, each firm, given the aggregate demand shock, the productivity of its technology, and the distribution of technology productivities in the economy, (i) chooses whether to remain in the industry or to exit to sell its resources to an entrant; and (ii) an active firm chooses how much capital and labor to employ, and hence output to produce. To characterize the intertemporal evolution of the distribution of firms, we discuss in particular how exit decisions, aggregate output, profits and distributions of firm productivities vary, (a) *across* different demand realization paths; (b) *along* a demand history path, detailing the effects of continued good or bad market conditions; and (c) for different *anticipated future* market conditions. Sufficient conditions are provide for worse demand realizations to lead to increased exit of low-productivity firms and then to improved distributions of firms at all future dates and states. Finally, it is shown that a downturn in demand can raise welfare due to the impact on exit decisions.

Keywords: stochastic heterogeneity, aggregate shocks, exit, thin markets, demand uncertainty.

JEL: E32, L16

1 Introduction.

This paper integrates aggregate demand uncertainty into a dynamic stochastic model of firm entry and exit, and derives the consequences both for the evolution of the distribution of firms and for individual firm decision making. Our model builds on the dynamic stochastic equilibrium model of firm entry and exit developed by Hopenhayn (1990, 1992a, 1992b). In these papers, Hopenhayn analytically characterizes the individual exit and production decisions of firms according to their age, size and productivity in the unique invariant steady state equilibrium.

We extend these analyses to an environment with aggregate demand uncertainty. We contrast individual firm investment and exit decisions and their consequences for aggregate output, profits and productivity distributions, (a) *across* different demand realization paths; (b) *along* a demand history path, examining the effects of continued good or bad market conditions for future distributions of firms; and (c) for different *anticipated future* market conditions. We provide conditions under which the theoretical model can reconcile empirical regularities regarding counter-cyclical exit, correlations of exit rates with future GDP growth, and the relative length and extent of recessions and expansions.

Incorporating aggregate uncertainty together with individual stochastic heterogeneity — both necessary features of a rich model of industry dynamics — introduces formidable technical and modeling challenges. A significant contribution of this paper is to characterize the distribution of firms, rather than simply calculate selected higher order moments. In particular, we characterize the *evolution* of the distribution of firms following arbitrary histories of demand shocks. To do this, we derive conditions under which the distribution of firms can be ordered conditional on equilibrium exit decisions. In this context, we consider such key questions as: Does an economy with a better distribution of firms produce more output in all states? Will a more protracted period of high demand shocks lead to a better or worse distribution of firms?

A second contribution is to endogenize the value of exit. We do so, by building in an opportunity cost to exit: A firm can exit and sell its resources to another firm, but this requires that the firm's resources go un-utilized for a period while the resources are retooled so that they can be used by a potentially more-efficient entrant. The amount that an entrant is willing to pay for those resources reflects the profits that it expects to earn, and hence will vary with market conditions.

Endogenizing the value of exit complicates the characterization of the equilibrium evolution of the distribution of firms. We prove that the endogenous value of production and the endogenous value of exit vary procyclically, both rising with higher aggregate demand shocks. Two basic issues must then be addressed: (i) Does firm exit rise or fall with higher demand shocks?, and (ii) Is the immediate impact of these exit decisions on the distribution of firm productivities preserved over time? In particular, is the effect of reduced exit on the distribution of firms persistent, so that the distributions of firms is worse at all subsequent dates and states, or could this effect be reversed? Answering this second question is fundamental to addressing the impact of recessions on the long-run productivity of the economy.

Obtaining analytical answers to these questions is difficult. The standard analytical tool for this class of models is to prove that the competitive economy corresponds to the solution of a social planner's problem. Here, even when the solution to a social planner's problem characterizes equilibrium, it is of limited help: The fact that the equilibrium solves a surplus maximization problem, does not a priori ensure that an improvement in the distribution of firms adversely affects *all* firms because of the endogenous effects on exit. Consequently, the social planner characterization is not useful in investigation issues such as whether an improvement in the distribution of firms is preserved at all future dates and states.

This leads to the development of conditions on the transition process for a firm's technology for the aggregate distribution to be totally ordered. We consider environments where distributions can always be ordered in stochastic dominance terms, and use this to prove that an improvement in the distribution of firms is preserved along *every* future demand path. This yields a strong characterization result: Ceteris paribus, an economy which had greater past exit produces more output at every future date and state.

When aggregate distributions are comparable in stochastic dominance terms one can identify conditions under which exit rate are counter-cyclical. For standard production technologies (e.g., CES), the endogenous value of production varies more with market conditions than the endogenous value of exit, for any two-state Markov demand process. It follows that exit falls with higher demand. Combining this with the result that the effect of increased exit on future distributions of firm productivities is always preserved, yields the result that demand downturns increase future output and lead to better future distributions of firm productivities (ordered by stochastic dominance) at *every* future date and state.

We use these results to characterize how aggregate demand shocks affect industry dynamics. We examine how outcomes *across* different demand realization paths, contrasting investment and exit decisions and their consequences for aggregate output, profits and productivity distributions when one history of demand realizations is uniformly better than another. Following this, the consequences for outcomes *along* a demand history path are studied in order to describe the effect of continued good or bad market conditions on both firm investment and exit decisions, and on aggregate variables. In particular, it turns out that as a demand contraction continues, the distribution of firms grows ever better, setting the stage for greater future output once demand improves. Conversely, firms in booming economies 'rest on their laurels', so that the distribution of firms grows ever worse as a boom continues, sowing the seeds for a greater fall in output when the demand boom ends.

We then derive the consequences of an improvement in *anticipated future* market conditions on current and interim firm decisions and for the aggregate economy. We show that better anticipated future market conditions induce more exit and give rise to better distributions of firm productivities at all earlier dates.

Relating our predictions to the data, we find that the theoretical model generates the counter-cyclical exit found in the data (Davis and Haltiwanger 1992), the correlations of exit rates with future GDP growth that are positive, economically large, persistent and statistically significant (Campbell 1998), and the observation that recessions are shorter and sharper than expansions.

Finally, the paper considers how thin resale markets for an exiting firm's specialized resources affect equilibrium dynamics. Thin resale markets are economically important. Ramey and Shapiro (2001) find that "The process of selling capital results in significant declines in economic value (equipment sold for only one-third its inflation-adjusted book value, after accounting for normal annual depreciation).... Because of the large discounts experienced on the sale of capital, the option value of installed capital is very high". Hence, "Firms may rationally hold on to (under-utilized) capital for long periods of time." Thin markets drive a wedge between the social and private opportunity cost of the resources. A bad firm recognizing that it may not receive the full value of its resources, may continue to operate, tying up valuable assets that would only be released upon exit.

Thinner resale markets reduce the sensitivity of the endogenous value of exit to current market conditions. Importantly, we show that industries with more specialized inputs (thinner resale markets) will have more unproductive, but larger, firms and lower rates of entry and exit. This can explain the finding of Dunne et al. (1989a) that substantial and persistent differences in entry and exit rates across industries exist, and that industries with higher entry rates also have higher exit rates. We show constructively that a downturn in demand can actually *enhance* total welfare because it narrows the wedge between the social and private opportunity cost of the plant, increasing exit. Thus, the Darwinian cleansing effect of a downturn on exit can raise future expected welfare by more than the immediate reduction in welfare due to the downturn.

Finally, we highlight what this paper does not do. First, to focus on the impact of aggregate demand shocks, our model does not directly introduce productivity gains from adopting new and better methods of production (except to detail how incorporating such features into our model can generate predictions consistent with the empirical regularity that recessions are sharper and more asymmetric than booms). Levinsohn and Petrin (1999) find that the factors we model dominate. They empirically decompose aggregate industry productivity changes in Chile into the portion due to rationalization, i.e., the replacement of losers by winners that we model; and the portion due to the adoption over time of better methods of production. They find "that very little of the increase in productivity was accounted for by firms actually becoming more productive. Rather than firms becoming more productive, reallocation of market shares to firms that were already more productive and net entry typically explain the increase in aggregate productivity." Indeed, our model predicts that newer firms should tend to be smaller, less efficient and more likely to exit — precisely the features found in the data — and features that are hard to reconcile with environments in which dynamics are driven by entrants acquiring cutting edge technologies. Second, to focus on the dynamics of the output market, we take input prices as constant as in Hopenhayn (1992a), and allow firms to enter only through the acquisition of another firm's plant. We highlight when and how our qualitative findings extend if these assumptions are relaxed.

The outline of the paper is as follows. We next place our contribution in the literature. Section 2 describes the economic environment. Section 3 characterizes industry dynamics. Section 4 considers how the resale market depth affects outcomes. Section 5 concludes. Most proofs are in an appendix.

1.1 Related Literature.

Hopenhayn (1990, 1992a, 1992b) is most closely related to our work. Hopenhayn (1992a) characterizes the individual patterns of entry and exit in the invariant steady state of his economy, emphasizing the importance of (stochastic) heterogeneity across firms in explaining empirical regularities regarding the individual actions of firms. Dunne et al. (1989a, b) document that on an annual basis entering and exiting firms account for about 40 percent of manufacturing firms, are on average one-third the size of continuing firms, that exiting firms have higher costs and that the conditional probability of exit declines with both age and size. The degree of heterogeneity across firms is enormous: Davis et al. (1996) document that rates of job creation and destruction average about 10 percent a year, but are highly concentrated — only 23 percent of job destruction is accounted for by establishments that shrink by less than 20 percent over a span of one year. These findings highlight the importance of firm specific sources of uncertainty for firm survival and investment dynamics.

Jovanovic (1982) explores entry and exit dynamics when where firms learn about their profitability from past performance. In Jovanovic's model, the economy improves systematically over time, as better firms tend to be more successful, and hence remain in the industry, and there is no exit in the limiting economy. Jovanovic and MacDonald (1994b) explore the entry and exit dynamics of an industry following a theoretical innovation. Jovanovic and MacDonald (1994a) analyze a related environment in which there is no entry or exit, but firms choose how much to invest to try to acquire a superior technology.

Because of the analytical challenges involved much of the literature relies on numerical characterizations. Ishwaran (2000) numerically investigates a version of Hopenhayn's model in which demand evolves according to a two-state Markov process and there is a single input, capital. She calculates moments conditioned on firm age and demand state. Our analysis highlights the importance of the path-dependent evolution of the economy. The entire history of demand shocks determines the equilibrium distribution of firms, and this distribution, in turn, impacts exit decisions. Her paper highlights the fact that the properties of industry dynamics that we derive cannot be addressed numerically (e.g., when does increased past exit or reduced demand lead to greater output at every future date and state?)

Campbell (1998) numerically analyzes a general equilibrium model of industry dynamics, melding a version of Hopenhayn's (1992a,b) model with a vintage capital model that embodies aggregate uncertainty through innovations to the mean technology quality of new entrants. Campbell offers a complementary explanation to ours for the correlation between current exit and future GDP growth. He finds that greater future anticipated technical innovations lead to more firm exit in earlier periods because consumers respond by increasing savings and reducing current consumption.

Other papers turn to deterministic models. Caballero and Hammour (1994) simulate a model in which demand follows an exogonously-specified cyclical path, new entrants are more productive, and there is a fixed cost of entry that depends exogenously on the measure of entrants. With sufficient entry cost externalities, when demand falls, the exit of old, unproductive firms rises by more than the entry of new productive firms falls, in which case average technology quality rises. Caballero and Hammour (1996) consider a variant with costly search.

Other papers assume that *all* prices are constant, which simplifies the analysis greatly, as the distribution of firms in the economy does not affect an individual firm's profits, and hence decision-making. In such settings, Monge (2001) and Cooley and Quadrini (2001) explore the impact of interest rate shocks on the entry and exit of firms and aggregate output. In their models, firms borrow to finance capital, and firm productivities evolve stochastically. Cooley and Quadrini showing that their model is consistent with the empirical regularities that smaller/newer firms have higher and more variable growth rates because they are credit rationed, yet they are more likely to exit.

Bergin and Bernhardt (2005) adopt the constant-price assumption to derive how dynamics are affected by the time-to-build feature of capital investment. Incorporating this feature of capital allows us to distinguish between large firms and productive firms, and to explain distinctly the size, productivity and age patterns of profit, output and exit. Capital in place "slows" exit responses by firms: Firms first tend to downsize and then exit in response to low demand or productivity shocks. As a result, the distribution of firm productivities evolves more "sluggishly" in response to demand changes. Relatedly, Lambson (1991) highlights how sunk costs dampen the responsiveness of entry and exit to market conditions.

Finally, Ericson and Pakes (1995, 1998) develop a parsimonious model of the industry dynamics of a small, imperfectly competitive industry in which firms' investments have stochastic outcomes. In their reduced form model, a firm's profit depends on the relative success of its investment decisions. Their goal is to develop a flexible framework for empirical work, one that incorporates firm heterogeneity.

2 The Model.

We consider the dynamics of a single industry with a continuum of risk neutral firms that discount future period profits using a common discount factor, $\beta \in (0,1)$. Inverse demand in a period is given by $p(Y,\theta)$, where Y is industry output and θ is a random demand shock. This market price is a continuous function, declining in Y and increasing in θ, with $p(0,\theta) > 0$ and $p(\cdot,\theta) \geq 0$. The demand shock follows a Markov process: Given θ, the demand shock in the next period is drawn according to $\Theta(\cdot \mid \theta)$. We assume that $\Theta(\cdot \mid \theta)$ is continuous in θ.[1]

A firm requires a plant to produce output according to the production function $f(\ell,\alpha,k)$, where $k \geq 0$ is capital, $\ell \geq$ is labor, and α captures the quality or productivity of its technology. The production function, $f(\ell,\alpha,k)$ is strictly monotone increasing in its arguments, strictly concave in k and ℓ with complementary inputs, and with $f(\ell,\alpha,k) = 0$ if either ℓ or k is 0. Without loss of generality, we normalize α to be in $[0,1]$. Let μ be the distribution over technologies, α, in the economy.

Timing of events: At the beginning of a period, the aggregate shock, θ, determines demand, $p(Y,\theta)$, and an operating firm with technology α receives a new realization of its technology, $\tilde{\alpha}$, drawn from a distribution $P(\cdot \mid \alpha)$ that is continuous in α. Given θ, and μ each firm α decides whether to remain in the market and produce that period, or exit to search for a buyer. The time-line for decisions is:

$\longrightarrow$ $\theta,\ \alpha$ realized $\longrightarrow$ Exit decisions made $\longrightarrow$ Capital and Labor choices $\longrightarrow$ Production and Sale $\longrightarrow$ $\tilde{\theta},\ \tilde{\alpha}$ realized $\longrightarrow$

Exit: We assume that plants are in fixed supply, and without loss of generality we normalize the measure

[1] Throughout, we use the weak* topology on measures, so that for any continuous function $h : \Theta \to R$, $\int h(\tilde{\theta})\Theta(d\tilde{\theta} \mid \theta^n) \to \int h(\tilde{\theta})\Theta(d\tilde{\theta} \mid \theta)$, as $\theta^n \to \theta$.

of plants in the economy and, hence the measure of possible firms, to one. Because plants are in fixed supply, new, potentially more productive, entrants must purchase a plant from an exiting firm.[2] We assume that the opportunity cost to continued production is that a firm cannot sell its plant immediately to a potentially more efficient user. That is, an exiting firm's plant must be idled for the one period that it takes to find a buyer that can better employ plant and for the purchaser to re-tool the plant for its own use.[3] To capture a potentially thin market for an exiting firm's plant, we assume that in negotiations with the buyer, an exiting firm receives only a share $1-\gamma \in (0,1]$ of the discounted profits that a new entrant expects given current market conditions when the entrant makes production and exit decisions optimally.

These assumptions capture the empirical findings of Ramey and Shapiro (2001), that "there (is) a time cost to restructuring. The process of winding down operations before selling capital results in significant periods of under-utilization. It is only at times when firms cease operations that they sell significant portions of capital." They also find that a firm which sells its capital generally receives far less than the capital's value, especially for more specialized capital. The value of γ captures the degree to which the plant is highly specialized so that exiting firm less than its full value from the firm that buys it.[4]

The findings are reinforced if it takes a new user longer to retool, so that upon exit a plant must remain idle for more than one period. Also, if there is positive probability that an exiting firm does not find a buyer in the period following exit, then the qualitative impacts are (a) to reduce the sensitivity of the value of exit to current market conditions, and (b) to sever the tight link between the mass of exiting firms in the previous period and the mass of entrants.[5]

Entry: Entry is determined by the exit in the previous period, and that, in turn depends on market conditions. What complicates the analysis is that not only does the value of a plant varies with market conditions, but so too do the respective values of producing and exiting. When demand is higher, the foregone cost of production is higher, but persistence in demand implies that the value of a more productive future technology is also higher, so that a prospective buyer is willing to pay more. We derive conditions under which weaker demand leads to more exit.

Firm decisions: If firm type α stays in the market, it chooses capital and labor to maximize profit,

$$\max_{\ell,k} pf(\ell,k,\alpha) - w\ell - rk,$$

where p is the market-clearing equilibrium price, $w > 0$ is the wage rate and $r > 0$ is the unit price of capital. Later, in remark 2, we highlight conditions under which our analysis extends when factor prices vary with the aggregate shock, θ. The solution to this profit maximization problem gives input demands, $\ell(p,\alpha)$ and $k(p,\alpha)$, and supply function $y(p,\alpha) = f(\ell(p,\alpha), k(p,\alpha), \alpha)$.

[2] After our analysis, we discuss how allowing for entry by creation of new firms in addition to entry by acquisition affects the results. Dunne et al. (1989a,b), and Baldwin and Gorecki (1987) document the importance of entry through purchase of existing firms. Baldwin (1995) finds that "Acquisition entry is (as) important a force in changing the composition of an industry as greenfield entry.", and that acquired plants exhibit increased productivity and acquire greater market shares.

[3] We could alternatively assume that firms can continue to operate while searching for a buyer, but that they received a reduced profit relative to what they would receive were they not searching.

[4] One can endogenize γ by assuming that a random number of potential users of an exiting firm's plant compete in a second-price sealed bid auction. If there is only one bidder in a period, it makes an offer that leaves the exiting firm indifferent between accepting the offer and waiting a period in the hope of attracting more bidders.

[5] In the current model, plants may be idle for multiple periods, if an exiting firm is replaced by a new firm that decides to exit after observing its draw of α and the random demand shock, θ.

Market Clearing: With measure μ on technologies, given an exit rule α^* (all firms with technologies less productive than α^* exit), let μ_{α^*} denote the measure on firms remaining in the market. Then aggregate supply is

$$Y = \int y(p,\alpha)\mu_{\alpha^*}(d\alpha) = \int_{\alpha^*}^{1} y(p,\alpha)\mu(d\alpha) = Y(p,\mu_{\alpha^*}).$$

Price is determined by θ and μ_{α^*} via the market-clearing condition $Y(p,\mu^{\alpha^*}) = D(p,\theta)$, where $D(p,\theta)$ is the inverse of $p(Q,\theta)$ for each θ. With price $p(\theta,\mu_{\alpha^*})$ determined by (θ,μ_{α^*}), a firm α earns profit

$$\pi(\theta,\mu_{\alpha^*},\alpha) = \max_{\ell,k} p(\theta,\mu_{\alpha^*})f(\ell,k,\alpha) - w\ell - rk.$$

Evolution of Technologies: If a firm's current technology is α, then next period's technology is drawn from $P(\cdot \mid \alpha)$, a conditional distribution over technologies given α. To capture the fact that a firm with a better technology in one period is likely to have a better technology the next period, we assume that

$$P(\cdot \mid \alpha) = w(\alpha)F(\cdot) + [1 - w(\alpha)]G(\cdot),$$

where $F \succeq_c G$ (F conditionally stochastically dominates G), $F \neq G$, $w(\alpha) \in [0,1]$ is continuous and strictly increasing in α.[6] That is, the new technology of a firm with current productivity parameter α is drawn from a weighted distribution of a good distribution, $F(\cdot)$, and a bad distribution $G(\cdot)$, where the weight, $w(\alpha)$, on the good distribution is an increasing function of the firm's current productivity. For simplicity, we assume that the technology quality of a new firm is drawn from the distribution $P(\cdot \mid \bar{\alpha})$, where $\bar{\alpha} \in (0,1)$ is sufficiently large that the worst firm type always chooses to exit. As a result, in equilibrium, firms with technology below some exit threshold, $\alpha^* < \bar{\alpha}$, exit; and those above α^* remain in the market.

The weighted-average transition kernel ensures that technology distributions are ordered by a single parameter and avoids difficulties that exit can create for comparing distributions over time. To see the issue, suppose that technology distribution μ stochastically dominates μ' due to the fact that μ' has many very bad technologies. Then, when the same exit rule is applied to both, the resulting distribution determined by μ' may dominate the distribution determined by μ, thereby reversing the ordering of distributions. For example, this would occur if μ puts all mass between α^* and $\bar{\alpha}$, ($\alpha^* < \bar{\alpha}$) so that no firms exit; while μ' puts all mass below α^*, so that every firm exits and receives a technology drawn from $P(\cdot \mid \bar{\alpha})$, which is better than a draw from any distribution $P(\cdot \mid \alpha)$, for $\alpha < \bar{\alpha}$.

The only other structure that we impose is that there is some learning by doing in the evolution of a firm's technology:[7]

$$\int P(\cdot,\alpha)P(d\alpha,\bar{\alpha}) \succeq P(\cdot,\bar{\alpha}).$$

That is, firms tend to improve over time: The technology of a firm in its second period of operation is drawn from a stochastically better distribution than the initial distribution governing the technology of entering firms.

[6] Given two probability distributions F and G on $[0,1]$, say that F conditionally first order stochastically dominates G ($F \succeq_c G$) if given α^*, for $\alpha \geq \alpha^* : F(\alpha \mid \alpha \geq \alpha^*) \leq G(\alpha \mid \alpha \geq \alpha^*)$. That is, given any α^*, the conditional distribution on $[\alpha^*,1]$ determined by F first order dominates that of G.

[7] For the results we use a far weaker condition, $\int w(\tilde{\alpha})dF(\tilde{\alpha}) \geq w(\bar{\alpha})$, which is implied by $\int P(\cdot,\alpha)P(d\alpha,\bar{\alpha}) \succeq P(\cdot,\bar{\alpha})$.

2.1 Equilibrium Exit

We next turn to characterizing equilibrium exit decisions. We first describe how the productivity of a firm's future technology is related (stochastically) to its current technology, and then detail how the productivity of an entering firm is determined. We then characterize valuation functions and the optimal exit decision of an individual firm.

In the study of the dynamics, two distributions are of special interest: The residial distribution of operating firms at time t after exit decisions have been made, and the distribution of firms in the next period induced by exit decisions. Recall that μ_{α^*} denotes the measure obtained from μ after all firms with technologies that are less productive than α^* exit: $\mu_{\alpha^*}(X) = \mu(X \cap [\alpha^*, 1])$. An exit rule α^* combined with μ determines the distribution over technology productivities in the next period,

$$\mu^{\alpha^*}(\cdot) = \int_{[0,\alpha^*)} P(\cdot \mid \bar{\alpha})\mu(d\alpha) + \int_{[\alpha^*,1]} P(\cdot \mid \alpha)\mu(d\alpha).$$

Exit over time: In equilibrium, a firm's exit decision maximizes expected profits given $(\alpha_t, \mu_t, \theta_t)$, and the distribution of firms over time is consistent with the optimization by almost all firms, for almost all θ_t. At any date t, equilibrium is characterized by an exit threshold that depends on μ and θ, $\alpha(\mu, \theta)$. This fully determines the evolution of the aggregate distribution μ over time along any path of demand realizations, $(\theta_1, \theta_2, \ldots)$. Consequently, the exit rule determines the market-clearing price sequence facing firms, and hence the present value of any firm α. The exit rule, $\alpha(\mu, \theta)$, is an equilibrium exit rule if and only if it determines a valuation function, v, that supports the exit rule: It must be that at (μ, θ) firms wish to exit if and only if their technology is below $\alpha(\mu, \theta)$. Let $\alpha^* = \alpha(\theta, \mu)$. Then, the expected value to a firm α from operating in the current period (and acting optimally thereafter) is:

$$v^c(\theta, \mu, \alpha) = \pi(\theta, \mu_{\alpha^*}, \alpha) + \beta \int \int v(\tilde{\theta}, \mu^{\alpha^*}, \tilde{\alpha})\Theta(d\tilde{\theta} \mid \theta)P(d\tilde{\alpha} \mid \alpha),$$

and the expected value to exit is

$$v^e(\theta, \mu, \alpha) = (1 - \gamma)\beta \int \int v(\tilde{\theta}, \mu^{\alpha^*}, \tilde{\alpha})\Theta(d\tilde{\theta} \mid \theta)P(d\tilde{\alpha} \mid \bar{\alpha}).$$

Here "v^c" denotes the value of continuing to operate, and "v^e" denotes the value of exiting. The value of an operating firm α facing market conditions (θ, μ) is equal to the sum of its maximized operating profits plus the discounted expected value of continuing to operate given that it chooses inputs optimally and makes future operation-exit decisions optimally. The expression for v^c reflects the potentially thin resale market for an exiting firm's plant: An exiting firm only receives fraction $1 - \gamma$ of the full discounted expected value of the plant to a new firm whose technology quality is drawn according to $\bar{\alpha}$. Since firms choose optimally whether to exit or to continue in operation, the value of a firm α is given by

$$v(\theta, \mu, \alpha) = \max\{v^c(\theta, \mu, \alpha), v^e(\theta, \mu, \alpha)\}.$$

Since v^e is independent of α, while v^c is increasing in α, there is a unique value, $\hat{\alpha}$, at which v^c and v^e are equal. Firms with technologies above $\hat{\alpha}$ wish to continue and firms with technologies below $\hat{\alpha}$ wish to exit. A necessary condition for equilibrium is that $\alpha^* = \hat{\alpha}$: The exit rule $\alpha(\theta, \mu)$ must satisfy

$$v^c(\theta, \mu, \alpha(\theta, \mu)) = v^e(\theta, \mu, \alpha(\theta, \mu)), \forall(\theta, \mu).$$

Clearly, $\alpha(\theta, \mu) < \bar{\alpha}$, because of the opportunity cost of exiting and transfering the plant to the entrant. In summary, for each (μ, θ) an equilibrium exit rule is characterized by $\alpha^* = \alpha^*(\mu, \theta)$ where $v^c(\theta, \mu, \alpha^*) = v^e(\theta, \mu, \alpha^*)$: Firms with $\alpha > \alpha^*(\mu, \theta)$ remain in the market, while firms with $\alpha < \alpha^*(\mu, \theta)$ exit. That is, it the less productive/higher cost firms that find it optimal to exit so that their plant can be reallocated to better uses. This result is consistent with Dunne et al.'s (1989b) finding that higher cost plants exit first, and Baldwin's (1995) finding that entrants, while relatively unproductive compared to the average firm, are more productive than the exiting firms that they replace.

Bergin and Bernhardt (1995) provide sufficient conditions for the existence of an equilibrium in economies with a continuum of agents, aggregate shocks and idiosyncratic shocks to agent types, in which an agent's payoff depends on his own type and action, the aggregate shock and the distribution over agent types and actions in the economy. The mild continuity assumptions on the transition functions and payoffs that are required for an equilibrium to exist are satisfied here.

3 Dynamics.

Our goal is to characterize how demand fluctuations affect future distributions of firm productivities and profits, as well as aggregate variables such as prices and industry output. We first investigate the possibility of characterizing dynamics in the competitive economy using a social planner's characterization. We then use a more direct approach to characterize equilibrium dynamics.

3.1 Social Planner's Characterizations.

The standard approach to characterizing industry dynamics is to show first that the competitive equilibrium corresponds to the solution of a social planner's problem, and then to solve that social planner's problem (e.g., see Hopenhayn (1992a)). We now detail when the competitive equilibrium to our economy can be characterized as the solution to a social planner's problem. We then explain why this social planner's characterization is of limited help in facilitating an analysis of industry dynamics.

If $\gamma = 0$ (thick resale markets for plants), then exit decisions in the competitive equilibrium correspond to the exit rule of a social planner who seeks to maximize discounted social surplus. Period social surplus can be represented as the area between the demand and supply curves. Let $P_s(Y, \mu')$ denote the aggregate supply curve when the distribution of firms in operation is μ'. Then if total output is Y^*, social surplus is $S(Y^*, \theta, \mu') = \int_{[0,Y^*]}[P(Y, \theta) - P_s(Y, \mu')]dY$. The social planner program is the optimization of the present value of the social surplus stream by choice of continuation (and hence exit) distribution. The functional equation for the social planner's problem is:

$$V(\theta, \mu) = \max_{\alpha}\{\max_{X} \int_0^X [P(Y, \theta) - P_s(Y, \mu_\alpha)]dY + \beta \int_\Theta V(\tilde{\theta}, \mu^\alpha)\Theta(d\tilde{\theta} \mid \theta)\}.$$

Here, $P_s(Y, \mu_\alpha)$ is the aggregate supply function, reflecting that the labor choices of operating firms correspond to those made by the social planner. That is, if at price P, firm α supplies $y(P, \alpha)$, then total output is $Y(P, \mu_\alpha) = \int y(P, \tilde{\alpha})\mu_\alpha(d\tilde{\alpha})$. Inverting for P gives $P_s(Q, \mu_\alpha)$. The solution to this program yields an exit rule, $\alpha(\theta, \mu)$, at each (θ, μ), which determines the evolution of the aggregate distribution.

Theorem 1 *If $\gamma = 0$, then the exit rule in the competitive economy is unique and corresponds to the solution to the social planner's problem.*

Proof: See the appendix. ■

The social planner characterization only holds when $\gamma = 0$. When $\gamma > 0$, the thin market for a firm's plant means that some of the value of the firm is extracted by the purchaser, reducing the incentive to exit relative to the true market value of the firm. In such circumstanes, the level of exit is socially inefficient, less than the socially optimal level of exit, so that no social planner's characterization holds.

Theorem 1 asserts that if $\gamma = 0$, then the exit rule in the competitive economy is unique and corresponds to the solution to the social planner's problem. However, the social planner's approach is of limited help. It does not *a priori* follow that an improvement in the distribution adversely affects *all* firms because of the endogenous effects on exit. A change in the distribution from μ to $\hat{\mu}$ also alters the equilibrium exit threshold, and both determine next period's distribution. In particular, there may be no ordered relation between the one-period-ahead distributions arising in equilibrium from μ and $\hat{\mu}$ respectively, so that it may not be possible to determine which leads to higher profit for a specific firm. That is, some firm types may benefit from an increase in μ if the resulting impact on exit of other firms in some future states goes the "wrong" way (e.g., if exit is greater in some future period where a firm type expected to be better).

3.2 Competition and Payoff Monotonicity

Fluctuations in demand directly affect exit decisions, and hence future distributions of firm productivities. In turn, differences in the distribution of firm productivities influence future exit decisions. To analyze the impact of demand fluctuations, we consider the role played by differences in the distribution of firm productivities. We first prove that (i) individual valuation functions are always monotone decreasing in μ (a better distribution of competitors always reduces the expected profits of *all* firm types); and (ii) an important dominance condition is satisfied: In equilibrium, better technology distributions are preserved over time, so that if $\mu_t \succ \hat{\mu}_t$, then along any future common demand shock path, at any future date $\tau > t$, $\mu_\tau \succ \hat{\mu}_\tau$.

In Theorem 2, we consider a finite horizon version of the economy. Let $v_n^c(\theta, \mu, \alpha)$ and $v_n^e(\theta, \mu, \alpha)$ be the (equilibrium) values to firm α from continuing and exiting respectively, when the current distribution is μ and the current state is θ, and there are n periods remaining. When the horizon is finite, the equilibrium exit rule can be computed by backward induction. Indeed theorem 2 proves that, *independent of whether the social planner formulation is applicable*, the equilibrium exit rule is unique. That is, at each (μ, θ), there is a unique equilibrium threshold. Further, v_n^c and v_n^e are monotone decreasing in μ. If they converge as n increases, monotonicity is preserved.

Theorem 2 *Payoffs are monotonically decreasing in the technology distribution and, in equilbrium, better distributions on technologies are maintained over time. Formally:*

1. *The functions $v_n^c(\theta, \mu, \alpha)$ and $v_n^e(\theta, \mu, \alpha)$ are continuously decreasing in μ for all n.*

2. *If $\hat{\mu}_t \succeq \mu_t$, then $\hat{\mu}_{t+1} \succeq \mu_{t+1}$.*

Finally, if v_n^c and v_n^e converge pointwise as $n \to \infty$, then the limiting functions are continuously decreasing in μ. When $\gamma = 0$, v_n^c and v_n^e converge since they are derived from a social planner program which converges as $n \to \infty$.

Proof: See the appendix. ■

The result that $v_n^c(\theta, \mu, \alpha)$ and $v_n^e(\theta, \mu, \alpha)$ are continuously decreasing in μ is both subtle and important. The key is to prove that comparing two aggregate distributions $\hat{\mu}_t$ and μ_t at period t, if $\hat{\mu}_t \succeq \mu_t$, then $\hat{\mu}_{t+1} \succeq \mu_{t+1}$ (so that result 1 follows from 2). To do this, we show that while exit may be less in the economy with the better distribution, it cannot be *so much less* that it reverses the ordering in distributions. Thus, dominance is inherited in subsequent periods. Were the dominance property not preserved over time, then some firm type may prefer to face a better distribution of competitors in t if that better distribution implied a worse distribution in $t+1$, when the firm type expected to have a higher α.

This result is far from immediate. Indeed, if capital choices are made before the demand and productivity shocks are realized, more structure is required to ensure this monotonicity result. This is because capital and technology productivities have distinct effects on exit decisions (see Bergin and Bernhardt (2005)).

The fact that a better distribution of technology productivities is preserved along *every* future demand path is crucial for the analysis that follows. Theorem 3 exploits this directly, showing that along a common demand path, in an otherwise identical economy with a better initial distribution of firms, future output in every future demand state is greater. Consequently, prices are always lower in the economy with the better initial distribution of firms, hurting all firms.

Theorem 3 *Consider two period t distributions, μ_t and $\hat{\mu}_t$ over firm technologies. Suppose that $\hat{\mu}_t \succeq \mu_t$ implies that $\hat{\mu}_{t+1} \succeq \mu_{t+1}$. Then along a common demand path, for $\tau \geq t$, output is higher in the hat economy, $\hat{Y}_\tau > Y_\tau$, so that prices and firm profits are lower: $\hat{p}_\tau < p_\tau$ and $\hat{\pi}(\alpha)_\tau < \pi(\alpha)_\tau$.*

Proof: By assumption an improvement in the distribution at time t ($\mu \uparrow \mu'$) improves next period's distribution (taking into account the resultant change in the current exit rule), and increases current period's output. Proceeding inductively, the improvement in next period's distribution leads to an increase in that period's output and an improvement in the distribution for the following period. And so on. Thus, the initial distribution in every period (prior to the exit decision) is better, and the output in every period greater. The patterns for prices and profits follow.

■

Thus, improved competition raises current output with associated consequences for prices and profitability. Improved competition may raise exit, but not by enough to offset the output consequences of the improved firm quality. Again, Bergin and Bernhardt (2005) show that more structure is required if firms choose capital prior to demand and technology shock realizations. This is because firms respond to a worse distribution over technologies by increasing capital, and capital and technology productivities have distinct impacts on output and exit decisions. However, analogous results follow if demand is sufficiently elastic.

3.3 Cyclical Fluctuations.

We have derived the impact of different initial distributions of firms for productivity, output, price and profitability along a *given* demand path, $\{\theta^{\tau-1}\}_{\tau\geq t}$. We next study the impact of fluctuations in θ on exit decisions, output, and profitability of firms. This allows us to characterize the consequences of demand shocks for current and future output, and current and future aggregate productivity. What complicates the analysis is that when demand improves, there are competing influences on a firm's decision to exit: operating is more profitable, but the firm is also worth more if sold. That is, improved demand raises incentives to remain in the market which would reduce the average efficiency of firms in the market next period. However, the persistence of improved demand also raises the benefit to having a better technology next period, producing a "counter-incentive" for inefficient firms to exit.

3.3.1 Cyclical Exit

We first derive conditions under which, ceteris paribus, downturns in demand induce more (unproductive) firms to exit (worsening the immediate effect of the downturn), to be replaced by firms that are stochastically better. We then combine this result with Theorem 2 to derive that worse current demand conditions always imply better future distributions of firm technologies. That is, not only do recessions have the cleansing effect of weeding out more firms with unproductive technologies, but past recessions also reduce the number of unproductive technologies at each future date and state. Exit decisions thus mitigate the impact of a prolonged downturn in demand.

The equilibrium exit threshold, α^*, is determined by

$$\pi(\theta,\mu_{\alpha^*},\alpha^*)+\beta\int\int v(\tilde{\theta},\mu^{\alpha^*},\tilde{\alpha})\Theta(d\tilde{\theta}\mid\theta)P(d\tilde{\alpha}\mid\alpha^*)=(1-\gamma)\beta\int\int v(\tilde{\theta},\mu^{\alpha^*},\tilde{\alpha})\Theta(d\tilde{\theta}\mid\theta)P(d\tilde{\alpha}\mid\bar{\alpha}).$$

Let $\bar{v}(\theta,\mu^{\alpha^*},\hat{\alpha})\stackrel{\text{def}}{=}\int v(\tilde{\theta},\mu^{\alpha^*},\tilde{\alpha})P(d\tilde{\alpha}\mid\hat{\alpha})$, $\forall\hat{\alpha}$, and rearrange the expression to give:

$$\pi(\theta,\mu_{\alpha^*},\alpha^*)=\beta\int[[(1-\gamma)\bar{v}(\tilde{\theta},\mu^{\alpha^*},\bar{\alpha})-\bar{v}(\tilde{\theta},\mu^{\alpha^*},\alpha^*)]\Theta(d\tilde{\theta}\mid\theta).$$

With α^* fixed, raising the current demand shock raises current profit, but if there is persistence in demand shocks, raising the current shock also makes higher future shocks more likely, raising the relative future payoff from exit (and being replaced by a stochastically better technology). Whether α^* rises with θ (i.e., whether there is less exit), then depends on which rises more.

Inspection reveals that quite generally, the value of remaining in the industry rises more rapidly with θ than does the value of exit as long as there is sufficient mean reversion in the demand process. In particular, this is so if demand shocks are independently distributed.[8] For a broad class of production technologies, we now prove that there is always "enough" mean reversion in *any* two-state Markov demand process for the relative value of remaining in the industry to rise with demand.

Specifically, we now focus attention on technologies that give rise to multiplicatively separable profit functions, so that $\pi(\alpha, p(\mu,\theta)) = g(p)h(\alpha)$ for some functions g and h. This class of production functions includes such standard production technologies as the CES and quadratic production functions.

Consider $\theta \in \{\bar{\theta}, \underline{\theta}\}$ with $\underline{\theta} \leq \bar{\theta}$. Let $\bar{\mu}$ and $\underline{\mu}$ be, respectively, the distributions of firm qualities induced by an arbitrarily long sequence of $\theta = \underline{\theta}$ and $\theta = \bar{\theta}$ realizations starting from μ_∞, the unique stationary distribution when demand is independent of θ. The next theorem establishes that in this environment, there is more exit if demand is low than if it is high.

Theorem 4 *Let profit functions be multiplicatively separable. Then for $\theta \in \{\underline{\theta}, \bar{\theta}\}$, for any distribution of firm technologies $\mu \in [\underline{\mu}, \bar{\mu}]$, there is more exit if demand is low than if it is high: $\alpha(\mu, \underline{\theta}) > \alpha(\mu, \bar{\theta})$.*

Proof: See the appendix. ■

In the proof we show that the exit rule does not depend on the initial demand realization if demand is perfectly persistent: the 'demand' shock enters current and continuation profits in the same multiplicative way. Persistence in demand does not affect current profits, but it does affect future expected profits. When the current demand is low, reducing demand persistence raises the probability of a future high demand shock and hence raises future expected prices. This raises the value of a better technology in the future, and hence the attraction of exit. Conversely, reducing demand persistence when the current demand shock is high, reduces future expected prices, reducing the value of a better future technology, lowering the relative value of exit. Combining these observations reveals that there is more exit when demand is low than when demand is high.

3.3.2 Output, price and profit movement.

We next combine the implications of theorem 3 (better distributions are preserved along a demand path, implying lower prices) and theorem 4 (exit is counter-cyclical) to characterize the evolution of key variables as demand shocks persist for a longer period of time.

[8] A necessary and sufficient condition for counter-cyclical exit is

$$\frac{\partial \pi(\theta, \mu_{\alpha^*}, \alpha^*)}{\partial \theta} > \lim_{\theta' \to \theta} \left[\frac{1}{\theta' - \theta}\right] \beta \int \int \left[[(1-\gamma)\bar{v}(\tilde{\theta}, \mu^{\alpha^*}, \bar{\alpha}) - \bar{v}(\tilde{\theta}, \mu^{\alpha^*}, \alpha^*)][\Theta(d\tilde{\theta} \mid \theta') - \Theta(d\tilde{\theta} \mid \theta)]\right.$$

However, this condition is difficult to verify.

Theorem 5 *Let profit functions be multiplicatively separable. Then for* $\theta \in \{\underline{\theta}, \bar{\theta}\}$*, for* $\mu_{t-1} \in [\underline{\mu}, \bar{\mu}]$*:*

1. Aggregate output is higher in period t *if demand is high than if it is low:* $Y_t(\bar{\theta}, \mu) > Y_t(\underline{\theta}, \mu), \forall \mu$*.*

2. If a period of high demand begins in period t *and ends* $\tau + 1$ *periods later then aggregate output first rises and then falls,* $Y_{t-1} < Y_t > Y_{t+1} > \cdots > Y_{t+\tau+1}$*, so that prices rise throughout:* $p_t < p_{t+1} < \cdots < p_{t+\tau}$*. Hence, the output and profits of a firm of type* α *rise:* $y_{t-1}(\alpha) < y_t(\alpha) < \cdots < y_{t+\tau}(\alpha)$ *and* $\pi_{t-1}(\alpha) < \pi_t(\alpha) < \cdots < \pi_{t+\tau}(\alpha)$*.*

3. If a low demand downturn begins in period t *and ends* $\tau + 1$ *periods later, then* $Y_{t-1} > Y_t < Y_{t+1} < \cdots < Y_{t+\tau+1}$ *so that prices fall as the downturn continues:* $p_{t-1} > p_t > \cdots > p_{t+\tau}$*. Hence, the output and profits of a firm type* α *fall as the downturn continues:* $y_{t-1}(\alpha) > y_t(\alpha) > \cdots > y_{t+\tau}(\alpha)$ *and* $\pi_{t-1}(\alpha) > \pi_t(\alpha) > \cdots > \pi_{t+\tau}(\alpha)$*.*

Proof: Part 1 follows because there is less exit when $\theta = \bar{\theta}$. Part 2 follows because the distribution worsens over time as the boom progresses (see the proof to theorem 4), implying that output falls (theorem 3). The implication of falling output for prices, firm output and profits is immediate. Part 3 is the downturn analogue to part 2.

∎

A more prolonged period of high demand leads to an increasingly inefficient distribution of firms, which, in turn, implies rising prices and hence higher individual output and profits. It is worth stressing that this result holds no matter how persistent the demand shock process is. With two demand states, $\{\bar{\theta}, \underline{\theta}\}$, in a demand boom, if viewed as a growth in demand occurs over one time period, output rises during the growth stage (as in 2. above) and subsequently tails off due to the worsening distribution of firm productivities as demand remains high.

Importantly, were we to modify the model to incorporate an exogenous systematic growth in technological opportunities, then the model can generate predictions consistent with the empirical regularity that recessions are sharper and more asymmetric than booms. In particular, if, in periods of high demand, the exogenous improvement in the distribution of technology productivities is enough to offset the increasingly inefficient (endogenous) distribution of firm productivities, then output grows gradually as long as demand remains high. Conversely, any systematic improvements in the distributions from which technologies are drawn sharply reinforces the endogenous improvement in firm productivities caused by a downturn in demand, so that recessions are short.

Also note that incorporating stickiness into capital investment, either with a capital-in-place formulation or by modeling the putty-clay feature of capital, prolongs the industry response to downturns in demand. Bergin and Bernhardt (2005) show that firms respond to a decrease in demand by first downsizing, and then subsequently exiting. Consequently, output may fall for multiple periods following the onset of a period of low demand.

Combining the insights of theorems 3 and 5, it follows immediately that past downturns lead to greater output, once demand has 'recovered'. Consider two economies that differ only in that one economy had a past period in which demand was lower in previous periods. Say that demand has 'recovered' in the economy that had lower shocks if both economies have the same current demand shock. Then:

Corollary 1 *Consider two economies with identical current demand, but one economy had weakly lower demand shocks in past periods. Then the economy which has 'recovered' from a past history of lower demand has a better distribution of firms, greater output and lower prices.*

Qualitatively, given theorem 3, the key to the corollary is that there is more exit in bad times. That is, as long as *exit rates are counter-cyclical* — $\theta < \theta'$ then $\alpha(\mu, \theta) > \alpha(\mu, \theta')$ — (as Campbell (1998) documents empirically), then the result holds without further structure on the θ process.

We have shown that exit is counter-cyclical in a two-state separable environment. So, too, (without imposing any other structure) exit will also be counter-cyclical as long as demand is not too persistent. As long as exit is counter-cyclical, we can contrast industry dynamics across economies that start out with the same distribution of firms, but one receives higher aggregate demand shocks than the other:

Theorem 6 *Suppose that the sufficient conditions hold for exit to be counter-cyclical. Let $\hat{\mu}_0 = \bar{\mu}_0$. Consider two aggregate shock histories, $\hat{\theta}^t, \bar{\theta}^t$ where the hat economy has higher demand realizations than the bar economy: $\hat{\theta}_0 > \bar{\theta}_0$, $\hat{\theta}_\tau \geq \bar{\theta}_\tau, 0 \leq \tau \leq t$. Then past lower demand:*

1. *Leads to better distributions of firms: $\bar{\mu}_{\tau+1} \succ \hat{\mu}_{\tau+1}, \quad 0 \leq \tau \leq t$.*

2. *Raises future output, thereby reducing future prices and firm profits: $\bar{Y}_\tau > \hat{Y}_\tau$, $\bar{p}_\tau < \hat{p}_\tau$, and $\bar{\pi}_\tau(\alpha) < \hat{\pi}_\tau(\alpha), \quad 0 \leq \tau \leq t$.*

Proof: Result 1 follows immediately from counter-cyclical exit and theorem 2. Consider the first period τ in which $\hat{\theta}_\tau > \bar{\theta}_\tau$. Then, counter cyclical exit ensures the higher demand shock implies less exit, and hence a worse distribution of firms. From theorem 2, even if subsequent demand shocks are identical, the worse distribution is preserved; and subsequent higher demand shocks reinforce the result.

Result 2 follows since the distribution of firms on the "bar" path is better than the distribution on the "hat" path, price will be lower on the "bar" path unless there is (substantially) more exit in the "bar" economy. So suppose that $\bar{\alpha}_t > \hat{\alpha}_t$. But then,

$$\begin{aligned}
\pi(\hat{\theta}_t, \hat{\mu}_t^{\hat{\alpha}_t}, \hat{\alpha}_t) < \pi(\hat{\theta}_t, \hat{\mu}^{\hat{\alpha}_t}, \bar{\alpha}_t) &< \pi(\bar{\theta}_t, \bar{\mu}_t^{\bar{\alpha}_t}, \bar{\alpha}_t) \\
&= \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v_{t+1}^*(\tilde{\theta}, \bar{\mu}_{\bar{\alpha}_{t+1}}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \bar{\theta})[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \bar{\alpha}_{t+1})] \\
&< \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v_{t+1}^*(\tilde{\theta}, \hat{\mu}_{\hat{\alpha}_{t+1}}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \hat{\theta})[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \hat{\alpha}_{t+1})],
\end{aligned}$$

a contradiction. The implications for output and firm profits are immediate.

■

Here, for example, $\bar{\mu}_\tau$ is the distribution on technology along the $\bar{\theta}$ sequence. Weak demand has the immediate effect of inducing more inefficient firms to exit, which leads to an improved distribution of firm efficiencies, which persists at all future dates (1). The higher productivity caused by a past downturn implies greater future output once demand 'recovers' sufficiently (2), illustrating the cleansing effect of recessions for future output. Indeed, if there is sufficiently little persistence in demand, then a lower current demand shock must *raise* expected future discounted total surplus. In turn, this increased future competition implies lower future prices and hence lower profits for a firm of a given technology quality. The greater the stress of a demand downturn (i.e., the lower are the demand realizations and the longer the lower demand realizations persist), the more fit are the survivors, and hence the more productive is the entire industry. These results are consistent with the findings in the empirical literature that correlations of exit rates with future GDP growth are positive, large, persistent and statistically significant (Campbell (1998)).

Our theorems state the results in terms of the impact of a more prolonged boom in demand. The results can also be stated in terms of their converse — more prolonged recessions lead to greater output when demand finally improves, and more sustained high demand leads to steeper declines to lower levels of output when demand finally falls.

Remark 1: The focus of our model is on a specific industry: we take factor prices as given. However, it is worthwhile to consider the extent to which our results remain valid were factor prices to vary with market conditions. Firm α's period production problem becomes, $\max_{\ell,k} p(\mu,\theta)f(\ell,k,\alpha)-w(\mu,\theta)\ell-r(\mu,\theta)k$. This determines firm α's profit, $\pi(\theta,\mu,\alpha)$. So, as when factor prices are constant, α's profit depends on the same three variables. The feature that our analysis exploits is $\frac{\partial\pi}{\partial\theta}>0$: better times raise period profits for *all* firm types. This is unequivocal if θ affects only demand. More generally, an application of the envelope theorem reveals that it is true if $\frac{\partial\pi}{\partial\theta}=\frac{\partial p(\mu,\theta)}{\partial\theta}\cdot f(\ell,k,\alpha)-\frac{\partial w(\mu,\theta)}{\partial\theta}\ell-\frac{\partial r(\mu,\theta)}{\partial\theta}k>0,\forall\alpha$. A sufficient condition for this to be so is that technologies be Cobb Douglas or quadratic, so that the profit function is a multiplicatively separable function of the firm's productivity parameter: if an increase in θ helps one firm type, it helps all firm types, and with a Cobb-Douglas technology, $f(k,\ell)=k^b\ell^c$, it does so if $\frac{p}{w^c r^b}$ rises with θ. Our theorems all extend to general equilibrium if, in addition, (a) factor prices (weakly) increase with industry input demand (so reduced past exit further raises period profits), and (b) re-interpreting the demand shocks as industry period profit shocks (that take into account the equilibrium impacts on factor prices), there is persistence in the θ 'profit' shocks. Theorem 2 extends because worse distributions of firms, ceteris paribus imply lower factor prices. In turn, persistence in 'profit' shocks, plus counter cyclical exit, ensure that the other theorems extend, including a generalized version of theorems 4 and 5 (given counter cyclical exit holds). ■

Remark 2: Although modeling entry is not a focus of this paper, here the number of new entrants corresponds to the measure of firms in their first period of production. Entry is greater when the economy leaves a recession (as is found in the data), both because there are more exiting firms due to the past downturn and because the increase in demand implies that more firms find it optimal to produce.

One can augment the model to allow for entry "by birth", so that a potential entrant can enter the market by creating a new firm rather than taking over an existing one. Both forms of entry are important. In Canadian manufacturing, Baldwin and Gorecki found that entrants after 1970 accounted for 26.2% of total sales by 1979. Those that entered by creating a firm accounted for 14% of total sales, while those that entered by acquiring another firm accounted for over 12% of total sales. Modifying our model to accommodate entry through the creation of new firms would appear to reinforce the counter-cyclical movement in the distribution of firm quality. With demand persistence, a high demand shock should increase the expected payoff of an entrant, and hence raise entry. Because, on average, firm productivities rise with age, the immediate impact of additional entry would be to lower the average efficiency of firms in operation, reinforcing our results. An associated implication is that contemporaneous entry and exit will tend to be negatively correlated, as is observed in the data. ■

3.4 Dynamics and Expectations.

We next consider the impact of an anticipated future increase in demand on current exit decisions and hence future distributions of firm productivities. We show that, ceteris paribus, better anticipated future market conditions leads to more exit at all earlier dates. Fix a future point in time, T, and consider the impact of replacing the transition kernel $\Theta(\theta_T \mid \theta_{T-1})$ with one of the alternative transition kernels, $\bar{\Theta}(\theta_T \mid \theta_{T-1})$ or $\hat{\Theta}(\theta_T \mid \theta_{T-1})$. If $\bar{\Theta}(\theta_T \mid \theta_{T-1}) \succ \hat{\Theta}(\theta_T \mid \theta_{T-1})$ for each θ_{T-1}, then from the perspective of time $t < T$, $\bar{\Theta}(\theta_T \mid \theta_{T-1})$ represents the expectation of better demand that $\hat{\Theta}(\theta_T \mid \theta_{T-1})$ — an anticipated increase in future demand relative to $\hat{\Theta}$.

Theorem 7 *Consider two economies that differ solely in anticipated future demand conditions at date T:*

$$\bar{\Theta}(\theta_T \mid \theta_{T-1}) \succ \hat{\Theta}(\theta_T \mid \theta_{T-1}),\ \forall\, \theta_{T-1}.$$

(For $t \neq T$, the two economies agree with transition kernel $\Theta(\theta_t \mid \theta_{t-1})$.) Then for any given demand shock θ_{t-1}, and distribution μ_{t-1}, an anticipated increase in future demand at date T gives rise to a better distribution of firm productivities at all intermediate dates: $\bar{\mu}_\tau \succ \hat{\mu}_\tau, \forall \tau \in \{t, \dots T\}$.

Proof: Because the period T distribution of demand shocks is better in the bar economy,

$$\int\int \bar{v}_T(\tilde{\theta}_T, \mu, \tilde{\alpha})\bar{\Theta}(d\tilde{\theta}_T \mid \theta_{T-1})P(d\tilde{\alpha} \mid \alpha) > \int\int \hat{v}_T(\tilde{\theta}_T, \mu, \tilde{\alpha})\hat{\Theta}(d\tilde{\theta}_T \mid \theta_{T-1})P(d\tilde{\alpha} \mid \alpha).$$

Thereafter, demand realizations are drawn from the same distribution, so $\bar{v}_T(\cdot) = \hat{v}_T(\cdot)$. For any given μ_{T-1}, current profits for any marginal exiter $\pi(\theta_{T-1}, \mu_{T-1\alpha^*}, \alpha^*)$ are the same in both the hat and bar economies, but the value to exit is greater in the bar economy. Hence, for any given μ_{T-1}, there must be more exit in the bar economy than the hat economy, so that $\bar{\mu}_T(\mu_{T-1}) \succ \hat{\mu}_T(\mu_{T-1})$. In turn, this increased exit in the bar economy implies that for any given μ_{T-1} and θ_{T-1}, date $T-1$ prices are higher in the bar economy than the hat economy. In turn, this implies that

$$\int\int \bar{v}_{T-1}(\tilde{\theta}_{T-1}, \mu, \tilde{\alpha})\Theta(d\tilde{\theta}_{T-1} \mid \theta_{T-2})P(d\tilde{\alpha} \mid \alpha) > \int\int \hat{v}_{T-1}(\tilde{\theta}_{T-1}, \mu, \tilde{\alpha})\Theta(d\tilde{\theta}_{T-1} \mid \theta_{T-2})P(d\tilde{\alpha} \mid \alpha).$$

Hence, for any given μ_{T-2}, there is more exit in the bar economy than the hat economy, so that $\bar{\mu}_{T-1} \succ \hat{\mu}_{T-1}$. Monotonicity of μ_T in μ_{T-1} then implies that $\bar{\mu}_T(\bar{\mu}_{T-1}(\mu_{T-2}), \theta) \succ \bar{\mu}_T(\hat{\mu}_{T-1}(\mu_{T-2}), \theta)$.

Repeating the argument, observe that since for any given μ_{T-2} there is more exit in the bar than hat economy, the distribution of prices must be higher in the bar than hat economy at date $T-2$, so that date

$T-2$ continuation payoffs must be greater for any given μ_{T-2} in the bar economy than the hat economy. Hence, for any given μ_{T-3}, there must be more exit in the bar economy than the hat economy. Repeating this argument, it follows that $\bar{\mu}_t \succ \hat{\mu}_t, 0 < t \leq T$.

■

The key to Theorem 7 is that an anticipated improvement in future demand does not directly affect current profits; it only raises the future value of having a better technology, and hence the value of exit. If the anticipated improvement in demand is only one period ahead, then the future value of being productive is increased, raising current exit and lowering current output, thereby indirectly raising current prices and profits. When the anticipated demand increase is further in the future, its anticipation raises the distribution of productivities at all earlier dates in the stochastic dominance sense. This is because to increase future productivity, exit must be raised, increasing the distribution of prices prior to the productivity improvement. In particular, suppose to the contrary that exit first increased at the last date prior to the improvement in demand, $T-1$. But this increased exit reduces period $T-1$ output, raising period $T-1$ prices and hence the value of exit at date $T-2$. Thus, anticipation of increased future exit in advance of the demand increase, drives exit and productivity increases at earlier dates. Hence, anticipation of different future market conditions can generate the positive correlations of exit rates with future GDP growth found by Campbell (1998).

4 Thin Resale Markets.

So far we have not considered how industry dynamics vary across industries distinguished by different depths of the market for the plants of exiting firms. That is, we have not considered how variations in γ affect outcomes. The degree of plant specialization varies significantly across industries. We now provide conditions under which the smaller is γ, i.e., the thicker is the market for the plant of an exiting firm, the greater is exit. Finally, we show by construction that when $\gamma > 0$ so that the social and private incentives to exit do not coincide, then not only can a recession weed out 'bad' firms, but a downturn in demand can actually *raise* welfare (total surplus) by inducing more efficient exit.

The *direct* effect of a thinner market is to reduce exit. To see this suppose there is a one-time increase in thinness in the market for a bankrupt firm's plant, so that γ increases to γ', an increase that is unanticipated at earlier dates. This increase to γ' reduces the value of exit relative to the value of remaining in the industry, leading to a worse distribution of firms at all future dates.

The net effect of a thinner resale market is more subtle because of the *indirect* effect of reduced exit on future competition. If $\gamma < \gamma'$, it does not follow that for all (θ, μ, α), that $v_\gamma(\theta, \mu, \alpha) > v_{\gamma'}(\theta, \mu, \alpha)$. While sufficiently unproductive firms are worse off facing γ', because bankrupt firms sell their plants for less, the *indirect* consequence of reduced exit is that the future distribution of firms is worse. In turn, this reduced future competition implies higher future prices, so that high α firms that are unlikely to enter bankruptcy may be *helped* if firms are more reluctant to exit. Consequently, for $\gamma < \gamma'$, it may not be that $v^c_\gamma(\theta, \mu, \alpha) - v^e_\gamma(\theta, \mu, \alpha) > v^c_{\gamma'}(\theta, \mu, \alpha) - v^e_{\gamma'}(\theta, \mu, \alpha)$, for *all* (θ, μ). It must be true for *some* (θ, μ), else the future distribution of firms would always be better in the γ' economy, so that both the direct and indirect effects would encourage more exit in the γ' economy, a contradiction of the premise.

If there is no aggregate uncertainty, so that $p(Y, \theta) = p(Y)$, then we can derive the effects of an increase in γ on industry dynamics:

Theorem 8 *Suppose there is no aggregate uncertainty, so that $p(Y, \theta) = p(Y)$. Then for all γ, given μ_0, there is a unique equilibrium. The distributions of firm productivities converge monotonically over time to the unique stationary equilibrium: either $\mu_t^\gamma \succeq \mu_{t-1}^\gamma, \forall t$ or $\mu_{t-1}^\gamma \succeq \mu_t^\gamma, \forall t$. In the limiting stationary economies, for $\gamma > \gamma'$,*

1. *There is less exit in the economy with a thinner resale market, $\alpha^{*\gamma}(\mu_\infty^\gamma) < \alpha^{*\gamma'}(\mu_\infty^\gamma) < \alpha^{*\gamma'}(\mu_\infty^{\gamma'})$.*
2. *The limiting distribution is better in the economy with the thicker resale market, $\mu_\infty^{\gamma'} \succ \mu_\infty^\gamma$.*
3. *Any given α operates at a larger and more profitable scale in the economy with thinner markets.*

The characterization of the evolution of the distribution of firms to the invariant steady state extends Hopenhayn (1992a) who analyzes the behavior of individual firms in the limiting economy. In particular, this theorem shows that the economy does not cycle over time as it approaches the limiting steady state.

If γ is interpreted as reflecting the degree to which inputs are specialized in an industry, then inter-industry comparisons can be made. Industries with more specialized inputs will tend to have more unproductive, but larger, firms and lower rates of exit. This is consistent with the finding of Dunne et al. (1989a) and Asplund and Nocke (2003) that substantial and persistent differences in entry and exit rates across industries exist, and that industries with higher entry rates also have higher exit rates.[9]

Further insights on the effects of thin markets for a bankrupt firm's plant can be gleaned from the polar case of perfectly elastic demand, $p(Y, \theta) = p(\theta)$. The qualitative findings hold as long as demand is 'sufficiently' elastic. If $p(Y, \theta) = p(\theta)$, then $v_\gamma(\theta, \mu, \alpha) = v_\gamma(\theta, \hat{\mu}, \alpha)$, $\forall \mu, \hat{\mu}$: only the direct negative effect of γ on exit remains. When $p(Y, \theta) = p(\theta)$, the marginal exiting firm equates

$$\pi(\theta, \alpha^*(\theta)) = \beta \int v_\gamma(\tilde{\theta}, \tilde{\alpha}) \Theta(d\tilde{\theta}|\theta)[(1-\gamma)P(d\tilde{\alpha}|\bar{\alpha}) - P(d\tilde{\alpha}|\alpha^*)].$$

Since the right-hand side is decreasing in γ, it follows immediately that a reduction in γ raises exit, and hence the distribution of firm technology qualities. Individual firms exit according to their private incentives, $1-\gamma$, so there is too little exit from a total surplus maximization perspective. The exit decision corresponds to the exit choice made by a social planner when the exiting firm is destroyed with probability γ; the fact that next period's distribution of firms does not reflect this destruction is irrelevant because the distribution does not affect a firm's payoffs. The greater is γ, the greater is the wedge between equilibrium exit and efficient exit.

We now show constructively that when $\gamma > 0$, so that firms are too reluctant from a surplus-maximizing standpoint to exit, *downturns in demand can increase welfare* by inducing inefficient firms to sell their

[9] Asplund and Nocke (2003) offer a sunk cost explanation for this empirical regularity.

plants to firms that could use them more productively. Because downturns in demand raise the attraction of exit, they induce more inefficient firms to act in accordance with maximizing social surplus, so that privately optimal actions are more closely aligned with socially optimal actions.

Example: Let technologies be given by $\alpha f(\ell)$, where $\alpha \in \{\underline{\alpha}, \hat{\alpha}\}, \underline{\alpha} < \hat{\alpha}$. Suppose that the productivity of a firm's technology does not vary with time, $\alpha_t = \alpha_{t-1}$, and that entrants are equally likely to draw each technology: $Pr(\alpha = \hat{\alpha}|\text{entry}) = .5$. Demand is perfectly elastic: $p(Y, \theta) = \theta, \theta \in \{\underline{\theta}, \bar{\theta}\}, \underline{\theta} < \bar{\theta}$, and demand shocks are independently and identically distributed. Markets for a bankrupt firm's plant are thin: $1 > \gamma > 0$. The discount factor is $\beta > .5$. There are as many good firms as bad.

Suppose that $\underline{\alpha}$ is sufficiently small relative to $\hat{\alpha}$ that total surplus is maximized when $\underline{\alpha}$ firms exit. Let γ be such that firm type $\underline{\alpha}$ is just indifferent between continuing to operate and exiting. Such a γ exists because the value of exiting varies continuously with γ from 0 when $\gamma = 1$ to the expected value of the new entrant when $\gamma = 0$. Consider the consequence for total surplus of a one-time marginal reduction in $\underline{\theta}$. This drop in demand induces all unproductive, $\underline{\alpha}$, firms to exit. The marginal loss in profits to a good firm from this reduced consumer demand is $\hat{\alpha} f(\ell^*(\hat{\alpha}, \underline{\theta}))$. The marginal loss in expected discounted profits to a bad firm is zero, since it now exits and previously was indifferent between exiting and not. If we let $z = \frac{\beta E[\pi(\theta,\alpha)]}{(1-\beta)}$, then the value of exit is $\frac{(1-\gamma)z}{(1-\gamma\beta)}$. The social gain from inducing exit of inefficient $\underline{\alpha}$ firms is therefore $\frac{\gamma(1-\beta)z}{(1-\beta\gamma)}$. Since there are just as many good firms as bad firms, the one-time reduction in θ from $\underline{\theta}$ increases social surplus if and only if $\frac{\gamma(1-\beta)z}{(1-\beta\gamma)} > \hat{\alpha} f(\ell^*(\hat{\alpha}, \underline{\theta}))$. But note that $\hat{\alpha} f(\ell^*(\hat{\alpha}, \underline{\theta}))$ does not depend on $\bar{\theta}$. Let $\gamma(\bar{\theta})$ be the value of γ that leaves $\underline{\alpha}$ just indifferent to exit when $\theta = \underline{\theta}$: $\frac{d\gamma}{d\theta} > 0$. As $\bar{\theta}$ is increased, it must be the case that $\frac{\gamma(1-\beta)z}{(1-\beta\gamma)}$ eventually exceeds $\hat{\alpha} f(\ell^*(\hat{\alpha}, \underline{\theta}))$, so the one-time recession of a lower θ increases total surplus.

■

If γ is sufficiently large, bad firms fail to internalize the social cost of their failure to exit and thereby have their technologies replaced by stochastically better ones. Downturns cause more bad firms to internalize these social costs and exit. If this gain from increased exit outweighs the foregone period surplus from lower consumer demand then total surplus is raised.

5 Conclusion.

This paper explores the dynamics of an industry when there is both aggregate demand uncertainty and idiosyncratic uncertainty about the productivity of an individual firm's technology. We characterize the intertemporal evolution of the distribution of firms, where firms are distinguished by the productivity of their technology. We contrast industry dynamics *across* different demand shock histories, *along* a particular demand shock history, and detail how *anticipation* of future demand shocks affects exit decisions and the industry's evolution. Our theoretical predictions about cyclical patterns in exit and productivity offer a coherent explanation for the cyclical patterns exhibited in the data.

6 Appendix

Proof of Theorem 1: Fix the distribution μ on $[0,1]$ and let $\mu(d\alpha) = g(\alpha)d\alpha$. If firms with technology lower than α exit then let μ_α denote the truncated distribution. If the price is P the supply of firm α is $q(P,\alpha)$ and total supply is $Y_s(P,\mu_\alpha) = \int_\alpha^1 q(P,\alpha)\mu(d\alpha)$. We can invert this to get $P_s(Y,\mu_\alpha)$. Since the exit threshold uniquely defines the distribution μ_α, we may write α in place of μ_α, with corresponding supply and inverse supply functions: $Y_s(P,\alpha)$ and $P_s(Y,\alpha)$.

The current period social surplus is given by $S(\theta,Y,\alpha) = \int_0^Y [P(Q,\theta) - P_s(Q,\alpha)]dQ$. Maximizing this over Y gives $Y^* = Y(\theta,\alpha)$ satisfying $P(Y^*,\theta) - P_s(Y^*,\alpha) = 0$, so that

$$S(\theta,Y^*,\alpha) = \int_0^{Y^*} [P(Q,\theta) - P_s(Q,\alpha)]dQ.$$

The variation of the surplus with respect to α is given by

$$\frac{dS(Y(\theta,\alpha),\alpha)}{d\alpha} = \frac{\partial S(Y^*,\alpha)}{\partial\alpha} = -\int_0^{Y^*} \frac{\partial P_s(Y,\alpha)}{\partial\alpha}dY = -\frac{\partial}{\partial\alpha}\int_0^{Y^*} P_s(Q,\alpha)dQ$$

Since $\int_0^{Y^*} P_s(Y,\alpha)dY = P^*Y^* - \int_0^{P^*} Q_s(P,\alpha)dP$,

$$\begin{aligned}
\frac{\partial}{\partial\alpha}\int_0^{Y^*} P_s(Q,\alpha)dQ &= -\frac{\partial}{\partial\alpha}\int_0^{P^*} Y_s(P,\alpha)dP \\
&= -\int_0^{P^*} \frac{\partial}{\partial\alpha}Y_s(P,\alpha)dP \\
&= -\int_0^{P^*} [\frac{\partial}{\partial\alpha}\int_\alpha^1 q(P,\tilde\alpha)g(\tilde\alpha)d\tilde\alpha]dP \\
&= \int_0^{P^*} q(P,\alpha)g(\alpha)dP \\
&= [\int_0^{P^*} q(P,\alpha)dP]g(\alpha) \\
&= \pi(\theta,\mu,\alpha)g(\alpha)
\end{aligned}$$

Thus,

$$\frac{\partial S(Y^*,\alpha)}{\partial\alpha} = -\pi(\theta,\mu,\alpha)g(\alpha).$$

Now, let V satisfy the recursion:

$$V(\theta,\mu) = \max_\alpha\{\max_Y \int_0^Y [P(Q,\theta) - P_s(Q,\alpha)]dQ + \delta\int_\Theta V(\tilde\theta,\mu^\alpha)\Theta(d\tilde\theta \mid \theta)\}$$

where $\mu^\alpha(\cdot) = \int_\alpha^1 P(\cdot \mid \alpha)\mu(d\alpha) + \int_0^\alpha P(\cdot \mid \bar\alpha)\mu(d\alpha)$ and $\mu(d\alpha) = g(\alpha)d\alpha$. Let $v(\alpha,\mu^\alpha)$ be the continuation payoff to a firm with technology α, when the aggregate distribution is μ^α.

Suppose that

$$\frac{\partial}{\partial\alpha}V(\theta,\mu^\alpha) = -\int_\Theta\int_0^1 v(\theta,\hat\alpha,\mu^\alpha)P(d\hat\alpha \mid \alpha)g(\alpha)\Theta(d\tilde\theta \mid \theta) + \int_\Theta\int_0^1 v(\theta,\hat\alpha,\mu^\alpha)P(d\hat\alpha \mid \bar\alpha)g(\alpha)\Theta(d\tilde\theta \mid \theta)$$

Then

$$\frac{\partial}{\partial \alpha}\Big\{\int_0^{Y^*}[P(Q,\theta)-P_s(Q,\alpha)]dQ+\delta\int_\Theta V(\tilde\theta,\mu^\alpha)\Theta(d\tilde\theta\mid\theta)\Big\}$$

$$=-\pi(\theta,\mu,\alpha)g(\alpha)-\int_\Theta\int_0^1 v(\tilde\theta,\hat\alpha,\mu^\alpha)P(d\hat\alpha\mid\alpha)g(\alpha)\Theta(d\tilde\theta\mid\theta)$$

$$+\int_\Theta\int_0^1 v(\tilde\theta,\hat\alpha,\mu^\alpha)P(d\hat\alpha\mid\bar\alpha)g(\alpha)\Theta(d\tilde\theta\mid\theta)$$

When this is 0, at say α^*,

$$[\pi(\alpha^*)+\int_\Theta\int_0^1 v(\tilde\theta,\hat\alpha,\mu^{\alpha^*})P(d\hat\alpha\mid\alpha^*)\Theta(d\tilde\theta\mid\theta)]g(\alpha^*)=[\int_\Theta\int_0^1 v(\tilde\theta,\hat\alpha,\mu^\alpha)P(d\hat\alpha\mid\bar\alpha)\Theta(d\tilde\theta\mid\theta)]g(\alpha^*)$$

So, the exit threshold that maximizes the social surplus, is the technology threshold at which a firm is indifferent between exiting and remaining in the market.

■

Remark: The calculations below confirm that the differential condition used above is satisfied recursively.

Put $\mu(\gamma)(\cdot)\overset{\text{def}}{=}\mu^\gamma(\cdot)=\int_\gamma^1 P(\cdot\mid\alpha)\mu(d\alpha)+\int_0^\gamma P(\cdot\mid\bar\alpha)\mu(d\alpha)$ where $\mu(d\alpha)=g(\alpha)d\alpha$. Then

$$\mu(\gamma)^\alpha(\cdot)=\int_\alpha^1 P(\cdot\mid\alpha)\mu(\gamma)(d\alpha)+\int_0^\alpha P(\cdot\mid\bar\alpha)\mu(\gamma)(d\alpha).$$

Collecting terms:

$$\begin{aligned}\mu(\gamma)^\alpha(\cdot)[=(\mu^\gamma)^\alpha] \quad = \quad & \int_\alpha^1 P(\cdot\mid\hat\alpha)\int_\gamma^1 P(d\hat\alpha\mid\alpha)\mu(d\alpha)+\int_\alpha^1 P(\cdot\mid\hat\alpha)\int_0^\gamma P(d\hat\alpha\mid\bar\alpha)\mu(d\alpha)\\ &+\int_0^\alpha P(\cdot\mid\bar\alpha)\int_\gamma^1 P(d\hat\alpha\mid\alpha)\mu(d\alpha)+\int_0^\alpha P(\cdot\mid\bar\alpha)\int_0^\gamma P(d\hat\alpha\mid\bar\alpha)\mu(d\alpha).\end{aligned}$$

Rearranging:

$$\begin{aligned}\mu(\gamma)^\alpha(\cdot) \quad &= \quad \int_\gamma^1\int_\alpha^1 P(\cdot\mid\hat\alpha)P(d\hat\alpha\mid\alpha)g(\alpha)d\alpha+\int_\gamma^1\int_0^\alpha P(\cdot\mid\bar\alpha)P(d\hat\alpha\mid\alpha)g(\alpha)d\alpha\\ &= \quad \int_0^\gamma\int_\alpha^1 P(\cdot\mid\hat\alpha)P(d\hat\alpha\mid\bar\alpha)g(\alpha)d\alpha+\int_0^\gamma\int_0^\alpha P(\cdot\mid\bar\alpha)P(d\hat\alpha\mid\bar\alpha)g(\alpha)d\alpha.\end{aligned}$$

Or

$$\begin{aligned}\mu(\gamma)^\alpha(\cdot) \quad = \quad & \int_\gamma^1\{\int_\alpha^1 P(\cdot\mid\hat\alpha)P(d\hat\alpha\mid\alpha)+\int_0^\alpha P(\cdot\mid\bar\alpha)P(d\hat\alpha\mid\alpha)\}g(\alpha)d\alpha\\ &+\int_0^\gamma\{\int_\alpha^1 P(\cdot\mid\hat\alpha)P(d\hat\alpha\mid\bar\alpha)+\int_0^\alpha P(\cdot\mid\bar\alpha)P(d\hat\alpha\mid\bar\alpha)\}g(\alpha)d\alpha.\end{aligned}$$

In this case:

$$\frac{\partial}{\partial\gamma}V(\theta,\mu(\gamma)^\alpha)=$$

$$-\int_\Theta \Big[\int_0^1 v(\tilde\theta,\tilde\alpha,\mu(\gamma)^\alpha)\{\int_\alpha^1 P(d\tilde\alpha \mid \hat\alpha)P(d\hat\alpha \mid \gamma) + \int_0^\alpha P(d\tilde\alpha \mid \bar\alpha)P(d\hat\alpha \mid \gamma)\}g(\gamma)\Big]\Theta(\tilde\theta \mid \theta)$$
$$+\int_\Theta \Big[\int_0^1 v(\tilde\theta,\tilde\alpha,\mu(\gamma)^\alpha)\{\int_\alpha^1 P(d\tilde\alpha \mid \hat\alpha)P(d\hat\alpha \mid \bar\alpha) + \int_0^\alpha P(d\tilde\alpha \mid \bar\alpha)P(d\hat\alpha \mid \bar\alpha)\}g(\gamma)\Big]\Theta(\tilde\theta \mid \theta).$$

Or,

$$\frac{\partial}{\partial\gamma}V(\theta,\mu(\gamma)^\alpha) =$$
$$-\Big[\{\int_\alpha^1\int_0^1 v(\tilde\theta,\tilde\alpha,\mu(\gamma)^\alpha)P(d\tilde\alpha \mid \hat\alpha)P(d\hat\alpha \mid \gamma) + \int_0^\alpha\int_0^1 v(\tilde\theta,\tilde\alpha,\mu(\gamma)^\alpha)P(d\tilde\alpha \mid \bar\alpha)P(d\hat\alpha \mid \gamma)\}g(\gamma)\Big]\Theta(\tilde\theta \mid \theta)$$
$$+\Big[\{\int_\alpha^1\int_0^1 v(\theta,\tilde\alpha,\mu(\gamma)^\alpha)P(d\tilde\alpha \mid \hat\alpha)P(d\hat\alpha \mid \bar\alpha) + \int_\alpha^1\int_0^\alpha v(\theta,\tilde\alpha,\mu(\gamma)^\alpha)P(d\tilde\alpha \mid \bar\alpha)P(d\hat\alpha \mid \bar\alpha)\}g(\gamma)\Big]\Theta(\tilde\theta \mid \theta).$$

Turning to the first period, the distribution is given by $\mu(\gamma)$. The impact of a variation in γ is given by:

$$\int_0^{P^*}[\frac{\partial}{\partial\gamma}\int_\alpha^1 q(P,\alpha)\mu(\gamma)(d\alpha)]dP$$

Substituting for $\mu(\gamma)$

$$\int_0^{P^*}[\frac{\partial}{\partial\gamma}\int_\alpha^1 q(P,\hat\alpha)(\int_\gamma^1 P(d\hat\alpha \mid \alpha)g(\alpha)d\alpha + \int_0^\gamma P(d\hat\alpha \mid \bar\alpha)g(\alpha)d\alpha)]dP.$$

This is equal to:

$$\begin{aligned}
&\int_0^{P^*}[-\int_\alpha^1 q(P,\hat\alpha)P(d\hat\alpha \mid \gamma)g(\gamma) + \int_\alpha^1 q(P,\hat\alpha)P(d\hat\alpha \mid \bar\alpha)g(\gamma)]dP \\
&\quad= -\int_\alpha^1[\int_0^{P^*} q(P,\hat\alpha)dP]P(d\hat\alpha \mid \gamma)g(\gamma) + \int_\alpha^1[\int_0^{P^*} q(P,\hat\alpha)dP]P(d\hat\alpha \mid \bar\alpha)g(\gamma) \\
&\quad= -\int_\alpha^1 \pi(\theta,\mu(\gamma),\hat\alpha)P(d\hat\alpha \mid \gamma)g(\gamma) + \int_\alpha^1 \pi(\theta,\mu(\gamma),\hat\alpha)P(d\hat\alpha \mid \bar\alpha)g(\gamma) \\
&\quad= [-\int_\alpha^1 \pi(\theta,\mu(\gamma),\hat\alpha)P(d\hat\alpha \mid \gamma) + \int_\alpha^1 \pi(\theta,\mu(\gamma),\hat\alpha)P(d\hat\alpha \mid \bar\alpha)]g(\gamma).
\end{aligned}$$

Collecting all terms:

$$\frac{\partial}{\partial\gamma}V(\theta,\mu(\gamma)) =$$
$$[-\int_\alpha^1 \pi(\theta,\mu(\gamma),\hat\alpha)P(d\hat\alpha \mid \gamma) + \int_\alpha^1 \pi(\theta,\mu(\gamma),\hat\alpha)P(d\hat\alpha \mid \bar\alpha)]g(\gamma)$$
$$-\delta\int_\Theta\Big[\{\int_\alpha^1\int_0^1 v(\tilde\theta,\tilde\alpha,\mu(\gamma)^\alpha)P(d\tilde\alpha \mid \hat\alpha)P(d\hat\alpha \mid \gamma)+\int_0^\alpha\int_0^1 v(\tilde\theta,\tilde\alpha,\mu(\gamma)^\alpha)P(d\tilde\alpha \mid \bar\alpha)P(d\hat\alpha \mid \gamma)\}g(\gamma)\Big]\Theta(\tilde\theta \mid \theta)$$
$$+\delta\int_\Theta\Big[\{\int_\alpha^1\int_0^1 v(\tilde\theta,\tilde\alpha,\mu(\gamma)^\alpha)P(d\tilde\alpha \mid \hat\alpha)P(d\hat\alpha \mid \bar\alpha)+\int_\alpha^1\int_0^\alpha v(\tilde\theta,\tilde\alpha,\mu(\gamma)^\alpha)P(d\tilde\alpha \mid \bar\alpha)P(d\hat\alpha \mid \bar\alpha)\}g(\gamma)\Big]\Theta(\tilde\theta \mid \theta).$$

Rearranging positive and negative terms, this yields:

$$\frac{\partial}{\partial\gamma}V(\theta,\mu(\gamma)) =$$

$$-\{\int_{\alpha}^{1}[\pi(\theta,\mu(\gamma),\hat{\alpha})+\delta\int_{\Theta}\int_{0}^{1}v(\tilde{\theta},\tilde{\alpha},\mu(\gamma)^{\alpha})P(d\tilde{\alpha}\mid\hat{\alpha})\Theta(d\tilde{\theta}\mid\theta)]P(d\hat{\alpha}\mid\gamma)+$$
$$\delta\int_{\Theta}\int_{0}^{\alpha}\int_{0}^{1}v(\tilde{\theta},\tilde{\alpha},\mu(\gamma)^{\alpha})P(d\tilde{\alpha}\mid\bar{\alpha})P(d\hat{\alpha}\mid\gamma)\Theta(d\tilde{\theta}\mid\theta)\}g(\gamma)$$
$$+\{\int_{\alpha}^{1}[\pi(\theta,\mu(\gamma),\hat{\alpha})+\delta\int_{\Theta}\int_{0}^{1}v(\tilde{\theta},\tilde{\alpha},\mu(\gamma)^{\alpha})P(d\tilde{\alpha}\mid\hat{\alpha})\Theta(d\tilde{\theta}\mid\theta)]P(d\hat{\alpha}\mid\bar{\alpha})+$$
$$\delta\int_{\Theta}\int_{0}^{\alpha}\int_{0}^{1}v(\tilde{\theta},\tilde{\alpha},\mu(\gamma)^{\alpha})P(d\tilde{\alpha}\mid\bar{\alpha})P(d\hat{\alpha}\mid\bar{\alpha})\Theta(d\tilde{\theta}\mid\theta)\}g(\gamma)$$
$$=-\int_{0}^{1}\int_{\Theta}v(\tilde{\theta},\hat{\alpha},\mu(\gamma))\Theta(d\tilde{\theta}\mid\theta)P(d\hat{\alpha}\mid\gamma)g(\gamma)+\int_{0}^{1}\int_{\Theta}v(\tilde{\theta},\hat{\alpha},\mu(\gamma))\Theta(d\tilde{\theta}\mid\theta)P(d\hat{\alpha}\mid\bar{\alpha})g(\gamma).$$

Thus, the derivative of the valuation function preserves the difference of valuation between a new firm and a continuing firm.

■

The discussion in theorem 2 below utilises properties of the class of distributions generated by the truncation procedures used in the paper. The following lemma establishing that with the weighted-average kernel transition, either increasing the measure of exiting firms, i.e., increasing the exit threshold α^* for a fixed distribution μ, or improving the current distribution over firm technologies μ for a fixed exit rule α, leads to an improved distribution of firm technology qualities in the next period. This result is used subsequently in the proof of theorem 2.

Lemma 1 *The measure $\mu^{\alpha^*}(\cdot)$ satisfies:*

1. *Raising the exit threshold improves next period's distribution:*

$$\hat{\alpha}^* \geq \alpha^* \textit{ implies } \mu^{\hat{\alpha}^*}(\cdot) \succ \mu^{\alpha^*}(\cdot).$$

2. *For a given exit threshold, an improvement in the current distribution improves next period's distribution:*

$$\hat{\mu},\mu \in \mathcal{W}(F,G) = \{\mu \mid \exists\beta\in[0,1], \mu=\beta F+(1-\beta)G\}, \hat{\mu}\succ\mu \textit{ implies } \hat{\mu}^{\alpha^*}(\cdot)\succ\mu^{\alpha^*}(\cdot).$$

Proof : We first show that for $\mu = \beta F + (1-\beta)G$, $\mu' = \beta' F + (1-\beta')G$, with $\beta' > \beta$. Then $F \succeq G$ implies $\mu' \succeq \mu$. To see this, it is sufficient to show that $\phi_\beta(\alpha)$ is increasing in β for each $\alpha \geq \alpha^*$, where:

$$\phi_\beta(\alpha) = \mu([\alpha,1] \mid \alpha \geq \alpha^*) = \frac{\mu([\alpha,1])}{\mu([\alpha^*,1])}.$$

Now,

$$\phi_\beta(\alpha) = \mu([\alpha,1] \mid \alpha \geq \alpha^*) \quad = \quad \frac{\beta F([\alpha,1]) + (1-\beta)G([\alpha,1])}{\beta F([\alpha^*,1]) + (1-\beta)G([\alpha^*,1])}$$

$$\begin{aligned}
&= \frac{\beta[F([\alpha,1]) - G([\alpha,1])] + G([\alpha,1])}{\beta[F([\alpha^*,1]) - G([\alpha^*,1])] + G([\alpha^*,1]} \\
&= \frac{\beta k(\alpha) + d(\alpha)}{\beta k(\alpha^*) + d(\alpha^*)},
\end{aligned}$$

where $k(\tilde{\alpha}) = [F([\tilde{\alpha},1]) - G([\tilde{\alpha},1])]$ and $d(\tilde{\alpha}) = G([\tilde{\alpha},1])$. Therefore,

$$\begin{aligned}
\frac{\partial \phi_\beta}{\partial \beta} &= \frac{[\beta k(\alpha^*) + d(\alpha^*)]k(\alpha) - [\beta k(\alpha) + d(\alpha)]k(\alpha^*)]}{[\beta k(\alpha^*) + d(\alpha^*)]^2} \\
&= \frac{[d(\alpha^*)k(\alpha) - d(\alpha)k(\alpha^*)]}{[\beta k(\alpha^*) + d(\alpha^*)]^2}.
\end{aligned}$$

The numerator is

$$F([\alpha,1])G([\alpha^*,1]) - G([\alpha,1])G([\alpha^*,1]) - [F([\alpha^*,1])G([\alpha,1]) - G([\alpha^*,1])G([\alpha,1])].$$

Canceling $G([\alpha,1])G([\alpha^*,1])$ gives the numerator as

$$F([\alpha,1])G([\alpha^*,1]) - F([\alpha^*,1])G([\alpha,1]) = F([\alpha^*,1])G([\alpha^*,1]) \left[\frac{F([\alpha,1])}{F([\alpha^*,1])} - \frac{G([\alpha,1])}{G([\alpha^*,1])} \right].$$

Thus, since $\left[\frac{F([\alpha,1])}{F([\alpha^*,1])} - \frac{G([\alpha,1])}{G([\alpha^*,1])} \right] > 0$ from conditional first order stochastic dominance, $\mu' \succeq \mu$.

With this preliminary result in hand, consider

$$\begin{aligned}
\mu^{\alpha^*}(\cdot) &= \int_{[0,\alpha^*)} P(\cdot \mid \bar{\alpha})\mu(d\alpha) + \int_{[\alpha^*,1]} P(\cdot \mid \alpha)\mu(d\alpha) \\
&= F(\cdot) \int_{[\alpha^*,1]} w(\alpha)\mu(d\alpha) + G(\cdot) \int_{[\alpha^*,1]} (1 - w(\alpha))\mu(d\alpha) \\
&\qquad + \mu([0,\alpha^*))w(\bar{\alpha})F(\cdot) + \mu([0,\alpha^*))(1 - w(\bar{\alpha})G(\cdot) \\
&= [\int_{[\alpha^*,1]} w(\alpha)\mu(d\alpha) + \mu([0,\alpha^*))w(\bar{\alpha})]F(\cdot) + \\
&\qquad [\int_{[\alpha^*,1]} (1 - w(\alpha))\mu(d\alpha) + \mu([0,\alpha^*))(1 - w(\bar{\alpha}))]G(\cdot).
\end{aligned}$$

Let

$$x_{\alpha^*}(\alpha) = \chi_{[0,\alpha^*)}(\alpha)w(\bar{\alpha}) + \chi_{[\alpha^*,1]}(\alpha)w(\alpha),$$

so that $E_\mu\{x_{\alpha^*}\} = \int x_{\alpha^*}(\alpha)\mu(d\alpha)$ gives the weight on $F(\cdot)$ in the distribution $\mu^{\alpha^*}(\cdot)$:

$$\mu^{\alpha^*}(\cdot) = E_\mu\{x_{\alpha^*}\}F(\cdot) + [1 - E_\mu\{x_{\alpha^*}\}]G(\cdot).$$

To prove 1, note that $\alpha^* < \bar{\alpha}$, and since w is monotone increasing, the function x_{α^*} is constant and equal to $w(\bar{\alpha})$ on $[0,\alpha^*)$, has a "downward jump" at α^* (from $w(\bar{\alpha})$ to $w(\alpha^*)$), and is monotone increasing on $[\alpha^*,1]$. If $\bar{\alpha} \geq \hat{\alpha}^* \geq \alpha^*$, then $x_{\hat{\alpha}^*} \geq x_{\alpha^*}$, so that

$$E_\mu\{x_{\hat{\alpha}^*}\} \geq E_\mu\{x_{\alpha^*}\}.$$

Thus 1 holds. Now we prove 2.

Let $\mu = \beta F(\cdot) + (1-\beta)G(\cdot)$. We need to show that as β increases, $E_\mu(x_{\alpha^*})$ increases. Note that

$$E_\mu(x_{\alpha^*}) = \mu([0,\alpha^*))w(\bar{\alpha}) + \mu([\alpha^*,1])E_\mu\{x_{\alpha^*} \mid \alpha \geq \alpha^*\}.$$

Since

$$\int_{[\alpha^*,1]} w(\alpha)\mu(d\alpha) = \beta\int_{[\alpha^*,1]} w(\alpha)F(d\alpha) + (1-\beta)\int_{[\alpha^*,1]} w(\alpha)G(d\alpha) = \\ \beta F([\alpha^*,1])E_F\{w(\alpha) \mid \alpha \geq \alpha^*\} + (1-\beta)G([\alpha^*,1])E_G\{w(\alpha) \mid \alpha \geq \alpha^*\}.$$

Thus,

$$E_\mu\{x_{\alpha^*}\} = [\beta F([0,\alpha^*)) + (1-\beta)G([0,\alpha^*))]w(\bar{\alpha}) + \\ \beta F([\alpha^*,1])E_F\{w(\alpha) \mid \alpha \geq \alpha^*\} + (1-\beta)G([\alpha^*,1])E_G\{w(\alpha) \mid \alpha \geq \alpha^*\},$$

or

$$E_\mu\{x_{\alpha^*}\} = \beta[F([0,\alpha^*))w(\bar{\alpha}) + F([\alpha^*,1])E_F\{x_{\alpha^*} \mid \alpha \geq \alpha^*\}] + \\ (1-\beta)[G([0,\alpha^*))w(\bar{\alpha}) + G([\alpha^*,1])E_G\{x_{\alpha^*} \mid \alpha \geq \alpha^*\}].$$

Using stochastic dominance (x_{α^*} is increasing on $[\alpha^*,1]$),

$$E_F\{x_{\alpha^*} \mid \alpha \geq \alpha^*\} \geq E_G\{x_{\alpha^*} \mid \alpha \geq \alpha^*\}.$$

Provided $E_F\{x_{\alpha^*} \mid \alpha \geq \alpha^*\} \geq w(\bar{\alpha})$, $E_\mu\{x_{\alpha^*}\}$ is increasing in β. A sufficient condition for this to be so is that $E_F\{w\} \geq w(\bar{\alpha})$. But $\int P(\cdot,\alpha)P(d\alpha,\bar{\alpha}) \succeq P(\cdot,\bar{\alpha})$ implies that $E_F\{w\} \geq w(\bar{\alpha})$. But this holds if and only if

$$[\int w(\alpha)P(d\alpha,\bar{\alpha})]F(\cdot) + [1 - \int w(\alpha)P(d\alpha,\bar{\alpha})]G(\cdot) \succeq P(\cdot \mid \bar{\alpha}) = w(\bar{\alpha})F(\cdot) + [1-w(\bar{\alpha})]G(\cdot).$$

Because $\int w(\alpha)F(d\alpha) > \int w(\alpha)P(d\alpha,\bar{\alpha})$,

$$\int w(\alpha)F(d\alpha)]F(\cdot) + [1 - \int w(\alpha)F(d\alpha)]G(\cdot) \succ \int w(\alpha)P(d\alpha,\bar{\alpha})]F(\cdot) + [1 - \int w(\alpha)P(d\alpha,\bar{\alpha})]G(\cdot),$$

so that $\int w(\alpha)F(d\alpha) = E_F\{w\} \geq w(\bar{\alpha})$.

■

Proof of Theorem 2: We prove the result inductively. Let $v_n^c(\theta,\mu,\alpha)$ be the payoff (present value) to agent α in an n-period problem, when the current aggregate shock is θ, the current aggregate distribution is μ and the agent chooses to stay in the market with n periods remaining. Similarly, $v_n^e(\theta,\mu,\alpha)$ is the payoff to agent α when the current aggregate shock is θ, the current aggregate distribution is μ and the agent chooses to exit the market with n periods remaining. The maximum of these functions is: $v_n(\theta,\mu,\alpha) = \max\{v_n^c(\theta,\mu,\alpha), v_n^e(\theta,\mu,\alpha)\}$.

At $n = 1$, they are defined: $v_1^c(\theta,\mu,\alpha) = \pi(\theta,\mu,\alpha)$ and $v_1^e(\theta,\mu,\alpha) = 0$. From the properties of the profit function, these are continuously decreasing in μ. Now, assume that the result holds for $n-1$:

both $v^c_{n-1}(\theta,\mu,\alpha)$ and $v^e_{n-1}(\theta,\mu,\alpha)$ are continuous and decreasing in μ. This implies that $v^*_{n-1}(\theta,\mu,\alpha)$ is decreasing in μ.

Consider the variation $\mu \uparrow \hat{\mu}$. The impact effect of this (with no change in the exit rule) is to lower both current profit and future expected profit:

$$\pi' = \pi(\theta, \hat{\mu}_{\alpha_n(\theta)}, \alpha_n(\theta)) \le \pi(\theta, \mu_{\alpha_n(\theta)}, \alpha_n(\theta)) = \pi$$

and

$$\begin{aligned}\int'_{\tilde{\theta}\tilde{\alpha}} &= \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \hat{\mu}^{\alpha_n(\theta)}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta)[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \alpha_n(\theta))] \\ &\le \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \mu^{\alpha_n(\theta)}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta)[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \alpha_n(\theta))] = \int_{\tilde{\theta}\tilde{\alpha}}\end{aligned}$$

There are two cases to consider: (1) $\pi' < \int'_{\tilde{\theta}\tilde{\alpha}}$ and (2) $\pi' > \int'_{\tilde{\theta}\tilde{\alpha}}$.

Case (1): $\pi' < \int'_{\tilde{\theta}\tilde{\alpha}}$

In case (1), to restore equilibrium we require that $\pi' \uparrow$ and $\int'_{\tilde{\theta}\tilde{\alpha}} \downarrow$. To raise π', raise $\alpha_n(\theta)$ (say $\hat{\alpha}_n(\theta)$ is the exit value that restores equilibrium: continuity of period profits in the exit rule, $\alpha_n(\theta)$, follows because output and hence price are, so that there exists such an $\hat{\alpha}_n(\theta)$). This increase in exit increases the efficiency of $\hat{\mu}^{\alpha_n(\theta))}$ as the distribution moves to $\hat{\mu}^{\hat{\alpha}_n(\theta))}$. As a result $\int'_{\tilde{\theta}\tilde{\alpha}}$ falls, and this is reinforced by the "fall" in $[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \alpha_n(\theta))]$ as $\alpha_n(\theta) \uparrow \hat{\alpha}_n(\theta)$. These changes result in a continuous fall in $\int'_{\tilde{\theta}\tilde{\alpha}}$ to $\hat{\int}_{\tilde{\theta}\tilde{\alpha}}$, say. Now, write

$$\hat{\int}_{\tilde{\theta}\tilde{\alpha}} = \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \hat{\mu}^{\hat{\alpha}_n(\theta)}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta)[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \hat{\alpha}_n(\theta))]$$

and from the calculations, $\hat{\int}_{\tilde{\theta}\tilde{\alpha}} < \int'_{\tilde{\theta}\tilde{\alpha}} \le \int_{\tilde{\theta}\tilde{\alpha}}$. Write $\hat{\pi} = \pi(\theta, \hat{\mu}_{\hat{\alpha}_n(\theta)}, \hat{\alpha}_n(\theta))$ for the new equilibrium current profit. Now,

$$\hat{\pi} = \pi(\theta, \hat{\mu}_{\hat{\alpha}_n(\theta)}, \hat{\alpha}_n(\theta)) = \hat{\int}_{\tilde{\theta}\tilde{\alpha}} < \int'_{\tilde{\theta}\tilde{\alpha}} \le \int_{\tilde{\theta}\tilde{\alpha}} = \pi(\theta, \mu_{\alpha_n(\theta)}, \alpha_n(\theta)) = \pi$$

current profit has fallen. Since $\hat{\alpha}_n(\theta) > \alpha_n(\theta)$,

$$\pi(\theta, \hat{\mu}_{\hat{\alpha}_n(\theta)}, \alpha_n(\theta)) < \pi(\theta, \hat{\mu}_{\hat{\alpha}_n(\theta)}, \hat{\alpha}_n(\theta)) < \pi(\theta, \mu_{\alpha_n(\theta)}, \alpha_n(\theta)).$$

Thus,

$$\forall \alpha,\ \pi(\theta, \hat{\mu}_{\hat{\alpha}_n(\theta)}, \alpha) < \pi(\theta, \mu_{\alpha_n(\theta)}, \alpha).$$

Therefore, price is lower and output higher. Also, because $\hat{\mu}^{\hat{\alpha}_n(\theta)} \succ \hat{\mu}^{\alpha_n(\theta)} \succ \mu^{\alpha_n(\theta)}$ and v_{n-1} is monotonic in μ,

$$\int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \hat{\mu}^{\hat{\alpha}_n(\theta)}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta) < \int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \mu^{\alpha_n(\theta)}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta)$$

Consequently, $v^c_n(\theta,\hat{\mu},\alpha) < v^c_n(\theta,\mu,\alpha)$. Similarly, $v^e_n(\theta,\hat{\mu},\alpha) < v^e_n(\theta,\mu,\alpha)$. Since both π' and $\int'_{\tilde{\theta}\tilde{\alpha}}$ are continuous in μ and $\alpha_n(\theta)$, so are $v^c_n(\theta,\mu,\alpha)$ and $v^e_n(\theta,\mu,\alpha)$.

Case (2): $\pi' > \int'_{\tilde{\theta}\tilde{\alpha}}$

In this case, we want to reduce π' and raise $\int'_{\tilde{\theta}\tilde{\alpha}}$. As $\pi(\theta, \hat{\mu}_{\alpha_n(\theta)}, \alpha_n(\theta))$ is too high, reduce $\alpha_n(\theta)$ to $\hat{\alpha}_n(\theta)$ (say, the new equilibrium exit rule) where $\hat{\mu}_{\hat{\alpha}_n(\theta)} \succ \mu_{\alpha_n(\theta)}$. This reduces current profit further, with greater output and lower price. As $\alpha_n(\theta)$ declines, this reduces the efficiency of $\hat{\mu}_{\alpha_n(\theta)}$, so that $\int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \hat{\mu}^{\alpha_n(\theta)}, \tilde{\alpha}) P(d\tilde{\theta} \mid \theta)$ increases. In addition $[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \alpha_n(\theta))]$ increases. At the new solution:

$$\pi(\theta, \hat{\mu}_{\hat{\alpha}_n(\theta)}, \hat{\alpha}_n(\theta)) = \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \hat{\mu}^{\hat{\alpha}_n(\theta)}, \tilde{\alpha}) Q(d\tilde{\theta} \mid \theta)[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \hat{\alpha}_n(\theta))]$$

Since

$$\pi(\theta, \mu_{\alpha_n(\theta)}, \alpha_n(\theta)) > \pi(\theta, \hat{\mu}_{\alpha_n(\theta)}, \alpha_n(\theta)) > \pi(\theta, \hat{\mu}_{\hat{\alpha}_n(\theta)}, \alpha_n(\theta)) > \pi(\theta, \hat{\mu}_{\hat{\alpha}_n(\theta)}, \hat{\alpha}_n(\theta))$$

this implies

$$\beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \mu^{\alpha_n(\theta)}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta)[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \alpha_n(\theta))]$$
$$> \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \hat{\mu}^{\hat{\alpha}_n(\theta)}, \tilde{\alpha}) Q(d\tilde{\theta} \mid \theta)[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \hat{\alpha}_n(\theta))].$$

And, since

$$[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \hat{\alpha}_n(\theta))] \succeq [(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \alpha_n(\theta))]$$

we get that for a set of positive measure of α's that

$$\beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \mu^{\alpha_n(\theta)}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta) > \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \hat{\mu}^{\hat{\alpha}_n(\theta)}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta)$$

Were it the case that $\mu^{\alpha_n(\theta)} \succ \hat{\mu}^{\hat{\alpha}_n(\theta)}$, this could hold for no α. Thus, this cannot be the case, and since the set of measures is totally ordered (from the weighted average assumption), the reverse is true:

$$\hat{\mu}^{\hat{\alpha}_n(\theta)} \succ \mu^{\alpha_n(\theta)}.$$

Thus,

$$\beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \mu^{\alpha_n(\theta)}, \tilde{\alpha}) \Theta(d\tilde{\theta} \mid \theta) > \beta \int_{\tilde{\alpha}} \int_{\tilde{\theta}} v_{n-1}(\tilde{\theta}, \hat{\mu}^{\hat{\alpha}_n(\theta)}, \tilde{\alpha}) \theta(d\tilde{\theta} \mid \theta) \forall \tilde{\alpha}$$

This implies that for all α, $v_n^c(\theta, \hat{\mu}, \alpha) < v_n^c(\theta, \mu, \alpha)$. Similarly, for all α, $v_n^e(\theta, \hat{\mu}, \alpha) < v_n^e(\theta, \mu, \alpha)$. Continuity again follows immediately. Note that in both cases, $\hat{\mu}^{\hat{\alpha}_n(\theta)} \succ \mu^{\alpha_n(\theta)}$.

■

Proof of Theorem 4: Let $Pr(\theta = \bar{\theta} | \bar{\theta}) = \rho$; $Pr(\theta = \underline{\theta} | \underline{\theta}) = \phi$. Suppose that $\rho = \phi = 1$, so that for some $\theta \in \{\underline{\theta}, \bar{\theta}\}$, $\theta_t = \theta_0 = \theta$, and the marginal exiting firm, $\alpha^*(\theta, \mu)$, is determined by the solution to

$$\pi(\theta, \mu_{\alpha^*}, \alpha^*) = \beta \int_{\tilde{\alpha}} v(\theta, \mu^{\alpha^*}, \tilde{\alpha})[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \alpha^*)].$$

With $\rho = \phi = 1$ the state is constant over time and equal to θ. It is straightforward (see Bergin and Bernhardt (2005), Theorem 4) to show that the aggregate distribution converges (monotonically) to some distribution $\mu_\infty(\theta)$: $lim_{t\to\infty}\mu_t \to \mu_\infty(\theta)$, with associated price sequence, $p(\mu_t,\theta) \to p(\mu_\infty,\theta)$. From the multiplicative decomposition of profits, asymptotically, the value functions are multiplicative functions of price, so that the exit rule, α^*, is asymptotically independent of θ (and only relative prices matter). Hence, $\mu_\infty(\underline{\theta}) = \mu_\infty(\bar{\theta}) = \mu_\infty$ with associated exit rule α_∞.

Let $\alpha^{\rho\phi}(\theta,\mu)$ be the equilibrium exit threshold at state θ when the aggregate distribution is μ and the transition probabilities on θ given by ρ and ϕ. Note that $\alpha^{1\phi}(\bar{\theta},\mu_\infty) = \alpha_\infty = \alpha^{\rho 1}(\underline{\theta},\mu_\infty)$ because at $\rho = 1$ ($\phi = 1$) the exit rule at $(\bar{\theta},\mu_\infty)$ $((\underline{\theta},\mu_\infty))$ is independent of ϕ (ρ).

For any (θ,μ), observe that $\alpha^{\rho\phi}(\theta,\mu)$ is (a) increasing in ρ and (b) decreasing in ϕ. (Raising ρ raises the future payoff leaving current payoff unchanged and therefore leads to more exit, increasing ϕ reduces the future payoff leaving current payoff unchanged and therefore leads to less exit.) Thus,

$$\alpha^{1\phi}(\underline{\theta},\mu_\infty) > \alpha^{\rho\phi}(\underline{\theta},\mu_\infty) > \alpha^{\rho 1}(\underline{\theta},\mu_\infty) = \alpha_\infty = \alpha^{1\phi}(\bar{\theta},\mu_\infty) > \alpha^{\rho\phi}(\bar{\theta},\mu_\infty) > \alpha^{\rho 1}(\bar{\theta},\mu_\infty).$$

Next, fix some ρ,ϕ pair. Since $\alpha^{\rho\phi}(\underline{\theta},\mu_\infty) > \alpha_\infty > \alpha^{\rho\phi}(\bar{\theta},\mu_\infty)$, at the distribution μ_∞, more exit occurs at $\underline{\theta}$ than at $\bar{\theta}$. Starting at μ_∞, denote the next period's distribution following $\bar{\theta}$ by $\mu_1^{\bar{\theta}}$ and following $\underline{\theta}$ by $\mu_1^{\underline{\theta}}$. Noting that $\mu_\infty = \mu_\infty^\infty$, $\mu_1^{\bar{\theta}} \prec \mu_\infty \prec \mu_1^{\underline{\theta}}$.

To make dependence on μ_∞ explicit, write $\mu_1^{\bar{\theta}}(\mu_\infty) \prec \mu_\infty \prec \mu_1^{\underline{\theta}}(\mu_\infty)$. Since for fixed θ, improving (worsening) the current distribution improves (worsens) next period's distribution, if the aggregate shock at $\mu_1^{\bar{\theta}}(\mu_\infty)$ is $\bar{\theta}$, then next period's distribution $\mu_2^{\bar{\theta},\bar{\theta}}(\mu_\infty) \prec \mu_1^{\bar{\theta}}(\mu_\infty)$. Letting $\bar{\theta}(t)$ denote t realizations of $\bar{\theta}$, and $\mu_t^{\bar{\theta}(t)}(\mu_\infty)$ the associated distribution in t periods, the distribution satisfies: $\mu_{t+1}^{\bar{\theta}(t+1)}(\mu_\infty) \prec \mu_t^{\bar{\theta}(t)}(\mu_\infty)$. Let $\underline{\mu}$ denote the limit. Similar reasoning gives $\mu_{t+1}^{\underline{\theta}(t+1)}(\mu_\infty) \succ \mu_t^{\underline{\theta}(t)}(\mu_\infty)$, where $\mu_t^{\underline{\theta}(t)}(\mu_\infty)$ is the distribution t periods later following a history of t-$\underline{\theta}$ shocks. let $\bar{\mu}$ denote the limit.

Now, fix some $\mu \in (\underline{\mu},\mu_\infty)$. Given $\bar{\theta}$, let $\mu^{\bar{\theta}}(\mu)$ be next period's distribution. From the previous reasoning, $\mu^{\bar{\theta}}(\mu) \prec \mu$. Next, consider μ and the shock $\underline{\theta}$. In this case, $\mu^{\underline{\theta}}(\mu) \succ \mu$. To see this, note that as ϕ increases, the future payoff is lower and this reduces exit. However, at $\underline{\theta}$ with $\phi = 1$, the system stays in state $\underline{\theta}$ and the aggregate distribution increases to μ_∞ asymptotically. In particular, $\mu^{\underline{\theta}}(\mu) \succ \mu$ when $\phi = 1$. However, when $\phi < 1$ the future payoff increases and this increases exit (recall $\alpha^{\rho\phi}(\theta,\mu)$ is decreasing in ϕ so exit increases as ϕ falls). But more exit improves next period's distribution, so that for the given $\phi\rho$ pair, $\mu^{\underline{\theta}}(\mu) \succ \mu$. Thus, $\mu^{\underline{\theta}}(\mu) \succ \mu \succ \mu^{\bar{\theta}}(\mu)$. This implies that exit at $(\underline{\theta},\mu)$ is greater than exit at $(\bar{\theta},\mu)$, or $\alpha^{\rho\phi}(\underline{\theta},\mu) > \alpha^{\rho\phi}(\bar{\theta},\mu)$. This completes the argument.

■

Proof of Theorem 8: Fix γ. We first argue that because distributions can be ordered over time, so that either we are in the stationary economy, or that $\mu_t^\gamma \succ \mu_{t-1}^\gamma, \forall t$, or that $\mu_{t-1}^\gamma \succ \mu_t^\gamma, \forall t$. Suppose to the contrary that $\mu_t^\gamma \succ \mu_{t-1}^\gamma$, but that $\mu_t^\gamma \succeq \mu_{t+1}^\gamma$. This implies $\alpha_{t-1}^{*\gamma} > \alpha_t^{*\gamma}$ (if $\mu \succ \mu'$ the same exit threshold applied to each preserves the ordering at the next round) and $p_{t-1}^\gamma > p_t^\gamma$. But then $\pi(\mu^\gamma_{t-1\cdot\alpha^{*\gamma}_{t-1}}, \alpha^{*\gamma}_{t-1}) > \pi(\mu^\gamma_{t-1\cdot\alpha^{*\gamma}_t}, \alpha^{*\gamma}_t)$. But this implies that

$$\int v_\gamma(\mu_{t+1}^\gamma,\tilde{\alpha})[(1-\gamma)P(d\tilde{\alpha} \mid \bar{\alpha}) - P(d\tilde{\alpha} \mid \alpha_t^{*\gamma})]$$

$$< \int v_\gamma(\mu_t^\gamma,\tilde{\alpha})[(1-\gamma)P(d\tilde{\alpha}\mid\bar{\alpha}) - P(d\tilde{\alpha}\mid\alpha_{t-1}^{*\gamma})] < \int v_\gamma(\mu_t^\gamma,\tilde{\alpha})[(1-\gamma)P(d\tilde{\alpha}\mid\bar{\alpha}) - P(d\tilde{\alpha}\mid\alpha_t^{*\gamma})].$$

But this implies that for some α, $v_\gamma(\mu_t^\gamma,\alpha) > v_\gamma(\mu_{t+1}^\gamma,\alpha)$, but $\mu_t^\gamma \succeq \mu_{t+1}^\gamma$, a contradiction. An analogous contradiction is derived if we assume that $\mu_{t-1}^\gamma \succ \mu_t^\gamma$, but $\mu_{t+1}^\gamma \succeq \mu_t^\gamma$.

The monotonic improvement or decline in the distribution of firm qualities over time implies that $lim_{t\to\infty} p_t^\gamma = p_\infty^\gamma$: The limiting economy is stationary.

We now argue that there is a unique equilibrium path. Suppose the equilibrium path is not unique. There are two cases to consider — where the distributions move in opposite directions and where they move in the same direction. Suppose that there were at least two equilibrium paths, indexed by hat and dot, such that $\dot{\mu}_1 = \mu_0^{\dot{\alpha}_0^*} \succeq \mu_0 \succ \hat{\mu}_1 = \mu_0^{\hat{\alpha}_0^*}$. Dropping the γ index, the argument above implies that $\hat{p}_t > \dot{p}_t$, $\forall t \geq 1$. Further, since $\hat{\alpha}_0^* < \dot{\alpha}_0^*$, $\hat{p}_0 < \dot{p}_0$. But then,

$$\pi_0(\mu_{0\cdot\hat{\alpha}_0^*},\hat{\alpha}_0^*) < \pi_0(\mu_{0\cdot\dot{\alpha}_0^*},\dot{\alpha}_0^*)$$

$$= \beta\int \dot{v}(\dot{\mu}_1,\tilde{\alpha})[(1-\gamma)P(d\tilde{\alpha}\mid\bar{\alpha}) - P(d\tilde{\alpha}\mid\dot{\alpha}_0^*)] < \beta\int \hat{v}(\hat{\mu}_1,\tilde{\alpha})[(1-\gamma)P(d\tilde{\alpha}\mid\bar{\alpha}) - P(d\tilde{\alpha}\mid\hat{\alpha}_0^*)],$$

a contradiction.

One can derive a similar contradiction to the possibility of two equilibrium paths, indexed by hat and dot, such that $\dot{\mu}_1 = \mu_0^{\dot{\alpha}_0^*} \succ \hat{\mu}_1 = \mu_0^{\hat{\alpha}_0^*} \succeq \mu_0$, or the converse. Again, abusing notation,

$$\dot{\mu}_1 \succ \hat{\mu}_1 \to \dot{\pi}_0 > \hat{\pi}_0 \to \int \dot{v}_1 > \int \hat{v}_1.$$

But $\int \dot{v}_1 > \int \hat{v}_1$ requires that there must be a future date where $\hat{\mu}_t \succ \dot{\mu}_t$. Since $\dot{\mu}_\infty \succ \hat{\mu}_\infty$ there is a greatest date T such that $\hat{\mu}_T \succeq \dot{\mu}_T$. But $\hat{\mu}(\hat{\mu}_T) \succeq \hat{\mu}(\dot{\mu}_T)$. Again,

$$\dot{\mu}_{T+1} \succ \hat{\mu}(\dot{\mu}_T) \to \dot{\pi}_T > \hat{\pi}_T \to \int \dot{v}_{T+1}(\dot{\mu}_{T+1},\tilde{\alpha}) > \int \hat{v}_{T+1}(\hat{\mu}(\dot{\mu}_T,\tilde{\alpha}),$$

a contradiction, since prices from $T+1$ on must be higher in the hat economy.

Now consider $\gamma > \gamma'$, and suppose that at the stationary distribution of firms in the no prime economy, μ_∞^γ, there is less exit than in the prime economy: $\alpha^{*\gamma}(\mu_\infty^\gamma) \geq \alpha^{*\gamma'}(\mu_\infty^\gamma)$, so that $\mu_\infty^\gamma \succeq \mu_\infty^{\gamma'}$ and hence, $p_\infty^\gamma \leq p_\infty^{\gamma'}$. But then $v_\gamma(\alpha,\mu_\infty^\gamma) < v_{\gamma'}(\alpha,\mu_\infty^\gamma)$. Consequently,

$$\pi(\mu^\gamma_{\infty\cdot\alpha^{*\gamma'}(\mu_\infty^\gamma)},\alpha^{*\gamma'}(\mu_\infty^\gamma)) \leq \pi(\mu^\gamma_{\infty\cdot\alpha^{*\gamma}(\mu_\infty^\gamma)},\alpha^{*\gamma}(\mu_\infty^\gamma)) = \beta\int v_\gamma(\mu_\infty^\gamma,\tilde{\alpha})[(1-\gamma)P(d\tilde{\alpha}\mid\bar{\alpha}) - P(d\tilde{\alpha}\mid\alpha^{*\gamma}(\mu_\infty^\gamma)]$$

$$< \beta\int v_{\gamma'}(\mu_\infty^{\gamma\cdot\alpha^{*'}(\mu_\infty^\gamma)},\tilde{\alpha})[(1-\gamma)P(d\tilde{\alpha}\mid\bar{\alpha}) - P(d\tilde{\alpha}\mid\alpha^{*\gamma'}(\mu_\infty^\gamma)],$$

a contradiction. Hence, $\mu_\infty^{\gamma'} \succ \mu_\infty^\gamma$ and $p_\infty^{\gamma'} \leq p_\infty^\gamma$. In turn, $p_\infty^{\gamma'} < p_\infty^\gamma$ implies that any given α in the no prime economy operate at a larger and more profitable scale. ■

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