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# Optimality of Workfare with Heterogeneous Preferences

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# Optimality of Workfare with Heterogeneous Preferences

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## Abstract

Using the standard nonlinear income taxation framework with heterogeneity of preferences, this paper examines the optimality of workfare as a screening tool. It is assumed that workfare does not serve as a human capital investment, participation is mandatory, and administrative costs are negligible. Imposing alternative cardinalizations on individuals utilities, allows for the possibility that the government optimally redistributes income to or from high disutility of labour individuals. Under either case, workfare is never optimal to impose on these individuals. It is also shown that non-productive workfare can be an efficient policy tool, in contrast to the results found in Besley and Coate (1995), Brett (1997), and Beaudry and Blackorby (1997).

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# 1 Introduction

During the past few years, governments have increasingly instituted workfare into their welfare programs. Workfare is the conditioning of welfare benefits, either cash or in-kind, on the fulfillment of some obligation by the welfare recipients. The type of obligation can vary substantially. It can include community service jobs or similar work requirements, subsidized employment, job search activities, education, and/or job-training. In addition, workfare programs can be voluntary, in which case individuals receive extra benefits for participating in the program, or workfare can be mandatory, in which case individuals lose benefits when they do not participate. Likewise, the arguments for workfare can take many forms. The most common argument is that workfare reduces the cost of redistributive programs, either by preventing long-term dependency by preserving and enhancing skills, or by reducing the number of people on welfare by effectively screening individuals.

In this paper, the optimality of workfare as a screening tool is examined. To do so, a narrow definition of workfare is adopted by assuming it does not serve as a human capital investment and that participation is mandatory. Second, administrative costs of the redistributive program are assumed to be negligible. Third, it is assumed that there is an informational asymmetry between individuals and the government.<sup>1</sup> The government can only observe individuals private income. It is assumed that the government designs a tax/transfer schedule to achieve its redistributive objective. The informational asymmetry implies that the government must satisfy incentive compatibility constraints. Individuals must prefer or be indifferent between the tax/transfer bundle intended for them and all other bundles. In effect, the government takes into account that individuals act optimally by taking the tax/transfer schedule as given when they make their work/leisure. As

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<sup>1</sup>Nichols and Zeckhauser (1982) first suggested that under imperfect information the government can increase the target efficiency of redistributive programs by conditioning transfers on non-means tests.

shown in the optimal taxation literature, the government is able to weaken these self-selection constraints by using nonlinear taxation.<sup>2</sup> The question this paper investigates is whether the self-selection constraints can also be weakened by using a non-means test, i.e., conditioning taxes/transfers on workfare.

In the standard nonlinear income taxation literature<sup>3</sup>, individuals only differ in ability and the government adopts either a welfarist or a non-welfarist objective function. Which objective is more realistic in the context of redistributive programs intended to help the poor is a matter of debate. However, the approach the government chooses, and the specific objective function it adopts, has implications for the design of the optimal nonlinear tax scheme. In this paper, a welfarist approach is taken. The government ‘cares’ only about the utility of individuals. It selects a tax/transfer schedule to maximize a quasi-concave social welfare function defined over individuals’ utilities.<sup>4</sup> This can be contrasted to a non-welfarist approach, commonly adopted in the literature (Besley and Coate (1992,1995) and Kanbur, Keen, and Tuomala (1994)), where the government does not give any weight to the leisure individuals forgo when they work. The government ‘cares’ only about individuals total consumption or income and any function of these variables can be a nonwelfarist objective function.

When individuals have the same preferences and their abilities are drawn from the same distribution, their welfare is an increasing function of their ability level. Therefore, a government with a symmetric quasi-concave welfarist objective function (i.e. one that bases redistribution on individual utilities)<sup>5</sup> will want to redistribute towards individuals

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<sup>2</sup>Stiglitz (1982) shows this in the case when the government adopts a welfarist objective function.

<sup>3</sup>This framework was initiated by Mirrlees (1971).

<sup>4</sup>Varying the degree of quasi-concavity of the social welfare function allows one to trace out all the relevant points on the pareto efficiency frontier.

<sup>5</sup>An example of a symmetric quasi-concave objective function is an isoelastic social welfare function. It takes the form of  $W = \left(\sum_i^N U_i\right)^{1-\rho} / \rho$ , with the coefficient of aversion to inequality,  $\rho \in [0, \infty]$ . If  $\rho$  is zero,  $W$  is a utilitarian social welfare function and as  $\rho$  tends to infinity,  $W$  becomes a maxi-min social welfare function.

at the bottom of the earning distribution.<sup>6</sup> In this case, four qualitative conclusions have emerged from the existing literature. First, marginal tax rates on all individuals are nonnegative. Second, the marginal tax rate of the highest earner is zero. Third, the marginal tax rate of the lowest earner is zero if all individuals are working at the optimum; otherwise it is positive.<sup>7</sup> Fourth, the progressivity of the optimal nonlinear tax schedule is ambiguous. Kanbur, Keen, and Tuomala (1994) show that the first and third results can be overturned if the government minimizes an income-based poverty index.<sup>8</sup> In this paper, it is shown that modifying the standard optimal nonlinear income taxation framework with a welfarist government by allowing for heterogeneous preferences overturns both the first and the fourth result.<sup>9</sup> When individuals have different preferences, it is possible that some individuals face a negative marginal tax rate and the optimal tax schedule is regressive. The issue addressed in this paper is: given the optimal tax system is in place and individuals differ with respect to both their abilities and their preferences, is it efficient to impose workfare?

The optimality of imposing workfare in the standard nonlinear income taxation framework with a welfarist government has been examined by Besley and Coate (1995), Brett (1997), and Beaudry and Blackorby (1997). The first two papers show that it is efficient to use workfare to separate individuals of differing abilities only if work requirements are productive. Under the additional assumption that individuals also have different unobservable home sector productivities, Beaudry and Blackorby (1997) show this result

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<sup>6</sup>This continues to hold if individuals also have differing preferences for work, represented by  $s$ , when individual utility takes the form  $u(c, L) = g(c) + sh(L)$ . However, the lowest income earner need not be the individual with the lowest ability. See Tarkiainen and Tuomala (1997).

<sup>7</sup>It might be optimal for some individuals at the bottom of the wage/ability distribution not to work. See Mirrlees (1971) and Chambers (1988). Note that if there are a discrete number of ability types then the marginal tax rate on the lowest ability type that works will always be positive. See Stiglitz (1982).

<sup>8</sup>In their paper, the lowest income earner with some earning potential has a negative marginal tax rate. However, the sign of their marginal tax rate is ambiguous when they are unable to work.

<sup>9</sup>Tarkiainen and Tuomala (1997) allow for differing tastes for work, but assume that individuals' utilities have the same general form. Section 3.1 is a discrete version of their continuous two-dimensional problem.

continues to hold only if individuals are more productive at workfare than they are in the formal labour market. In this paper, it is shown that non-productive workfare can be efficient to use when individuals are allowed to have heterogeneous preferences.

When individuals have different preferences, which reflect their different disutilities of labour, their welfare is no longer an increasing function of their ability level. The first question to address is: which individuals will the government want to redistribute towards? It is known that an individual with a high disutility of labour will work less than an individual with a lower disutility of labour who earns the same wage, that is, individuals with a high disutility of labour will be lower income earners. Does this mean the government should redistribute towards these individuals, that is, do these individuals also have a lower welfare? It is argued in this paper that the government may or may not redistribute towards such individuals. On the one hand, the government could believe that low income earners deserve the support of the state, irrespective of the factors that determined their low income status. One way to think about this case is to view a high disutility of labour as some form of disability, where a disability is defined as some ‘general activity limitation, possibly mental or emotional’ that does not preclude participation in the labour market.<sup>10</sup> In this case, the government wants to redistribute income to individuals with low income. On the other hand, it seems equally plausible that the government believes that some low income earners do not necessarily deserve the support of the state. It might argue that some low income earners are responsible for their low income status and therefore, the state should not transfer income to them. This case can be supported by re-interpreting a high disutility of labour as a high taste for leisure. Individuals are capable of working more, but have chosen not to, that is, they are ‘lazy’. In this case, the government might not want to redistribute income to these individuals.

To analyze the optimality of workfare under either of these two interpretations, the disutility of labour is embedded into a general form of utility function which allows the

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<sup>10</sup>Harkness (1993). In his paper, Harkness investigates the participation decision of disabled males in the Canadian labour market.

disutility of labour to be interpreted as either disability or laziness depending on how the utility function is cardinalized. By imposing alternative cardinalizations on individual utilities, the government optimally redistributes income to or from high disutility of labour individuals depending on how it interprets the disutility of labour. To highlight the results, a maxi-minimum social welfare function is adopted.

The second question can then be addressed: is workfare an optimal screening tool? Under either interpretation, it is shown that it is never optimal to impose workfare on individuals with a high disutility of labour. However, it can be optimal to impose workfare on individuals with a lower disutility of labour even if the work requirements are not productive. In addition, it is shown that when there is no workfare, individuals can have negative marginal tax rates and the tax structure can be unambiguously regressive in a specific region. This is in contrast to the standard results of the optimal non-linear income taxation framework.

The structure of the paper is as follows. Section 2 sets out the assumptions of the model and characterizes individuals' behaviour. Section 3 then characterizes the government's problem and examines the optimality of workfare under the different government objectives. Section 4 discusses the model's results and Section 5 concludes.

## 2 The Model

Individuals in this economy are represented by two characteristics indexed by  $i$  and  $j$ . The productive capability index  $i \in \{1, 2\}$  is reflected by individuals' wage rates,  $w_i$  and the disutility of labour index  $j \in \{\ell, h\}$  is reflected both by individuals disutility of labour function,  $g^j(\cdot)$  and by their utility function  $U^j(\cdot)$ . The total population in this economy is given by,  $N = \sum_{i=1}^2 \sum_j^{\ell, h} N_i^j$ .<sup>11</sup>

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<sup>11</sup>Essentially, there are four types of individuals in this economy. This makes the problem tractable and allows us to gain insight into the optimality of workfare given that individuals differ with respect to two unobservable characteristics, ability and taste.

Individual preferences are represented by:

$$u^j(c, L, d) = U^j(c - g^j(L + d)) \quad (1)$$

where  $c$  is consumption,  $L$  is labour supplied in the private market, and  $d$  is labour required for workfare. The function  $U^j(\cdot)$  is strictly quasiconcave implying positive marginal utility,  $MU^j = U^{j'}(\cdot) > 0$ . This general functional form allows for the possibility of different laissez-faire utility rankings depending on the measuring scheme used for  $U^j$ . Note that differing the cardinalization of  $U^j$  will not affect the individual's underlying preferences; they will only affect how  $U^h$  compares to  $U^\ell$ . The argument in the utility function has a quasilinear functional form to make the results comparable to those of Besley and Coate (1995).<sup>12</sup> The disutility of labour function,  $g^j(\cdot)$  is assumed to be strictly convex and twice differentiable.

Leisure is a good and the marginal rate of substitution between consumption and total labour supply is greater than zero,  $MRS_{c,L}^j = g^{j'}(L + d) > 0$ . It is assumed that type 2 individuals are more productive in the private labour market than type 1 individuals, that is  $w_2 > w_1$ . It is also assumed that individuals with preferences  $h$  have a higher disutility of labour or a higher taste for leisure,  $g^h(\bar{L}) > g^\ell(\bar{L})$  and  $g^{h'}(\bar{L}) > g^{\ell'}(\bar{L})$ , for any  $\bar{L}$ .

The individual's budget constraint is:

$$c = y + b = w_i L + b \quad (2)$$

where  $y = w_i L$  is income from labour supplied in private market and  $b$  is a benefit transfer, conditional on private income and possibly also on workfare.

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<sup>12</sup>Besley and Coate (1995) assume a linear utility function, so  $U(c - g(L + d)) = c - g(L + d)$  and  $MU = 1$ . If this linear form was used, then individuals with a lower disutility of labour ( $g^\ell(\cdot) < g^h(\cdot)$ ) would be better off than similarly abled individuals with a higher disutility of labour at every point in consumption/income space. A welfarist government that maximizes a symmetric quasi-concave social welfare function will always transfer income to the low-ability, high-disutility of labour individuals.

By substituting for private labour supply, the individual's utility in terms of consumption and private income or the indirect utility function can be written as:

$$V_i^j(c, y, d) \equiv U^j \left( c - g^j \left( \frac{y}{w_i} + d \right) \right) \quad (3)$$

where the marginal rate of substitution between consumption and private income is equal to one over the wage rate times the marginal rate of substitution between consumption and total labour supply,  $MRS_{c,y}^j = g^{j'}(\frac{y}{w_i})\frac{1}{w_i}$ <sup>13</sup>, and the partial derivative of the individual's indirect utility function with respect to workfare is  $-MU^j g^{j'}(\cdot) < 0$ .

The wage differential assumption and the assumptions on  $U^j(\cdot)$  and  $g^j(\cdot)$  ensure that the indifference curves of individuals who differ only in ability, or who differ only in tastes, exhibit the single-crossing property in consumption-income space. These assumptions are also sufficient to ensure that high ability individuals are better off than their low ability counterparts at any point in consumption-income space. However, additional assumptions on  $U^j(\cdot)$  must be made to assert that individuals with one type of preferences are better off at every point in consumption-income space than similarly abled individuals with different tastes.

If there is no welfare/workfare system in place, individuals choose consumption and private labour supply to maximize their utility (1) subject to their budget constraint (2), given  $b = d = 0$ . Let  $\delta$  be the Lagrange multiplier or the marginal utility of private income. The Lagrangian of the individual's problem is:

$$\mathcal{L} = U^j \left( c - g^j(L) \right) - \delta(c - w_i L)$$

The first-order conditions imply:

$$MRS_{c,L}^j = g^{j'}(\hat{L}_i^j) = w_i \rightarrow MRS_{c,y}^j = 1$$

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<sup>13</sup>The partial derivatives are  $\partial V_i^j / \partial c = MU^j$  and  $\partial V_i^j / \partial y = MU^j g^{j'}(\frac{y}{w_i})\frac{1}{w_i}$ .

From their budget constraints, the individuals' laissez-faire level of consumption is  $\hat{c}_i^j = \hat{y}_i^j = w_i \hat{L}_i^j$  and their maximized utility is  $V_i^j(\hat{c}_i^j, \hat{y}_i^j)$ . This laissez-faire outcome is illustrated in the consumption-income space in Figure 1.<sup>14</sup> From the wage differential assumption,  $\hat{L}_2^j > \hat{L}_1^j$ , for  $j = \ell, h$ . This implies high ability individuals are better off than their low ability counterparts,  $V^j(\hat{c}_1^j, \hat{y}_1^j) < V^j(\hat{c}_2^j, \hat{y}_2^j)$  for  $j = \ell, h$ . From the strict convexity assumption on  $g^j(\cdot)$ ,  $\hat{L}_i^\ell > \hat{L}_i^h$ , for  $i = 1, 2$ . Individuals with the same ability will work more and earn a higher income when they have a lower taste for leisure. Which type of individual is better off in the laissez-faire outcome depends on the assumptions made about  $U^j$ .

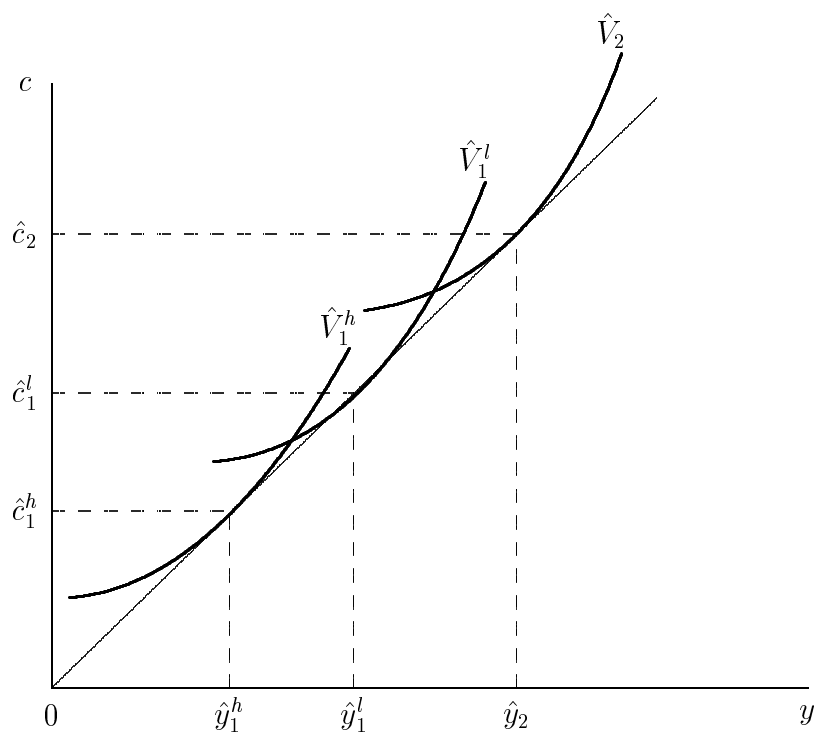
### 3 Government Policy

It is assumed that the government 'cares' about the welfare of individuals and adopts a maxi-minimum social welfare function.<sup>15</sup> However, this model departs from the standard framework by allowing individuals to differ with respect to tastes and by introducing workfare as a policy tool. Under the first assumption and without workfare, it is shown that the marginal tax rates on less abled individuals can be negative and that the optimal nonlinear income tax schedule can be regressive in a specific region. It is then shown how the assumption of heterogeneity in preferences changes the results of Besley and Coate (1995) and Brett (1997) and how the results derived in this paper relate to those in Beaudry and Blackorby (1997) who have allowed for unobservable home sector productivities.

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<sup>14</sup>For simplicity, it is assumed there is only one type of high ability individuals. Figure 1 is independent of the type of high ability individuals and therefore, the superscript on their indirect utility has been ignored.

<sup>15</sup>This is to simplify the analysis, but the results will carry through with a more general social welfare function.



**Figure 1**  
*Laissez-Faire Outcome*

The objective of the government is to maximize the welfare of the worst-off individual subject to the self-selection constraints, non-negativity constraints on individuals private income, and its revenue/resource constraint.

The revenue/resource constraint is given by:

$$\sum_i \sum_j N_i^j (b_i^j + \bar{w} d_i^j) \leq \bar{R}$$

where  $\bar{R}$  is the government's required revenue,  $b_i^j (= y_i^j - c_i^j)$  is the individuals' income tax/transfer, and  $\bar{w} \in [0, w_1]$  is the productivity of labour being supplied for work requirements.<sup>16</sup> It is assumed that the government can observe the amount of labour supplied for the given work requirements since it is imposing them. Effectively, the government can condition tax/transfer bundles on both income and the level of work requirements. If individuals differ in their ability at workfare, the government could use work requirements to identify individuals' ability. Therefore, it is assumed that individuals are equally productive at workfare. To make this assumption realistic, the productivity of workfare is given an upper bound. It is further assumed that any resources created by workfare are redistributed back into the economy.

How the government interprets individuals' preferences for labour determines how it will redistribute resources in this economy. Two alternatives are considered. First, the government can interpret a high disutility of labour as a form of disability. In this framework, the worst-off individual in the laissez-faire outcome is the low-ability, high taste for leisure individuals, so  $V^\ell(\hat{c}_i^\ell, \hat{L}_i^\ell) > V^h(\hat{c}_i^h, \hat{L}_i^h)$ , for  $i = 1, 2$ .<sup>17</sup> Second, the government can interpret a high disutility of labour as a high taste for leisure and as a form of 'laziness' on the part of those individuals. In this case, the individual with the lowest

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<sup>16</sup>Non-productive workfare means  $\bar{w} = 0$ .

<sup>17</sup>The ranking of maximized utilities in the laissez-faire outcome becomes  $V^h(\hat{c}_1^h, \hat{y}_1^h) < V^\ell(\hat{c}_1^\ell, \hat{y}_1^\ell) < , > , = V^h(\hat{c}_2^h, \hat{y}_2^h) < V^\ell(\hat{c}_2^\ell, \hat{y}_2^\ell)$ .

utility in the laissez-faire outcome is the low-ability, low taste for leisure individual, so  $V^\ell(\hat{c}_i^\ell, \hat{L}_i^\ell) < V^h(\hat{c}_i^h, \hat{L}_i^h)$ , for  $i = 1, 2$ .<sup>18</sup>

The question this paper addresses is: can the imposition of workfare increase the welfare of the worst-off individuals? In each of the cases considered, the government will transfer income from the high ability individuals to the worst-off individuals. As a result, high-ability individuals have an incentive to mimic. If the government imposed work requirements on these individuals, it would both increase their incentive to mimic, and reduce the potential revenue the government could raise. Therefore, the government will never optimally impose work requirements on the high-ability individuals. In addition, the non-negativity constraint on the high ability individuals' private income will never bind.

To examine the optimality of work requirements for the low ability individuals, it is initially assumed that the government takes the levels of workfare as given ( $d_1^\ell \geq 0$  and  $d_1^h \geq 0$ ) and maximizes the utility (3) of the worst-off individuals with respect to the level of offered benefits,  $b_i^j$ , subject to the government's constraints. Since all individuals have a variable labour supply, by choosing  $b_i^j$ , the government effectively determines private income and consumption,  $y_i^j$  and  $c_i^j$ . By the Envelope Theorem, differentiating the maximized Lagrangian with respect to the given level of workfare determines the effect of a slight increase in work requirements on the maximized welfare of the worst-off individual.<sup>19</sup> Workfare is optimal to impose if this expression is positive when evaluated at  $d = 0$ . Likewise, the optimality of non-productive or productive workfare can be determined by evaluating these expressions at  $d = 0$  and  $\bar{w} = 0$  or  $d = 0$  and  $\bar{w} > 0$ . To simplify the algebra, it is assumed that there are only three types of individuals, low ability individuals with high and low tastes for leisure and a high ability individual with a

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<sup>18</sup>The ranking of maximized utilities in the laissez-faire outcome becomes  $V^\ell(\hat{c}_1^\ell, \hat{g}_1^\ell) < V^h(\hat{c}_1^h, \hat{g}_1^h) < V^\ell(\hat{c}_2^\ell, \hat{g}_2^\ell) < V^h(\hat{c}_2^h, \hat{g}_2^h)$ .

<sup>19</sup>Welfare benefits have been chosen optimally.

given taste for leisure. For notational simplicity, the superscript on the high ability individuals' preferences is suppressed. Their preferences are specified only when they become important for the results.

### 3.1 High Disutility of Labour as a Form of Disability

Under the interpretation that a high disutility of labour is some form of disability, the government's objective is to maximize the utility of the low-ability, high disutility of labour individuals, subject to its resource constraint, non-negativity constraints on the low ability individuals' private income, and the self-selection constraints. In this case, the self-selection constraint between the high ability individuals and the worst-off individuals will never bind and can be excluded from the government's problem. Let  $\delta$ ,  $\tau_j$  for  $j = \ell, h$ , and  $\phi_2$  be the corresponding Lagrange multipliers on the remaining constraints. The Lagrangian for the government's problem is:

$$\begin{aligned} \max_{c_1^h, y_1^h, c_1^\ell, y_1^\ell, c_2, y_2} \quad & \mathcal{L} = V_1^h(c_1^h, y_1^h, d_1^h) \\ & + \lambda \left( N_1^h(y_1^h - c_1^h + \bar{w}d_1^h) + N_1^\ell(y_1^\ell - c_1^\ell + \bar{w}d_1^\ell) + N_2(y_2 - c_2) - \bar{R} \right) \\ & + \tau_\ell y_1^\ell + \tau_h y_1^h + \phi_1 \left( V_1^\ell(c_1^\ell, y_1^\ell, d_1^\ell) - V_1^\ell(c_1^h, y_1^h, d_1^h) \right) \\ & + \phi_2 \left( V_2(c_2, y_2) - V_2(c_1^\ell, y_1^\ell, d_1^\ell) \right) \end{aligned}$$

From the first-order conditions on  $c_2$  and  $y_2$  (shown in the Appendix), the standard result that the high ability individuals face a zero marginal tax rate,  $MRS_{c_2, y_2} = 1$ , is derived. However, depending on the level of work requirement imposed on them, the low-ability individuals can face a positive, negative, or zero marginal tax rate. Let  $\widehat{MRS}_i^j(c, y)$  be the marginal rate of substitution between consumption and income of an individual with ability  $i$  and preferences  $j$  who is mimicking the type of individual who consumes  $c$  and earns income  $y$ . Then, from the first-order conditions,

$$MRS_{c_1, y_1}^\ell = 1 - \frac{\phi_3 MU}{\phi_1 MU^\ell} [1 - \widehat{MRS}_2(c_1^\ell, y_1^\ell)] + \frac{\tau_\ell}{\phi_1 MU^\ell}$$

and

$$MRS_{c_1, y_1}^h = 1 - \frac{\phi_1 MU^\ell}{MU^h} [1 - \widehat{MRS}_1^\ell(c_1^h, y_1^h)] + \frac{\tau_h}{MU^h}$$

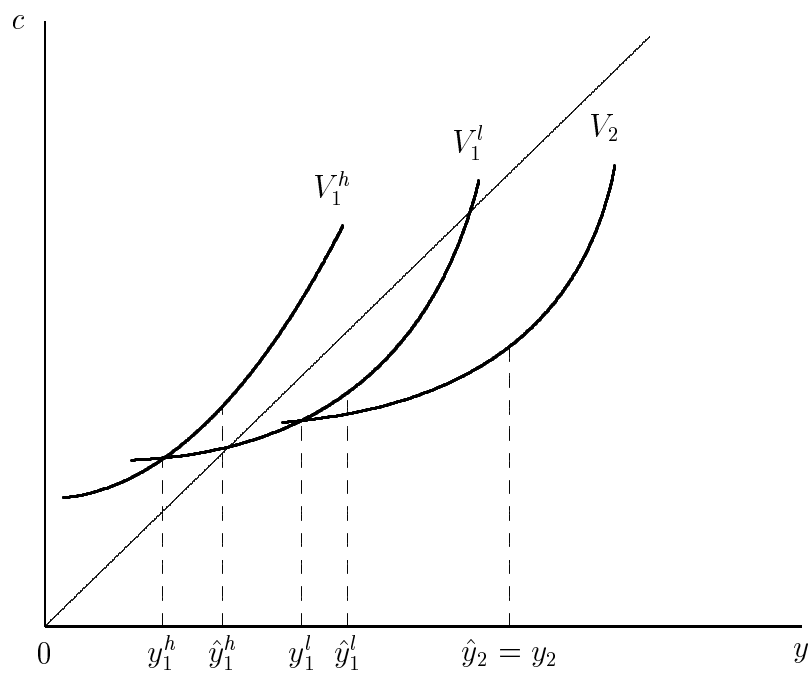
If there are no work requirements, then from the strict convexity of the disutility of labour function, the high-ability individuals supply less than their laissez-faire amount of labour to mimic the low-ability individuals with a low taste for leisure, so  $\widehat{MRS}_2(c_1^\ell, y_1^\ell) < 1$ . This implies that if the low-ability, low taste for leisure individuals earn a positive private income,  $\tau_\ell = 0$ , they will have a positive marginal tax rate,  $MRS_{c_1, y_1}^\ell < 1$ . Likewise, if the non-negativity constraint on the low-ability, disabled individuals' private income does not bind and  $y_1^h \leq \hat{y}_1^\ell$ , then these individuals will also have a positive marginal tax rate.<sup>20</sup> If both non-negativity constraints are slack, there can either be a separating equilibrium, or a low-ability pooling equilibrium, i.e., both type of low-ability individuals receive the same bundle. The separating outcome is illustrated in Figure 2.

It can also be shown that if the non-negativity constraint on the low-ability, low taste for leisure individual binds, so must the non-negativity constraint on the other type of low-ability individuals.<sup>21</sup> If both bind, there will be a pooling equilibrium with the high-ability individuals being indifferent between working their laissez-faire amount of labour or not working at all. This is shown in Figure 3. It is also possible that the high disutility of labour, low-ability individuals optimally do not work and the low-taste for leisure, low-ability individuals do work. The actual outcome will depend on the relative size of the different types of individuals in the population.

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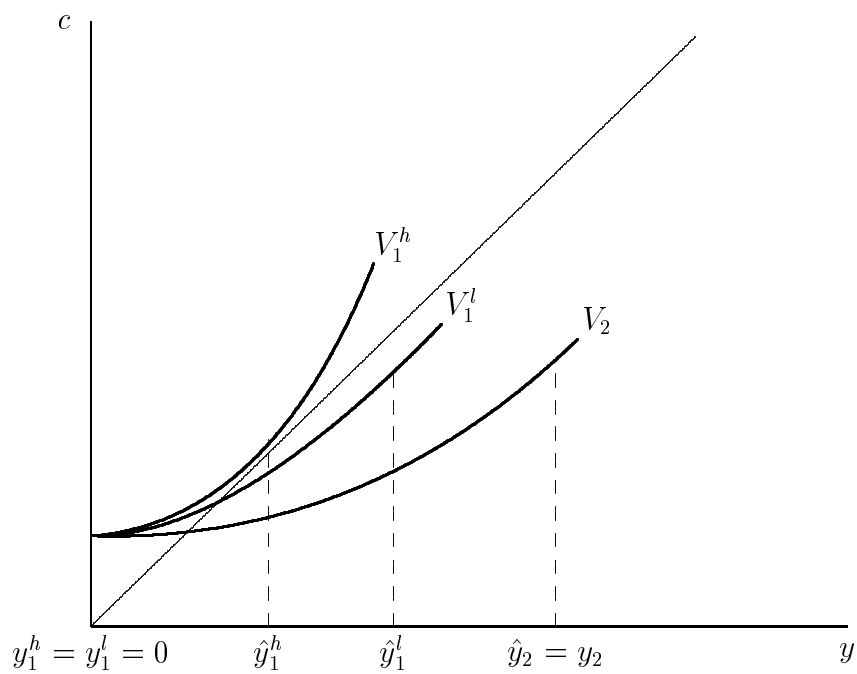
<sup>20</sup>In this case,  $\widehat{MRS}_1^\ell(c_1^h, y_1^h) < 1$  and  $\tau_h = 0$ .

<sup>21</sup>Given  $\phi_1 > 0$  and assuming  $\tau_l > 0$  and  $\tau_h = 0$ , then  $U^l(c_1^l - g^l(0)) = U^l(c_1^h - g^h(\frac{y_1^h}{w_1}))$  and  $U^h(c_1^h - g^h(\frac{y_1^h}{w_1})) > U^h(c_1^l - g^h(0))$ . These two expressions imply  $g^l(\frac{y_1^h}{w_1}) - g^l(0) > g^h(\frac{y_1^h}{w_1}) - g^h(0)$ , which is a contradiction, by the assumptions on the disutility of labour function.



**Figure 2**

*Subsection 3.1:* Separating Equilibrium ( $\tau_h = \tau_l = 0$ )



**Figure 3**

*Subsection 3.1: Pooling Equilibrium ( $\tau_h > 0, \tau_l > 0$ )*

However, in all possible outcomes when  $d = 0$ , the low-ability individuals have a marginal rate of substitution less than one, so they have positive marginal tax rates. As well, the average tax rates can be increasing, decreasing, or constant across individuals of differing tastes and/or ability as it is in the standard case.

For any given  $y_1^\ell > 0$ , the imposition of work requirements on the low-ability, low taste for leisure individuals increases the amount of labour high-ability individuals have to supply to mimic, by the amount of the work requirements.<sup>22</sup> It is possible that  $\widehat{MRS}_2(c_1^\ell, y_1^\ell) > MRS_{c_2, y_2} = 1$  and the low-ability, low disutility of labour individuals face a negative marginal tax rate.

By the Envelope Theorem, differentiating the Lagrangian with respect to the work requirement gives us the effect on social welfare of a change in the level of workfare, given that the level of transfers are optimal. Initially, it is assumed that the government imposes workfare only on the disabled, low-ability individuals whose welfare is being maximized,  $d_1^l = 0$  and  $d_1^h > 0$ .<sup>23</sup> Then,

$$\frac{\partial \mathcal{L}}{\partial d_1^h} = -MU^h g^{h'}\left(\frac{y_1^h}{w_1} + d_1^h\right) + \lambda N_1^h \bar{w} + \phi_1 MU^l g^{l'}\left(\frac{y_1^h}{w_1} + d_1^h\right) \quad (4)$$

To examine the optimality of imposing a positive work requirement, evaluate (4) at  $d_1^h = 0$  and substitute in the first-order condition on  $y_1^h$  to find:

$$\frac{\partial \mathcal{L}}{\partial d_1^h} = \lambda N_1^h (\bar{w} - w_1) - w_1 \tau_h. \quad (5)$$

Regardless of the preferences of the high ability individuals, (5) can never be positive, given the assumptions made. Only if the worst-off individuals were more productive at

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<sup>22</sup>Given that the government can observe the labour supplied at workfare, the high-ability individuals will have to supply  $d_1^\ell + \frac{y_1^\ell}{w_2}$  to mimic.

<sup>23</sup>Throughout the analysis, the optimality of imposing workfare on one type of low-ability individual is examined while assuming that there is no workfare being imposed on the other type. However, the results carry through if there is positive amount being imposed on the other type.

workfare than they are in the private market,  $\bar{w} > w_1$ , could their welfare be increased by imposing work requirements on them.

Suppose instead that the government is initially requiring workfare from the low ability individuals with a low disutility of labour. The government's objective function is unchanged, except  $d_1^h = 0$  and  $d_1^\ell > 0$ . To examine the effect of a change of  $d_1^\ell$  on the welfare of the high disutility of labour, low-ability individuals, differentiate the Lagrangian with respect to  $d_1^\ell$ :

$$\frac{\partial \mathcal{L}}{\partial d_1^\ell} = \lambda N_1^\ell \bar{w} - \phi_1 MU^\ell g^{\ell'}\left(\frac{y_1^\ell}{w_1} + d_1^\ell\right) + \phi_2 MU g'\left(\frac{y_1^\ell}{w_2} + d_1^\ell\right) \quad (6)$$

To examine the optimality of imposing a non-productive work requirement, evaluate (6) at  $d_1^\ell = \bar{w} = 0$ :

$$\frac{\partial \mathcal{L}}{\partial d_1^\ell} = -\phi_1 MU^\ell g^{\ell'}\left(\frac{y_1^\ell}{w_1}\right) + \phi_2 MU g'\left(\frac{y_1^\ell}{w_2}\right) \quad (7)$$

The sign of (7) depends on the preferences of the high-ability individuals. From the first-order condition on  $c_1^\ell$ ,  $\phi_1 MU^\ell = \lambda N_1^\ell + \phi_2 MU \geq \phi_2 MU$ . If the high ability individuals have a low disutility of labour, then  $g^{\ell'}(\frac{y_1^\ell}{w_1}) > g^{\ell'}(\frac{y_1^\ell}{w_2})$ , and (8) is necessarily negative. However, if the high ability individuals have a high disutility of labour, then it is uncertain which term is larger, and therefore, the sign of (7) is ambiguous. Imposing a non-productive work requirement on the low disutility of labour, low-ability individuals can improve the welfare of the worst-off individuals when the high ability individuals also have a high disutility of labour.

To determine the optimality of a productive work requirement, use the first-order condition on  $y_1^\ell$  and evaluate (6) at  $d_1^\ell = 0$  and  $\bar{w} > 0$ :

$$\frac{\partial \mathcal{L}}{\partial d_1^\ell} = \lambda N_1^\ell (\bar{w} - w_1) + \phi_2 MU g'\left(\frac{y_1^\ell}{w_2}\right) \left(1 - \frac{w_1}{w_2}\right) - w_1 \tau_\ell \quad (8)$$

Provided the low-ability, low taste for leisure individuals earn a positive private income,  $\tau_\ell = 0$ , then there will be a critical value of  $\bar{w}$  such that workfare is optimal. The critical value,  $\bar{w}^*$ , is the level that ensures the above expression is zero, and is given by:

$$\bar{w}^* = w_1 - \frac{\phi_2 MU g'(\frac{y_1}{w_2})(1 - \frac{w_1}{w_2})}{\lambda N_1^\ell} \quad (9)$$

The second term on the right-hand side of (9) represents the effect of the imposition of workfare on the self-selection constraint between the high ability and low ability, low disutility of labour individuals and is necessarily non-negative, so  $\bar{w}^* < w_1$ . This is also the necessary condition for workfare to be welfare-improving under the assumption of differing abilities only. Brett (1997) derives this condition in the two-ability case when the government maximizes a general, quasiconcave social welfare function.<sup>24</sup> However, if it is optimal not to have the low-ability, low disutility of labour individuals working in the private labour market, then this critical level will necessarily be higher and possibly greater than  $w_1$ .

In summary, imposing non-productive or productive work requirements on the low-ability individuals with a high disutility of labour makes them worse off. However, they can be made better off if non-productive workfare is required from the other type of low ability individuals when high ability individuals have a high disutility of labour. They can also be made better off, regardless of the preferences of the high ability individuals, when workfare is more productive than some critical level, which can be less than the productivity level of low-ability individuals in the private labour market.

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<sup>24</sup>Following Besley and Coate (1995), Brett assumes  $U$  is linear. He also allows private labour and work requirements to be imperfect substitutes in the disutility of labour function. The condition on the productivity of workfare become more (less) stringent the more (less) onerous work requirements are relative to work in the private market.

### 3.2 High-Taste for Leisure as a Form of ‘Laziness’

In this case, individuals with a high taste for leisure are assumed to be better off in the laissez-faire outcome than individuals of the same ability, but with a lower taste for leisure. The government maximizes the welfare of the low-ability, low-taste for leisure individuals with respect to the three transfer bundles and subject to the resource constraint, the non-negativity constraint on the private income of the low-ability individuals and the self-selection constraints. In this case, the self-selection constraint between the high ability individuals the low-ability, high taste for leisure individuals is not binding and can be ignored. The Lagrangian for the government’s problem is:

$$\begin{aligned} \max_{c_1^\ell, y_1^\ell, c_1^h, y_1^h, c_2, y_2} \quad & \mathcal{L} = V_1^\ell(c_1^\ell, y_1^\ell, d_1^\ell) \\ & + \lambda \left( N_1^h(y_1^h - c_1^h + \bar{w}d_1^h) + N_1^\ell(y_1^\ell - c_1^\ell + \bar{w}d_1^\ell) + N_2(y_2 - c_2) - \bar{R} \right) \\ & + \tau_\ell y_1^\ell + \tau_h y_1^h + \phi_1 \left( V_1^h(c_1^h, y_1^h, d_1^h) - V_1^\ell(c_1^\ell, y_1^\ell, d_1^\ell) \right) \\ & + \phi_2 \left( V_2(c_2, y_2) - V_2(c_1^\ell, y_1^\ell, d_1^\ell) \right) \end{aligned}$$

As in the previous case, the standard result is derived from the first-order conditions on  $c_2$  and  $y_2$  (as shown in the Appendix) that the high ability individuals face a zero marginal tax rate,  $MRS_{c_2, y_2} = 1$ . The expressions for the marginal rate of substitutions of the low-ability individuals are:

$$MRS_{c_1, y_1}^h = 1 + \frac{\tau_h}{\phi_2 MU^h}$$

and

$$MRS_{c_1, y_1}^\ell = 1 - \frac{\phi_1 MU^h}{MU^\ell} [1 - \widehat{MRS}_1^h(c_1^\ell, y_1^\ell)] - \frac{\phi_2 MU}{MU^\ell} [1 - \widehat{MRS}_2(c_1^\ell, y_1^\ell)] + \frac{\tau_\ell}{MU^\ell}$$

If there are no work requirements and the low-ability, high taste for leisure individuals earn a positive private income,  $\tau_h = 0$ , they will also have a zero marginal tax rate. If  $\tau_h > 0$ , then their marginal rate of substitution between consumption and income will be greater than one. However, by definition their marginal rate of substitution evaluated at  $L = 0$  is less than one. They optimally supply their laissez-faire amount of labour when  $d_1^h = 0$ . As in the previous case, if the non-negativity constraint is binding for the low taste for leisure, low-ability individuals, then it must also bind for the high taste for leisure, low-ability individuals.<sup>25</sup> This means the worst-off individuals optimally supply some labour in the private market and the sign of their optimal marginal tax rate faced is ambiguous. It can be positive, negative, or zero. In Figure 4, the case when the low-ability, low-taste for leisure individuals have a negative marginal tax rate is shown. Irrespective of the sign of the marginal tax rate faced by these individuals, the average tax rate for the low-ability individuals is increasing in taste. That is, individuals with a higher taste for leisure have a higher average tax rate. The optimal tax schedule is regressive in this region.

The imposition of workfare increases the marginal rate of substitution of the mimickers and thereby, increases the likelihood that the worst-off individuals face a negative marginal tax rate.

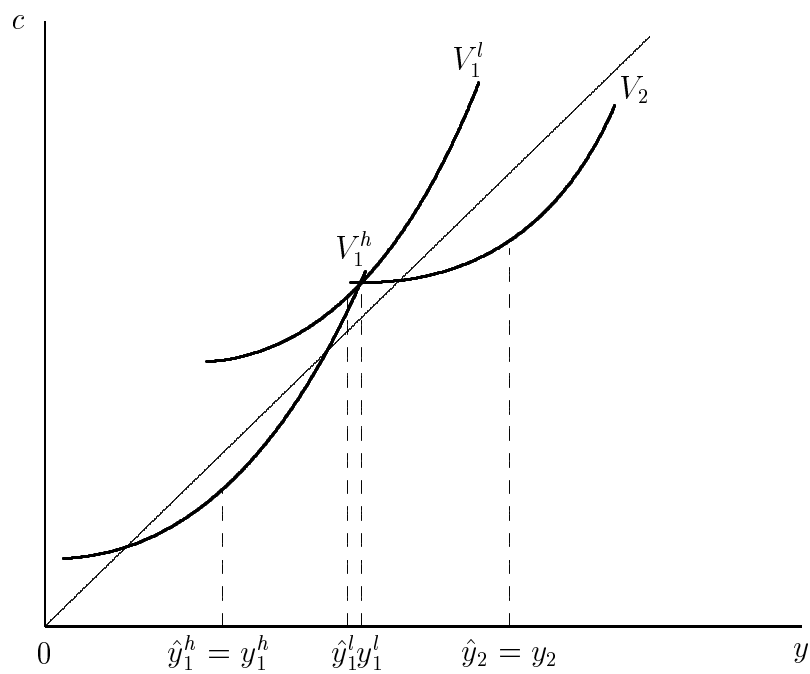
Initially, it is assumed the government imposes workfare only on the worst-off individuals,  $d_1^\ell > 0$  and  $d_1^h = 0$ . Using the Envelope Theorem,

$$\frac{\partial \mathcal{L}}{\partial d_1^\ell} = -MU^\ell g^{\ell'}\left(\frac{y_1^\ell}{w_1} + d_1^\ell\right) + \lambda N_1^\ell \bar{w} + \phi_1 MU^h g^{h'}\left(\frac{y_1^\ell}{w_1} + d_1^\ell\right) + \phi_2 MU g'\left(\frac{y_1^\ell}{w_2} + d_1^\ell\right) \quad (10)$$

Evaluating (10) at  $d_1^\ell = \bar{w} = 0$  determines whether non-productive work requirements

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<sup>25</sup>The proof is given in the previous section.



**Figure 4**

*Subsection 3.2: Separating Equilibrium ( $\tau_h = \tau_l = 0$ )*

can be welfare improving:

$$\frac{\partial \mathcal{L}}{\partial d_1^\ell} = -MU^\ell g^{\ell'}\left(\frac{y_1^\ell}{w_1}\right) + \phi_1 MU^h g^{h'}\left(\frac{y_1^\ell}{w_1}\right) + \phi_2 MU g'\left(\frac{y_1^\ell}{w_2}\right) \quad (11)$$

The sign of (11) is ambiguous, regardless of the preferences of the high ability individuals. From the first-order on  $c_1^\ell$ ,  $\phi_1 MU^h + \phi_2 MU = MU^\ell - \lambda N_1^\ell \leq MU^\ell$ . If the high ability individual has a low taste for leisure, then  $g^{h'}\left(\frac{y_1^\ell}{w_1}\right) > g^{\ell'}\left(\frac{y_1^\ell}{w_1}\right) > g^{\ell'}\left(\frac{y_1^\ell}{w_2}\right)$ . If the high ability individuals have a high disutility of labour, then either  $g^{\ell'}\left(\frac{y_1^\ell}{w_1}\right) \leq g^{h'}\left(\frac{y_1^\ell}{w_2}\right)$ , or  $g^{\ell'}\left(\frac{y_1^\ell}{w_1}\right) \geq g^{h'}\left(\frac{y_1^\ell}{w_2}\right)$ . Non-productive work requirements can be optimal to impose.

This result differs from the one derived by Besley and Coate (1995). In their model, individuals differ with respect to ability only and non-productive workfare is never optimal to impose.<sup>26</sup> Their results can be generated in this model by assuming that the self-selection constraint between the two low-ability individuals does not bind,  $\phi_1 = 0$ , and that the high ability individuals have a low taste for leisure. Expression (11) becomes:

$$\frac{\partial \mathcal{L}}{\partial d_1^\ell} = -MU^\ell g^{\ell'}\left(\frac{y_1^\ell}{w_1}\right) + \phi_2 MU^\ell g'\left(\frac{y_1^\ell}{w_2}\right) \quad (12)$$

which is necessarily negative. The disutility low ability individuals receive from fulfilling positive work requirements, as represented by the first term on the left-hand side of (12), is greater than the utility they receive from the additional resources created by the weakening of the self-selection constraint between individuals of differing ability, as represented by the second term on the left-hand side of (12). Non-productive workfare is never optimal to impose. However, in general, when individuals have different preferences non-productive workfare can be optimal. Work requirements also serve to weaken the self-selection constraint between individuals of differing tastes, as represented by the middle term on the left-hand side of (11).

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<sup>26</sup>They use a utility maintenance model. This is equivalent to assuming a utilitarian social welfare function.

Suppose instead that workfare is productive. To examine the optimality of imposing productive workfare on the low-ability, low taste for leisure individuals, use the first-order condition on  $y_1^\ell$ , and evaluate (10) at  $\bar{w} > 0$  and  $d_1^\ell = 0$ .

$$\frac{\partial \mathcal{L}}{\partial d_1^\ell} = \lambda N_1^\ell (\bar{w} - w_1) + \phi_2 MU g'(\frac{y_1^\ell}{w_2})(1 - \frac{w_1}{w_2}) - w_1 \tau_\ell \quad (13)$$

Given the low-taste for leisure, low-ability individuals optimally supply some private labour,  $\tau_\ell = 0$ , then imposing workfare on them increases their welfare when  $\bar{w} > \bar{w}^*$ , where  $\bar{w}^*$  is given by (9). Productive workfare can be optimal if it is weakening the self-selection constraint between individuals of differing ability.

Suppose the government is initially requiring workfare from the other type of low-ability individuals. The problem is identical to the one above, except  $d_1^\ell = 0$  and  $d_1^h > 0$ . In order to examine the optimality of this policy, the Envelope Theorem is used. By differentiating the Lagrangian with respect to  $d_1^h$ , the effect of a change of  $d_1^h$  on the welfare of the low-ability, low-taste-for-leisure individuals is derived.

$$\frac{\partial \mathcal{L}}{\partial d_1^h} = \lambda N_1^h \bar{w} - \phi_1 MU^h g^{h'}(\frac{y_1^h}{w_1} + d_1^h) \quad (14)$$

Evaluating (14) when  $d_1^h = 0$  and substituting in the first-order condition on  $y_1^h$ , the expression becomes:

$$\frac{\partial \mathcal{L}}{\partial d_1^h} = \lambda N_1^h (\bar{w} - w_1) - w_1 \tau_h \quad (15)$$

Work requirements imposed on the low ability individuals with a high-taste for leisure can never be welfare improving when  $\bar{w} \leq w_1$ .

In summary, imposing non-productive or productive work requirements on the worst-off individuals can improve their welfare. However, they will never be made better off if non-productive or productive work requirements are imposed on the low-ability individuals with a high taste for leisure.

## 4 Discussion

The innovation in this paper is the allowance for individuals to differ with respect to their preferences, as well as with respect to their abilities. What implications does heterogeneity of preferences have for the optimality of workfare in a nonlinear income tax scheme? First is the issue of the interpretation of preferences. It is possible that the government might want to redistribute towards or away from the lowest income earner. To allow for these two possibilities, the government is seen to interpret a high disutility of labour as either a type of disability or a form of ‘laziness’. In each case, the government redistributes between the two types of low-ability individuals with the direction of redistribution being determined by the government’s interpretation of preferences. Blackorby and Donaldson (1988) showed that along the second-best pareto efficiency frontier,<sup>27</sup> when individuals differ with respect to their preferences, self-selection constraints between the two types of individuals can only bind in one direction. They also show it can bind in either direction along the pareto efficiency frontier. However, in order to select the optimal point on the frontier some interpersonal comparison of the individuals utility must be made. The different interpretations of preferences is used to motive the comparison of utilities and therefore, the direction of redistribution.

Second is the issue of who should be required to work. As argued above, the government will never optimally impose workfare on the high ability individuals. When individuals differ in ability only, the choice facing the government is whether or not to impose workfare on the low-ability individuals. When low-ability individuals have different preferences, the choice then becomes whether or not to impose workfare on low-ability individuals, and on which type of low-ability individual to impose it upon. It was shown

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<sup>27</sup>This is constructed by maximizing the utility of one type of individual subject to giving the other type some level of utility and given imperfect information about individuals’ types. By varying this constraint, one can trace out the second-best pareto efficiency frontier.

that requiring workfare from low-ability, high-disutility of labour individuals is never optimal, but that it can be optimal to impose workfare on low-ability individuals with a low disutility of labour.

Insight into the above results can be gained by considering what workfare is effectively doing. Recall, any increase in the amount of resources available to be redistributed to the worst-off individuals always makes them better off. There are two ways workfare can make this happen. First, workfare is productive and creates additional resources. Second, workfare weakens the self-selection constraints between different types of individuals, allowing the government to extract more revenue from the high-ability persons. This implies that if workfare is not productive, then the only way it can increase welfare is if the second condition holds.

The third implication of heterogeneity of preferences is that workfare is never optimal when all individuals have the same ability, unless they are more productive at workfare than they are in the private market. This is the result derived by Beaudry and Blackorby (1997). In their model, individuals differ with respect to two unobservables, home sector and formal sector productivities,<sup>28</sup> and the government can observe both income earned and hours worked in the formal (tax-paying) sector. Effectively, the only unobservable is home sector productivities, which can be given the interpretation as taste for leisure. They show that work requirements can only be optimal if individuals are more productive at workfare than they are in the formal sector.

To see this result, assume that the self-selection constraints between the high ability and low ability individuals never bind,  $\phi_2 = 0$  in subsection 3.1 and 3.2. Then the Lagrangians with respect to the levels of workfare when the government imposes workfare on either type of the low-ability individuals, and in both subsections reduce to:

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<sup>28</sup>Individuals are endowed with a given amount of time. In the laissez-faire outcome, they spend all of this time in the sector for which they have a higher productivity.

$$\frac{\partial \mathcal{L}}{\partial d_1^j} = \lambda N_1^j (\bar{w} - w_1) - w_1 \tau_j \quad (16)$$

These expressions hold for any level of workfare  $d_1^j \geq 0$ . A necessary condition for them to be positive is that the low ability individuals have a higher productivity in workfare than they do in the private market. Workfare is never optimal to impose when individuals differ only with respect to their tastes for labour, irrespective of the amount of private labour they supply. In this case, all individuals have the same opportunity cost of supplying labour for workfare and workfare cannot weaken the self-selection constraint between the two types of individuals.

This paper shows that workfare can separate individuals of the same taste, when it is also being used to separate individuals of differing ability. From (11), it can be seen that imposing non-productive workfare on low-ability, low taste for leisure individuals reduces the incentive of both the low-ability, high taste for leisure individuals and the high-ability individuals to mimic them.

In summary, when individuals differ with respect to both ability and preferences, and the government interprets a high disutility of labour as some form of disability, then imposing non-productive or productive workfare on the worst-off individuals never makes them better off. On the other hand, if the government interprets a high disutility of labour as a form of ‘laziness’, then imposing non-productive work requirements on the low-ability, low-taste for leisure individuals can increase their welfare. However, their welfare will decrease if non-productive or productive work requirements are imposed on the low-ability individuals with a high taste for leisure.

Using either interpretation of disutility of labour, it was shown that productive workfare can be optimal to use in a non-linear, income taxation framework only if it imposed on individuals with a low disutility of labour, or if low-ability individuals are more productive at workfare than they are in the private labour market.

## 5 Conclusion

When the government cares about the disutility that individuals receive from working and individuals have different preferences, then workfare can be optimal only if it is imposed on individuals with a lower disutility of labour. It was assumed that workfare can never be more productive than the private sector productivity of the individuals it is imposed upon. If this was not the case, then workfare would always be optimal to implement even under perfect information.<sup>29</sup>

The results derived in this paper suggest greater investigation is needed into how individuals actually differ. If one could argue that the majority of potential welfare recipients have similar educational backgrounds, then workfare will not screen individuals with different preferences. In this case, instituting workfare in the welfare system will only increase administrative costs. Likewise, individuals will not be made better off under the realistic assumption that individuals are not any more productive in workfare programs, which typically involve menial or community service jobs, than they are in the private market.

The issue of the interpretation of disutility of labour has also been raised. When individuals have different preferences and the government is welfaristic then some form of interpersonal comparison of utilities must be made. Instead of assuming a complete ordering of utilities, this paper used the possible interpretations of a high disutility of labour to motivate the cardinalization of laissez-faire utilities. By assuming a complete aversion to inequality, the extreme case of redistribution was examined under the different interpretations of preferences. However, adopting any other quasi-concave objective function would not change the results. These results are interesting, especially in lieu of the publicized belief that individuals on welfare are ‘lazy’ and requiring them to work will force them back into the formal sector. This paper showed that even when the government

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<sup>29</sup>In this case, forcing people to work generates a greater amount of resources in the economy. Therefore, the government will always want to impose workfare.

does not want to redistribute income to such individuals it is never optimal to impose workfare on them.

On the other hand, if these individuals are believed to be disabled, the government would also not want to impose workfare on them. However, this paper has ignored the existence of some formal ‘tagging’ mechanism to identify disabled individuals. In most welfare systems there exists such a mechanism and it would be interesting to model it within a workfare program to see if ‘tagging’ individuals reduces the cost of the transfer system and increases welfare. In addition, it would be interesting to see if the results change by allowing for workfare that enhances participants’ earning ability through job-training and education.

The possibility that individuals earn unobservable income, or collect more than one welfare cheque has also been ignored. This is typical of the fraudulent behaviour in the welfare system that the government is concerned about, and it would be illuminating to have a model that could account for these activities.

## 6 Appendix

### Subsection 3.1: Government's Problem

$$\begin{aligned}
\max_{c_1^h, y_1^h, c_1^\ell, y_1^\ell, c_2, y_2} \mathcal{L} = & U^h \left( c_1^h + g^h \left( \frac{y_1^h}{w_1} + d_1^h \right) \right) \\
& + \lambda \left( N_1^h (y_1^h - c_1^h + \bar{w}d_1^h) + N_1^\ell (y_1^\ell - c_1^\ell + \bar{w}d_1^\ell) + N_2 (y_2 - c_2) - \bar{R} \right) \\
& + \tau_h y_1^h + \tau_\ell y_1^\ell + \phi_1 \left( U^\ell \left( c_1^\ell - g^\ell \left( \frac{y_1^\ell}{w_1} + d_1^\ell \right) \right) - U^h \left( c_1^h - g^h \left( \frac{y_1^h}{w_1} + d_1^h \right) \right) \right) \\
& + \phi_2 \left( U \left( c_2 - g \left( \frac{y_2}{w_2} \right) \right) - U \left( c_1^\ell - g \left( \frac{y_1^\ell}{w_2} + d_1^\ell \right) \right) \right)
\end{aligned}$$

The first-order conditions are:

$$\begin{aligned}
c_1^h : MU^h - \lambda N_1^h - \phi_1 MU^\ell &= 0 \\
y_1^h : -MU^h g^{h'} \left( \frac{y_1^h}{w_1} + d_1^h \right) \frac{1}{w_1} + \lambda N_1^h + \phi_1 MU^\ell g^{\ell'} \left( \frac{y_1^\ell}{w_1} + d_1^\ell \right) \frac{1}{w_1} + \tau_h &= 0 \\
c_1^\ell : -\lambda N_1^\ell + \phi_1 MU^\ell - \phi_2 MU &= 0 \\
y_1^\ell : \lambda N_1^\ell - \phi_1 MU^\ell g^{\ell'} \left( \frac{y_1^\ell}{w_1} + d_1^\ell \right) \frac{1}{w_1} + \phi_2 MU g' \left( \frac{y_2}{w_2} + d_1^\ell \right) \frac{1}{w_2} + \tau_\ell &= 0 \\
c_2 : -\lambda N_2 + \phi_2 MU &= 0 \\
y_2 : \lambda N_2 - \phi_2 MU g' \left( \frac{y_2}{w_2} + d_1^\ell \right) \frac{1}{w_2} &= 0
\end{aligned}$$

### Subsection 3.2: Government's Problem

$$\begin{aligned}
\max_{c_1^\ell, y_1^\ell, c_1^h, y_1^h, c_2, y_2} \mathcal{L} = & U^\ell \left( c_1^\ell + g^\ell \left( \frac{y_1^\ell}{w_1} + d_1^\ell \right) \right) \\
& + \lambda \left( N_1^h (y_1^h - c_1^h + \bar{w}d_1^h) + N_1^\ell (y_1^\ell - c_1^\ell + \bar{w}d_1^\ell) + N_2 (y_2 - c_2) - \bar{R} \right) \\
& + \tau_\ell y_1^\ell + \tau_h y_1^h + \phi_1 \left( U^h \left( c_1^h - g^h \left( \frac{y_1^h}{w_1} + d_1^h \right) \right) - U^\ell \left( c_1^\ell - g^\ell \left( \frac{y_1^\ell}{w_1} + d_1^\ell \right) \right) \right) \\
& + \phi_2 \left( U \left( c_2 - g \left( \frac{y_2}{w_2} \right) \right) - U \left( c_1^\ell - g \left( \frac{y_1^\ell}{w_2} + d_1^\ell \right) \right) \right)
\end{aligned}$$

*The first-order conditions are:*

$$c_1^\ell : MU^\ell - \lambda N_1^\ell - \phi_1 MU^h - \phi_2 MU = 0$$

$$y_1^\ell : -MU^\ell g^{\ell'}\left(\frac{y_1^\ell}{w_1} + d_1^\ell\right)\frac{1}{w_1} + \lambda N_1^\ell + \phi_1 MU^h g^{h'}\left(\frac{y_1^\ell}{w_1} + d_1^\ell\right)\frac{1}{w_1} + \phi_2 MU g'\left(\frac{y_1^\ell}{w_2} + d_1^\ell\right)\frac{1}{w_2} + \tau_\ell = 0$$

$$c_1^h : -\lambda N_1^h + \phi_1 MU^h = 0$$

$$y_1^h : \lambda N_1^h - \phi_1 MU^h g^{h'}\left(\frac{y_1^h}{w_1} + d_1^h\right)\frac{1}{w_1} + \tau_h = 0$$

$$c_2 : -\lambda N_2 + \phi_2 MU = 0$$

$$y_2 : \lambda N_2 - \phi_2 MU g'\left(\frac{y_2}{w_2}\right)\frac{1}{w_2} = 0$$

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