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Queen's Economics Department Working Paper No. 1004

## Social Insurance and Redistribution

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4-2002

# Social Insurance and Redistribution \*

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July 5, 2001

## Abstract

This paper studies optimal linear income taxation and redistributive social insurance when the former has the traditional labor distortion and the latter generates both *ex ante* and *ex post* moral hazard. Private insurance is available and individuals differ in labor productivity and in loss probability. We show that government intervention in insurance markets is welfare-improving, and social insurance is generally desirable particularly when there is a negative correlation between labor productivity and loss probability.

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\*Prepared for 'Public Finances and Public Policy in the New Millennium', a conference on the occasion of Richard Musgrave's 90th and CES's 10th birthdays, University of Munich, January 12-13, 2001. We are grateful for comments by Gabrielle Demange, Dominique Demougin and two referees. This paper presents research results of the Belgian Program on Interuniversity Poles of Attraction initiated by the Belgian State, Prime Minister's Office, Science Policy Programming. The scientific responsibility is assumed by the authors.

# 1 Introduction

One of the most compelling and lasting methodological insights of Richard Musgrave's (1959) classic *The Theory of Public Finance* was the conceptual separation between the Allocative and Distributive branches of government. It represented an operationalization of the First and Second Theorems of Welfare Economics. In ideal circumstances, the Allocative Branch should be concerned with taking the economy to the society's utility possibilities frontier by exploiting all gains from trade, while the Redistributive Branch alone need be concerned with choosing the ethically preferred point. From a policy perspective, the ability to separate efficiency and equity considerations is of enormous importance. To the extent that the Allocative Branch can go about its business of ensuring that resources are allocated efficiently, willingness-to-pay can be used as the benchmark for public project evaluation, and interpersonal welfare comparisons can be set aside. Much influential normative public economics has revolved around investigating the circumstances under which this separation applies, and the consequences of its not applying.

Two types of reasons have been stressed in the literature as to why efficiency and equity might not be separable. The first devolves from the theory of second best formalized by Lipsey and Lancaster (1956).<sup>1</sup> This literature focused initially on the agnostic implications of exogenously given second-best distortions for the use of market prices as signals of efficiency. Subsequently, with the advent of the optimal commodity tax literature, the existence of second-best distortions was found to make it necessary to incorporate equity weights into shadow pricing rules for public projects.<sup>2</sup> However, the mere existence of commodity tax distortions did not vitiate the Musgravian separation of Branches. Indeed, arguably the most important result of Diamond and Mirrlees' (1971) seminal contribution to optimal commodity tax analysis was their so-called Production Efficiency Theorem. According to this theorem, if commodity taxes were set optimally and all pure profits were taxed away, public sector shadow prices would be producer prices, at least for private commodities. This essentially revitalized the Musgravian

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<sup>1</sup>The idea of the theory of second best had been around for a long time. Elements of it may be found, for examples, in the work on public sector pricing by Boiteux (1956), in the taxation literature by Hotelling (1932) and Harberger (1964), and in the trade literature by Meade (1955). For a survey of the theory of second best and its relation to public economics, see Boadway (1997).

<sup>2</sup>The most complete summary of this can be found in Drèze and Stern (1987).

separation result after the onslaught of the theory of second best. Unfortunately, the Production Efficiency Theorem only applied with respect to private commodities, and only then if taxes were in fact set optimally. In the case of public goods, the Samuelson Rule had to be modified not only to include the effect of (perhaps optimal) linear tax distortions, but also to incorporate equity considerations.<sup>3</sup>

The second reason why equity and efficiency considerations might not be separable is in a sense more profound. It is because of an imperfectly informed government. The classic work of Mirrlees (1971) implied that if the government cannot observe private attributes of households, the Second Theorem of Welfare Economics would be violated, and economic outcomes would be restricted to the second-best utility possibilities frontier. Effectively, this theory supplied a fully endogenous explanation for why lump-sum redistributive taxation was not optimal: second-best tax distortions were useful as a way of eliciting information, albeit in a costly way.<sup>4</sup> Even here, however, it is conceivable that efficiency considerations alone might be used to determine public sector allocation rules. Indeed, the Production Efficiency Theorem survives: with optimal non-linear income taxes in place, public sector shadow prices for private commodities are still producer prices. With public goods, matters are slightly more complicated. Unlike with linear taxes, the Samuelson Rule for public goods applies with optimal non-linear taxes as long as leisure is separable from public and private goods (Boadway and Keen (1993)).

While the above literature is concerned with public spending on goods and services, this paper focuses on another prominent sort of spending — that on social insurance. Why do we have social insurance and not private insurance for such things as health care and disability? In a perfect Musgravian world, one might expect that private insurance based on market efficiency principles would suffice. Yet, in most countries social insurance takes a much larger share of GDP than private insurance, and in fact often preceded private insurance.

Traditionally, there are three types of reasons for public intervention in the field of insurance: transactions costs, market failures and redistribution.

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<sup>3</sup>The modified Samuelson Rule reflecting linear tax distortions was obtained by Atkinson and Stern (1974). The further modification to incorporate equity considerations may be seen in Atkinson and Stiglitz (1980).

<sup>4</sup>Guesnerie (1995) provides the most comprehensive account of the relationship between asymmetric information and distortionary taxation.

In the health care sector, private insurance exhibits higher transaction costs than social insurance. This is partly because of high administrative costs.<sup>5</sup> Market failures, the second reason, arise primarily from asymmetric information, such as that between insurers and insurees (adverse selection and moral hazard), and that between health care providers and health care consumers. In keeping with the Musgravian tradition, our interest is in the third reason, that is, the role of social insurance as a redistributive device.

In a full-information world of first best, there is little reason for redistribution using social insurance. The Distributive and Allocative functions of the government can be separated, so one would expect income taxation to achieve all the desired redistribution, and social insurance to operate according to the market rule of actuarial fairness. However, in a second best world of distortionary taxation, we will show that social insurance can be a powerful device for redistribution, complementing the tax-transfer system.

It has been established in the literature that if risks are negatively related to income so that the poor face higher risks on average, then we have an obvious redistributive argument for social insurance. As shown by Rochet (1989) and Cremer and Pestieau (1996), social insurance combined with a standard distortionary income tax can redistribute more effectively. The reason is that redistributing through social insurance does not involve the same distortion, and this is even more so when social insurance is less administratively costly than private insurance.<sup>6</sup>

This result has been developed in a setting where the risk probability is given and any loss can be compensated for without restriction. In other words, *ex ante* and *ex post* moral hazard were assumed away. When either one is taken into account, it appears that the case for social insurance is not as strong, and that, unlike in the above analyses, full coverage is no longer necessarily socially desirable. The purpose of this paper is to study those two types of moral hazard in an economy in which a linear income tax and a social insurance can be used jointly along with actuarially fair, but possibly costly, private insurance.<sup>7</sup>

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<sup>5</sup>These costs are linked to the small scale of private insurance firms and to their advertisement costs. On this, see Diamond (1992) and Mitchell (1998). The point goes back to Arrow (1963).

<sup>6</sup>See also Petretto (1999).

<sup>7</sup>Blomqvist and Horn (1984) bears some similarities with our paper even though it is not concerned with moral hazard. These authors also examine the case for public insurance when actuarially fair private insurance is available and individuals differ in both labour productivity and illness probability. No labour is supplied when ill, and public insurance

The paper is organized as follows. Section 2 presents the basic model and assumptions. Sections 3 and 4 consider two benchmark cases. In the first, there is perfect information and the government can make lump-sum redistributive transfers. In this case, Musgravian separation applies: despite moral hazard, actuarially fair insurance can be provided by the private sector, and all redistributive objectives can be accomplished by the tax-transfer system. In the second benchmark, there is no moral hazard, but the public sector is restricted to distortionary taxation—linear progressive taxation for simplicity. In this case, full social insurance is provided, crowding out private insurance. Section 5 then considers *ex post* moral hazard along with linear progressive taxation, and Section 6 *ex ante* moral hazard. In each case, there is generally a redistributive role for public intervention in private insurance markets, though the direction of intervention is ambiguous. Section 7 extends the *ex post* moral hazard case to a setting in which there are extra administrative costs associated with private insurance provision. Finally, Section 8 offers some concluding remarks.

## 2 Model and assumptions

The economy consists of three types of decision-makers—households, insurance firms and the government. Households face an idiosyncratic risk of accident, but might be able to take hidden actions that affect the size of the loss in the event of an accident—*ex post* moral hazard—or that affect the probability of the accident occurring—*ex ante* moral hazard. Households differ both in productivity and in accident risk. Insurance companies can observe household risk, and provide insurance competitively and—except in Section 7 where administrative costs are introduced—actuarially fairly.<sup>8</sup> The government’s objective is to redistribute income among households, but because it cannot observe productivities, it is restricted to using distortionary policy instruments (except in Section 3). Decision-making can be thought of as occurring sequentially. The government chooses its policies first, followed by the insurance firms, and then households. In each case, the outcomes of

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consists of a uniform lump-sum benefit to the ill.

<sup>8</sup>That is, there is no adverse selection. Our assumptions are generally designed to ensure that private insurance firms can provide insurance efficiently, thereby eliminating insurance market failure as a reason for government intervention.

subsequent stages are fully anticipated, so that equilibria of interest will be sub-game perfect.

To be more specific, we use as an example the case of health insurance, though the analysis would apply more generally to other types of personal risks faced by households. We consider two states of the world, denoted by 0 for good health and 1 for ill health. There are  $n$  types of individuals indexed by  $i = 1, \dots, n$ , each characterized by a wage rate and a risk characteristic. The wage rate for a type- $i$  person is exogenously given by  $w_i$ . In the absence of *ex ante* moral hazard, his or her exogenous probability of illness is  $\pi_i$ . Thus, all households with a given wage have the same probability of illness, which simplifies the analysis considerably. The proportion of households of type  $i$  is given by  $f_i$ , where  $\sum f_i = 1$ . With *ex ante* moral hazard, type  $i$  households can affect the probability of illness according to the function  $\pi_i(x)$ , where  $x$  is preventive spending which takes place before the state of health is revealed to the household. The function  $\pi_i(x)$  is decreasing in  $x$  with  $\pi_i(\infty) > 0$ .<sup>9</sup>

In the good state, health status is exogenously given as  $h^0$ . In the bad state, health status is  $h^1 = \bar{h} + m(z)$ , where  $z$  is curative expenditure on health improvement, and  $m'(z) > 0, m''(z) < 0$ . Expenditures  $z$  that are chosen by the household in case of *ex post* moral hazard are undertaken after the state of health is revealed to the household. In this case, we assume that  $h^1 = \bar{h} + m(z) < h^0$  for all values of  $z$  (i.e.  $m(\infty) < h^0 - \bar{h}$ ), so treatment cannot bring health status if ill to a level as high as health status if not ill (we depart from this assumption in Section 4). Notice that the parameters  $h^0$  and  $\bar{h}$ , as well as the function  $m(z)$ , are the same for all types of households. Only the probabilities of good health differ.

Households have identical state-independent utility functions:

$$u(c_i^j, h_i^j, \ell_i^j) \quad (1)$$

where  $c_i^j$  is consumption and  $\ell_i^j$  is labour supply of a type- $i$  household in state  $j$ . In some cases, we shall assume that utility takes the quasi-linear form:  $u(c_i^j + h_i^j - g(\ell_i^j))$ , where  $g(\ell_i^j)$  is increasing and strictly convex. In this case, labour supply depends only on the after-tax wage rate and  $z$  on

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<sup>9</sup>With *ex ante* moral hazard, we could as well assume that  $\pi_i(x)$  is the same for all households, as long as  $x$  is a normal good.



its out-of-pocket price: there are no income or cross-price effects. In particular, labour supply is then state-independent. With a more general utility function, labour could be higher in the bad state if the individual has to compensate for private health care spending or lower if ill health increases the disutility of labour.<sup>10</sup> Naturally, households maximize expected utility, weighted by the probabilities  $\pi_i$  for state 1 (ill health) and  $1 - \pi_i$  for state 0 (good health). Households take government policies and private insurance premiums as given. They choose  $x$  before the state of health is determined, and  $c, \ell, z$  after the state is determined.

Insurance firms are perfectly competitive. They offer insurance policies  $\{p_i, P_i\}$  to households of type  $i$ , where  $p_i$  is the proportion of health expenditures  $z_i$  covered (reimbursed) and  $P_i$  is the total premium. Insurance companies anticipate the effect of their insurance policies on curative expenditures  $z_i$  in the case of *ex post* moral hazard, and on preventive expenditures  $x_i$  in the case of *ex ante* moral hazard. Initially we ignore administrative costs, in which case competition entails that premiums are given by:

$$P_i = \pi_i(x_i)p_iz_i \quad i = 1, \dots, n \quad (2)$$

In a later section, we let there be a loading factor equal to  $k \geq 0$ . Then premiums for type- $i$  households are  $P_i = (1 + k)\pi_i(x_i)p_iz_i$ .

The government has two sorts of policy instruments—tax-transfer policies and social insurance. Except in the following section where the government can impose lump-sum taxes and transfers on households according to their types, tax-transfer policy consists of a linear progressive income tax with marginal tax rate of  $t$  and a lump-sum poll subsidy of  $a$  per household. Social insurance covers a proportion  $s$  of curative expenditures  $z_i$ , financed out of general tax revenues. Notice that the same rate of social insurance applies to all households. However, in the full information case considered in the next section, the government is able to offer a separate social insurance rate  $s_i$  to each household type. Denote total insurance coverage by  $\sigma_i = p_i + s$  (or  $p_i + s_i$  in the full information case).

As mentioned, there are three main stages of decision-making in this economy representing the sequence in which the decisions occur:

*Stage 1:* The government chooses its policies  $\{t, a, s\}$ . It cannot observe individual types or individual demands for goods, leisure or insurance, but

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<sup>10</sup>A natural extension of this modeling would be to have the labor supply falling to 0 in the bad state of nature.

can observe incomes. It knows preferences and the distribution of individuals by type  $i$ . The government anticipates the effect of its policies both on the insurance market and subsequently on households.

*Stage 2:* The competitive insurance industry sells private insurance to households. Market equilibrium (competition for customers, with zero profits) determines  $p_i$  and  $P_i$ . The insurance industry is assumed to be able to observe household risk types, so there is no adverse selection problem. Thus, insurance firms are better informed than the government since they can observe  $\pi_i$ . In this stage,  $\{t, a, s\}$  are taken as given, and household behaviour is correctly anticipated.

*Stage 3:* Households select  $\{x_i, c_i^1, \ell_i^1, z_i, c_i^0, \ell_i^0\}$ . Preventive expenditures  $x_i$  are chosen before the state of health is revealed. All other variables are state-specific since they are chosen after the state is revealed ( $z_i$  is chosen only in the bad state). Households take  $\{t, a, s, p_i, P_i\}$  as given from the previous two stages.

The equilibrium is assumed to be sub-game perfect, so we proceed to solve it by backward induction. The method of solution can best be illustrated by considering as a benchmark the full information case.

### 3 The full information benchmark

In this benchmark, the government can observe individual types  $i$ , so all policies can be type-specific. The government gives a lump-sum transfer of  $a_i$  to households of type  $i$ , as well as an individualized social insurance coverage rate of  $s_i$ .<sup>11</sup> Total coverage is then  $\sigma_i = s_i + p_i$ . We begin by analyzing household choice and proceed backwards to earlier stages.

#### Stage 3: Household choice

Households of type  $i$  face the following budget constraints in the bad and good states:

$$c_i^1 = w_i \ell_i^1 + a_i - x_i - (1 - \sigma_i) z_i^1 - P_i, \quad c_i^0 = w_i \ell_i^0 + a_i - x_i - P_i,$$

where the household can choose  $\ell$  and  $z$  after the state has been revealed. Given that no  $z$  will be chosen in the good state, the problem for household

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<sup>11</sup>An extension here and elsewhere in the paper could involve imposing a subsidy on preventive expenditures as well.

$i$  is:

$$\begin{aligned} \text{Max}_{\{x_i, \ell_i^j, z_i\}} \quad & \pi_i(x_i)u(w_i\ell_i^1 + a_i - x_i - (1 - \sigma_i)z_i^1 - P_i, \bar{h} + m(z_i), \ell_i^1) \\ & + (1 - \pi_i(x_i))u(w_i\ell_i^0 + a_i - x_i - P_i, h^0, \ell_i^0). \end{aligned}$$

The first-order conditions are (using self-evident notation for partial derivatives):

$$\ell_i^j : \quad w_i u_{c_j}^i + u_{\ell_j}^i = 0, \quad j = 0, 1$$

$$z_i : \quad -(1 - \sigma_i)u_{c1}^i + m'(z_i)u_{h1}^i = 0,$$

$$x_i : \quad \pi'_i(x_i)(u^i(1) - u^i(0)) - \pi_i(x_i)u_{c0}^i - (1 - \pi_i(x_i))u_{c1}^i = 0,$$

where  $u^i(j)$  is the utility level achieved in state  $j = 0, 1$ . The solution to this problem yields  $\ell_i^0(a_i - P_i)$ ,  $\ell_i^1(a_i - P_i, s_i + p_i)$ ,  $z_i(a_i - P_i, s_i + p_i)$ ,  $x_i(a_i - P_i, s_i + p_i)$ , and the indirect utility function  $v_i(a_i - P_i, s_i + p_i)$ . Applying the envelope theorem gives:

$$v_a^i = -v_P^i = \pi_i u_{c1}^i + (1 - \pi_i)u_{c0}^i = E[u_c^i], \quad v_s^i = v_p^i = \pi_i z_i u_{c1}^i \quad (3)$$

## Stage 2: Insurance market equilibrium

Insurance firms are perfectly competitive and compete in insurance policies. Firms take as given the policies offered by other firms and the level of utility that households can achieve by those policies. Each firm then offers households of type  $i$  a combination  $\{p_i, P_i\}$  to maximize profits, given the utility level achieved elsewhere and anticipating household behaviour in Stage 3. Thus the problem of a representative insurance firm with respect to each type  $i$  can be written as:

$$\text{Max}_{\{P_i, p_i\}} \quad P_i - \pi_i(x_i(a_i - P_i, s_i + p_i))p_i z_i(a_i - P_i, s_i + p_i)$$

$$\text{s.t.} \quad v_i(a_i - P_i, s_i + p_i) \geq \bar{v}^i$$

where  $\bar{v}^i$  is given by the industry as a whole. In market equilibrium all firms behave identically and profits are driven to zero by free entry. In effect, the

industry-wide utility level  $\bar{v}^i$  is competed up until profits equal zero. Thus, equilibrium in the insurance industry can be characterized as the solution to the following problem using the zero-profit condition, which is dual to the individual firm's problem:

$$\begin{aligned} \text{Max}_{\{P_i, p_i\}} \quad & v_i(a_i - P_i, s_i + p_i) \\ \text{s.t.} \quad & P_i = \pi_i(x_i(a_i - P_i, s_i + p_i))p_i z_i(a_i - P_i, s_i + p_i) \end{aligned}$$

The Lagrangean expression is:

$$\mathcal{L} = v_i(a_i - P_i, s_i + p_i) + \lambda_i[P_i - \pi_i(x_i(a_i - P_i, s_i + p_i))p_i z_i(a_i - P_i, s_i + p_i)].$$

The first-order conditions for this problem are:

$$\begin{aligned} P_i : \quad & v_P^i + \lambda_i[1 - \pi_i p_i z_P^i - p_i z_i \pi_i' x_P^i] = 0, \\ p_i : \quad & v_p^i - \lambda_i[\pi_i z_i + \pi_i p_i z_p^i + p_i z_i \pi_i' x_p^i] = 0, \end{aligned}$$

where  $v_P^i$  and  $v_p^i$  are given by (3) in anticipation of Stage 3. The solution to this problem gives  $P_i(a_i, s_i)$  and  $p_i(a_i, s_i)$ , and the value function is defined as  $V_i(a_i, s_i)$ . Note that because of the moral hazard problem,  $p_i < 1$ , since  $p_i = 1$  leads to  $z_i$  being indefinitely high. Also, as long as  $s_i < 1$ ,  $p_i > 0$  generally. Indeed at  $p_i = 0$ ,  $d\mathcal{L}/dp_i = \pi_i z_i(u_{c_1}^i - Eu_c^i)$  and it is plausible to assume that  $u_{c_1}^i > u_{c_0}^i$  since one can expect that  $c_i^1 < c_i^0$ . It is noteworthy that if  $s_i < 1$ , inequality  $p_i > 0$  always holds with a quasi-linear specification of the utility function:  $u(c + h - g(\ell))$ . Again applying the envelope theorem to this problem, we obtain:

$$\begin{aligned} V_a^i &= v_a^i - \lambda_i[\pi_i p_i z_a^i + p_i z_i \pi_i' x_a^i] = \lambda_i, \\ V_s^i &= v_s^i - \lambda_i[\pi_i p_i z_s^i + p_i z_i \pi_i' x_s^i] = \lambda_i \pi_i z_i, \end{aligned} \tag{4}$$

where we have used (3) and the first-order conditions for  $P_i$  and  $p_i$ .

### Stage 1: Government policy

The government chooses lump-sum taxes  $a_i$  and public insurance  $s_i$  to maximize the sum of utilities subject to its budget constraint, anticipating the outcomes of the subsequent two stages. Thus, its objective function is  $\sum f_i V_i(a_i, s_i)$ , and its budget constraint is  $\sum f_i \{a_i + s_i \pi_i(x_i(a_i - P_i(a_i, s_i), s_i + p_i(a_i, s_i))) z_i(a_i - P_i(a_i, s_i), s_i + p_i(a_i, s_i))\} = 0$ . The Lagrangean expression is:

$$\mathcal{L} = \sum_{i=1}^n f_i V_i(a_i, s_i) - \gamma \sum_{i=1}^n f_i \{a_i + s_i \pi_i(x_i(a_i - P_i(a_i, s_i), s_i + p_i(a_i, s_i))) z_i(a_i - P_i(a_i, s_i), s_i + p_i(a_i, s_i))\}.$$

The first-order conditions are, using the envelope results (4) from Stage 2:

$$a_i : \lambda_i - \gamma [1 + s_i \pi_i dz_i/da_i + s_i z_i \pi'_i dx_i/da_i] = 0,$$

$$s_i : \lambda_i \pi_i z_i - \gamma [\pi_i z_i + s_i \pi_i dz_i/ds_i + s_i z_i \pi'_i dx_i/ds_i] = 0.$$

Note that the total effects of  $a_i$  and  $s_i$  on  $z_i$  and  $x_i$  take into account the effect that government policies will have on private insurance coverage and premiums. Combining these two conditions, we obtain:

$$\gamma s_i \left[ \pi_i \left( \frac{dz_i}{ds_i} - \pi_i z_i \frac{dz_i}{da_i} \right) + \pi'_i z_i \left( \frac{dx_i}{ds_i} - \pi_i z_i \frac{dx_i}{da_i} \right) \right] = 0.$$

Therefore,  $s_i = 0$ : *There is no role for public insurance in the full-information benchmark.*<sup>12</sup> That also means there would be no role for social insurance  $s$  that would not discriminate among households of different types.<sup>13</sup> Blomqvist and Horn (1984) reach the same conclusion in a framework similar to ours where there is however no moral hazard.<sup>14</sup> Also from the first-order

<sup>12</sup>Social insurance is not needed but cannot be excluded. Indeed what matters in the present setting is total coverage  $\sigma_i$  which can result from any combination of  $s_i$  and  $p_i$ . Any imposition of  $s_i$  would be offset by a reduction in  $p_i$ . Of course, lump-sum subsidy  $a_i$  must be adjusted according to each combination.

<sup>13</sup>The result that  $s_i = 0$  runs counter to a standard result in the insurance literature that there will be market failure under *ex ante* moral hazard, though not necessarily under *ex post*. See, for example, Pauly (1974) and Marshall (1976). See also Gaynor et al (2000) who show that imperfect competition does not alleviate the moral hazard problem.

<sup>14</sup>See Proposition 2 in their paper.

condition on  $a_i$ ,  $s_i = 0$  implies  $\lambda_i = \gamma$  for all households. Therefore, from the first-order conditions on  $P_i$  from Stage 2, and using (3), we obtain:

$$E[u_c^i] = \gamma [1 + \pi_i p_i z_a^i + p_i z_i \pi'_i x_a^i].$$

Thus, the government does not equalize expected utilities in the full information case because of the moral hazard problem.

We turn now to the case where the government is imperfectly informed, and is restricted to pursuing its redistribute objectives using a linear progressive tax.

## 4 The case without moral hazard

It is useful also to consider the case where there is no moral hazard of either type. Assume for simplicity that, unlike in the previous case, there is only one value of curative expenditures  $\hat{z}$  and that it fully restores health status in the ill health state. That is,  $\hat{z}$  is such that  $h^1 = \bar{h} + m(\hat{z}) = h^0$ . Assume also that  $\pi_i$  is exogenously fixed for all  $i$ . There is a private insurance market that offers households coverage  $p_i$  and charges a premium  $P_i$  adjusted to their illness probability, so that  $P_i = \pi_i p_i \hat{z}$ .

Suppose first that there is public insurance that covers a proportion  $s$  of expenditures  $\hat{z}$ , and that households can purchase a private insurance freely. We omit explicit consideration of the insurance industry here because the absence of moral hazard makes the solution of the Stage 2 problem straightforward. It is clear that a competitive insurance industry would replicate the extent of coverage most preferred by each type of household. With a linear income tax, we can now write the expected utility of each household of type  $i$  as:

$$\begin{aligned} U^i &= \pi_i u((1-t)w_i \ell_i^1 + a - \pi_i p_i \hat{z} - (1-s-p_i)\hat{z}, h^1, \ell_i^1) \\ &\quad + (1-\pi_i)u((1-t)w_i \ell_i^0 + a - \pi_i p_i \hat{z}, h^0, \ell_i^0). \end{aligned}$$

Focusing first on the choice of  $p_i$ , we obtain by differentiating  $U^i$ :

$$\frac{\partial U^i}{\partial p_i} = \pi_i \hat{z} (u_{c1}^i - E[u_c^i]).$$

As long as there is less than full insurance, we have  $u_{c1}^i > E[u_c^i]$ . This implies that  $\partial U^i / \partial p_i > 0$  for any value of  $p_i < 1 - s$ . Therefore, as is well-known, in the absence of moral hazard it is optimal for all households to choose full insurance coverage,  $p_i = 1 - s$  for any  $s$ . In addition to their private insurance coverage, households choose their labour supplies conditional on the two health states:  $\ell_i^j(t, a, s)$ ,  $j = 0, 1$ . If there is full insurance, consumption and labour supply are identical in the two states of nature. However to keep the analysis as general as possible we continue to distinguish the two states of nature.

Let us now look at the optimal behaviour of the public sector. It will implement a linear progressive tax for redistributive reasons. The question is whether it will also want to intervene in insurance markets. If it does, we know that whatever the value of  $s$ , households choose their private insurance coverage so that their health expenditures are fully reimbursed ( $p_i = 1 - s$ ).

Given the tax parameters  $t$  and  $a$  and social insurance coverage  $s$ , the government revenue constraint is simply:

$$\sum f_i \{ \pi_i t w_i \ell_i^1(t, a, s) + (1 - \pi_i) t w_i \ell_i^0(t, a, s) - \pi_i s \hat{z} - a \} = 0. \quad (5)$$

The Lagrangean expression for the government's problem can then be written:

$$\begin{aligned} \mathcal{L} = & \sum f_i \{ \pi_i u(w_i(1-t)\ell_i^1(\cdot) + a - \pi_i p_i(\cdot)\hat{z} - (1-s-p_i(\cdot))\hat{z}, h^1, \ell_i^1(\cdot)) \\ & + (1 - \pi_i) u(w_i(1-t)\ell_i^0(\cdot) + a - \pi_i p_i(\cdot)\hat{z}, h^0, \ell_i^0(\cdot)) \\ & + \gamma [\pi_i t w_i \ell_i^1(\cdot) + (1 - \pi_i) t w_i \ell_i^0(\cdot) - a - \pi_i s \hat{z}] \}, \end{aligned}$$

where  $\gamma$  is the Lagrange multiplier associated with the revenue constraint and  $(\cdot) = (t, a, s)$ .

Using the envelope theorem, the first-order conditions can be written:

$$\frac{\partial \mathcal{L}}{\partial s} = \sum f_i \left\{ \pi_i \hat{z} u_{c1}^i + \gamma \pi_i t w_i \frac{\partial \ell_i^1}{\partial s} \right\} - \gamma \bar{\pi} \hat{z} = 0, \quad (6)$$

$$\frac{\partial \mathcal{L}}{\partial a} = \sum f_i \left\{ E[u_c^i] + \gamma \left( t w_i E \left[ \frac{\partial \ell_i}{\partial a} \right] - 1 \right) \right\} = 0, \quad (7)$$

$$\frac{\partial \mathcal{L}}{\partial t} = \sum f_i \left\{ -E[w_i \ell_i u_c^i] + \gamma \left( t w_i E \left[ \frac{\partial \ell_i}{\partial t} \right] + w_i E[\ell_i] \right) \right\} = 0, \quad (8)$$

where  $\bar{\pi} = \sum f_i \pi_i$ . We can rewrite (7) as follows:

$$\bar{b} \equiv \sum f_i E[b_i] = 1$$

where

$$b_i^j = \frac{u_{c_j}^i}{\gamma} + t w_i \frac{\partial \ell_i^j}{\partial a}$$

is the so-called *net marginal social utility of income* of type  $i$  households in state of the world  $j$ . Note that in case of complete insurance,  $b_i^1 = b_i^0$  (since health status, consumption and labour supply are identical in the two states) and  $\bar{b}^1 = \bar{b} = 1$ . Using these definitions and subtracting (7) multiplied by  $\bar{\pi}\hat{z}$  from (6) yields<sup>15</sup>:

$$\frac{\partial \mathcal{L}}{\partial s} \frac{1}{\gamma \hat{z}} = \text{cov}[b_i^1, \pi_i]. \quad (9)$$

Whether it is optimal to have some public coverage of health expenditures therefore depends upon the sign of  $\text{cov}[b_i^1, \pi_i]$ . Suppose that  $\pi_i$  and  $w_i$  are negatively correlated. Then,  $\pi_i$  and  $b_i^1$  are positively correlated, and as a consequence it is desirable to push  $s$  up to its ceiling value, namely unity.<sup>16</sup> When  $s = 1$ , there is no need for private insurance. This result, which is the polar opposite of the full information case, is that obtained by Rochet (1989) and Cremer and Pestieau (1996).

Even though this is not our main concern, one can also derive the optimal tax formula from (7) and (8):

$$t = \frac{-\text{cov}[E[b_i], E[w_i \ell_i]]}{\sum f_i w_i E[\partial \ell_i / \partial \omega_i]}, \quad (10)$$

where  $\omega_i = w_i(1 - t)$ , and  $\partial \ell_i^j / \partial \omega_i$  is the compensated derivative of labour supply of a type- $i$  household in state  $j$ . This expression is standard, with the numerator being the equity term and the denominator the efficiency term. Note that since health expenditures are fully reimbursed, (10) could be simplified by dropping the expected value operator.

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<sup>15</sup>Given that  $s + p_i = 1$ ,  $\ell_i = \ell_i(t, a, 1)$  and  $\frac{\partial \ell_i}{\partial s} = 0$ .

<sup>16</sup>It is worth noting that even with identical  $w_i$  social insurance is desirable as the only way from “good” to “bad” risks.



The weakness of the above analysis is that it implicitly assumes that both social and private insurance have no influence on the size of the loss  $z$  to be compensated, nor on the probability  $\pi$  of loss. We now turn to these two possibilities. Now, the amount of the loss that can be recouped depends on each agent's behaviour, and the probability of the loss is also the responsibility of each agent. With these two additions, we will see that full insurance is no longer desirable. It is useful to treat the two sorts of moral hazard separately.

## 5 *Ex post* moral hazard

Here we assume that the  $\pi_i$ 's are given (and either negatively or positively correlated with wages  $w_i$ ), but that individuals can influence their health status following an illness. By investing in curative expenditures  $z$ , they can reach a health status  $h^1 = \bar{h} + m(z) < h^0$ . A proportion of expenditures on health improvement is covered by social insurance,  $s$ , and private insurance,  $p_i$ . As before, we solve for the sub-game perfect equilibrium by backward induction.

### Stage 3: Household choice

Households of type  $i$  take as given government policies  $a$ ,  $t$  and  $s$ , and private insurance policy parameters  $p_i$  and  $P_i$ . The budget constraints in the two states of health are now:

$$\begin{aligned} c_i^1 &= (1-t)w_i\ell_i^1 + a - (1-\sigma_i)z_i - P_i, \\ c_i^0 &= (1-t)w_i\ell_i^0 + a - P_i. \end{aligned}$$

The problem for a type- $i$  household is:

$$\begin{aligned} \text{Max}_{\{\ell_i^j, z_i\}} \quad & \pi_i u((1-t)w_i\ell_i^1 + a - (1-\sigma_i)z_i - P_i, \bar{h} + m(z_i), \ell_i^1) \\ & + (1-\pi_i)u((1-t)w_i\ell_i^0 + a - P_i, h^0, \ell_i^0). \end{aligned}$$

The first-order conditions are:

$$\begin{aligned} \ell_i^j : \quad & (1-t)w_i u_{\ell_j}^i + u_{\ell_j}^i = 0, \quad j = 0, 1 \\ z_i : \quad & -(1-\sigma_i)u_{c1}^i + m'(z_i)u_{h1}^i = 0. \end{aligned}$$

The solution to this problem yields  $\ell_i^0(t, a - P_i)$ ,  $\ell_i^1(t, a - P_i, s + p_i)$ ,  $z_i(t, a - P_i, s + p_i)$ , and the indirect utility function  $v_i(t, a - P_i, s + p_i)$ . Applying the envelope theorem gives:

$$v_t^i = -E[w_i \ell_i^i u_c^i], \quad v_a^i = -v_P^i = E[u_c^i], \quad v_s^i = v_p^i = \pi_i z_i u_{c1}^i \quad (11)$$

## Stage 2: Insurance market equilibrium

As before, insurance industry equilibrium can be characterized as the outcome from choosing private coverage  $p_i$  and premiums  $P_i$  to maximize household expected utility by type subject to a type-specific zero profit (or actuarial fairness) condition, and anticipating the consequences for Stage 3. The insurance equilibrium for a type- $i$  household is the solution to the maximization of the following Lagrangean:

$$\mathcal{L} = v_i(t, a - P_i, s + p_i) + \lambda_i [P_i - \pi_i p_i z_i(t, a - P_i, s + p_i)].$$

The first-order conditions for this problem are:

$$P_i : \quad v_P^i + \lambda_i [1 - \pi_i p_i z_P^i] = 0,$$

$$p_i : \quad v_p^i - \lambda_i [\pi_i z_i + \pi_i p_i z_p^i] = 0,$$

where  $v_P^i = -E[u_c^i]$  and  $v_p^i = \pi_i z_i u_{c1}^i$  from (11). The solution to this problem gives  $P_i(t, a, s)$  and  $p_i(t, a, s)$ . As already mentioned in Section 3, it is plausible that  $p_i > 0$  as long as  $s < 1$ . The maximum value function for this problem is defined as  $V_i(t, a, s)$ . By the envelope theorem, we obtain its properties:

$$V_t^i = -E[w_i \ell_i^i u_c^i] - \lambda_i \pi_i p_i z_t^i, \quad V_a^i = \lambda_i, \quad V_s^i = \lambda_i \pi_i z_i, \quad (12)$$

where we have used the first-order conditions on  $P_i$  and  $p_i$  as well as (11).

## Stage 1: Government policy

The government chooses the linear tax parameters  $t$ ,  $a$  and the level of social insurance  $s$  to maximize the sum of utilities subject to its budget constraint, anticipating the outcomes of the subsequent stages. The Lagrangean expression is:

$$\begin{aligned} \mathcal{L} = & \sum f_i V_i(t, a, s) + \gamma \sum \{ t w_i [\pi_i \ell_i^1(t, a - P_i(\cdot), s + p_i(\cdot)) \\ & + (1 - \pi_i) \ell_i^0(t, a - P_i(\cdot))] - a - s \pi_i z_i(t, a - P_i(\cdot), s + p_i(\cdot)) \} \end{aligned}$$

where  $P_i(t, a, s)$  and  $p_i(t, a, s)$  are determined in Stage 2.

The first-order conditions are:

$$t : \sum f_i V_t^i + \gamma \sum f_i \{w_i E[\ell_i] + tw_i E[d\ell_i/dt] - s\pi_i dz_i/dt\} = 0,$$

$$a : \sum f_i V_a^i + \gamma \sum f_i \{-1 + tw_i E[d\ell_i/da] - s\pi_i dz_i/da\} = 0,$$

$$s : \sum f_i V_s^i + \gamma \sum f_i \{-\pi_i z_i + tw_i E[d\ell_i/ds] - s\pi_i dz_i/ds\} = 0.$$

Using (11), these can be rewritten as:

$$\sum f_i \left\{ \frac{E[w_i \ell_i u_c^i] + \lambda_i \pi_i p_i z_i^i}{\gamma} - w_i E[\ell_i] - tw_i E\left[\frac{d\ell_i}{dt}\right] + s\pi_i \frac{dz_i}{dt} \right\} = 0, \quad (13)$$

$$\sum f_i \left\{ \frac{\lambda_i}{\gamma} - 1 + tw_i E\left[\frac{d\ell_i}{da}\right] - s\pi_i \frac{dz_i}{da} \right\} = 0, \quad (14)$$

$$\sum f_i \left\{ \frac{\lambda_i}{\gamma} \pi_i z_i - \pi_i z_i + tw_i E\left[\frac{d\ell_i}{ds}\right] - s\pi_i \frac{dz_i}{ds} \right\} = 0. \quad (15)$$

From (14), we obtain:

$$\sum f_i E[b_i] = \bar{b} = 1,$$

where  $b_i^j = \lambda_i/\gamma + tw_i[d\ell_i^j/da] - s\pi_i[dz_i/da]$  is the net marginal social utility of income for a type- $i$  person in state of the world  $j$ . Next, combining (13) and (15), we obtain:

$$\begin{aligned} & E[b_i \pi_i z_i] - \bar{b} E[\pi_i z_i] - s \sum f_i \pi_i \left[ \frac{dz_i}{ds} - \pi_i z_i \frac{dz_i}{da} \right] \\ & + t \sum f_i w_i E \left[ \frac{d\ell_i}{ds} - \pi_i z_i \frac{d\ell_i}{da} \right] = 0 \end{aligned}$$

or

$$s = \frac{\text{cov}[E[b_i], \pi_i z_i]}{\sum f_i \pi_i d\tilde{z}_i/ds} + \frac{t \sum f_i w_i E[d\tilde{\ell}_i/ds]}{\sum f_i \pi_i d\tilde{z}_i/ds}, \quad (16)$$

where  $d\tilde{z}_i/ds = dz_i/ds - \pi_i z_i dz_i/da$  is a compensated total demand derivative, and similarly for  $d\tilde{\ell}_i/ds$ . These are total demand derivatives since, for

example,  $dz_i/ds$  is the total derivative of  $z_i$  with respect to  $s$ , meaning that  $z_i$  is a function not only of  $s$  but also of  $P_i$  and  $p_i$ , which are in turn functions of  $s$ . Indeed an increase in  $s$  causes  $p_i$  and  $P_i$  to fall, which is accounted for in these total derivatives.

Equation (16) consists of two terms. The first term is analogous to the standard expression for the optimal marginal tax rate as in (10) above. The denominator is the efficiency term and gives the compensated effect of an increase in  $s$  (financed by a state-invariant lump-sum tax) on curative expenditures. We expect this term to be positive, though it is not necessarily the case. The larger it is, the smaller is the value of  $s$ . The numerator—the covariance between the marginal social utility of income,  $b_i$ , and  $\pi_i z_i$ —is the equity term. If, as in Rochet (1989), the covariance between  $b_i$  and  $\pi_i$  is positive, we still have to verify whether taking  $\pi_i z_i$  instead of  $\pi_i$  changes the sign. If we assume that  $\pi_i$  and  $w_i$  are ‘sufficiently’ negatively correlated and that  $z_i$  does not increase much with  $w_i$ , then the covariance term is positive.

The second term is related to a second-best effect. Changes in  $s$  induce indirect changes in the deadweight loss due to the distortion imposed by the marginal tax rate  $t$ . If  $\ell$  increases with  $s$ , an increase in  $s$  will indirectly increase tax revenues. Since the social value of an additional unit of tax revenues is greater than one, this would enhance the case for social insurance.<sup>17</sup>

In general, it is difficult to say whether equation (16) yields  $s$  greater or less than zero,<sup>18</sup> despite the fact that in the absence of moral hazard we earlier obtained  $s = 1$ . More precise results can be obtained only by using specific functional forms. Consider, for example, the quasi-linear case introduced earlier,  $u(c_i^j + h_i^j - g(\ell_i^j))$ . In this case, labour supply is independent of  $s$ . Moreover,  $z_i$  depends only on  $\sigma_i = s + p_i$ : more precisely,  $dz_i/d\sigma_i = -1/m''(z_i) > 0$ . Thus, (16) reduces to:

$$s = \frac{\text{cov}[E[b_i], \pi_i z_i]}{\sum f_i \pi_i d\tilde{z}_i/ds} = \frac{\text{cov}[E[b_i], \pi_i z_i]}{-\sum f_i \pi_i (1 + \partial p_i/\partial s - \pi_i z_i \partial p_i/\partial a)/m''(z_i)} \quad (17)$$

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<sup>17</sup>By the same token, an expression for the optimal tax rate  $t$  would include an interaction effect of the tax rate on curative expenditures. This would be analogous to the results of Arnott and Stiglitz (1986) who argued that an indirect way for government policy to address the moral hazard problem would be to tax commodities that are complementary with the moral hazard activity.

<sup>18</sup>A negative  $s$  could be interpreted as a tax on curative spending, which is an imperfect way of taxing private insurance premiums.

In the Appendix, we show that for this quasi-linear utility function,  $\partial p_i / \partial s - \pi_i z_i \partial p_i / \partial a > -1$ , so that the denominator of (17) is positive. Then, assuming that the correlation between  $\pi_i$  and  $w_i$  is negative enough for the covariance term to be positive, the optimal value of  $s$  will be positive. However, unlike in Rochet (1989),  $s < 1$  because of moral hazard. This can be seen from the first-order condition on  $s$  above. As  $s$  approaches 1,  $z_i$  goes to  $\infty$ , so that  $\partial \mathcal{L} / \partial s$  becomes negative when  $s$  approaches 1.

## 6 *Ex ante* moral hazard

*Ex ante* moral hazard involves preventive expenditures that can affect the probability of illness. For simplicity, we assume that, as in Section 4, curative expenditures are fixed at the level  $\hat{z}$ , and thus the good health status  $h^0$  is also attained in the ill health state. We can thus exclude the health status variable from the utility function. We proceed as usual by backward induction, looking first at the household's choices, then at private insurance market equilibrium, and finally at the government's optimization.

### Stage 3: Household choice

Given government policies  $a$ ,  $t$  and  $s$  and private insurance policies  $p_i$  and  $P_i$ , each type- $i$  household solves the following problem:

$$\begin{aligned} \text{Max}_{\{x_i, \ell_i^j\}} \quad & \pi_i(x_i) u((1-t)w_i \ell_i^1 + a - x_i - (1-\sigma_i)\hat{z} - P_i, \ell_i^1) \\ & + (1 - \pi_i(x_i)) u((1-t)w_i \ell_i^0 + a - x_i - P_i, \ell_i^0) \end{aligned}$$

The first-order conditions are assumed to be interior and are given by:

$$\begin{aligned} \ell_i^j : \quad & (1-t)w_i u_{\ell_j^i}^i + u_{\ell_j^i}^i = 0 \quad j = 0, 1 \\ x_i : \quad & \pi_i'(x_i)(u^i(1) - u^i(0)) - E[u_c^i] = 0 \end{aligned}$$

where  $u^i(j)$  is the utility level achieved in state  $j = 0, 1$ . This yields supply functions  $\ell_i^0(t, a - P_i)$ ,  $\ell_i^1(t, a - P_i, s + p_i)$ ,  $x_i(t, a - P_i, s + p_i)$ , and the indirect

utility function  $v_i(t, a - P_i, s + p_i)$ . Applying the envelope theorem gives Roy's identities:

$$v_t^i = -E[w_i \ell_i u_c^i], \quad v_a^i = -v_P^i = E[u_c^i], \quad v_s^i = v_p^i = \pi_i(x_i) \hat{z} u_{c1}^i \quad (18)$$

## Stage 2: Insurance market equilibrium

In equilibrium, insurance policies offered by the private sector to type- $i$  households maximize:

$$\mathcal{L} = v_i(t, a - P_i, s + p_i) + \lambda_i [P_i - \pi_i(x_i(t, a - P_i, s + p_i)) p_i \hat{z}]$$

The first-order conditions for this problem are:

$$P_i : v_P^i + \lambda_i [1 - p_i \hat{z} \pi_i'(x_i) x_P^i] = 0$$

$$p_i : v_p^i - \lambda_i [\pi_i(x_i) \hat{z} + p_i \hat{z} \pi_i'(x_i) x_p^i] = 0$$

where  $v_P^i = -E[u_c^i]$  and  $v_p^i = \pi_i(x_i) \hat{z} u_{c1}^i$  from (18). The solution to this problem gives  $P_i(t, a, s)$  and  $p_i(t, a, s)$ , where as before it is plausible that  $p_i > 0$  as long as  $s < 1$ . The maximum value function is  $V_i(t, a, s)$ . By the envelope theorem and using (18), we obtain its properties:

$$V_t^i = -E[w_i \ell_i u_c^i] - \lambda_i p_i \hat{z} \pi_i'(x_i) x_t^i, \quad V_a^i = \lambda_i, \quad V_s^i = \lambda_i \pi_i(x_i) \hat{z} \quad (19)$$

## Stage 1: Government policy

As usual, the government chooses  $\{t, a, s\}$  to maximize a utilitarian social welfare function subject to its budget constraint and the reaction functions of the private sector. The Lagrangean expression is:

$$\mathcal{L} = \sum_{i=1}^n f_i V_i(t, a, s) + \gamma \sum_{i=1}^n f_i \{t w_i [\pi_i(x_i(\cdot)) \ell_i^1(\cdot) + (1 - \pi_i(\cdot)) \ell_i^0(\cdot)] - a - s \pi_i(x_i(\cdot)) \hat{z}\}$$

where  $(\cdot) = (t, a - P_i(t, a, s), s + p_i(t, a, s))$  in which  $P_i(t, a, s)$  and  $p_i(t, a, s)$  are determined in Stage 2. Proceeding exactly as before, rearrangement of the first-order conditions yields the analogue of (16):

$$s = \frac{\text{cov}[E[b_i], \pi_i(x_i) \hat{z}]}{\sum f_i \hat{z} \pi_i'(x_i) d \tilde{x}_i / ds} + \frac{t \sum f_i w_i E[d \tilde{\ell}_i / ds]}{\sum f_i \hat{z} \pi_i'(x_i) d \tilde{x}_i / ds} \quad (20)$$

where  $b_i^j = \lambda_i/\gamma + tw_i[d\ell_i^j/da] - s\hat{z}\pi'_i[dx_i/da]$ . Note that all indirect effects on the government budget of changing  $a$  are accounted for, including those through induced changes in the probability  $\pi_i$ .

The interpretation of the terms in this expression is identical to (16) for the *ex post* moral hazard case. The denominator—the efficiency term involving the effect of social insurance on preventive expenditures—is expected to be positive, since  $\pi'_i < 0$  and we expect that a compensated increase in  $s$  will reduce preventive expenditures. The equity effect, reflected in the covariance term, is positive provided that  $x_i$  is normal, given that  $\pi_i(x_i)$  decreases as  $w_i$  increases and that  $\pi_i$  is not positively correlated with  $w_i$ . The second term reflects the indirect, or second-best, effect. Its sign depends upon how labour supplies are affected by changes in the social insurance rate  $s$ .

In general, it is difficult to sign  $s$  from equation (20), especially given the fact that the derivatives of  $\ell_i$  and  $x_i$  are total ones.<sup>19</sup> It is instructive again to consider as a special case that of quasi-linear preferences, which are here given simply by  $u(c_i^j - g(\ell_i^j))$  since health status is the same in both states. In this case, (20) reduces to:

$$s = \frac{\text{cov}[E[b_i], \pi_i(x_i)\hat{z}]}{\sum f_i \hat{z} \pi'_i(x_i) d\tilde{x}_i/ds}$$

It can be shown by a comparative static analysis of household and insurance industry behaviour that a sufficient condition for  $s > 0$  is that private insurance is a normal good.

## 7 Administrative costs

As mentioned, it has been documented that there are administrative costs of operating a competitive insurance industry that may be avoided by a single-payer government system. Administrative costs effectively increase the cost of private insurance relative to a public scheme. There are two consequences of this. First, and most obviously, the attractiveness of social insurance is enhanced relative to private insurance, despite the informational disadvantages the public sector might face. Second, from the fact that the cost of private insurance is not actuarially fair, it is no longer the case that

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<sup>19</sup>It is, however, possible to show that the solution will generally be an interior one. It is impossible that  $s = 1$  as this would induce  $x_i$  to go to  $\infty$ . On the other hand, it is possible that  $s = 0$ , though this would be by chance only.

all households will necessarily purchase private insurance. To illustrate this, we employ the case of *ex post* moral hazard. The model is the same as in Section 5 above except for the administrative costs associated with private insurance. In particular, we assume that there is a loading factor equal to a proportion  $k > 0$  of insurance premiums.<sup>20</sup> The no-profit condition then becomes  $P_i = (1 + k)\pi_i p_i z_i$ .

The same three stages of decision-making apply. Household behaviour in Stage 3 is essentially the same as before, with Roy's identities from (11) applying. In Stage 2, the insurance market equilibrium for type- $i$  households solves:

$$\max_{\{P_i, p_i\}} v_i(t, a - P_i, s + p_i) \quad \text{s.t.} \quad P_i = (1 + k)\pi_i p_i z_i(t, a - P_i, s + p_i)$$

We can no longer be sure that there will be an interior solution for  $p_i$ , even if  $s = 0$ . That is, a non-negative constraint on coverage,  $p_i \geq 0$ , may be binding. Given that, the first-order conditions for this problem are:

$$P_i : v_P^i + \lambda_i[1 - (1 + k)\pi_i p_i z_P^i] = 0$$

$$p_i : v_p^i - \lambda_i(1 + k)[\pi_i z_i + \pi_i p_i z_p^i] \leq 0$$

where the inequality holds if the constraint  $p_i \geq 0$  is binding. We might expect that higher wage groups will demand greater coverage. In fact, a comparative static analysis on these first-order conditions reveals that, in general, it is not clear on which income groups the inequality constraint is binding. That is,  $\partial p_i / \partial w_i \lesseqgtr 0$ . It turns out that, as in the Appendix, a sufficient condition for  $\partial p_i / \partial w_i > 0$  is the familiar one that private insurance is a normal good. In any case, the solution to this problem gives  $P_i(t, a, s)$  and  $p_i(t, a, s)$ , along with the value function  $V_i(t, a, s)$ . By the envelope theorem, we obtain in the usual way:

$$V_t^i = -E[w_i \ell_i u_c^i] - \lambda_i(1 + k)\pi_i p_i z_t^i, \quad V_a^i = \lambda_i, \quad V_s^i \leq (1 + k)\lambda_i \pi_i z_i \quad (21)$$

where the inequality holds when  $p_i \geq 0$  is binding.

Let  $I_0$  be the set of household types  $i$  such that the constraint  $p_i \geq 0$  is binding, that is, the set that purchases no private insurance. Then

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<sup>20</sup>We are assuming that  $k > 0$ . Positive loading factors in private health insurance are well documented in the literature. See for instance Phelps (1992), ch. 10. In some sectors private insurance might be less costly than social insurance. The results should then be modified accordingly.



government optimization yields a set of conditions analogous to (13), (14), and (15). Solving them for  $s$  we obtain:

$$s = [\sum f_i \pi_i d \tilde{z}_i / ds]^{-1} \left[ \text{cov}[E[b_i], \pi_i z_i] + t \sum f_i w_i E[d\tilde{\ell}_i / ds] + k \sum_{i \notin I_0} f_i \pi_i z_i \lambda_i / \gamma + \sum_{i \in I_0} f_i \pi_i z_i (u_{c1}^i - E[u_c^i]) / \gamma \right] \quad (22)$$

The denominator on the right-hand side of (22) and first two terms in the numerator are the same as before. They capture respectively the efficiency, the equity and second-best indirect effects. As before, they have ambiguous signs except in special cases. We expect the denominator and the equity term to be positive, but in general these terms are all ambiguous. The second-best term reflecting the indirect effect of changes in  $s$  on the deadweight loss of taxation will disappear if the utility function is quasi-linear in consumption and health status. The last two terms of the numerator are related to the inefficiency of private insurance: they vanish if  $k = 0$ . The term involving  $k$  reflects the efficiency cost of having individuals purchase expensive private insurance. The term involving those households that are quantity constrained ( $i \in I_0$ ) reflects the benefits of providing social insurance to those households for whom private insurance coverage is too expensive. Overall, since the last two terms are both positive, the existence of administrative costs of private insurance tends to enhance the case for public insurance coverage  $s$ , which is not surprising.

## 8 Conclusion

The starting point of this paper was the finding of Rochet (1989) that with distortionary income taxation, social insurance is desirable as a redistributive device. The gist of his argument was the distortionary feature of income taxation. With a non-distortionary redistributive tax, there would be no need for social insurance as long as it is not cheaper than market-provided insurance. One of our purposes was to see how robust this finding was when introducing moral hazard.

We distinguished between *ex ante* and *ex post* moral hazard and showed that the case for public intervention in insurance markets remains. However, while in Rochet's analysis, optimal social insurance is complete and crowds out private insurance, in the presence of moral hazard, that is no longer the

case. Public and private insurance will generally exist side by side. Moreover, it is no longer necessarily the case that optimal social insurance rates be positive. That is even true in the case where there is a negative correlation between productivity and the expected value of spending incurred to correct for the loss. We also introduce the idea that social insurance could be less costly than private insurance. This clearly strengthens the case for social insurance, and increases the chances that it should be positive.

A number of extensions to the current analysis could be contemplated. First, it might be interesting to see whether or not an optimal non-linear tax would dampen the case for social insurance. Evidence from related literatures suggests that even when non-linear taxes are set optimally, the case for second-best policy instruments typically remains intact. Second, for the case of *ex ante* moral hazard, we could consider the possibility of subsidizing preventive spending. Third, the viewpoint adopted here was purely normative. It would be interesting to adopt a political economy approach with social insurance being determined by voting.<sup>21</sup> Finally, instead of treating both types of moral hazard separately, it would be useful to combine them in a single model, although that will certainly increase the complexity without resolving the ambiguity.

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<sup>21</sup>In that respect, see Hindriks and De Donder (2000).

## Appendix

In this appendix we assume that the utility function is quasi-linear and derive the comparative statics for  $p_i$  under *ex post* moral hazard. These will then be used to show the necessary conditions to have a positive value for  $s$ .

If the utility function is  $u(c_i^j + h_i^j - g(l_i^j))$ , the first-order conditions for the household's problem simplify to:

$$(1 - t)w_i - g'(l_i^j) = 0 \quad \text{and} \quad -(1 - \sigma_i) + m'(z_i) = 0$$

and solve as  $l_i^1 = l_i^0 = l((1 - t)w)$  and  $z_i = z(\sigma_i)$ . Thus labour supply depends only on net-of-tax wages and is state independent, and curative expenditures only depend on the coverage rate. Using these results, the Stage 2 first-order condition on  $p_i$  can then be written as:

$$\Delta \equiv u_{c1}^i z_i + Eu_c^i(z_i + p_i z_p) = 0$$

Differentiation of this expression yields:

$$\Delta_{p_i} dp_i + \Delta_{w_i} dw_i + \Delta_{\pi_i} d\pi_i + \Delta_a da + \Delta_t dt + \Delta_s ds = 0$$

with

$$\begin{aligned} \Delta_a &= u_{c1}^{ii} z_i - Eu_c^{ii}(z_i + p_i z_p^i) \\ \Delta_t &= -w_i l_i \Delta_a \\ \Delta_s &= \pi_i p_i z_p^i \Delta_a + z_i u_{c1}^{ii} [z_i - \pi_i (z_i + p_i z_p^i)] + Eu_c^i(z_p^i + p_i z_{pp}^i) - u_{c1}^i z_i p_i \\ \Delta_{w_i} &= (1 - t) l \Delta_a \\ \Delta_{\pi_i} &= -p_i z_i \Delta_a - (u_{c1}^i - u_{c0}^i) c(z_i + p_i z_p^i) \\ \Delta_{p_i} &= \Delta_s - \pi_i z_i \Delta_a - Eu_c^i z_p^i < 0 \quad (\text{by the SOC}) \end{aligned}$$

where  $u^{ii}$  denotes the second derivative of  $u$ .

In general, the effects of the exogenous variables on the sign of  $p_i$  are ambiguous. However, if we assume that private insurance is a normal good, i.e.,  $\partial p_i / \partial a > 0$ ,  $\Delta_a > 0$  and thus we have:

$$\frac{\partial p_i}{\partial a} > 0, \quad \frac{\partial p_i}{\partial t} < 0, \quad \frac{\partial p_i}{\partial s} > -1, \quad \frac{\partial p_i}{\partial w_i} > 0 \quad \text{and} \quad \frac{\partial p_i}{\partial \pi_i} > 0 \quad (\text{by the SOC})$$

Note that even though an increase in  $s$  may raise or lower  $p_i$ , total coverage  $\sigma_i(=s+p_i)$  increases: there is less than complete crowding-out of private insurance. As well, using the comparative static effects above:

$$1 + \frac{\partial p_i}{\partial s} - \pi_i z_i \frac{\partial p_i}{\partial a} = \frac{1}{-\Delta_{p_i}} \{Eu_c^i z_p^i\}$$

which is positive. Therefore the compensated effect of a change of  $s$  in  $z_i$  ( $d\tilde{z}_i/ds$ ) is also positive. Thus the efficiency term in expression (17) is positive, guaranteeing that, with a positive covariance term, social insurance coverage is never negative.

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