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Investment Coordination and Demand Complementarities

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Abstract

This paper analyzes the possibility of investment coordination leading to outcomes which dominate non-investment equilibria in the presence of monopolistic competition. We establish when complementarity leads to investment coordination failures and explore the welfare implications of coordinated investment. Our main results caution against demand complementarities as a motive for investment coordination. We find that: 1) generally, a strict notion of complementarity (Hicks) is necessary for the existence of an investment coordination problem and 2) that when the problem does exist, coordination often lowers social welfare.

JEL Classification: O14, O33, L13, L16

1. Introduction

Coordination amongst investors may make investments profitable even though they would not be when undertaken on their own. A recent literature has been concerned with establishing the existence of these problems in underdeveloped economies and exploring their implications.[1] The literature has focused on examples of coordination problems with comparatively little emphasis on conditions necessary or sufficient for such problems to exist. However it is clear that some factors are important in giving rise to coordination problems: almost all models of investment coordination failure involve both scale economies and complementarities between investing sectors. In this paper we take the first point as given and exogenously posit the existence of scale economies and then answer the following question: Given scale economies in production, what types of *demand* complementarity give rise to investment coordination problems?[2]

Externalities caused by complementarities in demand have been a focus of much recent work. These take place when an investing firm, by lowering its product's price, raises demand for producers of other (complementary) goods. The demand increase in other sectors raises investment returns there and, when reciprocal effects occur across sectors, can give rise to benefits from coordination.

[1]See Baland and Francois (1996a), Ciccone (1993), Ciccone and Matsuyama (1992), Gans (1995), Matsuyama (1992 a, b), Rodriguez (1993) and the survey by Matsuyama (1995).

[2]In this analysis we ignore technological externalities such as knowledge spillovers, network externalities and the use of shared technology for two main reasons. Firstly it is already well understood that these factors can give rise to the need for coordinated action. However, as Milgrom and Roberts (1992) and Matsuyama(1995) have argued these types of coordination problem are likely to lead to readily identifiable and localized benefits which provide strong incentives for firms to internalize through integration. As such they do not seem promising as explanations of macroecononomic coordination failure on a scale sufficient to concern development economists.

This paper uses a framework which is a modified and generalized form of Murphy Shleifer and Vishny (1989). The features of this framework are that the process of investment (entry) occurs within one sector within which goods are perfect substitutes. The relationship between the investing sector and others is allowed to vary. In the conclusion we discuss the contrast between this framework and the alternative approach based on the increasing product variety model (Dixit and Stiglitz (1977)) as surveyed in Matsuyama (1995). The main results of our analysis are:

- In a broad set of cases, Hicks complementarity is a necessary condition for coordination problems, gross complements alone do not lead to coordination failures.
- Hicks complementarity also ensures that there always exists a set of parameter values under which a coordination problem arises.
- Extending the standard framework to allow declining average variable costs or inferior goods relaxes the conditions required so that Hicks complementarity is no longer necessary.
- With symmetric mark-up rates over sectors, Hicks complementarity, though leading to a coordination problem, implies the coordinated outcome is dominated by the non-investment outcome.
- Benefits from coordination arise from differences in production technology across sectors and not from the mere existence of mutually profitable investments under coordination.

Section 2 of the paper develops the basic framework of analysis and establishes conditions necessary for a coordination problem to arise. Section 3 examines the welfare consequences of coordination. Finally Section 4 compares the framework used here, and results established, to one based on expanding product variety and other relevant literature.

2. The Model

We consider a representative agent economy where the agent supplies all labour and owns all profits. The utility function of this agent is denoted $u(x)$ where x denotes the vector of all $n+1$ goods in this economy. In this section we assume that preferences for leisure, the $n+1$th good, are such that:

Assumption 1. *The agent inelastically supplies one unit of labour.*

With the wage as numeraire, aggregate income is defined as:

$$y = 1 + \sum_{k=1}^{n} \Pi_k(p, y) \tag{2.1}$$

where p is the vector of prices and Π_i profits in sector i. The first term, 1, is labour income, since wage is numeraire and labour is supplied inelastically.[3] The maximization of (2.1) subject to the constraint that the value of purchases not exceed aggregate income yields the following Marshallian demands for any good i:

$$x_i = x_i(p, y). \tag{2.2}$$

We also assume that the preferences of the representative agent are such that:

Assumption 2. *All goods are normal;* $\frac{dx_j}{dy} \geq 0, \forall j$.

[3]Equivalently 1 can be thought of as an economy wide endowment of labour which can be converted to to leisure under constant returns or used in production. Under this specification, labour supply is endogenous and all results continue to hold. This interpretation is further explored in the discussion of Corollary 1.

Each of the n sectors produces a different good and labour is the only factor of production. Within each sector, subscripted by i, there exists a unique firm with access to a technology superior to that of its current competitors. We refer to this firm as the incumbent. Incumbents produce as follows:

Assumption 3. *Production uses labour as the only input. The marginal cost of production for an incumbent in sector i is a constant and is denoted γ_i^I units of labour.*

The incumbent's current competitors in sector i have access to a technology allowing production at a strictly higher constant marginal cost denoted γ_i^C, i.e. $\gamma_i^C > \gamma_i^I$.[4] In the absence of collusion between firms in an industry, and provided goods are not too substitutable across industries, perfect substitutability between incumbent and entrant will lead to limit pricing so that $p_i^I = \gamma_i^C$. The restriction on substitutability of goods across industries can be expressed in terms of own price elasticity of demand:

Assumption 4. $\left(\frac{p_i - \gamma_i}{p_i}\right) \frac{p_i}{x_i} \frac{dx_i}{dp_i} > -1,$

where $\frac{dx_i}{dp_i}$ denotes the total change in demand for good i taking into account adjustment in all other sectors.[5] There is a potential entrant in each industry, not the incumbent, with the opportunity to invest in a cost reducing technology.[6] Investment of

[4] One can alternatively think of the technological advantage as a higher quality product, as in Grossman and Helpman's (1991) quality ladders model. In that case, $1/\gamma$ denotes the quality of the good produced, so that quality increases in γ.

[5] The literature on coordination failures proceeds with a similar assumption, which is natural in this framework since the concern is with goods that are complementary. Moreover, this is a relatively weak assumption since the direct price elasticity is multiplied by the mark-up rate. Without this assumption, price in any one industry depends upon the pattern of prices in all others thus making equilibrium analysis intractable.

[6] If investments were available to incumbents, prices would not fall after investment and there would not exist demand complementarities.

a fixed cost, F_i, denominated in terms of labour input, in industry i allows the entrant to produce at marginal cost $\gamma_i^E < \gamma_i^I$. Upon entry, by Assumption (4), profits are maximized by displacing the current incumbent through limit pricing at $p_i^E = \gamma_i^I$. Thus the price fall in industry i is $\gamma_i^C - \gamma_i^I$.[7]

Entry in one sector can benefit potential entrants (investors) elsewhere since, by lowering price, entrants may increase demand for complementary goods produced in other sectors. We therefore define an investment coordination problem as follows:

Definition An *Investment coordination problem* is said to exist if and only if there exists a subset of z firms denoted K, with $z \leq n$:

$$\Pi_q(q\ invests\ alone) \leq 0, \forall q \in K \tag{2.3}$$

and

$$\Pi_q(all\ firms\ in\ K\ invest) \geq 0, \forall q \in K \tag{2.4}$$

with strict inequality for at least one q.

An investment coordination problem follows from the existence of multiple equilibria which in turn depends upon unprofitable investment in one sector raising returns to investment elsewhere. If only profitable investment increased benefits to investing elsewhere there would be a unique equilibrium since these investments are always undertaken, thereby ruling out the no-investment equilibrium. Therefore, in what follows, we examine the impact of investment in sector i on the demand and profits of another sector j under the most favourable conditions for an investment coordination problem, that is, when investment in i yields zero net profits. If it can be shown that, even in this case, $j's$ demand does not increase, then it will also not increase when investment in i

[7] Though price falls by a discrete amount after entry, we proceed by assuming the amount is small enough to be well approximated by a derivative taken at the old price.

makes a loss, since, for the same change in relative prices, aggregate income and demand will be lower (via assumption 2).

Denote the investing firm by i and suppose that this firm, upon investing alone, breaks even. The break-even assumption implies:

$$F_i = (\gamma_i^I - \gamma_i^E)x_i^E = \Pi_i^E \tag{2.5}$$

where x_i^E stands for the demand for industry $i's$ product and Π_i^E denotes profit after investment has taken place in industry i (in the following, superscript E denotes post-investment (Entrant) values and superscript I pre-investment (Incumbent) values). From equation (2.1), pre-investment income, y^I, is given by:

$$y^I = 1 + \sum_k \Pi_k^I, \tag{2.6}$$

After investment in sector i aggregate income is given by:

$$y^E = 1 + \sum_k \Pi_k^E - F_i \tag{2.7}$$

which, at the break-even point, by substituting from (2.5) yields:

$$y^E = 1 + \sum_{k \neq i} \Pi_k^E \tag{2.8}$$

The net effect on demand for good j resulting from an investment in industry i will be denoted Δx_j (sector i will denote the investing sector, unless otherwise specified). It can be decomposed into two effects: the change in demand for j due to the fall in $i's$ price and the change in demand for j due to the change in aggregate income:

$$\Delta x_j \approx \frac{\partial x_j}{\partial p_i}\Delta p_i + \frac{\partial x_j}{\partial y}\Delta y. \tag{2.9}$$

Using the Slutsky equation, we obtain the following approximation to change in demand:

$$\Delta x_j \approx \frac{\partial h_j}{\partial p_i}\Delta p_i - x_i \frac{\partial x_j}{\partial y}\Delta p_i + \frac{\partial x_j}{\partial y}\Delta y, \tag{2.10}$$

where h_j denotes the Hicks demand for good j. In the equations above and the rest of the paper we define $\Delta p_i = p_i^E - p_i^I$, as the price fall in sector i due to investment in i. We similarly use the notation Δx_j to denote the total change in demand for sector $j's$ good due to investment in i, Δy to denote the total change in income due to investment in i, and $\Delta\Pi_j$ to denote the change in $j's$ profits due to an investment in i.(Note again that the reference to the investing sector, i is omitted).

It follows from the discussion above that $p_i^E - p_i^I = \gamma_i^I - \gamma_i^C$. But, note that $x_i^I(\gamma_i^C - \gamma_i^I) = \Pi_i^I$ (this follows from the Bertrand interaction between firms) which implies that (2.10) becomes:

$$\Delta x_j \approx \frac{\partial h_j}{\partial p_i}\Delta p_i + \frac{\partial x_j}{\partial y}(\Delta y + \Pi_i^I). \tag{2.11}$$

Once again, by definition, we know that $\Delta y = y^E - y^I$ and from equations (2.6) and (2.8) $y^E - y^I = \sum_{k\neq i}(\Pi_k^E - \Pi_K^I) - \Pi_i^I$. Equation (2.11) then becomes:

$$\Delta x_j \approx \frac{\partial h_j}{\partial p_i}\Delta p_i + \frac{\partial x_j}{\partial y}(\sum_{k\neq i}(\Pi_k^E - \Pi_K^I) - \Pi_i^I + \Pi_i^I). \tag{2.12}$$

Profits in sectors $k \neq i$ after investment are given by:

$$\Pi_k^E = (\gamma_k^C - \gamma_k^I)x_k^E. \tag{2.13}$$

As a result, equation (2.12) can be written as:

$$\Delta x_j \approx \frac{\partial h_j}{\partial p_i}\Delta p_i + \frac{\partial x_j}{\partial y}[-x_i\Delta p_i - \Pi_i^I] + \frac{\partial x_j}{\partial y}(\sum_{k\neq i}(\Pi_k^E - \Pi_K^I). \tag{2.14}$$

Under Bertrand competition, entry by a new firm lowers the output price to the marginal cost of the incumbent. As a result the following relationship between price fall and initial mark-up of the incumbent holds.

Lemma 1. *Under Assumptions (3) and (4), the price fall of an investment in industry i just equals (in absolute value) the initial mark-up in that industry:*

$$\Delta p_i = -(\gamma_i^C - \gamma_i^M). \tag{2.15}$$

But pre-entry profit in sector i just equals $x_i^M(\gamma_i^C - \gamma_i^M)$. Lemma 1 therefore implies that the second term on the right hand side of (2.14) equals zero; the real income effect of a price fall just equals the incumbent's pre-entry profits. Moreover, profits in sectors $j \neq i$ after entry in sector i are given by:

$$\Pi_j^E = (\gamma_j^C - \gamma_j^M)x_j^E. \tag{2.16}$$

As as result equation (2.14) becomes:

$$\Delta x_j = \frac{\partial h_j}{\partial p_i}\Delta p_i + \frac{\partial x_j}{\partial y}(\sum_{q\neq i}(\gamma_q^C - \gamma_q^I)\Delta x_q). \tag{2.17}$$

The equation holds for all sectors $j \neq i$. We therefore have a system of $n-1$ equations, which describe the effects of a decline in price. We now establish a necessary condition for the existence of a coordination problem in this framework.

Lemma 2. *Under Assumptions 1, 3 and 4, there exists an investment coordination problem only if there exists a sector j such that:*

$$\frac{\Delta x_j}{\Delta p_i} \approx \frac{\left[\frac{\partial h_j}{\partial p_i}(1 - \sum_{q\neq i,j}^n(\gamma_q^C - \gamma_q^I)\frac{\partial x_q}{\partial y}) + \sum_{q\neq i,j}^n(\gamma_q^C - \gamma_q^I)\frac{\partial h_q}{\partial p_i}\frac{\partial x_j}{\partial y}\right]}{\left[1 - \sum_{q\neq i}(\gamma_q^C - \gamma_q^I)\frac{\partial x_q}{\partial y}\right]} < 0 \tag{2.18}$$

Proof: See appendix.

Proposition 1. *Under assumptions 1-4, Hicks complements are necessary for the existence of an investment coordination problem.*

Proof:

The derivative of the budget constraint with respect to y implies $1 = \sum_{q=1}^n \gamma_q^c \frac{\partial x_q}{\partial y}$ since $\gamma_q^C = p_q^I$, which implies that the term $\sum_{q\neq i}\gamma_q^C\frac{\partial x_q}{\partial y}$ is strictly smaller than 1 (since the ith term is not included in the summation and goods are normal). Thus since each mark-up rate $\frac{\gamma_q^C-\gamma_q^I}{\gamma_q^C}$ is clearly less than 1, the summation in the denominator is less than 0. As a result the denominator is positive.[8]

Turning to the numerator, we know that $\frac{\partial h_q}{\partial p_i} \geq 0 \;\; \forall i, q$ if there are no Hicks complements. As a result, the numerator is positive for all j and condition (2.18) is not satisfied. Q.E.D.

Under Assumptions 1-4, Hicks complementarity is therefore necessary for the existence of an investment coordination problem. The intuition behind this result is most easily understood in a two good world, denoted sectors 1 and 2.[9] Investment in sector 1 has two effects. The first is that the price of good 1 falls from γ_1^C to γ_1^I. The second is that, since the investing firm just breaks even and displaces the previous incumbent, the latter's profit disappears and aggregate income falls by the amount of that profit, i.e. Π_1^0 (a business stealing effect). Consider the net effect of these changes on the profits of sector 2. Since neither price nor marginal costs in 2 have changed, the only avenue of these effects is through a change in the demand for sector 2's good. From (2.9) the total change in demand for sector 2's good is given by:

$$\Delta x_2 = \frac{\partial x_2}{\partial p_1}\Delta p_1 + \frac{\partial x_2}{\partial y}\Delta y. \tag{2.19}$$

Since $\Delta y = -\Pi_1^0 + (\gamma_2^C - \gamma_2^I)\Delta x_2$, and using the Slutsky equation to expand the first term, yields:

$$\Delta x_2 = \frac{\partial h_2}{\partial p_1}\Delta p_1 - x_1\frac{\partial x_2}{\partial y}\Delta p_1 + \frac{\partial x_2}{\partial y}\Pi_1^0 + \frac{\partial x_2}{\partial y}(\gamma_2^C - \gamma_2^I)\Delta x_2$$

[8]Note that this is also a necessary condition for the stability of the equilibria defined by the demand equations. We will henceforth assume that, even when Assumption (2) is not satisfied, the denominator remains positive. Another way of ensuring this is to let the mark-up rate become arbitrarily small.

[9]Note that in such a world, Hick's complementarity is ruled out.

$$\Rightarrow \Delta x_2 = \frac{\frac{\partial h_2}{\partial p_1}\Delta p_1 - x_1 \frac{\partial h_2}{\partial y}\Delta p_1 - \frac{\partial x_2}{\partial y}\Pi_1^0}{1 - (\gamma_2^C - \gamma_2^I)\frac{\partial x_2}{\partial y}}. \tag{2.20}$$

Using lemma 1, equation (2.20) becomes:

$$\Delta x_2 = \frac{\frac{\partial h_2}{\partial p_1}}{1 - (\gamma_2^C - \gamma_2^I)\frac{\partial x_2}{\partial y}}\Delta p_1, \tag{2.21}$$

which is always non positive since i's price has fallen and goods can not be Hicks complements. In obtaining (2.21) the terms in the numerator which include y cancel out because the effect on 2's demand due to the income fall from the profit loss in sector 1 just equals the effect of the positive real income rise due to the price fall. Thus we find that gross complements do not lead to coordination problems because the income loss due to investment is just that amount required to place the consumer at her initial utility level, i.e. *the income loss just offsets the real income effect of the price fall.* Since income effects cancel, all that remains is the pure Hicks substitution effect. Without Hicks complements this is negative, so that Hicks complements are necessary to generate multiple equilibria in this economy.

It is, of course, impossible to specify sufficient conditions for the existence of an Investment Coordination problem since mutually beneficial coordination generally depends upon parameter values. It is, however, possible to specify a condition under which there always exists parameter values under which investment coordination is profitable.

Proposition 2. *Under Assumptions 1-4, and if there are Hicks complements, there always exist parameter values under which an investment coordination problem exists.*

Proof: Let mark-up rates in all sectors, other than i and j be arbitrarily small. Then from equation (2.20): $sgn\left\{\frac{\Delta x_j}{\Delta p_i}\right\} = sgn\left\{\frac{\Delta x_i}{\Delta p_j}\right\} = sgn\left\{\frac{\partial h_j}{\partial p_i}\right\}$. Q.E.D.

2.0.1. Extending the basic model

In the appendix we consider two extensions of the basic model obtained by relaxing Assumptions 3 and 2 in turn. Relaxing Assumption 3 considerably complicates the analysis and appears to sometimes weaken the conditions required for an investmentment coordination problem. We show, by example, that coordination problems may arise between Hicks substitutes, but relegate analysis to the appendix.

Remark 1. *With decreasing average variable costs, Hicks complementarity is not necessary for the existence of an Investment Coordination problem under Assumptions 1, 2 and 4.*

Allowing inferior goods also gives rise to coordination failures. But the profitability of coordination arises from the lowering of aggregate income which diverts demand towards inferior good producers, thus lowering welfare.

Remark 2. *When all goods are Hicks substitutes, an Investment Coordination Problem can exist between sectors i and j if both i and j are inferior goods under Assumptions 1, 3 and 4. However investment coordination between sectors producing inferior goods reduces welfare.*

The welfare implications of the general model are explored in the next section.

3. Welfare effects of Coordination

Does the existence of an investment coordination problem imply that coordination raises social welfare? In some frameworks, coordination allows the continued improvement of products, or continued introduction of products, which generate intertemporal externalities and lead to an increase in social welfare (e.g. Ciccone and Matsuyama (1992 Sect. 5)). Here we address the question of whether coordinated investment is welfare improving without additional external reasons favouring the coordinated outcome. We therefore examine the welfare consequences of a move from the non-investment to the investment equilibrium when investment coordination problems exist.

For simplicity, assume that only industries i and j coordinate and invest simultaneously. The propositions presented below extend directly to the case of more than two investment coordinating industries. Consider first the income effect of such coordinated investment. From equations (2.6) and (2.8) this is given by

$$\begin{aligned} \Delta y \;=\; & -\Pi_i^I - F_i + x_i^I\left(\gamma_i^I - \gamma_i^E\right) + \\ & \left(\sum_{q\neq i,j}^{n}\left(\gamma_q^C - \gamma_q^I\right)\frac{\Delta x_q}{\Delta p_i} + \frac{\Delta x_i}{\Delta p_i}\left(\gamma_i^I - \gamma_i^E\right) + \frac{\Delta x_j}{\Delta p_i}\left(\gamma_i^I - \gamma_i^E\right)\right)\Delta p_i \\ & -\Pi_j^I - F_j + x_j^I\left(\gamma_j^I - \gamma_j^E\right) + \\ & \left(\sum_{q\neq i,j}^{n}\left(\gamma_q^C - \gamma_q^I\right)\frac{\Delta x_q}{\Delta p_j} + \frac{\Delta x_i}{\Delta p_j}\left(\gamma_i^I - \gamma_i^E\right) + \frac{\Delta x_j}{\Delta p_j}\left(\gamma_i^I - \gamma_i^E\right)\right)\Delta p_j, \end{aligned} \tag{3.1}$$

where the first part depicts the change arising from sector $i's$ investment and the second that for sector j. Consider now the most favourable case to social optimality of coordinated investment which arises when investing alone each firm, i and j, just breaks even. This implies

$$F_i = x_i^I\left(\gamma_i^I - \gamma_i^E\right) + \left(\gamma_i^I - \gamma_i^E\right)\Delta x_i \tag{3.2}$$

and

$$F_j = x_j^I \left(\gamma_j^I - \gamma_j^E\right) + \left(\gamma_j^I - \gamma_j^E\right) \Delta x_j. \tag{3.3}$$

Equation 3.1 can then be re-expressed as

$$\Delta y = -\Pi_i^I + \left(\sum_{q \neq i,j}^{n} \left(\gamma_q^C - \gamma_q^I\right) \frac{\Delta x_q}{\Delta p_i} + \frac{\Delta x_j}{\Delta p_i} \left(\gamma_j^I - \gamma_j^E\right) \right) \Delta p_i$$

$$-\Pi_j^I + \left(\sum_{q \neq i,j}^{n} \left(\gamma_q^C - \gamma_q^I\right) \frac{\Delta x_q}{\Delta p_j} + \frac{\Delta x_i}{\Delta p_j} \left(\gamma_i^I - \gamma_i^E\right) \right) \Delta p_j. \tag{3.4}$$

Subtracting $\frac{\Delta x_j}{\Delta p_i}\left(\gamma_j^C - \gamma_j^I\right)$from the first term in the first large bracket and adding it to the second term in that bracket (and doing the same for $\left(\gamma_i^C - \gamma_i^I\right)$ in the second large bracket yields

$$\Delta y = -\Pi_i^I + \left(\sum_{q \neq i}^{n} \left(\gamma_q^C - \gamma_q^I\right) \frac{\Delta x_q}{\Delta p_i} + \frac{\Delta x_j}{\Delta p_i} \left(\gamma_j^I - \gamma_j^E - \gamma_j^C + \gamma_j^I\right) \right) \Delta p_i$$

$$-\Pi_j^I + \left(\sum_{q \neq j}^{n} \left(\gamma_q^C - \gamma_q^I\right) \frac{\Delta x_q}{\Delta p_j} + \frac{\Delta x_i}{\Delta p_j} \left(\gamma_i^I - \gamma_i^E - \gamma_i^C + \gamma_i^I\right) \right) \Delta p_j. \tag{3.5}$$

Using the envelope theorem, the net impact of investment in sectors j and i on the (indirect) utility of the representative agent is given by

$$dU = \lambda dy - \lambda x_i^I dp_i - \lambda x_j^I dp_j, \tag{3.6}$$

where λ stands for the Lagrange multiplier of the budget constraint. Introducing the expression for the income change from (3.5) and remembering that, in each sector, the extent of the price fall just equals the initial mark-up, one obtains

$$\Delta U = \lambda \left[\begin{array}{c} \left(\sum_{q \neq i}^{n} \left(\gamma_q^C - \gamma_q^I\right) \frac{\Delta x_q}{\Delta p_i} + \frac{\Delta x_j}{\Delta p_i} \left(\gamma_j^I - \gamma_j^E - \gamma_j^C + \gamma_j^I\right)\right) \Delta p_i \\ + \left(\sum_{q \neq j}^{n} \left(\gamma_q^C - \gamma_q^I\right) \frac{\Delta x_q}{\Delta p_j} + \frac{\Delta x_i}{\Delta p_j} \left(\gamma_i^I - \gamma_i^E - \gamma_i^C + \gamma_i^I\right)\right) \Delta p_j \end{array} \right]. \tag{3.7}$$

As a result, under Assumptions (1), (3) and (4), investment coordination leads to a Pareto preferred outcome if and only if the right hand side of equation (3.7) is positive. We summarize with the following proposition.

Proposition 3. *Investment coordination between i and j raises welfare if and only if*

$$\Phi = \left(\sum_{q \neq i}^{n} \left(\gamma_q^C - \gamma_q^I \right) \frac{\Delta x_q}{\Delta p_i} + \frac{\Delta x_j}{\Delta p_i} \left(\gamma_j^I - \gamma_j^E - \gamma_j^C + \gamma_j^I \right) \right) \Delta p_i +$$

$$\left(\sum_{q \neq j}^{n} \left(\gamma_q^C - \gamma_q^I \right) \frac{\Delta x_q}{\Delta p_j} + \frac{\Delta x_i}{\Delta p_j} \left(\gamma_i^I - \gamma_i^E - \gamma_i^C + \gamma_i^I \right) \right) \Delta p_j > 0 \tag{3.8}$$

Expression (3.8) thus makes clear the avenues through which investment coordination raises welfare. What ultimately matters is that i and $j's$ price falls direct demand away from sectors with low mark up rates to sectors with high ones, thereby leading to an increase in profits and income. To see this clearly, consider what happens when mark-up rates are identical.

Assumption 5. *(Symmetry) The incumbent's mark-up rate,*$\frac{\gamma_q^C - \gamma_q^I}{\gamma_q^C}$*, is identical across sectors and also equal to the entrant's mark-up rate* $\frac{\gamma_q^I - \gamma_q^E}{\gamma_q^I}$.

The following proposition confirms that coordination is only worthwhile with heterogeneity across sectors.

Proposition 4. *Under Assumption 5, Hicks complements, though leading to the possibility of multiple equilibria,* **always** *imply that the no investment equilibrium is socially preferred to the investment equilibrium.*

Proof: see Appendix.

If some goods are Hicks complements of a particular good, then utility maximization implies there must exist others that are Hicks substitutes. As a result, if a cost reducing investment in sector i increases the demand for the output of sector j, then, necessarily, demand for the output of sectors producing Hicks substitutes to i is correspondingly reduced. Thus two firms can be made better off by coordinating investments and price reductions only by reallocating demand towards themselves and away from some sectors which have not invested. That is, the gains to investing firms from coordination come at the cost of sectors that produce Hicks substitutes. From a welfare perspective, such gains and losses perfectly offset each other when sectors have symmetric mark-up rates. However, with differences in mark-up rates, demand reallocation from sectors with low rates to those with high rates generates real income effects. For a given unit of expenditure, demand reallocation to sectors with high mark-ups increases the income multiplier of expenditure, and thereby indirectly increases demand for other sectors thus raising welfare. Formally:

Corollary 1. *If the conditions for an investment coordination problem are satisfied, then there exists a distribution of mark-up rates across sectors such that coordinated investment is the social optimum.*

Proof: In condition (3.8), let $(\gamma_q^C - \gamma_q^I) = 0$ for all $q \neq i, j$. Then a sufficient condition for coordination to be an optimum is $\left(2\gamma_k^I - \gamma_k^E - \gamma_k^C\right) > 0$ for $k = i, j$. Under this condition $\Phi > 0$ since $\frac{\Delta x_j}{\Delta p_i}, \frac{\Delta x_i}{\Delta p_j}, \Delta p_j, \Delta p_i < 0$. Q.E.D.

Corollary 1 also sheds light on two possible causes of coordination failure in developing economies: 1) In a largely underdeveloped economy producing with a low mark-up, backstop technology (such as subsistence agriculture), coordinated entry of high mark-up, modern sectors which divert demand from the backstop sectors to one another can be welfare improving. 2) Relaxing assumption 1 we allow leisure to be a choice variable such that one unit is produced by one unit of labour effort. Clearly, the leisure producing sector will have a zero mark-up. Suppose also that there exists only two other sectors which have an investment coordination problem. Then the corollary above implies that solving the coordination problem leads to a Pareto improvement. In the coordinated outcome, individuals undertake more work and consume less leisure but receive higher utility since both production and real wages increase by more than enough to offset these increases, as mark-ups in the producing sectors exceed those in leisure production.

4. Previous literature

The precedents for this work extend back to the early work of Scitovsky (1954), Fleming (1955) and others concerned with the conditions under which coordination could ensure an outcome which was preferred to the non-coordinated outcome. Fleming's informal argument that horizontal complementarities, like the one's examined in this paper, never lead to coordination problems, is not far off the mark. Fleming however seems to have unknowingly assumed away the very condition which we show is necessary for a coordination problem:

> In what follows, we shall assume, in conformity with what appears to be the intention of the balanced-growth doctrine, that the influence exercised by an increase in the supply of A on the real demand for other products is randomly distributed among non-A industries......In other words those industries which are of a critical size in this respect are not, on the average, either specially complementary to or specially competitive with, industry A. (p. 245)

By assuming uniform complementarity between the investing industry A and all others, Fleming has assumed away the possibility of Hicks complementarity across industries, since consumer maximization rules out all goods being Hicks complements. In light of our analysis, Flemings conclusion that

> the installation of an unprofitable plant in industry A, even though its unit cost, at the least unprofitable output, is below that of pre-existing production, is likely to induce contraction rather than expansion in other consumer goods industries,......[rendering investment] not less but more unprofitable than if installed singly. (p. 246)

is as we would expect. Though we show it is possible for a coordination failure, when Hicks complementarity of goods is allowed, our welfare analysis shows that the existence of the coordination problem in itself does not imply welfare gains to coordination.

Early related work also arises from analyses of the optimality of a decentralized market equilibrium in the presence of non-price taking behaviour, such as Hart (1980) and Makowsky (1980). Hart shows that, even if the economy is arbitrarily large relative to firm size, the equilibrium is sub-optimal if products are sufficiently complementary unless (a) consumer's preferences are convex and can be represented by a differentiable utility function, or (b) each firm has access to the production set of the whole economy (or equivalently merger is costless). A key difference between Hart's analysis and the analysis here is Hart's assumption of Cournot-Nash competition between firms, in contrast to our assumption of Bertrand competition. The two main advantages of our approach are that it allows explicit restrictions to be obtained on the properties of demands for goods which give rise to investment coordination problems, and that our approach permits direct comparison with literature on coordination problems and growth traps which also utilizes Bertrand competition between incumbent and entrant. The major advance of our paper is that it goes beyond establishing that the decentralized market outcome *can* be inefficient, and determines conditions under which equilibria obtained are Pareto rankable. Furthermore it presents precise conditions under which complementarity leads to multiplicity.

The emphasis on Hicks complementarity as a necessary condition for the existence of a coordination problem has appeared elsewhere. Matsuyama (1995) in his survey of this literature, shows that in an increasing product variety framework, as in Dixit and Stiglitz (1977), a necessary condition for a coordination problem is again Hicks complementarity between goods. This is surprising since in our model an important factor mitigating the effects of complementarity is the profit loss of incumbent firms who are driven out by entry. In contrast increasing product variety assumes away incumbent firms within investing sectors. Instead, an entrant simply expands the existing variety of goods available, thereby lowering demand for current producers, each of which is equally substitutable with the entrant. However, this direct demand reduction may be offset by an increase in demand for goods taken as a whole. Matsuyama argues that only when goods are Hicks complements, can such entry lead to a rise in incumbents' profits. Since the necessity of Hicks complementarity in our framework arises because the real income effect of the price fall is just offset by the income effect of incumbent's profit loss, it is surprising that this should also be necessary in the increasing product variety framework of Matsuyama where there is no incumbent. In a related paper (Baland and Francois (1996b)), we explore the relationship between our model and Matsuyama's more fully.

4.1. Appendix

4.1.1. Proof of Lemma 2

Equation (2.17) holds for all sectors $j \neq i$, defining a system of $n-1$ equations which can be written in the following matrix form:

$$[\Delta x]_{(n-1)\times 1} = \left[\frac{\partial h}{\partial p_i}\right]_{(n-1)\times 1} \Delta p_i + \left[\frac{\partial x}{\partial y}\right]_{(n-1)\times 1} \left[\gamma^C - \gamma^I\right]_{1\times(n-1)} [\Delta x]_{(n-1)\times 1} \tag{4.1}$$

or equivalently,

$$\left[I - \left[\frac{\partial x}{\partial y}\right]\left[\gamma^C - \gamma^I\right]\right]_{(n-1)\times(n-1)} [\Delta x]_{(n-1)\times 1} = \Delta p_i \left[\frac{\partial h}{\partial p_i}\right]_{(n-1)\times 1}. \tag{4.2}$$

We can apply Cramer's rule to determine the sign of each term of $[\Delta x]_{(n-1)\times 1}$. The denominator is the following determinant:

$$\begin{vmatrix} 1-\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_1}{\partial y} & -\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_1}{\partial y} & \cdots & -\left(\gamma_n^C-\gamma_n^I\right)\frac{\partial x_1}{\partial y} \\ -\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_2}{\partial y} & 1-\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_2}{\partial y} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ -\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_n}{\partial y} & \cdots & \cdots & 1-\left(\gamma_n^C-\gamma_n^I\right)\frac{\partial x_n}{\partial y} \end{vmatrix}$$

In evaluating this determinant many terms cancel, this is clearly seen in a three good example, i.e., three sectors other than i, so that it becomes:

$$\begin{vmatrix} 1-\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_1}{\partial y} & -\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_1}{\partial y} & -\left(\gamma_3^C-\gamma_3^I\right)\frac{\partial x_1}{\partial y} \\ -\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_2}{\partial y} & 1-\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_2}{\partial y} & -\left(\gamma_3^C-\gamma_3^I\right)\frac{\partial x_2}{\partial y} \\ -\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_3}{\partial y} & -\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_3}{\partial y} & 1-\left(\gamma_3^C-\gamma_3^I\right)\frac{\partial x_3}{\partial y} \end{vmatrix},$$

expanding along any row or column, for example the first row, yields:

$$\begin{aligned} &\left(1-\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_1}{\partial y}\right)\left[\begin{array}{c}\left(1-\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_2}{\partial y}\right)\left(1-\left(\gamma_3^C-\gamma_3^I\right)\frac{\partial x_3}{\partial y}\right) \\ -\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_2}{\partial y}\left(\gamma_3^C-\gamma_3^I\right)\frac{\partial x_3}{\partial y}\end{array}\right] \\ &+\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_1}{\partial y}\left[\begin{array}{c}-\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_2}{\partial y}\left(1-\left(\gamma_3^C-\gamma_3^I\right)\frac{\partial x_3}{\partial y}\right) \\ -\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_3}{\partial y}\left(\gamma_3^C-\gamma_3^I\right)\frac{\partial x_2}{\partial y}\end{array}\right] \\ &-\left(\gamma_3^C-\gamma_3^I\right)\frac{\partial x_1}{\partial y}\left[\begin{array}{c}-\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_2}{\partial y}\left(-\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_3}{\partial y}\right)+ \\ \left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_2}{\partial y}\left(1-\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_2}{\partial y}\right)\end{array}\right] \end{aligned}$$

which simplifies to:

$$\begin{aligned} &\left(1-\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_1}{\partial y}\right)\left[\left(1-\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_2}{\partial y}-\left(\gamma_3^C-\gamma_3^I\right)\frac{\partial x_3}{\partial y}\right)\right] \\ &+\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_1}{\partial y}\left(-\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_2}{\partial y}\right)-\left(\gamma_3^C-\gamma_3^I\right)\frac{\partial x_1}{\partial y}\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_3}{\partial y}, \end{aligned}$$

yielding, after expansion once more:

$$1-\left(\gamma_1^C-\gamma_1^I\right)\frac{\partial x_1}{\partial y}-\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_2}{\partial y}-\left(\gamma_3^C-\gamma_3^I\right)\frac{\partial x_3}{\partial y},$$

which can be expressed more simply as:

$$1-\sum_{q=1}^{3}\frac{\left(\gamma_q^C-\gamma_q^I\right)}{\gamma_q^C}\gamma_q^C\frac{\partial x_q}{\partial y}.$$

Thus for the general case of n goods the solution is:

$$1-\sum_{q=1,q\neq i}^{n}\frac{\left(\gamma_q^C-\gamma_q^I\right)}{\gamma_q^C}\gamma_q^C\frac{\partial x_q}{\partial y}. \tag{4.3}$$

In sector 1, the numerator of Cramer's ratio is:

$$\Delta p_i\begin{vmatrix}\frac{\partial h_1}{\partial p_i} & -\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_1}{\partial y} & \cdots & -\left(\gamma_n^C-\gamma_n^I\right)\frac{\partial x_1}{\partial y}\\ \frac{\partial h_2}{\partial p_i} & 1-\left(\gamma_2^C-\gamma_2^I\right)\frac{\partial x_2}{\partial y} & \cdots & \vdots\\ \vdots & \vdots & \ddots & \vdots\\ \frac{\partial h_n}{\partial p_i} & \cdots & \cdots & 1-\left(\gamma_n^C-\gamma_n^I\right)\frac{\partial x_n}{\partial y}\end{vmatrix}. \tag{4.4}$$

Expanding this determinant leads to a similar sequence of cancellations, so that the numerator becomes:

$$\Delta p_i\left[\frac{\partial h_1}{\partial p_i}\left(1-\sum_{q=2,q\neq i}^{n}\frac{\left(\gamma_q^C-\gamma_q^I\right)}{\gamma_q^C}\gamma_q^C\frac{\partial x_q}{\partial y}\right)+\sum_{q=2,q\neq i}^{n}\left(\gamma_q^C-\gamma_q^I\right)\frac{\partial h_q}{\partial p_i}\frac{\partial x_1}{\partial y}\right]. \tag{4.5}$$

Q.E.D.

4.1.2. Declining average variable costs

Here we relax Assumption 3 and allow average variable costs to fall. With declining variable costs an incumbent can compete more aggresively, lowering price below initial average cost, provided demand for the good produced rises. As a result, Lemma 1 does not hold implying that the real income effect of a price fall can exceed the profit loss in the investing sector. This suggests Hicks complementarity is no longer a necessary condition for an investment coordination problem.

More formally consider the following modifications of the basic model in Section II. In all sectors the average cost parameters γ_j^C, γ_j^I and γ_j^E are replaced by, c_j^C, c_j^I and c_j^E respectively, with $\frac{\partial c_j(x_j)}{\partial x_j} \leq 0$. We also impose the following restriction:[10]

$$\frac{\partial \Pi_j^I}{\partial x_j} = \frac{\partial((c_j^C(x_j) - c_j^I(x_j))x_j)}{\partial x_j} > 0 \tag{4.6}$$

and

$$\frac{\partial \Pi_j^E}{\partial x_j} = \frac{\partial((c_j^I(x_j) - c_j^E(x_j))x_j)}{\partial x_j} > 0. \tag{4.7}$$

Relaxing Assumption 3 introduces a major complication as now prices in all sectors can vary with entry into one sector.

Bertrand competition between the entrant in sector i and incumbent implies a price for good i equal to the incumbent's average cost, that is:

$$p_i^E = c_i^I(x_i^E), \tag{4.8}$$

and

$$\begin{aligned} \Delta p_i &= -p_i^I + p_i^E \\ &= -\left[c_i^C(x_i^I) - c_i^I(x_i^I) + c_i^I(x_i^I) - c_i^I(x_i^1)\right] \\ &= -\frac{\Pi_i^I}{x_i^I} + \left[c_i^I(x_i^E - c_i^I(x_i^I)\right], \end{aligned} \tag{4.9}$$

or, approximating the change in costs through the first order derivative: $\Delta p_i \approx -\frac{\Pi_i^I}{x_i^I} - \frac{c_i^I(x_i^I)}{x_i^I}\Delta x_i$. Note that, with $c' < 0$, the price fall is bigger than the initial mark-up provided Δx_i is positive.

[10]These conditions ensure that scale economies benefit leaders relative to their competitors. This is essential for the coordination problem we are concerned with to be possible.

The model soon becomes unmanageable when we consider a large number of sectors. We therefore consider two illustrative examples in the two and three sector cases which demonstrate the basic effects at issue.

Example 1: A two sector economy. Consider an economy with two sectors, 1 and 2, and assume that entry takes place in sector 1. Since the entrant just breaks even, the change in sector 2's demand can be expressed as:

$$\begin{aligned} dx_2 &= \left[\frac{\partial h_2}{\partial p_1}dp_1 + \frac{\partial x_2}{\partial y}\frac{\partial \Pi_2}{\partial x_2}dx_2\right] + \left[\frac{\partial h_2}{\partial p_2}\frac{\partial c_2^C(x_2)}{\partial x_2}dx_2 - x_2\frac{\partial x_2}{\partial y}\frac{\partial c_2^C(x_2)}{\partial x_2}dx_2\right] \\ &\quad - \left[\frac{\partial x_2}{\partial y}x_1\frac{\partial c_1^I(x_1)}{\partial x_1}dx_1\right] \end{aligned} \tag{4.10}$$

in which the effects of variable average costs are represented by the last two bracketed expressions. With a change in x_2, the incumbent is forced by competitors to charge p_2. This has two effects, a substitution effect and an income effect. The second bracket is the change caused by the real income effect of the price fall in sector 1 being greater than the initial mark-up. Also, since:

$$\frac{\partial \Pi_2}{\partial x_2} = \frac{\partial c_2^C(x_2)}{\partial x_2}x_2 - \frac{\partial c_2^I(x_2)}{\partial x_2}x_2 + c_2^C(x_2) - c_2^I(x_2), \tag{4.11}$$

equation (4.10) can also be written as:

$$\begin{aligned} dx_2 &= \left[\frac{\partial h_2}{\partial p_2}\frac{\partial c_2^C(x_2)}{\partial x_2}dx_2\right] - \left[x_1\frac{\partial x_2}{\partial y}\frac{\partial c_1^I(x_2)}{\partial x_1}dx_1\right] \\ &\quad + \left[\frac{\partial h_2}{\partial p_1}dp_1 + (\frac{\partial x_2}{\partial y}(-x_2\frac{\partial c_2^I}{\partial x_2} + c^I(x_2) - c_2^I(x_2)\right] dx_2 \end{aligned} \tag{4.12}$$

The change in demand for good 1 is:

$$\begin{aligned} dx_1 &= \left[\frac{\partial h_1}{\partial p_2}\frac{\partial c_2^C(x_2)}{\partial x_2}dx_2 - x_2\frac{\partial x_1}{\partial y}\frac{\partial c_2^I(x_2)}{\partial x_2}dx_2\right] - \left[\frac{\partial x_1}{\partial y}x_1\frac{\partial c_1^I}{\partial x_1}dx_1\right] + \\ &\quad \left[\frac{\partial h_2}{\partial p_2}dp_1 + \frac{\partial x_2}{\partial y}\frac{\partial \Pi_2^I}{\partial x_2}dx_2\right]. \end{aligned} \tag{4.13}$$

Solving equations, (4.12) and (4.13), one obtains the following:

$$dx_2 \geq 0 \Leftrightarrow \frac{\partial h_2}{\partial p_1} + \frac{\partial h_2}{\partial p_1}\frac{\partial x_1}{\partial y}x_1\frac{\partial c_1^I(x_1)}{\partial x_1} - \frac{\partial h_1}{\partial p_1}\frac{\partial x_2}{\partial y}x_1\frac{\partial c_1^I(x_1)}{\partial x_1} \geq 0. \tag{4.14}$$

For any system of demand it must be true that: $\sum_q p_q \frac{\partial h_q}{\partial p_1} = 0$ and from the budget constraint $1 = \sum_q p_q \frac{\partial x_q}{\partial y}$. Condition (4.14) can therefore be written as:

$$dx_2 \geq 0 \Leftrightarrow \frac{\partial h_2}{\partial p_1}\left[1 + \frac{x_1}{p_1}\frac{\partial c_1^I(x_1)}{\partial x_1}\right] dp_1 \geq 0. \tag{4.15}$$

The derivative of the total cost function of sector 1's incumbent with respect to x_1 yields:

$$\frac{\partial(c_1^I(x_1)x_1)}{\partial x_1} = c_1^I(x_1) + x_1\frac{\partial c_1^I(x_1)}{\partial x_1} = c_1^I\left[1 + \frac{x_1}{c_1^I(x_1)}\right]\frac{\partial c_1^I(x_1)}{\partial x_1}. \tag{4.16}$$

For the system of demands to constitute an equilibrium, the derivative must be non-negative, we therefore assume that this is always true. As $p_1 > c_1^I(x_1)$, it follows from (4.15) that dx_2 is negative. On the other hand dx_1 can be shown to be positive. Consequently, in a 2 good world, the real income effect associated with a price fall exceeding the initial mark-up in the investing sector, 1, is never large enough to outweigh the direct substitution effect associated with the price fall. There is no investment coordination problem in this case.

In a three or more good world the result no longer holds. In this case, real income effects and substitution effects of a particular good are not systematically related.

Example 2: A three sector economy. Assume again that entry takes place in sector 1, and the entrant just breaks even. We also assume that sector 3 makes no profit $c_3^C(x_3) = c_3^I(x_3)$ and that $\frac{\partial c_3^C(x_3)}{\partial x_3} = \frac{\partial c_3^I(x_3)}{\partial x_3} = 0$, so that the price of the two goods remains constant. Then, using the expression for dx_2 from equation (4.10), and imposing the local stability conditions, one obtains the same equation as in (4.14).

$$dx_2 \geq 0 \Leftrightarrow \frac{\partial h_2}{\partial p_1} + \frac{\partial h_2}{\partial p_1}\frac{\partial x_1}{\partial y}x_1\frac{\partial c_1^I(x_1)}{\partial x_1} - \frac{\partial h_1}{\partial p_1}\frac{\partial x_2}{\partial y}x_1\frac{\partial c_1^I(x_1)}{\partial x_1} \geq 0. \tag{4.17}$$

However, the important point to note here is that even when $\frac{\partial h_2}{\partial p_1}$ tends to zero, $\frac{\partial h_1}{\partial p_1}$ no longer necessarily approaches zero as well, since good 3 can still be a substitute to good 1.

As a result, even if all goods are Hicks substitutes, the expression above can be positive and a coordination problem can exist. Remark 1 summarizes the discussion above.

4.1.3. Inferior goods

Here we relax normality of all goods, Assumption 2, and explore the possibility of investment coordination problems when there exist inferior goods. The possibility of such goods arises in developing countries where at low income levels much of expenditure goes to basic staples, such as grains, but at higher income levels substitution towards more expensive foods and/or other consumption goods occurs. Even though we now allow some $\frac{\partial x_j}{\partial y}$ to be negative we must again ensure that, for the stability of the system, the denominator in expression (2.18) is positive.

Even when all goods are Hicks substitutes, $j's$ profit can rise with entry occuring in sector i, provided sector j produces an inferior good. This can be seen directly from condition (2.18) which implies:

$$Sgn\left\{\frac{\Delta x_j}{\Delta p_i}\right\} = Sgn\left\{\frac{\partial h_j}{\partial p_i}\left(1 - \sum_{q\neq i,j}^{n}(\gamma_q^C - \gamma_q^I)\frac{\partial x_q}{\partial y}\right) + \sum_{q\neq i,j}^{n}\left((\gamma_q^C - \gamma_q^I)\frac{\partial h_q}{\partial p_i}\right)\frac{\partial x_j}{\partial y}\right\}.$$

When $\frac{\partial h_j}{\partial p_i}$ is sufficiently small, the sign of $\frac{\Delta x_j}{\Delta p_i}$ depends upon the sign of $\frac{\partial x_j}{\partial y}$. So that if negative, a fall in the price of good i raises demand for sector j. This occurs because the substitution effect between i and j is small compared to its magnitude relative to other goods. As the demand for these other goods falls, profits and aggregate income fall too, and so demand for the inferior good j rises. However by using condition (2.18) again, it follows that provided good i is a normal good, $\frac{\Delta x_i}{\Delta p_j} > 0$, so that even though sector j benefits from entry in sector i, sector i does not also benefit from entry in j, so that there can be no mutual benefits to coordination between i and j.

For an investment coordination problem to occur, both sectors must produce inferior goods. If this is the case, $\frac{\partial h_j}{\partial p_i}$ is small and $\frac{\partial x_j}{\partial y}$ is sufficiently negative then $\frac{\Delta x_i}{\Delta p_j} < 0$. Just as demand for j rises with a fall in p_i,so too does a fall in p_j, by attracting demand from other goods and lowering profits and income, raise the demand for good i. This can lead to an investment coordination problem between the inferior good producing sectors i and j even when all goods are Hicks substitutes.

Now, consider the welfare implications of coordinated investments. The change in aggregate income is decomposed into two components, the change in y associated with investment in sector j, which we denote $\Delta y(j)$, and that associated with investment in sector i, $\Delta y(i)$. If investment coordination with sector i is profitable to sector j, then, through equation (2.20)it follows that this is because

$$\frac{\partial h_i}{\partial p_j}\Delta p_i - x_i\frac{\partial x_j}{\partial y}\Delta p_i + \frac{\partial x_j}{\partial y}\Delta y(i) > 0 \tag{4.18}$$

which, since $\frac{\partial x_j}{\partial y} < 0$, implies

$$\Delta y(i) - x_i \Delta p_i < -\frac{\frac{\partial h_i}{\partial p_j}\Delta p_i}{\frac{\partial x_j}{\partial y}} \leq 0.$$

Similarly, for sector i,

$$\Delta y(j) - x_j \Delta p_j < -\frac{\frac{\partial h_j}{\partial pi}\Delta p_j}{\frac{\partial x_i}{\partial y}} \leq 0. \tag{4.19}$$

As the total change in income is the sum of the two changes, by using equation (3.6), it follows that $dU < 0$. Remark 2 summarizes the discussion above. Q.E.D.

4.1.4. Proof of proposition 4.

Since pre-investment technologies are the same across sectors, we denote the common mark-up rate $\frac{\gamma_q^C - \gamma_q^I}{\gamma_q^C} = \tau$. Substituting for τ, the second term in the first large bracket in

(3.8) becomes $\frac{\Delta x_j}{\Delta p_i}\left(\tau(\gamma_j^I - \gamma_j^C)\right)$ which after substituting for τ again yields $\frac{\Delta x_j}{\Delta p_i}\left(\tau^2\gamma_j^C\right)$. Similar substitution in the second large bracket implies equation (3.8) becomes

$$\Phi = \left(\tau\sum_{q\neq i}^{n}\gamma_q^C\frac{\Delta x_q}{\Delta p_i} - \frac{\Delta x_j}{\Delta p_i}\tau^2\gamma_j^C\right)\Delta p_i +$$

$$\left(\tau\sum_{q\neq j}^{n}\gamma_q^C\frac{\Delta x_q}{\Delta p_j} - \frac{\Delta x_i}{\Delta p_j}\tau^2\gamma_i^C\right)\Delta p_j. \tag{4.20}$$

Using equation (2.18) to express $\frac{\Delta x_q}{\Delta p_i}$ and $\frac{\Delta x_q}{\Delta p_j}$ in terms of derivatives of the Hicks demand functions and derivatives with respect to income yields

$$\left[\begin{array}{c}\tau\sum_{q\neq i}^{n}\gamma_q^C\left[1-\sum_{k\neq i}\frac{(\gamma_k^C-\gamma_k^I)}{\gamma_k^C}\gamma_k^C\frac{\partial x_k}{\partial y}\right]^{-1}\\ \left[\frac{\partial h_q}{\partial p_i}(1-\tau\sum_{z\neq q,i}^{n}\gamma_z^C\frac{\partial x_z}{\partial y})+\tau\sum_{z\neq q,i}^{n}\gamma_z^C\frac{\partial h_z}{\partial p_i}\frac{\partial x_q}{\partial y}\right] - \frac{\Delta x_j}{\Delta p_i}\left(\tau^2\gamma_j^C\right)\end{array}\right]\Delta p_i +$$
$$\left[\begin{array}{c}\tau\sum_{q\neq j}^{n}\gamma_q^C\left[1-\sum_{k\neq j}\frac{(\gamma_k^C-\gamma_k^I)}{\gamma_k^C}\gamma_k^C\frac{\partial x_k}{\partial y}\right]^{-1}\\ \left[\frac{\partial h_q}{\partial pj}(1-\tau\sum_{z\neq q,j}^{n}\gamma_z^C\frac{\partial x_z}{\partial y})+\tau\sum_{z\neq q,j}^{n}\gamma_z^C\frac{\partial h_z}{\partial pj}\frac{\partial x_q}{\partial y}\right] - \frac{\Delta x_i}{\Delta p_j}\left(\tau^2\gamma_i^C\right)\end{array}\right]\Delta p_j.$$

Now, consider the term in brackets which is multiplied by Δp_i. Ignoring for the moment the term raised to the power -1 and the term involving τ^2, expanding the summation over q and cancelling leads to all terms being eliminated except the $\gamma_q^C\frac{\partial h_q}{\partial p_i}$ terms. Thus equation (4.20) becomes

$$\Phi = \left[\tau\left[1-\sum_{k\neq i}\frac{(\gamma_k^C-\gamma_k^I)}{\gamma_k^C}\gamma_k^C\frac{\partial x_k}{\partial y}\right]^{-1}\sum_{q\neq i}^{n}\gamma_q^C\frac{\partial h_q}{\partial p_i} - \frac{\Delta x_j}{\Delta p_i}\left(\tau^2\gamma_j^C\right)\right]\Delta p_i +$$
$$\left[\tau\left[1-\sum_{k\neq j}\frac{(\gamma_k^C-\gamma_k^I)}{\gamma_k^C}\gamma_k^C\frac{\partial x_k}{\partial y}\right]^{-1}\sum_{q\neq j}^{n}\gamma_q^C\frac{\partial h_q}{\partial pj} - \frac{\Delta x_i}{\Delta p_j}\left(\tau^2\gamma_i^C\right)\right]\Delta p_j. \tag{4.21}$$

It is well known that a weighted sum of the derivatives of the Hicks demand with respect to the price of a particular good, say good i, equals

$$\sum_j p_j\frac{\partial h_j}{\partial p_j} = 0. \tag{4.22}$$

Using equation (4.22) in equation (4.24), remembering that $\gamma_q^C = p_q^C$, yields

$$\begin{aligned} \Phi &= \left[-\tau \left[1 - \sum_{k \neq i} \frac{(\gamma_k^C - \gamma_k^I)}{\gamma_k^C} \gamma_k^C \frac{\partial x_k}{\partial y} \right]^{-1} \gamma_i^C \frac{\partial h_i}{\partial p_i} - \frac{\Delta x_j}{\Delta p_i} \left(\tau^2 \gamma_j^C \right) \right] \Delta p_i + \\ & \left[-\tau \left[1 - \sum_{k \neq j} \frac{(\gamma_k^C - \gamma_k^I)}{\gamma_k^C} \gamma_k^C \frac{\partial x_k}{\partial y} \right]^{-1} \gamma_j^C \frac{\partial h_j}{\partial pj} - \frac{\Delta x_i}{\Delta p_j} \left(\tau^2 \gamma_i^C \right) \right] \Delta p_j. \end{aligned} \qquad (4.23)$$

Finally, replace the mark-up rates with τ and simplify using the derivative of the budget constraint with respect to income (which implies $1 = \sum_j \gamma_j^C \frac{\partial x_j}{\partial y}$) to obtain

$$\begin{aligned} \Phi &= \left[-\tau \left[1 - \tau \gamma_i^C \frac{\partial x_i}{\partial y} \right]^{-1} \gamma_i^C \frac{\partial h_i}{\partial p_i} - \frac{\Delta x_j}{\Delta p_i} \left(\tau^2 \gamma_j^C \right) \right] \Delta p_i + \\ & \left[-\tau \left[1 - \tau \gamma_j^C \frac{\partial x_j}{\partial y} \right]^{-1} \gamma_j^C \frac{\partial h_j}{\partial pj} - \frac{\Delta x_i}{\Delta p_j} \left(\tau^2 \gamma_i^C \right) \right] \Delta p_j. \end{aligned} \qquad (4.24)$$

As per equation (2.18) the term raised to power -1 in each bracket is positive, furthermore $\frac{\Delta x_j}{\Delta p_i}$ and $\frac{\Delta x_i}{\Delta p_j}$ are negative and own price derivatives of Hicks demands are always negative so that both terms in (4.24) are positive. Finally since prices fall with coordinated investment, this implies $\Phi < 0$. Investment coordination lowers representative agent's welfare. Q.E.D.

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