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Credit and Money in a Search Model with Divisible Commodities

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Abstract

This paper examines the competition between money and credit in a search model with divisible commodities. It is shown that fiat money can be valuable even though it yields a lower rate of return than the coexisting credit. The competition between money and credit increases efficiency. The monetary equilibrium with credit Pareto dominates the monetary equilibrium without credit whenver the two coexist. When a credit is repaid with money, the competition also bounds the purchasing power of money from below by that of credit. In so doing it eliminates the weak monetary equilibrium found in previous search models. With numerical examples, we rank three different monetary equilibria and examine the properties of the interest rate.

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to accommodate credit. He concluded that "(especially without credit) the analysis of monetary policy must be omitting some of the most important elements, and can not satisfactorily consider open market operations." As a theoretical concern, one may argue that credit can also alleviate the lack of double coincidence of wants. Fiat money may cease to be valuable in the presence of credit that delivers a higher rate of return.

Even if fiat money continues to be valuable in spite of return dominance, there is another reason for introducing credit in a search model. The Kiyotaki-Wright model possesses multiple monetary equilibria. Money can be either partially or fully accepted in the economy. As shown by Shi (1993) and Trejos and Wright (1993), similar multiplicity exists when prices are endogenous determined. There are a weak monetary equilibrium and a strong monetary equilibrium that differ in the purchasing power of money. It is interesting to see whether competing credit can eliminate the inefficient weak monetary equilibrium.

The two restrictions in the Kiyotaki-Wright model have been separately but not simultaneously removed in researches that have followed. Shi (1993) and Trejos and Wright (1993) introduced sequential bargaining to determine the purchasing power of money. Diamond (1990) introduced credit into a nonmonetary search economy. Hendry (1992) introduced credit into a monetary search economy but restricted trade to one-to-one swaps of inventories. As pointed out above, this restriction forces the interest rate to be zero. Moreover, Hendry restricted the competition between money and credit by assuming that money holders cannot choose to use credit once they are matched to producers.

This paper simultaneously removes the two restrictions in the Kiyotaki-Wright model. We allow two agents to conduct an *IOU* trade when they have only a single coincidence of wants. Beside price determination, the model improves upon Hendry's in two other respects. First, there is a direct competition between credit and money because a money holder can choose between using money and *IOU* to exchange for goods. Second, an *IOU* can be repaid with either money or goods. In contrast, an *IOU* is repaid with goods only in Hendry's model.

It is shown that monetary equilibria exist with or without credit. For suitable parameter values three monetary equilibria, an increase in the proportion of moneyholders increases the maturity and reduces the interest rate. We also illustrate that the monetary equilibrium with only nominal repayments Pareto dominates the monetary equilibrium with only real repayments.

The remainder of this paper is organized as follows. The next section describes the search economy with special tastes and technology that rule out barter. Section 3 establishes the existence of two monetary equilibria, one with credit and one without. It ranks the two equilibria and shows that credit dominates money in the rate of return when they coexist. Section 4 extends the model to allow barter and examines three monetary equilibria associated with different means of repayment. Section 5 concludes the paper and the appendix provides necessary proofs.

2. The Economy

We extend Diamond's (1984) coconut economy in several dimensions and motivate the modelling assumptions. A particular feature of the environment is the absence of barter. This simplifies discussions on the credit arrangement. Section 4 will incorporate barter.[2]

2.1. Tastes and endowments

Imagine an island on which there are many coconut trees. Coconuts are in N different shapes, lying in the set $\mathcal{N} = \{1, 2, \cdots, N\}$. Coconuts are perishable once picked from the trees. There are also N types of equally populated agents. We call an agent of type $i \in \mathcal{N}$ agent i, although many other agents have the same type. Agent i consumes only type $i+1$ coconuts (with modulus N), which we call agent i's *consumption good.* The utility of consuming q units of the consumption good is $u(q)$ where u is twice continuously differentiable with $u(0) = 0, u' > 0, u'' < 0, u'(0) = \infty$ and $u'(\infty) = 0$. All agents have the same rate of time preference $r > 0$.

Agents are specialized in production. Agent i only knows how to climb the trees that grow coconuts of shape i. We call coconut i agent i's *production good.* Since agents cannot produce their own consumption goods, exchange is necessary for consumption.[3] If $N \leq 2$, any two types

[2] I thank Nobu Kiyotaki and a referee for suggesting the simplification that eliminates barter.

[3] These assumptions on production and consumption are made to simplify the model. In Shi (1994a) and Burdett, et al. (1993), agents are allowed to produce their consumption goods. A monetary equilibrium exists in

exchanges described below. We restrict the analysis to the case where money is indivisible. That is, each moneyholder holds only one unit of money and exchanges the entire unit of money when exchange (see section 5 for a discussion).

2.2. Exchange

The island is large and there is no centralized market place. Agents meet each other according to a random matching process. In each period, trading partners arrive to an agent in exchange in a Poisson process with a constant rate $\beta > 0$. The number of agents with whom a given agent is matched is β times the total number of agents involved in exchange. That is, the matching technology exhibits constant returns to scale. There are no increasing returns to scale that are crucial to Diamond's (1984) results.

In general there are three types of agents in exchange. The first is moneyholders. The second is ordinary producers who have no contractual agreement. We reserve the term producer for such an agent. The third type is a debtor. A debtor is an agent who can produce but who has not repaid his debt. As we will see below, creditors do not involve in exchange. Because there is no double coincidence of wants, the possible exchanges are monetary trade and credit trade. They are summarized as follows.

	producers	debtors	moneyholders
producers	credit trade	————	monetary trade (credit trade)
debtors	————	————	monetary trade
moneyholders	monetary trade (credit trade)	monetary trade	————

There are two types of monetary trade. One is between a moneyholder and a producer; the other is between a moneyholder and a debtor. Both types of trade require that the agent who exchanges goods for money produce the moneyholder's consumption goods. After exchange the moneyholder consumes immediately and becomes a producer. In a monetary trade between a moneyholder and a producer, the producer produces q_m units of goods for money, where the quantity q_m is determined by bilateral bargaining described later. After exchange the producer becomes a (hungry) moneyholder. In a monetary trade between a moneyholder and a debtor, the commodity money. A debtor could accept an IOU from other agents and swap IOU's with his own creditor to regain his knife. With the cost ϵ, the IOU swap does not occur because it reduces the original creditor's utility by ϵ.

Several aspects of the credit trade are noteworthy. First, an agent can at most issue one IOU at a time. He can issue the second IOU only after repaying the first. This is because a producer has only one knife. The inability to issue multiple IOU's rules out trade between two debtors. For two debtors to trade, at least one of them must issue the second IOU.

With $N \geq 4$, the inability to issue multiple IOU's also rules out repayments with goods. To see why, let us use the example where agent i is the debtor and agent $i+1$ is the creditor. To repay with the creditor's consumption good $i+2$, the debtor must meet agent $i+2$ in exchange. Since agent i cannot issue the second IOU before repaying the first, the only possible way for him to exchange for agent $(i+2)'$s production good is to produce agent $(i+2)'$s consumption good $i+3$. Because agent i can only produce good i, this is possible if and only if $i = i+3$ with modulus N. That is, the exchange is possible only if $N = 3$, which is excluded by the assumption $N \geq 4$. In section 4, we will allow the possibility of barter and hence allow IOU's to be repaid with either money or goods.

Second, a credit trade enables an agent to consume twice consecutively. A would-be debtor enters a credit trade with enough energy to produce. By issuing an IOU, he does not produce immediately but consumes the second time. Thus a debtor can produce twice consecutively, once for a unit of money for the repayment and once after the repayment of debt. That is, after the repayment a debtor becomes a producer. The creditor receives money and becomes a (hungry) moneyholder.

Third, there is a direct competition between credit and money. Whenever a monetary trade is possible between a moneyholder and a producer, so is a credit trade. The moneyholder can issue an IOU instead of using money to exchange for goods. In the above table of exchange we included such an optional trade in the parentheses. If money is a preferable medium of exchange, moneyholders must prefer to use it to exchange for goods. The direct competition between credit moneyholder. If the moneyholder rejects the proposal, the two agents search for new partners in a fixed time interval Δ. If either of the two finds a new match that has a single coincidence of wants, the trade between the two is terminated. Bargaining continues when neither of the two finds a suitable new match in the interval Δ. In that case nature chooses the proposer again and the game continues. When $\Delta \to 0$, this sequential bargaining game delivers the solution for q_m which coincides with the above Nash bargaining solution. The bargaining game also induces an immediate agreement on the terms of trade.

A particular feature of the above sequential bargaining game is that the two agents search for new partners after a rejection of a proposal. In contrast, the framework in Rubinstein (1982) precludes search between bargaining rounds. Allowing search between rounds creates the possibility that the agents may be left out when the partner finds a suitable new match in the interval Δ. This possibility changes the agents' threat points. As publicized by Osborne and Rubinstein (1990, pp.54-63), equilibrium outcomes depend sensitively on the details of the bargaining process and in particular on how bargaining breaks down. While the same sensitivity may exist in a monetary model, the issue has been examined to some extent by Shi (1993) and Trejos and Wright (1993) and will not be the focus of this paper. These previous works have shown that fiat money is valuable with or without the assumption that agents search between bargaining rounds.

We further simplify the bargaining solution by restricting the bargaining weight a:

Assumption 1. *In a bilateral trade the agent who exchanges goods for money or credit has a bargaining weight $a = 0$.*

This assumption is equivalent to that the moneyholder in a monetary trade or the debtor in a credit trade makes a take-it-or-leave-it offer. It is made for simplifying the algebra. Simplification can also be achieved by the alternative assumption that the producer makes a take-it-or-leave-it offer ($a = 1$). The difficulty with this alternative assumption is that a moneyholder obtains zero surplus in the monetary trade, which will induce a zero value for holding money. That is, money will not be valuable when moneyholders have no bargaining power. To ensure existence of

Lemma 2.1. *An IOU is accepted and expected to be repaid if and only if (2.2) and the following conditions hold:*

$$V_p - V_d - cq_m \geq 0, \tag{2.5}$$

$$V_m \geq V_c. \tag{2.6}$$

Proof. To repay an IOU the debtor must trade for money. For a moneyholder to trade with a debtor, (2.2) is necessary. For a debtor to repay an IOU, (2.5) is necessary. If it were violated, the debtor would retain a higher value from not repaying the IOU, V_d, than the value from repaying the IOU, $V_p - cq_m$. In this case a creditor would not expect an IOU to be repaid and hence would not accept an IOU. In addition, for the creditor to accept the repayment, the surplus $(V_m - V_c)$ must be nonnegative. That is, (2.6) must hold. If all conditions in the lemma are satisfied, an IOU is accepted and anticipated to be repaid. ■

Since $V_d < V_p$ by (2.5), two agents without coincidence of wants at all do not carry out the credit trade. If they traded, the surplus to the debtor would be $V_d - V_p < 0$. Finally a moneyholder can in principle conduct a credit trade with a producer and receive a surplus $(V_d - V_m + u(q_c))$. For a moneyholder to choose a monetary trade, he must obtain a larger surplus from the monetary trade. Thus we require:

$$V_p - V_m + u(q_m) > V_d - V_m + u(q_c). \tag{2.7}$$

3. Monetary Equilibrium

3.1. Monetary equilibrium without credit

There are two types of monetary equilibria in this economy. In one only the monetary trade is conducted; in the other both the monetary trade and the credit trade are conducted. Let us first examine the monetary equilibrium where credit is not accepted. Suppose that the values (V_p, V_d, V_m) violate at least one of the two conditions (2.5) and (2.6). We later verify that these values can be delivered by equilibrium. Without credit trade, all agents are in the exchange.

debtors in exchange by $d = N_d/(1 - N_d)$. In a stationary equilibrium, the measures of different types of agents in exchange are constant. Then

$$0 = \dot{N}_d = \beta(1 - n - d)zN_p - \beta nzN_d.$$

The first term $\beta(1 - n - d)zN_p$ measures the producers who become new debtors. In particular, $\beta(1 - n - d)z$ is the rate at which a producer meets another producer who can produce his consumption goods. The second term βnzN_d in the above equation measures the debtors who repay their IOU's and become new producers.

From the definitions of n and d, we have

$$\begin{aligned} N_d &= 1 - M/n, \quad d = n/M - 1, \\ N_p &= 1 - M - 2N_d = 2M/n - M - 1. \end{aligned} \tag{3.3}$$

Substitute these relations into the equation for N_d, we have:

$$n(\frac{n}{M} - 1) = [2 - (1 + \frac{1}{M})n]^2. \tag{3.4}$$

There is a unique admissible solution $n \in [M, \frac{2M}{1+M}]$.[6] The solution is an increasing function of M. Also, d is a decreasing function of M.

To describe an equilibrium, the values V must also be determined. They are given by the following equations:

$$rV_m = \beta(1 - n)z(V_p - V_m + u(q_m)); \tag{3.5}$$

$$\begin{aligned} rV_p &= \beta nz(V_m - V_p - cq_m) + \beta(1 - n - d)z(V_d - V_p + u(q_c)) \\ &\quad + \beta(1 - n - d)z(V_c - V_p - cq_c); \end{aligned} \tag{3.6}$$

$$rV_d = \beta nz(V_p - V_d - cq_m); \tag{3.7}$$

$$rV_c = \beta nz(V_m - V_c). \tag{3.8}$$

[6]The other solution is greater than $2M/(1 + M)$ and hence implies $N_p < 0$.

The lemma is clear from Figure 1 and its proof is delayed to the appendix A. It does not exclude the possibility that there can be more than one admissible solution to (3.9) and (3.10). However for some special utility functions, there is only one positive solution. Thus we will consider only the smallest positive solution to (3.9) and (3.10).

Remark 1. *The solution satisfies (2.2), (2.4) and (2.6).*

Proof. First, (2.4)$\Leftrightarrow$ $V_p > 0$ $\Leftrightarrow$(3.11). (2.2) holds because $(1-n)(u(q_m) - cq_m) = (R + n)c(q_m - q_c) > 0$. The equality follows from (3.9) and the inequality from (3.11). Thus by (3.5), $V_m > 0$. (3.8)$\Rightarrow$ $V_m - V_c = \frac{R}{R+n}V_m > 0 \Rightarrow$ (2.6). ■

For an equilibrium with credit to exist, it suffices to verify (2.5) and (2.7). That is, debtors repay their IOU's and moneyholders use money to exchange. In fact, (2.5) implies $V_p - V_d > cq_m > 0$ and hence implies (2.7). Only (2.5) is left to be verified. Let $R = r/(\beta z)$ be the effective rate of time preference. The proof of the following proposition is left for the appendix B.

Proposition 3.5. *For sufficiently small R, there exists $M_0 \in (0,1)$ such that a monetary equilibrium with credit exists for $M \geq M_0$.*

The conditions for the existence of credit are quite intuitive. It takes time to repay the IOU. The length of time depends on the proportion of moneyholders in the economy. If there are few moneyholders, repayments take a long time because it is difficult for the debtor to find a suitable moneyholder to trade. Thus for credit to be valuable, M must be large. For the same reason, a high matching rate β and a high chance z also make the repayment faster. A small rate of time preference r also helps the existence of valuable credit by reducing the utility cost of time that is required for the repayment. These factors are summarized in Proposition 3.5 by a small $R = r/(\beta z)$. Section 4.4 will provides some numerical examples to illustrate the critical level M_0.

3.3. The interest rate and return dominance

Credit and money have different rates of return, although both are used in exchange. Let θ be the net interest rate or the internal rate of return to IOU. A creditor pays q_c units of goods for might suspect. The producer in a monetary trade is indifferent between producing for money or producing for an IOU. The surplus is zero in both cases under Assumption 1.

The second reason for a moneyholder to prefer money is that there are no strains attached. Money is a genuine medium of exchange but an IOU is not. After a monetary exchange the two agents can be matched to any other agents. In contrast, a credit trade ties the creditor and the debtor for some time. The IOU must be repaid to release the two agents from the contractual relation. Neither the creditor nor the debtor consumes in the period of repayment. Furthermore, from the table of exchange in section 2.2, a debtor has fewer suitable exchanges than a producer. With these two factors, a credit trade slows down the speed of exchange and consumption for the traders.

If money has such advantages over credit, one wonders why credit is still valued in this economy. Why would not the competition between money and credit drive out credit? The explanation relies on the lack of centralized marketplace in this economy. Although money is attractive, not everyone has it. In the match between two producers with a single coincidence of wants, the one who cannot supply the partner's consumption good could obtain a higher surplus if he had money. But costly search is necessary for acquiring money. In this case the two agents in such a match try to make the best out of the current match. That is, they are willing to carry out a credit trade as long as it gives a non-negative surplus. If it were costless to find money whenever there is a single coincidence of wants, money would be used in every trade.

At a general level, the above explanation shares the insight given by Hosios (1990) in a random matching model of unemployment. He explains why the wage rate does not approach the competitive equilibrium level when the supply and demand satisfy the competitive conditions. The reason is that matching takes time and takes place before bargaining. The terms of trade determined by bargaining have only limited allocative or signalling function. In the current model, two matched producers do not have an immediate access to money. The difference between the purchasing power of money q_m and the purchasing power of credit q_c is likely to remain as long as matching is random.

the nominal price of an IOU.

Similarly an increase in the matching rate β or the probability z reduces the effective rate of time preference and hence increases the nominal price of the IOU. This tends to reduce the expected interest rate. However, such an increase also reduces the maturity of the IOU and hence tends to increase the expected interest rate. The overall effect is analytically ambiguous.

An increase in the proportion M of moneyholders has ambiguous effects on both the nominal price and the maturity of the IOU. The effect on the maturity is ambiguous because more moneyholders in the economy speed up the repayment of the IOU faster but slows down the exchange of money for good. The ambiguous effects of M on the purchasing power q_c can be explained as follows. An increase in M can increase q_c because it generates a liquidity effect. It increases the proportion of money holders and makes IOU more easily repaid with money. Anticipating such easier repayments, the creditor may be willing to trade a larger quantity of goods for the IOU. That is, "loans" are easier to obtain. An increase in M can also decrease q_c because it generates a crowding-out effect. Having more moneyholders in the market crowds out producers. More moneyholders are chasing each producer. The purchasing power of money q_m tends to be lower. Since the ultimate goal for a creditor to accept an IOU is to use the repaid money to purchase consumption goods, a reduction in the purchasing power of money reduces the purchasing power of credit.

Numerical exercises show that for large M, an increase in M reduces the nominal price and increases the maturity of an IOU. The effect on the maturity dominates so that the expected interest rate falls. The following example is used:

Example 3.6. $u(q) = q^{\sigma}$, $\sigma = 2/3$, $c = 1$, $\beta = 5$, $z = 0.1$, $R = 0.01$.

For $M = 0.68$, $\ln(q_m/q_c) = 0.03$ and $E\theta = 0.003$. When M increases to 0.70, $\ln(q_m/q_c)$ increases to 0.0305 but $E\theta$ decreases to 0.0029.

4. Equilibrium with Barter

4.1. Extension of previous sections

We now modify the descriptions of tastes and production to allow barter. We show that the monetary equilibrium with credit continues to exist for suitable parameter values. The monetary equilibrium without credit also exists for some parameters but we omit the examination.

The economy has a continuum of types of goods identified by a circle with circumference 2. To economize on notation, use the same notation $\mathcal{N}$ to denote the points along the circle. There are also a continuum of types of agents with a unit mass. Identify the set of agents also by $\mathcal{N}$. Agent i is specialized in producing good i. We assume that agents do not consume their own products (see footnote 3 for clarification). The set of consumption goods of agent i is $\{j \in \mathcal{N} : \widehat{j\,i} \leq z\}$, where $\widehat{j\,i}$ is the length of the arc between goods j and i. Every consumption good is equally preferred. To make barter an imperfect means of exchange, assume $0 < z < 1$. In this case, two randomly selected producers can barter with probability z^2.

Since barter is possible, an *IOU* may be repaid with either money or goods or both, depending on the specific equilibrium. Let us refer to repayments with money as nominal repayments and the repayments with the creditor's consumption goods as real repayments. The general possibilities of trade are as follows:

	producers	debtors	moneyholders
producers	barter credit trade	[barter]	monetary trade (credit trade)
debtors	[barter]	[barter]	monetary trade
moneyholders	monetary trade (credit trade)	monetary trade	———

As before, the trade in ( ) is optional to the moneyholder. The trade in [ ] takes place only when creditors accept real repayments. The equilibria that generate real repayments are examined later in sections 4.2 and 4.3. In this subsection we examine only nominal repayments. Call the corresponding equilibrium a monetary equilibrium with only nominal repayments. In such an equilibrium, the only form of barter is between two producers.

The terms of trade in barter must be determined. Since the two agents in barter both produce obtain real repayments through barter with a producer or another debtor. Since a debtor has a different threat point from a producer, the terms of barter trade between a debtor and a producer are different from q_b. Similar to the simplification on the difference between q_d and q_m in section 2.3, we assume away such a difference. Thus the real repayment is q_b units of consumption goods. By accepting real repayments, the deviating creditor increases the expected surplus by $\beta(1-n-d)z^2(V_p - V_c + u(q_b))$; the deviating debtor increases the expected surplus by $\beta(1-n-d)z^2(V_p - V_d - cq_b)$. For real repayments not to be accepted in equilibrium, at least one of the following inequalities must hold:

$$V_p - V_c + u(q_b) < 0, \quad V_p - V_d - cq_b < 0. \tag{4.1}$$

The equations for q_m and q_c now are (3.9) and the following:

$$\left(\frac{1}{1-n-d} + \frac{2(1-z)}{R+n}\right) ncq_m = (1-z)u(q_c) + \left(1 - z + \frac{R+n}{1-n-d}\right) cq_c + z(u(q_b) - cq_b). \tag{4.2}$$

Similar to section 3.2, the following results can be established. The proofs are omitted.

Lemma 4.1. *For sufficiently small z, there exist an odd number of positive solution pairs (q_m, q_c) that satisfy (3.9) and (4.2). The smallest such pair satisfies*

$$q_c < \frac{n}{R+n} q_m - \left(\frac{1-n-d}{R+n}\right) \frac{z}{c}(u(q_b) - cq_b).$$

It also satisfies (2.2), (2.4) and (2.6).

Proposition 4.2. *Let z and R be sufficiently small. There exists $M_1 \in (0,1)$ such that a monetary equilibrium with only nominal repayments exists for $M \geq M_1$.*

As before the effective rate of time preference R must be small to induce a credit arrangement. The parameter z is required to be sufficiently small to restrict the usefulness of barter. Since there is no barter in section 3.2, Proposition 4.2 illustrates the continuity of the existence of the monetary equilibrium with credit when z is near zero. The equilibrium also extends return dominance to an economy with barter. The interest rate has the same definition as in section 3.3 and return dominance is evident from the inequality in Lemma 4.1. The intuition for return dominance is similar to that in section 3.3.

$$u(q_c) + \left[1 + \frac{R + (1-n)z}{(1-n-d)(1-z)}\right] cq_c = \frac{(1-n)z}{R + (1-n)z} \left\{u(q_b) + \left[1 + \frac{R + (1-n)z}{(1-n-d)(1-z)}\right] cq_b\right\}. \tag{4.10}$$

Lemma 4.3. *There are either two admissible solution pairs (q_m, q_c) or no solution to (4.9) and (4.10). The two pairs differ only in q_m. All possible solutions satisfy (2.2), (2.4), the inequality $u(q_b) + V_p - V_c > 0$ and*

$$q_c < \frac{(1-n)z}{R + (1-n)z} q_b. \tag{4.11}$$

The proof is in the appendix D. When the solutions to the equations (4.9) exist, we denote them by q_L and q_H with $q_L < q_H$. To shorten terminology, we refer to the pair (q_m, q_c) as an equilibrium if it satisfies the corresponding incentive constraints. To describe existence, denote

$$\alpha = \frac{u(q_b) - cq_b}{u(\frac{u(q_b)}{c}) - u(q_b)}, \quad z_0 = \frac{1}{2\alpha}\left[1 + \alpha - \sqrt{(\alpha - 1)(\alpha + 3)}\right].$$

Note that $\alpha > 1$ because the function $u(q) - cq$ is decreasing for $q > q_b$ and because $u(q_b)/c > q_b$. Also $z_0 \in (0, 1)$. The following proposition is proven in the appendix E.

Proposition 4.4. *For given z and $1 - M$ that are bounded strictly above zero, there exist R_0 such that the two solutions q_L and q_H exist for $R < R_0$. Moreover, $q_L < q_c < q_b < q_H$. (q_L, q_c) is a monetary equilibrium with only real repayments. The pair (q_H, q_c) is also such a monetary equilibrium only if $z < z_0$.*

As before, the effective discount rate R must be small to make credit valuable. Also, for an *IOU* to be repaid only with the creditor's consumption goods, the debtor must be able to barter for these goods relatively fast. Otherwise nominal repayments are a superior option. Not surprisingly Proposition 4.4 requires z and $1 - M$ to be bounded strictly above zero.

There can be two monetary equilibria with different purchasing powers of money. Multiple monetary equilibria are a typical feature of search models of money. The earlier model by Diamond (1984) generates multiple equilibria from the increasing returns to scale in the matching technology. With constant returns to scale, the Kiyotaki-Wright model generates multiple levels interest rate.

4.3. Equilibrium with both real and nominal repayments

It is possible that an IOU is repaid with either goods or money, whichever becomes available first to the debtor. In this case, the equation for n is revised to:

$$[n+(1-n)z](\frac{n}{M}-1)=(1-z)[2-(1+\frac{1}{M})n]^2.$$

The value V_p is still given by (4.4). The values (V_m, V_d, V_c) are now given by

$$rV_m=\beta(1-n)z[V_p-V_m+u(q_m)];$$

$$rV_d=\beta nz(V_p-V_d-cq_m)+\beta(1-n)z^2(V_p-V_d-cq_b);$$

$$rV_c=\beta nz(V_m-V_c)+\beta(1-n)z^2[u(q_b)+V_p-V_c].$$

The incentive constraints are (2.2), (2.4), (2.5), (2.6), (2.7) and (4.7).

The joint equation system for (q_m, q_c) is

$$cq_c=\frac{1-n}{R+n+(1-n)z}\left[zu(q_b)-u(q_m)+(1+\frac{R+n}{1-n})cq_m\right],$$

$$\frac{(2+k)n}{R+n+(1-n)z}cq_m=u(q_c)+(1+k)cq_c-\frac{(1-n)z}{R+n+(1-n)z}\left[u(q_b)+(1+k)cq_b\right],$$

where $k=[R+n+(1-n)z]/[(1-n-d)(1-z)]$. Similar to section 4.2, there are an even number of solutions to these two equations. However, it is difficult to verify the incentive constraints analytically. In the next subsection we provide a numerical example to show that such an equilibrium exists for large M and small but not too small z.

4.4. Coexistence of the equilibria

We give numerical examples to show the coexistence of the three monetary equilibria in subsections 4.1 to 4.3. Denote the equilibrium with only nominal repayments by E^N, the one with only real repayments by E^R and the one with both real and nominal repayments by E^{NR}. For equilibrium E^R, the purchasing power of money can be either high or lower. Use E^R_H and E^R_L to

5. Conclusion

Random matching generates inefficiency and distinguishes the search model of money from conventional monetary models that employ Walrasian markets. In this viewpoint, return dominance is a natural outcome of search models. Supporting this view, the present paper has shown that the existence of valuable fiat money in the Kiyotaki-Wright model is robust to the introduction of credit. When a credit is repaid with only money, the presence of credit bounds the purchasing power of money from below and eliminates the inefficient weak monetary equilibrium found in previous search models.

The search model captures the transaction cost in a specific way. When it is costly to find a suitable trading partner, agents try to make the best deal out of their current matches. The resulting terms of trade necessarily deviate from those in a Walrasian market. The deviation not only depends on the proportions of different types of agents in the market but also on how agents perceive the gains of trade. In the present paper, for example, the existence and the value of credit all depend on agents' beliefs. Therefore a search model might provide a novel approach to other issues such as the exchange rate and the purchasing power parity condition.

While this paper improves upon the previous ones by simultaneously introducing credit and price determination, it shares the restriction that moneyholders spend all of their holdings in each trade. To examine some policy issues such as money growth, it is more appropriate to allow moneyholders to spend only part of their holdings. The difficulty involved in this extension is that agents' strategies depend on their money holdings and their histories of trading. In general there will be a distribution of money holdings and hence of prices. The joint determination of the value of money and the distribution of money holdings presents a serious technical difficulty. Diamond and Yellen (1990) have attempted to solve the joint determination under the restrictions of cash-in-advance and indivisible goods. It remains to see whether their technique is tractable without these restrictions. Circumventing the problem by making some simplifications, Shi (1994b) has extended the search model to incorporate money growth and capital accumulation.

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Hence a sufficient condition for $q_f < q_g$ is

$$\frac{n-R}{n+R} > \frac{1-n+\frac{1-n}{1-n-d}R}{1-n+2R}.$$

This condition can be shown to be equivalent to

$$(1-n)(3n-2) - 2d(2n-1) - (2+\frac{1-n}{1-n-d})R > 0. \tag{B.1}$$

Consider the function $a(M) \equiv (1-n)(3n-2) - 2d(2n-1)$. With the features of n and d shown above, we have $a(1) = 0$ and $a'(1) = -1 < 0$. For M sufficiently close to 1, $a(M) > a(1) = 0$. Therefore for sufficiently small R, there exists $M_0 \in (0,1)$ such that (B.1) is satisfied. ■

C. Monetary Equilibrium with $q_d \neq q_m$

In this appendix we show that the monetary equilibrium with credit in section 3.2 exists even when a debtor exchanges a quantity q_d in a monetary trade that is different from q_m. The equation for n is the same as (3.4). Under Assumption 1, the moneyholder makes a take-it-or-leave offer in the trade with a debtor so that $q_d = (V_p - V_d)/c$ and $V_d = 0$. The equation for V_p and V_c are the same as (3.6) and (3.8). Omitting the zero surplus terms, V_m is given by

$$rV_m = \beta z(1-n-d)[u(q_m) - cq_m] + \beta zd[u(q_d) - cq_m].$$

The quantities q_m, q_c and q_d are given by the following equations:

$$u(q_c) = c(1+\frac{R}{1-n-d})q_d; \tag{C.1}$$

$$nq_m = Rq_d + (R+n)q_c; \tag{C.2}$$

$$(1-n-d)u(q_m) = (1+\frac{R}{n})(1+R-n)cq_c + (1+R)\frac{R}{n}cq_d - du(q_d). \tag{C.3}$$

(C.1) gives q_c as a convex function of q_d. Substituting this function into (C.2) and (C.3) we obtain two equations for q_m and q_d.

Lemma C.1. *There are an odd number of positive solution pairs (q_m, q_d) to (C.2) and (C.3). The smallest such pair satisfies*

$$q_m > q_c, q_d > q_c, u(q_c) > cq_d, u(q_m) - cq_m > 0;$$

$$\frac{q_d}{q_m} \in (\frac{n}{n+2R}, \frac{1-n+R}{1-n}). \tag{C.4}$$

if z and $1-M$ are bounded above zero, then at $q_m = q_c$ we have

$$\begin{aligned} LHS(4.9) \quad | \quad {}_{q_m=q_c} &= u(q_c) - (1+\frac{R}{1-n-d})cq_c \\ &= RHS(4.9) + \left(1 - \frac{(1-n)z}{1-n-d}\right)(u(q_c) - cq_c) > RHS(4.9). \end{aligned}$$

The solutions q_L and q_H exist. The same inequality implies $q_L < q_c < q_b < q_H$.

To qualify as a monetary equilibrium with only real repayments, the pair (q_m, q_c) must satisfy (2.7), the inequality $V_p - V_d - cq_b \geq 0$ and one of the inequalities in (4.8). For $R \to 0$, both (q_L, q_c) and (q_H, q_c) satisfy (2.7) and $V_p - V_d - cq_b \geq 0$ because

$$\begin{aligned} V_p + u(q_m) - V_d - u(q_c) &= \tfrac{(1-n)z}{R+(1-n)z}[u(q_b) + cq_b] - u(q_c) - cq_c + u(q_m) \\ &\underrightarrow{R \to 0} \; u(q_m) > 0, \end{aligned}$$

$$V_p - V_d - cq_b = \frac{(1-n)zu(q_b) - Rcq_b}{R+(1-n)z} - cq_c \quad \underrightarrow{R \to 0} \quad u(q_b) - cq_b > 0.$$

Since $V_m < V_c \Leftrightarrow q_m < q_c$ and since $q_L < q_c$, the pair (q_L, q_c) is a monetary equilibrium with only real repayments.

In contrast, the pair (q_H, q_c) violates the constraint $V_m < V_c$. To qualify as a monetary equilibrium with only real repayments, the pair must satisfy $V_p - V_d - cq_m < 0$. Because

$$\begin{aligned} V_p - V_d - cq_m &= \frac{(1-n)z}{R+(1-n)z}(u(q_b) + cq_b) - cq_c - cq_H \\ \underrightarrow{R \to 0} \quad u(q_b) - cq_H &> 0, \end{aligned}$$

it is necessary that $q_H < u(q_b)/c$. Since the left-hand side of (4.9) is decreasing in q_m at q_H, the last inequality is equivalent to

$$u(\frac{u(q_b)}{c}) - (1+\frac{R}{1-n-d})c(\frac{u(q_b)}{c}) < \frac{(1-n)z}{1-n-d}[u(q_b) - (1+\frac{R}{(1-n)z})cq_c].$$

When $R \to 0$, $q_c \to q_b$ and the above inequality is satisfied if and only if $d/(1-n) < 1 - \alpha z$. Substituting the solution for n, the condition is equivalent to $z < z_0$. ■