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The Consequences of Overlapping Tax Bases for Redistribution  
and Public Spending in a Federation

by

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# **The Consequences of Overlapping Tax Bases for Redistribution and Public Spending in a Federation**

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## **Abstract**

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Tax and expenditure policies are studied in a federation with imperfectly mobile households. States implement a linear progressive tax and supply a public good. A vertical fiscal externality, reflecting the effect of state policies on federal revenues, provides an incentive for state taxes to be too progressive. A horizontal fiscal externality causes non-optimal states taxes and expenditures because of the migration effect. The federal government implements its own linear progressive tax and makes transfers to the states. The federal government can nullify both externalities by appropriate fiscal policies, and redistributive taxation can be decentralized to the states.

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## 1. INTRODUCTION

It has long been recognized that in federal systems, lower-level (e.g., state) governments acting as Nash competitors with respect to one another may take tax and expenditure decisions that lead to non-optimal outcomes. This occurs because of the mobility of tax bases across state boundaries: if one state increases its tax rate on a given base, the state's tax base will fall not just because of elasticity in the supply of the base, but also because of cross-border mobility. This loss in tax base to neighboring states can be thought of as a horizontal fiscal externality. The marginal cost of public funds perceived by each state will be higher than that to the nation as a whole, so states will have a tendency to set tax rates and expenditure levels too low.<sup>1</sup> The result can either be inefficiency in the allocation of resources across and within jurisdictions or non-optimal redistributive policies. In these models of fiscal competition, one role for the federal government is to implement policies that undo the non-optimalities arising from decentralized state decision-making. For example, Wildasin (1991) constructs a simple model of redistribution from rich to poor within states, and shows that with costless mobility of the poor, inefficient state redistribution occurs. He shows how a system of matching grants from a federal government to state governments can neutralize the effect of migration on state redistribution policies.

What has been less recognized is that inter-jurisdictional spillovers can also result from the vertical interaction of governments in a federation. The existence of spillovers between, say, state and federal governments was first pointed out by Johnson (1988). He noted that part of the cost of an increase in the use of redistributive income taxation by a state will be borne indirectly by residents of other states by virtue of their being federal taxpayers. That is, when a state increases its rate of income tax, its tax base and therefore the federal tax liabilities of its residents will decline, thereby putting more of the tax burden on residents of other states. He uses this result to argue that state residents will therefore prefer redistribution to be undertaken by the state rather than the federal government. The implications of this phenomenon for the optimality of resource allocation in a federation were not addressed since he did not provide a full-blown equilibrium analysis

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<sup>1</sup> See for example, the general treatments in Wildasin (1986), Bewley (1981), Gordon (1983), and Rubinfeld (1987).

of decision-making in a federation.

That vertical policy spillovers can cause inefficient decision-making by state and federal governments has recently been recognized. Dahlby (1994, 1996) has argued that, in contrast with the case of horizontal fiscal externalities, both federal and state governments will underestimate the marginal cost of public funds associated with raising revenues because they will neglect the adverse effects of increases in their own tax rates on total revenues of the other level of government. The implication is that this will lead to the size of government being too large. Dahlby implicitly assumes that both levels of government behave as Nash competitors, and does not analyze in detail the interaction of the two levels of government. Boadway and Keen (1996) analyze the equilibrium outcome that would occur in a federation in which the federal and state governments each finance their own public good using taxes levied on the same income tax base. Their focus is on the optimal structure of federal-state transfers (especially the size of the ‘fiscal gap’), and they assume for the most part that the federal government is a Stackelberg leader.<sup>2</sup> However, their analysis focuses on efficiency issues alone by assuming that all individuals are identical. In that context, they obtain a result that is well-known in the fiscal federalism literature: horizontal externalities arising from the mobility of households (even perfect mobility) are nullified. This is because states that maximize the per capita utility of their own residents effectively also act as if they were maximizing per capita utility across the federation. Myers (1990) refers to this as ‘incentive equivalency’.

In actual economies, households are heterogeneous, and much of what governments do is redistributive in nature. Moreover, in federations both upper and lower levels of government implement policies that are redistributive in effect, if not in intent. Federal governments typically deploy progressive income tax-transfer systems. In many federations, state governments or their equivalent often co-occupy the income tax field, examples of which include Canada and the United States. Even if they do not levy progressive taxes,

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<sup>2</sup> Keen (1995) extends the analysis to the case where the government is a revenue-maximizing Leviathan rather than being a benevolent social welfare maximizer. Sato (1997) provides a general analysis of state and federal fiscal policies in a federal economy consisting of identical but imperfectly mobile households and heterogeneous states when governments are restricted to distorting taxes.

they nonetheless indirectly fulfill a redistributive role through their taxes and expenditures combined. Thus, states often supply quasi-private services in areas of education, health and welfare which, when financed by proportional taxation (e.g., sales or payroll taxes), have redistributive consequences analogous to linear progressive taxes. In these circumstances, both horizontal and vertical fiscal externalities will be effective. The purpose of this paper is to analyze state behavior in the face of these externalities, and to consider how the federal government might design its fiscal policies to counter the adverse incentives faced by the states.

We construct a model of a federation in which one of the main roles of government is to redistribute income, and in which that role is shared between federal and state levels. The model is essentially an extension of the standard linear income tax analysis to the setting of a federation, allowing for the fact that governments might also need to finance spending on public goods. We initially consider a federation consisting of several identical states, each of which has a given initial distribution of population by ability-type. Households are mobile between states, though subject to a psychic cost of moving that varies across households. The optimal policies characterizing the so-called unitary nation allocation are derived in Section 1 and serve as a benchmark in the following. Decision rules for taxation and public spending can be interpreted as variants of the standard marginal cost of public funds expressions, adjusted to include redistributive effects. We next characterize decentralized allocations, concentrating on the simple case in which states are homogeneous. State choices of a linear tax schedule and public spending levels are derived in Section 3 and compared with the unitary nation optimum. These will reflect both horizontal and vertical fiscal externalities. Then, the federal government's choice of its own policies is analyzed in Section 4. It is shown how the federal government can choose its tax rate and the level of transfers it makes to households and state governments in order to induce the unitary nation optimum. Concluding comments and extensions are given in Section 5.

## **2. THE MODEL AND THE UNITARY NATION BENCHMARK**

The federations we consider consist of two levels of government, a federal level and a state one, where each level of government has independent taxing and spending authority. We

assume that the states are identical in all relevant respects. In particular, the size and ability distributions of their populations are the same. We discuss later a case in which states are heterogeneous in their ability distributions; a more detailed treatment may be found in a background discussion paper available on request. This section sets out the basic features of the economy and the responsibilities of the public sector. It is useful to begin with the case in which there is only one level of government, the central one. The second-best allocation in this unitary nation will serve as a useful benchmark against which to compare the outcomes when both federal and state governments have fiscal powers.

Each state is endowed with  $N$  native individuals. As discussed below, individuals can move from one state to another, though at some non-pecuniary cost that can vary among individuals. Apart from these migration cost differences, individuals differ only in their ability to produce output, expressed in efficiency units of labour per hour of work. There are  $n$  such ability types. Let  $w^i$  denote the ability of type  $i$  individuals ( $i = 1, \dots, n$ ) with  $w^{i+1} > w^i$ , and  $N^i$  the number of type  $i$  individuals in each state (with  $\sum_i N^i = N$ ). Because we assume that states are homogeneous and the equilibria of interest are all symmetric, we can suppress the indexing of individuals by state. Individuals supply labour which is used in the production of a composite private good and of a state public good according to a linear technology transforming one efficiency unit of labour in one unit of either the private good or the public good.<sup>3</sup> With the private good used as numeraire, competition in the labour and good markets implies that ability  $w^i$  also stands for the wage rate earned by type  $i$  individuals.

An individual of type  $i$  earns gross income  $Z^i \equiv w^i L^i$ , where  $L^i$  denotes the number of hours spent at work. Earned income is subject to a linear progressive income tax whose parameters can depend on the state in which the individual resides. Consider individuals who are native of a representative state. Let  $T$  and  $A$  be the marginal tax rate and the lump-sum transfer that characterize the tax system in the representative state, and let  $\hat{T}$  and  $\hat{A}$  be the corresponding values of the tax parameters in another state to which

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<sup>3</sup> National public goods could be introduced, but this would complicate the analysis with little additional insight to the effects of decentralization examined in later sections. Our focus is on the inefficiency of state fiscal policies and the reaction of the federal government to that. National public goods have no particular role to play in that regard.

individuals might migrate. Since we are dealing with a symmetric equilibrium, we need not distinguish among the various other states. If an individual stays in the representative state, net income is equal to  $(1 - T)Z^i + A$ ; otherwise, it is equal to  $(1 - \hat{T})\hat{Z}^i + \hat{A}$ . All individuals have the same concave utility function,  $U(C^i, L^i) + B(g)$ , where  $C^i$  is the consumption of the private composite good and  $g$  is the provision of the public good in the state of residence ( $U_C > 0, U_L < 0, B' > 0$ ). So, individuals who reside in the representative state solve the following maximization problem:

$$\max_{\{L^i\}} U((1 - T)w^i L^i + A, L^i). \quad (1)$$

The solution to (1) determines their labour supply function,  $L^i(A, T)$ , and thus their labour income,  $Z^i(A, T) = w^i L^i(A, T)$ . Throughout the paper, we assume that leisure is non-inferior, implying  $Z_A^i \leq 0$ , and that the labour supply curves are upward-sloping, implying  $Z_T^i \leq 0$  and  $Z^{i+1} \geq Z^i$ . The maximum value function for problem (1) is the indirect utility function,  $V^i(A, T) + B(g)$ . Applying the envelope theorem yields:  $V_A^i = U_C^i > 0$  and  $V_T^i = -Z^i V_A^i < 0$ . If the individual were to migrate, indirect utility would be equal to  $V^i(\hat{A}, \hat{T}) + B(\hat{g})$ , with  $\hat{g}$  being the state public good supplied in the destination state.

To determine the households' state of residence, we use a variant of the 'attachment to home' model of Mansoorian and Myers (1993). Individuals have a psychological attachment to their native state which leads to some non-pecuniary disutility from migrating to any other state. This disutility varies across individuals within each ability group. Ranking individuals by increasing order of migration disutility, we denote by  $K_i(M^i)$  the disutility of the  $M^i$ th individual of type  $i$  leaving the representative state. We assume that  $K^i$  is strictly increasing in  $M^i$  ( $K_i' > 0$ ), with  $K_i(0) = 0$ , and that  $K_i(-M^i) = -K_i(M^i)$ . This allows us to state the migration equilibrium condition that determines  $M^i$ , the number of type  $i$  individuals migrating out of the representative state, as:

$$V^i(A, T) + B(g) = V^i(\hat{A}, \hat{T}) + B(\hat{g}) - K_i(M^i). \quad (2)$$

The value of  $M^i$  satisfying this equation can be negative, implying that there is a positive inflow into the representative state (recalling that  $K_i(M^i) = -K_i(-M^i)$ ). The solution



to (2) provides a migration function,  $M^i(A, T, g, \hat{A}, \hat{T}, \hat{g})$ , whose comparative statics can easily be derived. The effects of changes in the fiscal parameters of the home and other states on the outflow of migrants are:

$$M_A^i = -\frac{V_A^i}{K_i'} < 0, \quad M_T^i = -Z^i M_A^i > 0, \quad M_g^i = -\frac{B'}{K_i'} < 0 \quad (3.1)$$

$$M_{\hat{A}}^i = \frac{\hat{V}_A^i}{K_i'} > 0, \quad M_{\hat{T}}^i = -\hat{Z}^i M_{\hat{A}}^i < 0, \quad M_{\hat{g}}^i = \frac{\hat{B}'}{K_i'} > 0. \quad (3.2)$$

These properties, which all accord with intuition, will be used in the following sections.

We now turn to the fiscal decisions that a unitary central government would take in the absence of any fiscal responsibilities at the state level. Throughout the paper, public authorities are assumed to use the maxi-min (or Rawls) criterion, which here means that they simply maximize the utility of type 1 individuals. This simplifies the analysis without affecting the main results. From the perspective of a unitary nation government, all states are alike. The unitary nation optimum will therefore be a symmetric equilibrium with no migration. This implies that we can characterize the unitary nation optimum by focusing on the choice of optimal fiscal parameters for a representative state, deleting any reference to inter-state migration. The unitary nation optimum satisfies:

$$\max_{\{A, T, g\}} V^1(A, T) + B(g) \quad (4)$$

$$\text{subject to } N(T\bar{Z} - A) - g = 0, \quad (5)$$

where  $\bar{Z}$  denotes the average per capita before-tax income,  $\bar{Z} = N^{-1} \sum_i N_i Z^i$ .

The solution to this optimization problem yields, after straightforward substitution, the following necessary conditions:

$$Z^1 = \frac{\bar{Z} + T\bar{Z}_T}{1 - T\bar{Z}_A} \quad (6)$$

and

$$\frac{B'}{Z^1 V_A^1} = \frac{1}{N(\bar{Z} + T\bar{Z}_T)}. \quad (7)$$

Together with (5), these determine the optimal unitary nation fiscal policy, which we denote by  $A^*$ ,  $T^*$ , and  $g^*$ . This policy will be used as a benchmark in the next sections.

For given  $g$ , condition (6) can be given a geometrical representation which will be useful in what follows. To this end, let us first note that (for given  $g$ ) a *tax possibility frontier* relating  $A$  to  $T$  can be drawn in  $(A, T)$ -space. This frontier is the locus of pairs  $(A, T)$  that satisfy the public sector budget constraint (5):  $F(A, T, g) \equiv N(T\bar{Z} - A) - g = 0$ . Its slope is given by:

$$-\frac{F_T}{F_A} = \frac{\bar{Z} + T\bar{Z}_T}{1 - T\bar{Z}_A} \quad (8)$$

the denominator of which is positive by the assumption that leisure is non-inferior ( $\bar{Z}_A < 0$ ). As a result, the tax possibility frontier will either slope upwards or downwards according to the sign of  $\bar{Z} + T\bar{Z}_T$ . If the tax possibility frontier is strictly concave, it will initially have a positive slope in  $(A, T)$ -space and then a negative slope corresponding to whether  $(A, T)$  lies on the left-hand or right-hand side of the Laffer curve. Generally the frontier need not be concave, but Romer (1975) shows that it is concave for the Cobb-Douglas utility function. It is straightforward to show that it is also concave for a utility function that is quasi-linear in labour. Concavity of the tax possibility frontier is assumed for the analyses to follow. A concave tax possibility frontier, labelled TPF, is represented in Figure 1. We could also draw in this space social indifference curves, which here are simply indifference curves for type 1 individuals whose utility is being maximized under the Rawlsian criterion. Their slope is given by  $-V_A^1/V_T^1 = Z^1(A, T)$  and they are concave since

$$\left. \frac{dZ^1}{dT} \right|_{V^1} = \frac{\partial Z^1}{\partial T} + \frac{\partial Z^1}{\partial A} \frac{dA}{dT} \Big|_{V^1} = Z_T^1 + Z_A^1 Z^1 < 0$$

is the substitution effect, which is always negative.

Condition (6) indicates that, at the unitary nation optimum,  $A$  and  $T$  are chosen so as to equate the slope of the type 1 individuals' indifference curve (the left-hand side of (6)) with that of the tax possibility frontier (its right-hand side). The pair  $(A^*, T^*)$ , where a social indifference curve labelled SIC is tangential to the tax possibility frontier, satisfies

this requirement in Figure 1. It is necessarily located in the increasing portion of the tax possibility frontier. A similar geometric interpretation could be given to condition (7), now in the  $(T, g)$ -space for given  $A$ .

The optimality conditions (6) and (7) can be given a familiar interpretation we shall refer to later on. Condition (6) can be rewritten as follows:

$$\frac{\bar{Z}}{Z^1} = (1 - T\bar{Z}_A) \left(1 + T\frac{\bar{Z}_T}{\bar{Z}}\right)^{-1}. \quad (9)$$

The ratio on the left-hand side of this expression can be interpreted as a measure of the social benefit of further redistribution. For the most-deserving individuals (type 1's), the benefit of a rise in  $T$  is given by  $dA = \bar{Z}dT$ , while its cost amounts to  $Z^1dT$ . At the maxi-min optimum, the social benefit of further redistribution is set equal to its marginal cost. The latter is the net revenue that must be raised to finance a further increment of  $A$ ,  $(1 - T\bar{Z}_A)$ , multiplied by the standard marginal cost of public funds (MCPF),  $(1 + T\bar{Z}_T/\bar{Z})^{-1}$ . Assuming  $\bar{Z}_T < 0$  and  $\bar{Z}_A < 0$ , those factors are both greater than unity.

Turning to condition (7), it can be written as:

$$\pi^1 \frac{\bar{Z}}{Z^1} = \frac{1}{N} \left(1 + T\frac{\bar{Z}_T}{\bar{Z}}\right)^{-1} \quad (10)$$

where  $\pi^1 = B'/V_A^1$  is the type 1 individuals' marginal willingness to pay for the state public good  $g$ . This is a Samuelson-type formula with some non-standard features. Only the type 1 individuals' willingness to pay matters due to the use of the maxi-min criterion, and this willingness to pay is weighted by  $\bar{Z}/Z^1$  ( $> 1$ ) to reflect the fact that, in this model, public goods provision acts like a redistributive instrument. At the optimum, this weighted willingness to pay is set equal to the MCPF divided by  $N$  because the cost of the public good is shared among all  $N$  persons in each state.

### 3. DECENTRALIZATION OF RESPONSIBILITIES TO THE STATES

In the previous section, the optimal fiscal policy was characterized for a nation consisting of several identical states as if there were only one central government for the nation. Using this policy as a benchmark, we now examine the implications of decentralizing fiscal

responsibilities to the states and simultaneously giving some instruments to the federal government for influencing the outcome. We retain the assumption of identical states. The federal and state governments are assumed to finance their expenditures by means of a proportional tax on the same tax base. In our simple static model, taxing the same base could be given various interpretations. For example, it could be interpreted as a direct tax on labour income by the federal government and as an indirect tax on consumption by the state governments. Let  $t$  and  $\tau$  be the rates of this proportional tax at the federal and state levels respectively. At the federal level, tax revenue is assumed to be used for a lump-sum transfer to all individuals in the nation and a lump-sum transfer to every state, which we denote respectively by  $a$  and  $s$ . At the state level, tax revenue is used for a lump-sum transfer to every state resident, which we denote by  $\alpha$ , and for the provision of the local public good,  $g$ .<sup>4</sup> At the end of this section, we will restrict the use of the state tax proceeds to the supply of the public good. By definition,  $T \equiv t + \tau$  and  $A \equiv a + \alpha$  denote the total proportional tax rate and the total lump-sum transfer faced by individuals respectively.

In the present section, we investigate the behaviour of each state government, given the fiscal policies of the federal government and the other states. It is assumed throughout that there are enough states such that each one behaves as a Nash competitor vis-à-vis both all other states and the federal government. That is, they take all fiscal parameters of other governments as given. Of course, since all states are identical, so are their tax policies in equilibrium. As explained in the introduction, Nash behaviour of the states combined with the overlap of tax bases with the federal government creates vertical fiscal externalities of a sort that has been documented elsewhere in the literature (Johnson, 1988; Boadway and Keen, 1996). And, the mobility of households results in well-known horizontal fiscal externalities among states (Zodrow and Meiskowski, 1986; Wildasin, 1991; Keen and Kotsiogiannis, 1995). Vertical fiscal externalities provide an incentive for states to set their tax rates too high, while horizontal fiscal externalities provide the opposite incentive

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<sup>4</sup> Thus, we follow the convention of using upper-case Roman characters to refer to total tax parameters, while lower-case Roman and Greek characters refer to federal and state tax parameters respectively.

(Dahlby, 1996). Our analysis of state behaviour shows the consequences of these conflicting fiscal externalities for the extent of redistribution and public good supply in a decentralized federation. This sets the stage for the analysis of the optimal federal government behaviour in the next section. Since there is only one federal government alongside many states, the federal government will be assumed to act as a Stackelberg leader.

The assumption of identical states allows us to focus on a representative state government's optimal fiscal policy (given that the federal government does not need to differentiate its own policy across states). For given federal policy  $(a, t, s)$ , the problem for the representative state is:<sup>5</sup>

$$\max_{\{\alpha, \tau, g\}} V^1(a + \alpha, t + \tau) + B(g) \quad (11)$$

subject to

$$\sum_{i=1}^n (N^i - M^i)(\tau Z^i - \alpha) + s - g = 0, \quad (12)$$

where we have  $Z^i = Z^i(a + \alpha, t + \tau, g)$  and  $M^i = M^i(a + \alpha, t + \tau, g, a + \hat{\alpha}, t + \hat{\tau}, \hat{g})$  with  $(\hat{\alpha}, \hat{\tau}, \hat{g})$  being the other states' fiscal policies, taken as given by the representative state (Nash behaviour). Since, in equilibrium, all states behave identically, any one state will perceive  $(\hat{\alpha}, \hat{\tau}, \hat{g})$  as being the same for all other states.

The first-order conditions for the representative state's problem can be written as:

$$Z^1 = \frac{\sum_{i=1}^n (N^i - M^i)(Z^i + \tau Z_T^i) - \sum_{i=1}^n (\tau Z^i - \alpha) M_T^i}{\sum_{i=1}^n (N^i - M^i)(1 - \tau Z_A^i) + \sum_{i=1}^n (\tau Z^i - \alpha) M_A^i} \quad (13)$$

and

$$\frac{B'}{Z^1 V_A^1} = \frac{1 + \sum_{i=1}^n (\tau Z^i - \alpha) M_g^i}{\sum_{i=1}^n (N^i - M^i)(Z^i + \tau Z_T^i) - \sum_{i=1}^n (\tau Z^i - \alpha) M_T^i}. \quad (14)$$

Together with budget constraint (12), these two conditions define the representative state's reaction functions in terms of the federal and other states' fiscal policies. As states are identical, their policies will be alike in the Nash equilibrium among states. This Nash equilibrium yields the states' reaction functions with respect to the federal government's policy choice:  $\alpha(a, t, s)$ ,  $\tau(a, t, s)$ , and  $g(a, t, s)$ . And, with identical states, no migration

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<sup>5</sup> This can be thought of as the second-stage of a two-stage leader-follower procedure, where the first stage consists of the federal government setting its fiscal policies knowing how states are going to behave.

occurs in equilibrium ( $M^i = 0$ ,  $i = 1, \dots, n$ ). Using this observation and rearranging terms, conditions (13) and (14) can be written in the Nash equilibrium as:

$$Z^1 = \frac{(\bar{Z} + \tau \bar{Z}_T)}{1 - \tau \bar{Z}_A} + \frac{\sum_{i=1}^n (\tau Z^i - \alpha)(Z^i - Z^1)M_A^i}{N(1 - \tau \bar{Z}_A)} \quad (15)$$

and

$$\frac{B'}{Z^1 V_A^1} = \frac{1}{N(\bar{Z} + \tau \bar{Z}_T)} - \frac{\sum_{i=1}^n (\tau Z^i - \alpha) \left( \frac{Z^i V_A^i}{Z^1 V_A^1} - 1 \right) M_g^i}{N(\bar{Z} + \tau \bar{Z}_T)}. \quad (16)$$

These conditions can be compared with those for the unitary nation optimum, (6) and (7).

To obtain more insight into the meaning of the results in this and the following section, it is useful to define expressions for various migration effects arising from changes in state policy instruments  $\tau$ ,  $\alpha$  and  $g$ . Define  $M_{xy}^i$  as the migration that would result from an increase in policy instrument  $x$  accompanied by the change in policy instrument  $y$  which keeps constant the utility of the lowest-ability individuals. Using the results provided in equations (3.1), we have:

$$M_{\tau\alpha}^i \equiv \frac{dM^i}{d\tau} \Big|_{V^1, g} = M_T^i + M_A^i \frac{d\alpha}{d\tau} \Big|_{V^1} = M_T^i + Z^1 M_A^i = -(Z^i - Z^1)M_A^i, \quad (17)$$

$$M_{g\tau}^i \equiv \frac{dM^i}{dg} \Big|_{V^1, \alpha} = M_g^i + M_T^i \frac{d\tau}{dg} \Big|_{V^1} = M_g^i + \left( \frac{B'}{Z^1 V_A^1} \right) M_T^i = - \left( \frac{Z^i V_A^i}{Z^1 V_A^1} - 1 \right) M_g^i, \quad (18)$$

and

$$M_{\alpha g}^i \equiv \frac{dM^i}{d\alpha} \Big|_{V^1, \tau} = M_A^i + M_g^i \frac{dg}{d\alpha} \Big|_{V^1} = M_A^i - \left( \frac{V_A^1}{B'} \right) M_g^i = \left( 1 - \frac{V_A^1}{V_A^i} \right) M_A^i. \quad (19)$$

Note that  $M_{xy}^1 = 0$  for all pairs of policies  $x, y$ . Using this along with the definitions (17) and (18), conditions (15) and (16) can be re-written as:

$$Z^1 = \frac{(\bar{Z} + \tau \bar{Z}_T)}{1 - \tau \bar{Z}_A} - \frac{\sum_{i=2}^n (\tau Z^i - \alpha) M_{\tau\alpha}^i}{N(1 - \tau \bar{Z}_A)} \quad (15')$$

and

$$\frac{B'}{Z^1 V_A^1} = \frac{1}{N(\bar{Z} + \tau \bar{Z}_T)} - \frac{\sum_{i=2}^n (\tau Z^i - \alpha) M_{g\tau}^i}{N(\bar{Z} + \tau \bar{Z}_T)}. \quad (16')$$

To interpret the right-hand sides of (15') and (16') relative to those in (6) and (7), we proceed in two steps, looking first at the vertical fiscal externality and second at the horizontal one. To concentrate on the former, let us first abstract from any between-state migration (assuming infinite migration disutility for a while) and keep  $g$  constant. Then the right-hand side of condition (15') reduces to its first term. The difference between this and the right-hand side of (6), with  $\tau$  appearing rather than  $T$  in both the numerator and denominator, can be seen to reflect the vertical tax externality. When a state chooses its tax policy, it does so without taking into account that the labour supply response of its residents influences the tax revenues of the federal government. As Johnson (1988) pointed out, if labour supply falls with increased redistribution, then the cost of redistribution facing the state is reduced because it is partially borne by the federal government and therefore by residents of other states. In fact, using the same reasoning as in the last section, the first term on the right-hand side of (15') is the slope of the tax possibility frontier *as perceived by the state* when there is no migration. Nash policy equilibrium in the federation must involve a combination of  $A$  and  $T$  that lies on the tax possibility frontier. For given federal policy variables  $(a, t)$ , a point along the frontier will only be an equilibrium if the representative state has no incentive to change its tax rate. This requires from condition (15') that at the equilibrium point, the (higher) indifference curve of the lowest-ability individual be tangent to the tax possibility frontier as it is perceived by the state. Note that for any strictly positive value of  $t$ , our assumptions  $\bar{Z}_T < 0$  and  $\bar{Z}_A < 0$  imply:

$$\frac{\bar{Z} + \tau \bar{Z}_T}{1 - \tau \bar{Z}_A} > \frac{\bar{Z} + T \bar{Z}_T}{1 - T \bar{Z}_A}. \quad (20)$$

That is, the tax possibility frontier as perceived by the state will be steeper than the true national one, because the state neglects the effect of its tax changes on federal tax revenues. In particular, this is true in Figure 2 at the point  $(A^*, T^*)$  that corresponds to the optimal unitary nation policy. Therefore, if  $t > 0$ , the only possible equilibria along the true tax possibility frontier must lie to the right of point  $(A^*, T^*)$ . That is, there must be 'over-redistribution'. And the larger is  $t$ , the farther to the right the equilibrium will be (even possibly in the decreasing part of the tax possibility frontier). Such a point is represented

by  $(T_N, A_N)$  in Figure 2. Referring back to the interpretation given to condition (9), inequality (20) means that with  $t > 0$  the revenue requirement and the marginal cost of public funds are both underestimated by the representative state government.

The same kind of conclusion can be reached for the choice by states of their public good provision. For this purpose, let us compare the first term on the right-hand side of (16') with the right-hand side of (7) at the unitary nation optimum  $(A^*, T^*, g^*)$ . For  $t > 0$ , we have  $(\bar{Z} + \tau \bar{Z}_T)^{-1} < (\bar{Z} + T \bar{Z}_T)^{-1}$ . This implies that, given  $A^*$ , the states will 'overspend' on the local public good ( $g > g^*$ ); and, the larger is  $t$ , the stronger this tendency will be. As earlier, the representative state government underestimates the marginal cost of public funds.

Let us now return to the terms in (15') and (16') which are related to migration effects and so to horizontal fiscal externalities. These are the second terms on the right-hand sides of the above conditions. The numerators of these migration terms are easily interpreted using our definitions of  $M_{xy}^i$  in (17) and (18). Starting with condition (15'), the numerator can be interpreted as the change in the state's net tax revenue caused by the migration that would result from a rise of  $\tau$  accompanied by an increase in  $\alpha$ , keeping the utility of the lowest-ability individuals constant (i.e.,  $d\alpha = Z^1 d\tau$ ). If this change in tax revenue is negative, it reduces the slope of the tax possibility frontier as perceived by the states, and so discourages them from redistributing income. This is likely if the absolute value of  $M_A^i$  rises with ability (i.e. with  $i$ ) and if the net tax revenue is positive and large for high-ability individuals. A similar interpretation can be given to the numerator of the migration term in condition (16'). It can be interpreted as the change in the state's tax revenue due to the migration that would be caused by an increase in  $g$  accompanied by an increase in  $\tau$ , keeping the lowest-ability individuals on their original indifference curves (i.e.,  $d\tau = B'(Z^1 V_A^1)^{-1} dg$ ). If this change in tax revenue is positive, it reduces the marginal cost of  $g$  as perceived by the state and so stimulates the provision of the public good.

The upshot is that, in addition to the vertical fiscal externalities arising from federal and state co-occupation of the same tax base, the possibility for individuals to migrate across states induces horizontal fiscal externalities. In deciding its fiscal policy, each state takes account of the fact that migration affects its own budget, but it does not take account



of the fact that a positive or negative change in its own net revenue is matched in the other states as a whole by a change of identical magnitude but of opposite sign.

We can summarize our results for the implications of decentralizing fiscal responsibilities to the states when they behave as Nash competitors.

- A vertical fiscal externality results from federal and state co-occupation of the same tax base. If  $t > 0$ , this externality leads states to set  $T > T^*$  because they choose their tax policies without taking into account the effect of their choices on federal tax revenues. For a given  $g$ , this gives rise to over-redistribution and, for a given  $A$ , this gives rise to over-provision of the public good.
- A horizontal fiscal externality results from the migration of individuals in response to changes in  $\tau$  used to finance increases in  $\alpha$  or  $g$ . This externality may be positive or negative depending on whether migration increases or decreases net tax revenues of other states. For given  $g$ , a positive horizontal tax externality pushes towards too little redistribution, and for given  $A$ , it pushes towards too little public good provision. The opposite occurs if the externality is negative.

#### 4. OPTIMAL POLICY OF THE FEDERAL GOVERNMENT

The Nash equilibrium fiscal policy of the states resulting from the behaviour analyzed in section 3 clearly depends upon the policy adopted by the federal government. As mentioned earlier, the federal government is assumed to act as a Stackelberg leader and therefore anticipates the Nash equilibrium described above. Given that the federal government uses the same maxi-min welfare criterion as the state governments, it solves the following problem:

$$\max_{\{a,t,s\}} V^1(a + \alpha(a, t, s), t + \tau(\cdot)) + B(N\tau(\cdot)\overline{Z}(a + \alpha(\cdot), t + \tau(\cdot)) - N\alpha(\cdot) + s) \quad (21)$$

$$\text{subject to } Nt\overline{Z}(a + \alpha(\cdot), t + \tau(\cdot)) - Na - s = 0. \quad (22)$$

Using the envelope theorem, it is straightforward to obtain the solution to this problem. It yields the same outcome as the unitary nation optimum characterized by conditions (6) and (7). Not surprisingly, this implies that with  $t$ ,  $a$  and  $s$ , the federal government acting as a Stackelberg leader has enough instruments at its disposal to obtain the unitary nation optimum. In the rest of this section, our aim is to provide some qualitative characterizations of the policy to be applied by the federal government.

To obtain insight into optimal federal policy, it is useful to rewrite the states' first order conditions given in (15') and (16') in a way that makes them directly comparable to the unitary nation optimum conditions given in (6) and (7). Doing so gives:

$$Z^1 = \frac{(\bar{Z} + T\bar{Z}_T)}{1 - T\bar{Z}_A} + \frac{-Nt(\bar{Z}_T + Z^1\bar{Z}_A) - \sum_{i=2}^n (\tau Z^i - \alpha) M_{\tau\alpha}^i}{1 - T\bar{Z}_A} \quad (23)$$

and

$$\frac{B'}{Z^1 V_A^1} = \frac{1}{N(\bar{Z} + T\bar{Z}_T)} + \frac{Nt\bar{Z}_T \left( \frac{B'}{Z^1 V_A^1} \right) + \sum_{i=2}^n (\tau Z^i - \alpha) M_{g\tau}^i}{N(\bar{Z} + T\bar{Z}_T)}. \quad (24)$$

Comparing (23) and (24) with (6) and (7), it is then straightforward to infer that, for the state's equilibrium policy to lead to the unitary nation optimum with  $t + \tau = T^*$ ,  $a + \alpha = A^*$ , and  $g = g^*$ , the following two conditions must be satisfied by federal policies  $t$ ,  $a$ , and  $s$ :

$$Nt(\bar{Z}_T + Z^1\bar{Z}_A) + \sum_{i=2}^n ((T^* - t)Z^i - (A^* - a)) M_{\tau\alpha}^i = 0, \quad (25)$$

and

$$Nt\bar{Z}_T \left( \frac{B'}{Z^1 V_A^1} \right) + \sum_{i=2}^n ((T^* - t)Z^i - (A^* - a)) M_{g\tau}^i = 0. \quad (26)$$

These conditions form a system of two linear equations with two unknowns,  $a$  and  $t$ . Once the values of  $a$  and  $t$  have been obtained, the value of  $s$  can be inferred from the federal budget constraint:

$$Nt\bar{Z} - Na = s. \quad (27)$$

In these equations,  $Z^i(\cdot)$  and the derivatives of  $Z^i(\cdot)$  and  $M^i(\cdot)$  are all evaluated at  $A = A^*$ ,  $T = T^*$ , and  $g = g^*$ , that is, at the unitary nation optimal values of  $A$ ,  $T$ , and  $g$ .

The interpretation of (25) and (26) is straightforward. In each of these conditions, the first terms, which involve changes in federal tax revenues, represent the adjustments required in the states' first order conditions to offset the vertical fiscal externalities. The second terms, which involve changes in a state's net tax revenues resulting from induced migration, represent adjustments required to offset the horizontal fiscal externalities. The federal government's choice of  $t$  and  $a$  must be such as to satisfy (25) and (26) simultaneously, implying that the externalities cancel each other out. As we have seen, the vertical

and horizontal externalities in (25) can be interpreted in terms of the effects on federal tax revenues and migration-induced state tax revenues of states' changing  $\tau$  and  $\alpha$  so as to keep the lowest-ability individuals at the same level of utility, while in (26), they can be interpreted in terms of similar effects caused by the states changing  $\tau$  and  $g$ , keeping  $V^1$  constant. (These interpretations follow from the fact that condition (23) has been derived from the ratio of the first order conditions for  $\tau$  and  $\alpha$ , while (24) comes from the ratio of the conditions for  $\tau$  and  $g$ .)

Let us now make use of the conditions (25)–(27) to characterize the federal government's optimal policy  $(a, t, s)$ . The implications of vertical and horizontal fiscal externalities for government policy are sufficiently complicated that we focus on various special cases so as to enable us to shed light on the general case.

#### *4.1 No Migration*

If the migration disutility is infinite ( $M_T^i = 0$ ), conditions (25) and (26) imply that  $t = 0$ . That is, the federal government should impose no tax. The reason for this is straightforward. If there is no migration, the horizontal fiscal externality disappears, leaving only the vertical externality. Since the vertical externality arises because the states neglect the effect on the federal government's income tax revenue of changing their own tax rate, this can be avoided by having the federal government set  $t$  equal to 0. This is analogous to a similar result obtained in Boadway and Keen (1996) in a setting in which all households are identical. In the following, we assume that migration costs do not rule out the possibility of migration.

#### *4.2 Two Types of Migrants*

Suppose now that there are only two types of migrants,  $i = 1$  and, say, 2 ( $K'_i = \infty$  for all  $i \neq 1, 2$ ). A special case of this would be the familiar one where there are only two ability types in the population. With only two types of migrants, equations (25) and (26) can be written as:

$$Nt(\bar{Z}_T + Z^1\bar{Z}_A) + ((T^* - t)Z^2 - (A^* - a)) M_{\tau\alpha}^2 = 0 \quad (25')$$

$$Nt\bar{Z}_T\left(\frac{B'}{Z^1V_A^1}\right) + ((T^* - t)Z^2 - (A^* - a)) M_{g\tau}^2 = 0. \quad (26')$$

The only way for the federal government to satisfy these conditions is to set  $t = 0$  and  $a = A^* - T^*Z^2$ , which implies that  $\tau Z^2 - \alpha = 0$ . This policy allows each instrument to correct for a separate externality. Setting  $t = 0$  eliminates the vertical externality since federal tax revenues are not affected by changes in state policies. Choosing  $a$  to induce the states to set policies such that  $\tau Z^2 - \alpha = 0$  eliminates the horizontal externality since it implies that there are no net revenue consequences to the state of a type 2 person moving. Note that there is no need to worry about the migration of type 1 persons. This is because the per capita utility of these persons is being maximized by both the state and federal governments: the marginal policies that are considered keep type 1's on the same indifference curve. Equivalently, the principle of incentive equivalency (Myers, 1990) applies with respect to type 1 persons, so no horizontal externalities arise.

Given the optimal choice of  $t$  and  $a$ , the subsidy  $s$  is then set to satisfy the federal budget constraint, that is,  $s = -Na$ . This also ensures the optimal level of public good provision. To see this, note that when  $\tau = T^*$  the state budget constraint gives  $g = s + \sum N^i(T^*Z^i - \alpha) = -Na + T^*N\bar{Z} - N\alpha = N(T^*\bar{Z} - A^*)$ . This is just the unitary nation budget constraint (5) with  $T = T^*$  and  $A = A^*$ , and so  $g$  will be set at its optimal value  $g^*$ . The upshot of this case is that *redistribution policy is turned over completely to the states*. The federal government levies a poll tax ( $a < 0$ )<sup>6</sup>, the proceeds of which are used to finance transfers to the states, or to fill the fiscal gap. This assignment of the redistribution function is the complete reverse of the conventional one found in, say, Musgrave (1959) and Oates (1972), which holds that redistribution ought to be solely a federal government responsibility.

It is straightforward to show that the same result applies if we allow for heterogeneous states. To see this, suppose there are two states, 'home' and 'other', that differ in the ability distributions of their populations. Using the maxi-min criterion, the optimal policy chosen by a unitary nation government would be one that equalizes the utilities of the lowest-ability individuals across the two states by reallocating revenues between them. Thus, the

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<sup>6</sup> The result  $s > 0$ , and so  $a < 0$ , comes from:  $s = -N(A^* - T^*Z^2) = Ng + NT^*(Z^2 - \bar{Z}) > 0$ , where the second equality follows from  $NA^* = NT^*\bar{Z} - g$ .

problem for the unitary nation government can be viewed as:

$$\max \quad V^1(A, T) + B(g) \quad (28)$$

subject to:

$$V^1(A, T) + B(g) - V^1(\hat{A}, \hat{T}) - B(\hat{g}) = 0 \quad (29)$$

$$\sum_{i=1}^2 (N^i - M^i)(TZ^i - A) - g + \sum_{i=1}^2 (\hat{N}^i + M^i)(\hat{T}\hat{Z}^i - \hat{A}) - \hat{g} = 0, \quad (30)$$

with the decision variables being  $A$ ,  $T$ ,  $g$ ,  $\hat{A}$ ,  $\hat{T}$ , and  $\hat{g}$ , where a hat denotes variables of the ‘other’ state. The first constraint ensures that the lowest-ability persons are equally well off in either state while the second one is the nation-wide budget constraint.

In order to reproduce the unitary nation optimum in the decentralized state, the pair  $(a, t)$  applying to the ‘home’ state solves the following system of equations analogous to (25’) and (26’) above:

$$t \sum_i (N^i - M^i)(Z_T^i + Z^1 Z_A^i) + [\hat{T}^* \hat{Z}^2 - \hat{A}^* - (tZ^2 - a)] M_{\tau\alpha}^2 = 0 \quad (31)$$

and

$$t \sum_i (N^i - M^i) Z_T^i \left( \frac{B'}{Z^1 V_A^1} \right) + [\hat{T}^* \hat{Z}^2 - \hat{A}^* - (tZ^2 - a)] M_{g\tau}^2 = 0, \quad (32)$$

where  $\hat{T}^*$  and  $\hat{A}^*$  are the values taken by  $\hat{T}$  and  $\hat{A}$  at the unitary nation optimum, and the  $Z$ ’s,  $M$ ’s and their derivatives are evaluated at this optimum. Note once again that the principle of incentive equivalency applies with respect to type 1 persons, and so there is no need to worry about the migration of type 1 persons. Equivalent conditions to (31) and (32) must also be satisfied by the pair  $(\hat{a}, \hat{t})$  applying to the other state. Next, the federal transfers to the states  $(s, \hat{s})$  are determined as earlier:  $s = g^* + a - \tau \bar{Z}$ , and likewise for  $\hat{s}$ .

To satisfy conditions (31) and (32), the federal government sets  $t = 0$  and  $a = \hat{A}^* - \hat{T}^* \hat{Z}^2$ . This induces the home state to choose  $\alpha$  and  $\tau$  so that:

$$\begin{aligned} \tau Z^2 - \alpha &= T^* Z^2 - A^* + a \\ &= T^* Z^2 - A^* - \hat{T}^* \hat{Z}^2 + \hat{A}^* = \Delta R^2 \end{aligned}$$

where  $\Delta R^2$  is the effect on the nation's net tax revenues of a type 2 individual migrating to the home state. For the 'other' state,  $\hat{a}$  and  $\hat{t}$  are chosen so that  $\hat{\tau}\hat{Z}^2 - \hat{\alpha} = -\Delta R^2$ . As was the case with identical states, setting  $t = \hat{t} = 0$  eliminates the vertical externality since federal tax revenues are not affected by changes in state policies. Choosing  $a$  to induce the home state to set policies such that  $\tau Z^2 - \alpha = \Delta R^2$  and similarly for the 'other' state eliminates the horizontal externality since it implies that there are no net revenue consequences to the states of type 2 persons moving. Thus, provided the federal government is able to differentiate both the  $a$  and  $s$  across states, redistributive policy can once again be decentralized to the states. (Wildasin (1991), in a model in which only the poor are mobile and redistributive transfers are lump-sum, shows that differential transfers to the states are sufficient to allow for decentralized redistribution to be efficient.)

#### 4.3 States Provide Only a Public Good

The case where there are more than two types of migrants is more complicated because the federal government is unable to associate a particular policy instrument with each externality. Instead, it selects its tax policy mix  $(t, a)$  to nullify the *joint* influence of the two externalities on the state governments' behaviour. To characterize this policy mix, we begin with the particular case of our model where state governments are not allowed to use their revenue for redistributive purposes ( $\alpha = 0$  and so  $a = A^*$ ). This is an interesting case to consider on its own right. It corresponds to the situation where expenditure responsibilities are devolved to the states with the restriction that state governments levy proportional taxes, such as sales taxes. In this case, only conditions (26) and (27) apply, and the first of these conditions simplifies to:

$$tN\bar{Z}_T + (T^* - t)E = 0 \quad (33)$$

where

$$E \equiv \left( \frac{Z^1 V_A^1}{B'} \right) \sum_{i=2}^n Z^i M_{g\tau}^i. \quad (34)$$

Following the same logic as before,  $(T^* - t)E$  can be interpreted as the reduction in the state's tax revenue, or the horizontal fiscal externality, that results from migration if, at

the unitary nation optimum, the representative state changes  $\tau$  and  $g$  so as to keep  $V^1$  unchanged. This requires  $dg = (B')^{-1} Z^1 V_A^1 d\tau$ . Likewise,  $tN\bar{Z}_T$  is the vertical externality caused by these changes. From condition (33) we then infer that:

$$t = T^* \frac{E}{E - N\bar{Z}_T} \quad \text{and} \quad \tau = \frac{N\bar{Z}_T T^*}{N\bar{Z}_T - E}. \quad (35)$$

The relationships among  $t$ ,  $\tau$ , and  $E$  as derived from (35) are illustrated in Figure 3. Assume first that at the unitary nation optimum,  $E > 0$ . The above condition then implies that  $0 < t < T^*$ , and so  $\tau = T^* - t > 0$ : federal and state tax rates are both positive. This scenario corresponds to regime III in Figure 3. The reasoning behind this result is as follows. With  $E > 0$  and  $0 < t < T^*$ , condition (33) indicates that the above changes effected by the representative state create a negative vertical and a positive horizontal externality. If the sum of these two externalities were, say, positive, then the representative state would set its tax rate too low, and so  $T = t + \tau < T^*$ . By adjusting  $t$ , the federal government can control the relative magnitudes of the two externalities: increasing  $t$  causes the negative vertical externality to rise (in absolute value) and the positive horizontal one to fall. Thus, the federal government sets  $t$  at the level at which the two externalities nullify each other. In this case, since the federal government sets  $a = A^*$ , the sign of  $s$  can be either positive or negative by the federal budget constraint (27).

Suppose next that at the unitary nation optimum,  $E - N\bar{Z}_T < 0$  (and therefore  $E < 0$ ). Condition (35) then implies that  $t > T^*$ , and so  $\tau = T^* - t < 0$ , and we are thus in regime I in Figure 3. This is so because, if  $t$  and  $\tau$  were both positive at the unitary nation optimum, the changes  $d\tau > 0$  and  $dg = (B')^{-1} Z^1 V_A^1 d\tau$  would create horizontal and vertical revenue externalities that are both negative, and the representative state would be induced to set its tax rate too high. To avoid this, the sign of the horizontal revenue externality must become positive by setting  $t > T^*$ . So the state governments refund to individuals some constant proportion of the labour income taxes they have paid at the federal level, and the fiscal gap is met by the lump-sum transfer that each state receives from the federal government:  $s = g^* - \tau N\bar{Z} > g^*$ . It is useful to note that if  $\tau = T^* - t$  were constrained to be non-negative, the federal government would simply set  $a = A^*$ ,

$t = T^*$ , and  $s = g^*$ , and the constraint  $\tau \geq 0$  would be binding at the state level.

Finally, assume that  $E < 0$  and  $E - N\bar{Z}_T > 0$ . From condition (35), we then have  $t < 0$  and so  $\tau > 0$ , and we are thus in regime II in Figure 3. This case is similar to the previous one, except that, for the two externalities to compensate each other,  $t$  and  $\tau$  are of opposite signs to those obtained previously. The optimal policy for the federal government is then to *subsidize* labour income ( $t < 0$ ). The desired progressivity of the tax system would still be achieved because of the fact that the states react to the fiscal externalities by setting their tax rate relatively high. Of course, this would require that the states make transfers to the federal government ( $s < 0$ ). If this is not feasible, then the unitary optimum could not be achieved.

#### 4.4 The General Case

We now return to the general model where the state governments can spend their revenue on both the public good and redistribution. Conditions (25) and (26) are satisfied simultaneously by choosing the levels of  $a$  and  $t$  so that the horizontal and vertical externalities just cancel out. Recall that we interpreted these externalities in terms of changes in  $\tau$ ,  $\alpha$  and  $g$  that keep  $V^1$  constant, or  $d\alpha = Z^1 V_A^1 d\tau$   $dg = (B')^{-1} Z^1 V_A^1 d\tau$ . Conditions (25) and (26) yield a system of two linear equations in  $a$  and  $t$ , the solution of which is presented in the Appendix. We shall here focus on one set of hypotheses that are sufficient to characterize the federal government's optimal policy. Others are considered in the Appendix.

The tax rate to be chosen by the federal government is shown in the Appendix to be such that  $0 < t < T^*$  if the following two conditions are satisfied simultaneously:

$$E > 0 \quad \text{and} \quad H_a \equiv \left( \frac{Z^1 V_A^1}{B'} \right) \sum_{i=2}^n M_{g\tau}^i < 0, \quad (36)$$

where the second condition implies that, at the unitary nation optimum, the changes  $d\tau > 0$  and  $dg = (B')^{-1} Z^1 V_A^1 d\tau$  cause a positive inflow of individuals into the representative state. Furthermore, it can be shown that  $a > A^*$  (and so  $\alpha < 0$ ) if the following additional hypothesis holds:

$$\sum_i Z^i M_{\alpha g}^i < 0. \quad (37)$$



This condition means that, at the unitary nation optimum, the changes  $d\alpha > 0$  and  $dg = -(B')^{-1}V_A^1 d\alpha$  cause a fall in the tax base of other states, that is, they create a negative tax base externality. Thus, the above conditions imply that the federal government sets  $t$  and  $a$  in such a way that  $\tau > 0$  and  $\alpha < 0$  : state governments levy both a lump-sum tax and a proportional tax on labour income. The result that  $\alpha < 0$  can be understood as follows. Combining conditions (25) and (26) yields with  $\tau = T^* - t$  and  $\alpha = A^* - a$  :

$$tN\bar{Z}_A + \tau \sum_i Z^i M_{\alpha g}^i - \alpha \sum_i M_{\alpha g}^i = 0. \quad (38)$$

The interpretation of this is identical to that of each of (25) and (26), except that the changes are here  $d\alpha > 0$  and  $dg = -(B')^{-1}V_A^1 d\alpha$  and so concern the balance between redistribution and public good provision at the state level: the vertical externality caused by these changes (first term) ought to be offset by the horizontal externalities (last two terms). With  $\tau > 0$ , the above hypotheses imply that the first two terms of (38) are both negative and the factor multiplying  $\alpha$ ,  $\sum_i M_{\alpha g}^i$ , is positive, and is the outflow of individuals caused by the above changes. So for the externalities to compensate each other,  $\alpha$  must be negative.

To summarize, when fiscal responsibilities are decentralized to the states and households are mobile, the federal government can use its policy instruments  $a, t$  and  $s$  to offset the vertical and horizontal fiscal externalities. In so doing it induces the states to choose  $\alpha, \tau$  and  $g$  such that the unitary nation optimum is achieved. In the case where only one ability type is mobile in addition to type 1, the federal government sets  $t = 0$  to nullify the vertical externality, and chooses  $a < 0$  and  $s > 0$  such that the horizontal externality is offset and the states set the correct level of  $g$ . In the more general case, the signs of  $t, a$  and  $s$  depend on the parameter values of the model. Sufficient conditions exist such that at least some of the federal government's policy instruments can be signed.

## 5. CONCLUSIONS

State-level governments in many federations have significant taxing and expenditure responsibilities, and they can exercise these responsibilities in ways that have redistributive consequences. They may indeed have access to tax instruments which are explicitly redistributive, such as progressive income taxes, or to proportional taxes, like sales or payroll taxes, which when combined with spending on state public goods or services may result in overall fiscal stances that are redistributive. The standard theory of function assignment in a federation states that the redistributive function of government ought to be largely a responsibility of the federal government, with lower-level governments being restricted mainly to efficiency objectives. Even though redistribution might have some features of a local public good (Pauly, 1973), the mobility of households across state boundaries would result in state governments competing away redistributive objectives, a phenomenon now referred to as horizontal fiscal externalities.

Furthermore, when a federal government is brought into the picture, another form of externality emerges as a result of the interaction of the federal and state governments imposing taxes on the same, or similar, bases. State governments neglect the vertical externality they impose on the federal government: an increase in the state tax rate will affect the size of the tax base and therefore the amount of tax revenue collected by the federal government. This vertical externality induces states to underestimate the true marginal cost of public funds facing them.

Both sorts of externalities are present in the federation we have considered in this paper. It consists of many states and a single federal government in which states have access, along with the federal government, to a linear progressive tax whose proceeds they use to provide a state public good. Governments at both federal and state levels aim at maximizing the utility of the least-able individuals. When the federal government acts as a Stackelberg leader, it basically directs its policy instruments towards offsetting the fiscal externalities faced by the states, and otherwise lets the states assume responsibility for redistribution and provision of the state public good. In the simplest case in which only two ability types are mobile including the lowest-ability persons, this involves the federal government merely levying a poll tax and making transfer payments to the states. This

result turns the conventional assignment prescription on its head: the states are given full responsibility for redistribution. In more realistic cases, the federal government must also impose a proportional tax (or subsidy) on persons, but the size of this tax is directed to internalizing the fiscal externalities faced by the states rather than to redistribution.

We have conducted our analysis in the simplest of models in order to focus on the main issues. As well, we have restricted attention to the case in which the federal government has enough instruments at its disposal to achieve the unitary nation optimum. In this setting, the intuition for the results is most clearcut. Matters would obviously become more complicated if the assumptions of the model were relaxed. Some interesting ways that might be done include the following.

Perhaps the simplest extension would allow for an alternative social decision rule. For example, most of the results apply straightaway to the case where the policy outcome is chosen by a median voter provided this voter has below-average income. The analysis would parallel that of the maxi-min case except that the objective function would be the utility of the median voter rather than that of the lowest-ability individuals. Of course, given that the states have three policy instruments,  $\tau$ ,  $\alpha$  and  $g$ , problems of vote cycling might occur if all three variables were voted on at the same time. This could be overcome by voting over policies sequentially.

We have also assumed that the federal and state governments adopt the same social welfare objective. We could have considered the case where the federal and state governments have different social welfare functions, but this would complicate matters considerably. Moreover, the meaning of the unitary nation optimum would no longer be clear and the federal government would no longer be willing to turn over major fiscal responsibilities to the states because to do so would imply that state preferences determine the outcome.

We have assumed that the state governments act as Nash competitors, taking as given the levels of policy variables of all other governments, and that the federal government acts as a Stackelberg leader. Neither of these may capture reality. States presumably recognize at least to some extent the impact of their policies on the federal government and on neighbouring jurisdictions. And the federal government may not be able to commit to

policies announced as if it were a first mover.

Finally, systems of redistribution that are more efficient, such as optimal non-linear taxes, might have been considered. Though perhaps more realistic, such tax systems are likely to give rise to similar fiscal externalities as in the linear case. And, they would undoubtedly increase the conceptual and analytical complexity of the model considerably.

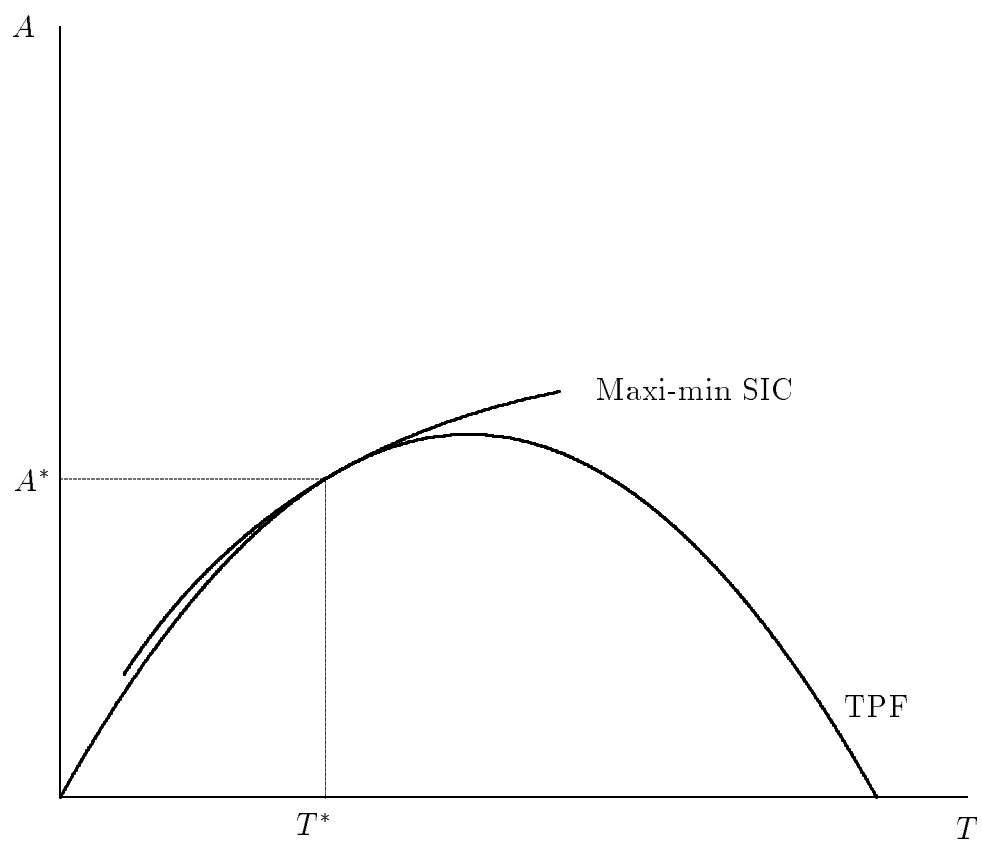
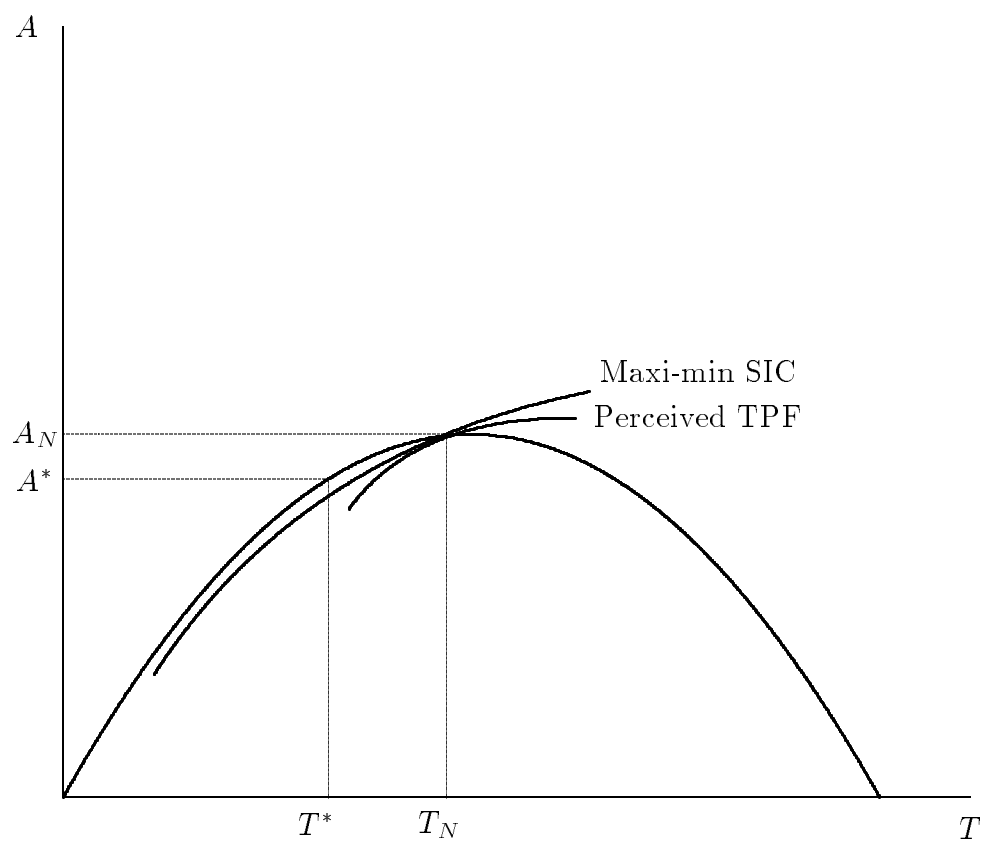


Figure 1



**Figure 2**

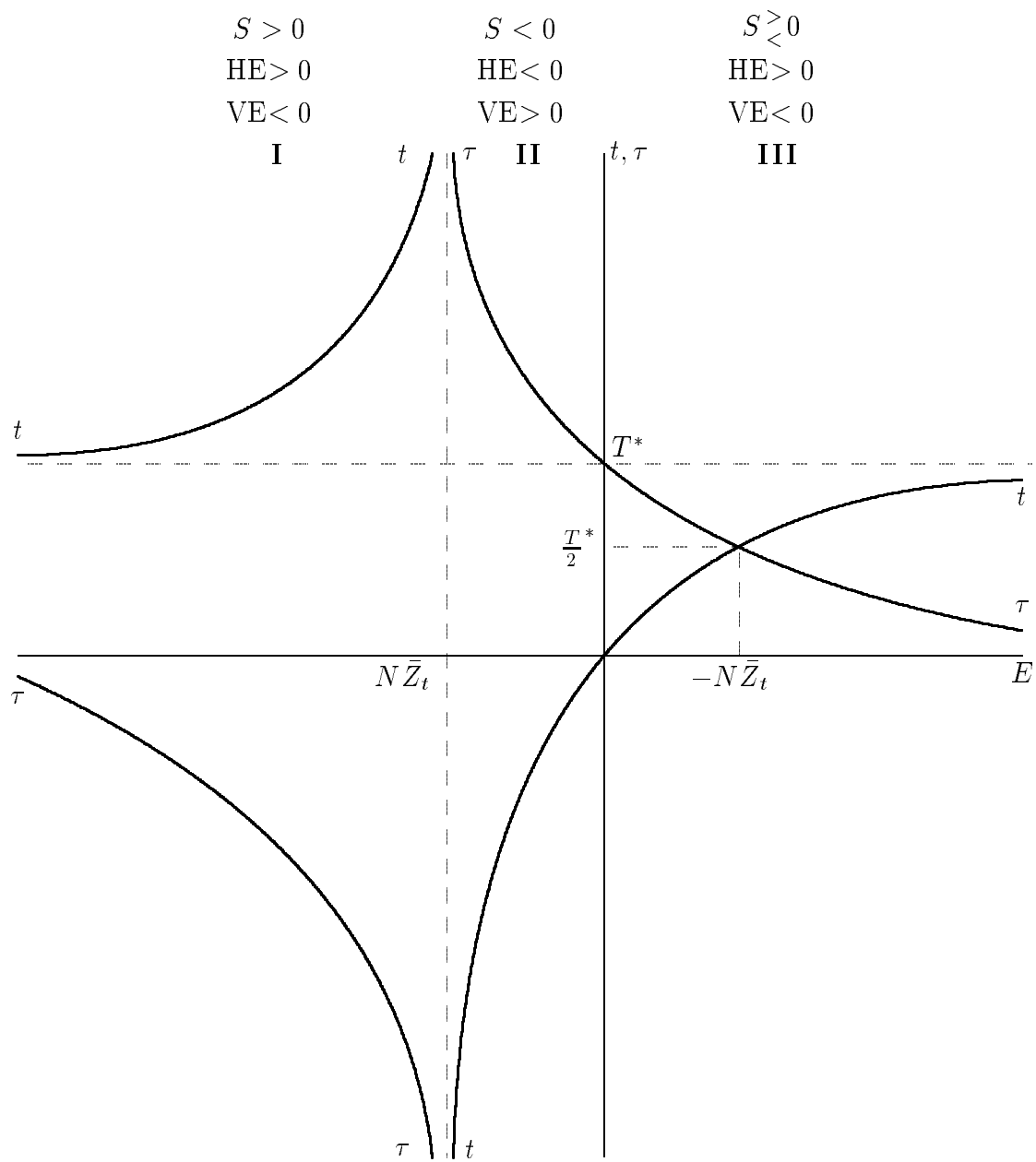


Figure 3

## Appendix

The system of two conditions, (25) and (26), can be written as:

$$t(N(\overline{Z}_T + Z^1 \overline{Z}_A) - K) + aF_a = -F_c \quad (A.1)$$

and

$$t(N \overline{Z}_T - E) + aH_a = -H_c. \quad (A.2)$$

with

$$K \equiv \sum_i Z^i M_{\tau\alpha}^i > 0; \quad F_a \equiv \sum_i M_{\tau\alpha}^i > 0;$$

$$F_c \equiv KT^* - F_a A^* \quad \text{and} \quad H_c \equiv ET^* - H_a A^*.$$

The definitions of  $E$  and  $H_a$  are provided in (34) and (36). The determinant of the matrix of the left-hand sides of (A.1) and (A.2) is:

$$D \equiv (N(\overline{Z}_T + Z^1 \overline{Z}_A) - K)H_a + (E - NZ_T)F_a. \quad (A.3)$$

The solution to the above system yields:

$$t = -D^{-1}T^*[KH_a - EF_a] \quad (A.4)$$

$$a = -D^{-1}[NZ^1 \overline{Z}_A H_c + N\overline{Z}_T(H_c - F_c) + A^*(KH_a - EF_a)], \quad (A.5)$$

from which one can infer by means of  $\tau = T^* - t$  and  $\alpha = A^* - a$ :

$$\tau = D^{-1}NT^*[(H_a - F_a)\overline{Z}_T + H_a \overline{Z} \overline{Z}_A] \quad (A.6)$$

and

$$\alpha = D^{-1}NT^*[(\overline{Z}_T(E - K) + Z^1 \overline{Z}_A E), \quad (A.7)$$

where

$$H_a - F_a = -Z^1 \sum_1 M_{\alpha g}^i \quad (A.8)$$



and

$$E - K = -Z^1 \sum_i Z^i M_{\alpha g}^i. \quad (\text{A.9})$$

Let us first assume that  $E > 0$  and  $H_a < 0$ , which imply that  $D > 0$ . From (A.4) and (A.6), we then conclude that  $t > 0$  and  $\tau > 0$ . To sign  $a$  and  $\alpha$ , we supplement the above hypotheses with  $E - K > 0$ . With this additional assumption,

$$H_c - F_c = (E - K)T^* - (H_a - F_a)A^* \quad (\text{A.10})$$

is positive. Using this in (A.5) and (A.7) yields the result that  $a > 0$  and  $\alpha < 0$ .

Assuming next that  $H_a > 0$  and  $E - N\bar{Z}_T < 0$  (which imply  $E < 0$ ), we infer from (A.3) that  $D < 0$  and from (A.4) that  $t > 0$ . Assuming further that

$$\sum_i M_{\alpha g}^i < 0, \quad (\text{A.11})$$

allows us to sign  $\tau > 0$ . Condition (A.11) means that, at the unitary nation optimum, a positive inflow of individuals into the representative state would arise if the changes  $d\alpha > 0$  and  $dg = -(B')^{-1}V_A^1 d\alpha$  were carried out. Rewriting the externality-offsetting condition (26) as

$$tN\bar{Z}_T + \tau E - \alpha H_a = 0. \quad (\text{A.12})$$

and using  $\tau > 0$ ,  $E < 0$  yields the result that  $\alpha < 0$ .

Let us finally assume that  $E < 0$ ,  $E - N\bar{Z}_T > 0$  and  $H_a < 0$ . From (A.3),  $D > 0$ . From (A.6), we conclude that  $\tau > 0$  and so  $t < T^*$ . However, it is impossible to find meaningful additional assumptions to sign  $t$  and  $a$ .

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