Queen's Economics Department Working Paper No. 996

Tax Implementability of Fair Allocations

Yukihiro Nishimura
Department of Economics, Queen's University

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

9-2000

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Yukihiro Nishimura

Department of Economics, Queen's University, Kingston, Ontario K7L 3N6, Canada.*†

September 25, 2000

Abstract

This paper examines the tax implementability of allocations based on Foley's (1967) concept of fairness as no-envy (or envy-free) and its alternatives. An allocation is tax implementable if there exists a tax schedule under which the allocation is realized as a result of agents' optimization. Tillmann (1984) and Bös and Tillmann (1985) showed that the class of fair allocations that are income tax implementable is quite limited. This paper examines the implementability of fair allocations by a tax schedule that depends on labor supply and gross income ((y, l)-implementability), whose availability is supported by Dasgupta and Hammond (1980), Tillmann (1989), Kolm (1997), Beaudry and Blackorby (1997) and others. A relevant incentive constraint is perishability of abilities, where agents can exert a lower ability level than they actually possess. We first show that in any economy, every envy-free allocation is (y, l)-implementable. On the other hand, we can show that perishability of abilities results in the impossibility of (y, l)-implementability of the egalitarian equivalence (Pazner and Schmeidler (1978a)), the l^*-equal budget allocation (Kolm (1996, 1997), Maniquet (1998)), and the balanced-envy allocation (Daniel (1975)). Among them, a special form of the egalitarian equivalence examined by Fleurbaey and Maniquet (1999) and others is (y, l)-implementable in a class of preference-skill distributions where the lazier agent has the higher skill.

*Tel: 1-613-533-6000, ext. 75540. Fax: 1-613-533-6668. E-mail address: nishimur@qed.econ.queensu.ca.

†The author acknowledges Professors Koichi Tadenuma, Hiromitsu Ishi, Eiji Tajika, Shinji Yamashige, Kotaro Suzumura, Robin Boadway, Georg Tillmann and anonymous referees for helpful comments.

Key Words: Tax Implementability, Envy-free

JEL Classification: D63, D78, H21

1 Introduction

This paper examines the tax implementability of allocations based on Foley's (1967) concept of fairness as no-envy (or envy-free) and its alternatives. An allocation is envy-free if nobody envies the consumption-labor bundle of any other agent.[1] It is tax implementable if there exists a tax schedule under which the allocation is realized as a result of optimization by the agents. Bös and Tillmann (1985) showed that, in the class of economies typically dealt with in the optimal taxation literature, the only envy-free allocation that is implementable by the income tax is 'no production' or a trivial allocation with huge dead-weight loss. This impossibility theorem results from the fact that a difference in abilities cannot be completely eliminated under income taxation. The same disincentive problem exists in most of the alternative fairness concepts, as shown by Tillmann (1984).

This impossibility theorem opens two avenues of further research. One is to consider alternative fairness concepts which behave well in the second-best context. This direction is examined in Fleurbaey and Maniquet (1998) and Bossert et al. (1999). The other is to consider more sophisticated tax-transfer systems than income taxation. Specifically, this paper examines an environment where an agent's labor supply is observable, so that the tax can be conditioned on the agent's ability.

[1] See Thomson and Varian (1985) and Arnsperger (1994) for comprehensive surveys.

Dasgupta and Hammond (1980), Tillmann (1989), Kolm (1997), Beaudry and Blackorby (1997) and others note that, very often, income and labor supply are known, implying that one's ability is revealed. It is reasonable to consider, as is typically assumed in most of the papers listed above, that *agents can exert a lower ability than they actually possess (perishability of abilities).* This behavior hampers some first-best allocations.

In Section 2, we examine the tax implementability of envy-free allocations in circumstance where an agent's labor supply is observed. Bös and Tillmann (1985) suggested some policy instruments such as 'suitably chosen lump-sum taxes or a tax on individual abilities'[2] (p.40) to implement envy-free allocations. However, this turns out not to be the case if agents' tastes are heterogeneous. Since lump-sum taxes are not distortionary, the resultant allocation is Pareto efficient. This outcome is not compatible with no-envy in some economies, as Pazner and Schmeidler (1974) showed. Moreover, even if Pareto efficient and envy-free allocations exist, they are not implementable through ability taxes in some economies (Lemma 1). Our first result is a generalization of Bös-Tillmann's statement. Suppose the labor supply and gross income of agents are observable. Then, by a suitable tax schedule that depends on *labor supply as well as gross income* (hereafter referred to as (y, l)-implementability, following Tillmann (1989)), in any economy, every envy-free allocation is tax implementable (Theorem 1).

In Section 3, we examine (y, l)-implementability of other allocations based on alternative fairness concepts. These alternative concepts were originally motivated to circumvent the problem that no-

[2] Or 'an egalitarian education policy', which is outside the scope of this paper.

envy is incompatible with Pareto efficiency in the first-best context (Pazner and Schmeidler (1974)). The most celebrated such concept is egalitarian equivalence (Pazner and Schmeidler (1978a)), where all agents are equally well-off between their actual bundle and a common hypothetical bundle. We show that in a broad class of economies examined in Fleurbaey and Maniquet (1996) and Tillmann (1999), no Pareto efficient egalitarian equivalent allocation is (y, l)-implementable (Theorem 2). A special form of egalitarian equivalence examined by Fleurbaey and Maniquet (1999) and others is (y, l)-implementable in a class of preference-skill distributions where the lazier agent has the higher skill (Theorem 3).

The second class of alternative fairness concepts is that of the l^*-equal budget, studied in Fleurbaey and Maniquet (1996), Kolm (1996, 1997) and Maniquet (1998). This concept includes two classic redistribution rules – Varian's (1974) equal non-labor income rule and Pazner-Schmeidler's (1978b) equal full income rule – as two extreme members. The disincentive problem of the equal full income rule is noted in Pazner (1977) and Dworkin (1981). Their observation, 'the slavery of the talented', is shown to apply to *any* l^*-equal budget allocation except for the equal non-labor income rule (Theorem 4).

Finally, we examine Daniel's (1975) balanced envy criterion, where the number of agents one envies is set equal to the number of agents who envy that agent. In an environment like Pazner and Schmeidler (1974), where Pareto efficiency is incompatible with no-envy, the Pareto efficient allocation satisfying Daniel's fairness concept entails mutual envy. Given perishability of abilities, this fact hampers (y, l)-implementability (Theorem 5).

2 Tax Implementability of Envy-Free Allocations

2.1 The Model

Consider an n-agent economy with one consumption good, c, and labor, l. Each agent i ($i = 1, ..., n$) has a preference relation represented by the utility function $u_i(c, l)$ defined over bundles $(c, l) \in X \equiv I\!R_+ \times [0, 1]$. The utility function is continuous, increasing in the first element and decreasing in the second, and quasi-concave. Let $\omega_i > 0$ be the actual ability level of agent i. Production exhibits constant marginal costs,[3] and firms are competitive, so that firms pay agents' revealed ability levels as a competitive wage. We also assume *perishability of abilities*: the revealed ability level could be lower than the actual one. If agent i reveals ability $w_i \leq \omega_i$ and works l_i hours, the agent obtains $w_i l_i$ as a pre-tax income level. Therefore, the set of *potentially feasible allocations* in an economy is given by $\{(c_i, l_i)_{i=1}^n \in X^n \,|\, \sum_{i=1}^n c_i \leq \sum_{i=1}^n w_i l_i, \;\; w_i \leq \omega_i \; \forall i\}$. By *feasible allocations* we mean the potentially feasible allocations satisfying production efficiency, i.e., $\{(c_i, l_i)_{i=1}^n \in X^n \,|\, \sum_{i=1}^n c_i = \sum_{i=1}^n \omega_i l_i\}$.

We assume that the government has statistical knowledge of all individual characteristics (abilities and preferences) but cannot identify who is who. This situation is called 'assignment uncertainty' (Roberts (1984)), commonly assumed in the optimal tax literature. Redistribution is implemented through a tax schedule, which is a function that maps from some signal (say, gross income) to some real number (tax or subsidy). Agents maximize their utility by sending the optimal signal for themselves. The resulting allocation is called tax implementable.

[3] The analysis can easily be extended to the case of a convex technology, commonly considered in both the fair allocation literature and the optimal tax literature. We need 100% profit taxation, typically assumed in the latter.

2.2 Income Tax, Lump-Sum Tax, and Ability Tax Implementability of Envy-Free Allocations

Definition 1 *(Foley (1967)) A feasible allocation* $(c_i, l_i)_{i=1}^n$ *is envy-free iff* $u_i(c_i, l_i) \geq u_i(c_j, l_j)$ *for all agents* i *and* j.

In this subsection we briefly discuss implementability of envy-free allocations by income tax, lump-sum tax, and ability tax.

Suppose that income is observable but neither labor supply nor ability is observable. Then the problem of income tax implementability applies. An *income tax* $T(w_i l_i)$ assigns a tax payment to an individual whose reported income is $w_i l_i$. The tax is not sensitive to either w_i or l_i, given the same amount of $w_i l_i$.

Definition 2 *A feasible allocation* $(c_i, l_i)_{i=1}^n$ *is income tax implementable iff there exists a tax schedule* $T : \mathbb{R}_+ \to \mathbb{R}$ *such that*[4] [5]*:*

$$\text{for all } i,\ c_i = \omega_i l_i - T(\omega_i l_i),\ u_i(c_i, l_i) = \max_{(c,l) \in X, w_i \leq \omega_i} u_i(c, l) \ s.t.\ c = w_i l - T(w_i l). \qquad (1)$$

Bös and Tillmann (1985) showed that, if everyone has the same preferences but different productivities, then in every envy-free and income tax implementable allocation, at most one agent supplies labor.[6] We can also show that, in an example of Pazner and Schmeidler (1974) where the

[4] In our definition of feasible allocation, free disposal is not allowed. Hence, the tax schedule is *ex post* purely redistributive: Let T_i be the amount of tax paid by agent i from his optimal choice. Then $\sum_{i=1}^n T_i = 0$.

[5] Given monotonicity of preferences, agents reveal $w_i = \omega_i$ at any income taxation. Hence, there is no incentive to hide true ability, unlike the ability tax or (y, l)-tax discussed below.

[6] They originally showed that, if the ability distribution is variable (the true economy is drawn from an ability distribution whose upper bound is infinity), the only income tax implementable envy-free allocation is 'no production'. We consider the case where the statistical properties of the true economy are known, typically assumed in the optimal

preferences of the agents are heterogeneous,

$$n = 2,\ u_1(c, l) = c - \frac{10}{11}l,\ u_2(c, l) = c - \frac{1}{2}l,\ \omega_1 = 1,\ \omega_2 = \frac{1}{10}, \tag{2}$$

the only income tax implementable envy-free allocation is $(c_i, l_i)_{i=1}^n = (0, 0)_{i=1}^n$ (the proof of the latter is available upon request to the author). The same disincentive problem exists in most of the alternative fairness concepts, as shown by Tillmann (1984).

Facing these impossibilities with income taxation, Bös and Tillmann suggested that *'suitably chosen lump-sum taxes or a tax on individual abilities'* can implement envy-free allocations. We now show that this statement does not apply when preferences are heterogeneous.

Formally, lump-sum tax and ability tax implementability are defined as follows.

Definition 3 *1. A feasible allocation $(c_i, l_i)_{i=1}^n$ is lump-sum tax implementable iff there exists $T : \{1, ..., n\} \to \mathbb{R}$ such that:*

$$for\ all\ i,\ c_i = \omega_i l_i - T(i),\ u_i(c_i, l_i) = \max_{(c,l) \in X, w_i \leq \omega_i} u_i(c, l)\ s.t.\ c = w_i l - T(i). \tag{3}$$

2. A feasible allocation $(c_i, l_i)_{i=1}^n$ is ability tax implementable iff there exists $T : \mathbb{R}_+ \to \mathbb{R}$ such that:

$$for\ all\ i,\ c_i = \omega_i l_i - T(\omega_i),\ u_i(c_i, l_i) = \max_{(c,l) \in X, w_i \leq \omega_i} u_i(c, l)\ s.t.\ c = wl - T(w). \tag{4}$$

Definition 4 *A feasible allocation $(c_i, l_i)_{i=1}^n$ is Pareto efficient iff there is no other feasible allocation $(\tilde{c}_i, \tilde{l}_i)_{i=1}^n$ such that $u_i(\tilde{c}_i, \tilde{l}_i) \geq u_i(c_i, l_i)$ for all i with at least one strict inequality.*

taxation literature. Tillmann (1989) extends this result to economies with heterogeneous preferences (p.34, Lemma 1.4), but his result is not exact. We can show the following. Assume there are at least two agents with identical preferences, but different productivity levels. Then *in any envy-free and income tax implementable allocation, at most the highest skilled individual among the homogeneous preference agents supplies labor* (correction in italics). See also Tillmann (1999).

Our first result is the following:

Lemma 1 *1. There exists an economy in which no envy-free allocation is either lump-sum tax implementable or ability tax implementable. 2. There exists an economy in which, even though there exist Pareto efficient and envy-free allocations, none of them is ability tax implementable.*

Proof: Applying the first theorem of welfare economics, every lump-sum tax implementable allocation and every ability tax implementable allocation are Pareto efficient. Pazner and Schmeidler (1974) showed that, in an economy represented by (2), no envy-free allocation is Pareto efficient. The result of the first part is implied by these facts. To prove the second part, consider the following economy: $n = 4$, $u_1(c,l) = u_3(c,l) = c + ln(1-l)$, $u_2(c,l) = u_4(c,l) = c + 2ln(1-l)$, $\omega_1 = \omega_2 = 4$, $\omega_3 = \omega_4 = 2$. One can show that the following is the only Pareto efficient and envy-free allocation: $c_1 = 2.36643$, $c_2 = 1.67329 = c_3$, $c_4 = 0.28699$, $l_1 = 0.75$, $l_2 = 0.5 = l_3$, $l_4 = 0$. Since $c_1 - \omega_1 l_1 \neq c_2 - \omega_2 l_2$, there is no ability tax satisfying (4). *Q.E.D.*

Lemma 1 shows that, even with full information about individual characteristics, we need in general to set policy instruments other than lump-sum taxes or ability taxes to implement envy-free allocations.

2.3 Tax Based on Labor Supply and Gross Income

Consider instead the taxation of labor supply and gross income, referred to as $T(y,l)$ (where y is the gross income wl) by Tillmann (1989).

Definition 5 *A feasible allocation* $(c_i, l_i)_{i=1}^n$ *is* (y,l)*-implementable iff there exists* $T : I\!R_+ \times I\!R_+ \rightarrow I\!R$ *such that:*

$$for\ all\ i,\ c_i = \omega_i l_i - T(\omega_i l_i, l_i),\ u_i(c_i, l_i) = \max_{(c,l)\in X, w_i \leq \omega_i} u_i(c,l)\ s.t.\ c = w_i l - T(w_i l, l). \tag{5}$$

The former condition says that, if agent i reveals the true ability level ω_i with labor supply l_i, then the agent can obtain c_i as post-tax income. The latter says that the agent maximizes the utility subject to the feasibility constraint under perishability of abilities.

Ability tax implementability implies (y,l)-implementability. Moreover, some Pareto inefficient allocations are (y,l)-implementable. It is easy to see that there is no set inclusion between lump-sum tax implementability and (y,l)-implementability.

The following lemma is useful:

Lemma 2 *A feasible allocation* $(c_i, l_i)_{i=1}^n$ *is* (y,l)*-implementable iff it satisfies:*

$$for\ all\ i\ and\ j,\ if\ \omega_i \geq \omega_j,\ u_i(c_i, l_i) \geq u_i(c_j, l_j). \tag{6}$$

Proof: Necessity is immediate. To prove sufficiency, let $(c_i, l_i)_{i=1}^n$ be an allocation satisfying (6). Consider the following $T(y,l)$: for all i, $T(\omega_i l_i, l_i) = \omega_i l_i - c_i$ and $T(y,l) = \infty$ if there is no i such that $y = \omega_i l_i$ and/or there is no i such that $l = l_i$. This $T(y,l)$ satisfies (5). *Q.E.D.*

Any envy-free allocation satisfies condition (6). As a result, we can say the following:

Theorem 1 *Every envy-free allocation is* (y,l)*-implementable.*

Lemma 1 and Theorem 1 clarify the information required to implement the first-best Pareto efficient allocations and the first-best envy-free allocations. According to the second theorem of welfare economics, identification of information about every individual is required to implement first-best Pareto efficient allocations. On the other hand, to implement envy-free allocations, the government must observe individual labor supply and income levels, as well as the statistical properties of all individual characteristics (assignment uncertainty). Perishability of abilities does not jeopardize implementability of envy-free allocations.

3 (y, l)-Implementability of Other Fair Allocations

In this section we examine (y, l)-implementability of other fair allocations advocated in the social choice literature.

3.1 Egalitarian Equivalence

The most celebrated alternative fairness concept, originally motivated by the first-best equity-efficiency trade-off shown by Pazner and Schmeidler (1974), is egalitarian equivalence:

Definition 6 *(Pazner and Schmeidler (1978a), Maniquet (1998)) Given* $l^* \in [0, 1]$*, a feasible allocation* $(c_i, l_i)_{i=1}^n$ *is called an* l^**-egalitarian equivalent allocation iff 1) it is Pareto efficient and 2) there exists* $c_0 \in \mathbb{R}_+$ *such that* $u_i(c_i, l_i) = u_i(c_0, l^*)$ *for all* i*.*

The solution can be characterized by the axioms of responsibility and compensation (Maniquet (1998)). In the special case where l^*=0, some characterization theorems are given by Roemer and

Silvestre (1987), Dutta and Vohra (1993), and Fleurbaey and Maniquet (1993, 1999) with the use of monotonicity and weak egalitarian axioms.

3.1.1 General Impossibility

Fleurbaey and Maniquet (1996) showed that there exists an economy where *any* l^*-egalitarian equivalent allocation violates 'no-envy among equally skilled', a necessary condition for (6). Hence, *there exists an economy where no* l^**-egalitarian equivalent allocation is* (y,l)*-implementable.*

We now show that the class of economies where this impossibility result holds is broad. For expositional simplicity, we assume in this subsection that the utility functions are differentiable in each element. Let $MRS((\tilde{c},\tilde{l}),u_i) \equiv -\frac{\partial u_i/\partial l}{\partial u_i/\partial c}|_{(c,l)=(\tilde{c},\tilde{l})}$.

Theorem 2 *1. In any economy where there exist agents j and k such that a)* $\omega_j \geq \omega_k$*, b)* $MRS((c,l),u_j) > MRS((c,l),u_k)$ *for all* $(c,l) \in X$*, c)* $MRS((\tilde{c},0),u_k) \geq \omega_k$ *for all* $\tilde{c} \geq 0$*, no* l^**-egalitarian equivalent allocation with* $l^* > 0$ *is* (y,l)*-implementable.*

2. In any economy where there exist agents m and p such that a) $\omega_m \leq \omega_p$*, b)* $MRS((c,l),u_m) > MRS((c,l),u_p)$ *for all* $(c,l) \in X$*, c)* $MRS((\tilde{c},0),u_m) < \omega_m$ *for all* $\tilde{c} \geq 0$*, no* 0*-egalitarian equivalent allocation is* (y,l)*-implementable.*

Therefore, in any economy satisfying 1.a)-c) and 2.a)-c), no l^**-egalitarian equivalent allocation is* (y,l)*-implementable.*

An environment considered by Fleurbaey and Maniquet (1996, Theorem 1) satisfies these conditions. Tillmann's (1999) economy satisfies conditions 1.a)-b) and 2.a)-b). Conditions 1.b) and 2.b) mean that agents can be ordered by laziness (formally, the marginal rate of substitution between leisure and consumption). This is true in many cases. $\omega_k = 0$ (Fleurbaey and Maniquet (1998), A1) is sufficient for condition 1.c). Condition 2.c) is weaker than $\omega_m > 0$ and $MRS((\tilde{c}, 0), u_m) = 0$ for all $\tilde{c} \geq 0$, a usual condition to exclude the corner solution.

Proof of Theorem 2: 1. Let $(c_i, l_i)_{i=1}^n$ be an l^*-equivalent allocation, $l^* > 0$. First, $l_k = 0$ by Pareto efficiency, condition c) and monotonicity of preferences. Second, by condition b), $MRS((c_0, l^*), u_j) > MRS((c_0, l^*), u_k)$ for c_0 such that $u_i(c_0, l^*) = u_i(c_i, l_i)$ for all i. Combining with $l_k = 0$, for $\tilde{c}$ such that $u_j(\tilde{c}, 0) = u_j(c_j, l_j)$, $\tilde{c} < c_k$. Therefore, $u_j(c_k, l_k) = u_j(c_k, 0) > u_j(\tilde{c}, 0) = u_j(c_j, l_j)$. By condition a), (6) is violated.

2. Let $(c_i, l_i)_{i=1}^n$ be a 0-equivalent allocation. Notice that, by condition b), for all $(c, l) \in X$, if $u_m(c, l) = u_m(c_m, l_m)$, then $u_p(c, l) \geq u_p(c_p, l_p)$ with strict inequality when $l > 0$. Since $l_m > 0$ by Pareto efficiency and condition c), $u_p(c_m, l_m) > u_p(c_p, l_p)$. By condition a), (6) is violated. *Q.E.D.*

Maniquet's (1998) discussion on the tax implementability of the l^*-egalitarian equivalent allocation (p.196) does not take into account its violation of (y, l)-implementability. Implementability requires other observable characteristics than ability.

3.1.2 A Possibility Result

In the case where l^*=0, the egalitarian-equivalent allocation is (y, l)-implementable in a class of economies. Consider a preference structure where *all* agents are ordered by laziness.

Definition 7 *1. Agents can be ordered by laziness iff for all i and for all j such that $i > j$,*

$MRS((c,l),u_i) < MRS((c,l),u_j)$ for all $(c,l) \in X$. 2. Given Definition 7.1, one's preferences and abilities are negatively correlated iff for all i and for all j such that $i > j$, $\omega_i < \omega_j$.

We consider the case of perfect correlation between preferences and skills, which is a natural starting point for examining an environment with two-dimensional characteristics. In the opposite environment, where $\omega_i < \omega_j$ in Definition 7.2 is replaced with $\omega_i \geq \omega_j$ (referred to as *positive correlation*), Pikkety (1994) showed compatibility between no-envy and Pareto efficiency. In the case of negative correlation, impossibility results (Pazner and Schmeidler (1974) and Tillmann (1989)).

We additionally assume that $MRS((\tilde{c},0),u_i) < w_i$ for all i and $\tilde{c} \geq 0$. We now show the following result:

Theorem 3 *Suppose that $MRS((\tilde{c},0),u_i) < w_i$ for all i and $\tilde{c} \geq 0$. When agents are ordered by laziness (Definition 7.1), a 0-egalitarian equivalent allocation is (y,l)-implementable iff one's working preference and abilities are negatively correlated (Definition 7.2).*

Proof: Let $(c_i,l_i)_{i=1}^n$ be a 0-equivalent allocation. Given Definition 7.1, for all $(c,l) \in X$ and all k and j such that $k \leq j$, if $u_j(c,l) = u_j(c_j,l_j)$, then $u_k(c,l) \leq u_k(c_k,l_k)$. Given Definition 7.2, $\omega_k \geq \omega_j$ iff $k \leq j$. Therefore, (6) is satisfied. The 'only if' part of the theorem has been shown in Theorem 2.2. *Q.E.D.*

Table I summarizes the analysis from combining Theorem 1, Theorem 3, and the earlier results. The third and the fourth row show the compatibility with Pareto efficiency and (y,l)-

implementability, respectively.

Table I: Theorem 3 and the related results

No-envy			0-egalitarian equivalence		
correlation	negative	positive	correlation	negative	positive
compatibility	*No*	*Yes*	compatibility	*Yes*	*Yes*
implementability	*Yes*	*Yes*	implementability	*Yes*	*No*

3.2 l^*-Equal Budget

In this section, we consider another class of allocation rules called l^*-equal budget, advocated by Fleurbaey and Maniquet (1996) and Kolm (1996, 1997) and again characterized by the responsibility and compensation axioms by Maniquet (1998).

Definition 8 *(Kolm (1996, 1997), Maniquet (1998)) Given* $l^* \in [0,1]$, *a feasible allocation* $(c_i, l_i)_{i=1}^n$ *is called an* l^**-equal budget allocation iff it satisfies:*

$$for\ all\ i,\ u_i(c_i, l_i) = \max_{(c,l)\in X} u_i(c,l)\ s.t.\ c - \omega_i l = l^*(\bar{\omega} - \omega_i), \tag{7}$$

where $\bar{\omega} \equiv \sum_{i=1}^n \omega_i / n$.

This is a competitive equilibrium where agent i's subsidy is $l^*(\bar{\omega}-\omega_i)$. The value of l^* represents the degree of redistribution.[7] When $l^* = 1$, the allocation corresponds to Pazner-Schmeidler's (1978b) income fairness rule. When $l^* = 0$, the allocation corresponds to Varian's (1974) wealth-fair rule: with linear technology this is just the laissez-faire allocation.[8]

[7] In Kolm's (1996, 1997) interpretation, l^* represents the degree of self-ownership of leisure. Consider the economy where every agent is endowed with a coupon entitling him or her to $1/n$ of a part l^* of each agent's labor time. Each agent i has to buy his or her leisure time l^*, whose market value is $l^*\omega_i$. Hence, $l^*\bar{\omega}$ is the revenue from this coupon. The l^*-equal budget allocation is the competitive equilibrium from this leisure ownership.

[8] An extension of this definition to the convex production technology is given in Maniquet (1998). There, the 0-equal budget allocation is the competitive equilibrium with equal division of the rent (profit). And more generally,

An undesirable feature of the 1-equal budget allocation (income fairness rule) is depicted by Pazner (1977) and Dworkin (1981): 'In general, the more able the person, ceteris paribus, the more penalized he is relative to an unable one' (Pazner (1977), p.461). In our context, perishability of abilities hampers the implementability of the first-best 1-equal budget allocations. In fact, this observation holds at *any* l^*-budget allocation with $l^* > 0$, which admits transfer from the higher abilities to the lower abilities.

Theorem 4 *There exists an economy where no l^*-equal budget allocation with $l^* > 0$ is (y, l)-implementable.*

Proof: Consider the following economy:

$$u_1(c, l) = c - (\omega_1 + \epsilon)l, \ u_2(c, l) = c - \omega_2 l, \ \epsilon > 0, \ \omega_2 > \bar{\omega} > \omega_1 \tag{8}$$

(there is no specification of other agents' characteristics). At any l^*-equal budget allocation $(c_i, l_i)_{i=1}^n$ with $l^* > 0$, we have $c_1 > 0$, $l_1 = 0$, and $u_2(c_2, l_2) < 0$. As $u_2(c_1, l_1) > u_2(c_2, l_2)$ and $\omega_2 > \omega_1$, this violates (6). *Q.E.D.*

Kolm (1996, 1997) recognized a potential disincentive problem of l^*-equal budget allocations, especially with high values of l^*, but he did not mention such a strong impossibility of attaining the first-best allocation. Roughly speaking, the limit of redistribution results when low ability agents are lazy (or handicapped)[9] and/or high ability agents have high Hicksian elasticities of labor (flat

the l^*-equal budget allocation is the competitive equilibrium from endowments of a leisure-coupon introduced in footnote 7 and equal division of the rent.

[9]When $l_i \geq l^*$ at an l^*-equal budget allocation, no agent j with $\omega_j > \omega_i$ will envy (mimic) agent i's bundle. Hence, the allocation is implementable if $l_i \geq l^*$ for all i.

indifference curves which strengthen the perishability constraint).

3.3 Daniel's (1975) Balanced Criterion

Finally we examine the following allocation:

Definition 9 *(Daniel (1975)) A feasible allocation* $(c_i, l_i)_{i=1}^n$ *is called a Daniel-balanced allocation iff it is Pareto efficient and satisfies* $\#\{j | u_i(c_j, l_j) > u_i(c_i, l_i)\} = \#\{j | u_j(c_i, l_i) > u_j(c_j, l_j)\}$.

In an environment like Pazner and Schmeidler (1974), where Pareto efficiency is incompatible with no-envy, the Pareto efficient allocation satisfying Daniel's fairness concept entails mutual envy. In the tax implementability context, higher ability individuals will not self-select the allocation intended for them, given perishability of abilities. Hence, implementability is hampered again.

Theorem 5 *There exists an economy where no Daniel-balanced allocation is* (y, l)*-implementable.*

Proof: In an economy in (2), at any Daniel-balanced allocation $(c_i, l_i)_{i=1}^n$, $u_1(c_2, l_2) > u_1(c_1, l_1)$ and $u_2(c_1, l_1) > u_2(c_2, l_2)$ holds, since Pareto efficiency and no-envy are incompatible. Since $\omega_1 > \omega_2$, this allocation violates (6). *Q.E.D.*

4 Conclusion

This paper has examined the tax implementability of various concepts of fair allocations under the assumption that the government can observe an agent's labor supply as well as gross income. We first show that observability of labor supply expands the possibility to implement no-envy allocations (Theorem 1). However, even with observability of labor supply, implementability of first-best Pareto efficient allocations satisfying alternative fairness criteria of no-envy is not obtained, when we take into account perishability of abilities or no-envy among equally skilled (Theorem 2, 4 and 5).

Our results invite us to consider *the three-fold-conflicts between efficiency, equity, and implementability.* Observability of labor supply gives a positive result for no-envy, but we still face Pazner-Schmeidler's (1974) result with respect to efficiency. Alternatives which circumvent this first-best equity-efficiency trade-off meet the obstacle of implementability even when labor supply is observable. The above considerations suggest three directions for further research. In each direction the contributions are emerging but still not completed.

1. Equity: construction of a concept of fairness, taking account of both implementability and efficiency (Fleurbaey and Maniquet (1998) and Bossert et al. (1999)).

2. Efficiency: characterization of the second-best Pareto efficient allocations given equity and implementability frameworks. The fair allocation literature often considers first-best Pareto efficiency. However, as a by-product of the analysis in the text, we find that there exists an economy where *no first-best Pareto efficient allocation in which the transfer goes from the higher skilled agent to the lower skilled agent is (y, l)-implementable* (eq. (8) with $n = 2$). This impossibility result motivates us to examine the second-best Pareto efficient allocations (Beaudry and Blackorby (1997)).

3. Implementability: implementability of some equitable and efficient allocations through various institutional frameworks (Guesnerie (1996), Pikkety (1993)).

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Table I: Theorem 3 and the related results

No-envy			0-egalitarian equivalence		
correlation	negative	positive	correlation	negative	positive
compatibility	*No*	*Yes*	compatibility	*Yes*	*Yes*
implementability	*Yes*	*Yes*	implementability	*Yes*	*No*