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## Trigger Strategies and the Cyclicalities of Markups

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by

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# Trigger Strategies and the Cyclicalities of Markups

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## Abstract

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An environment capable of generating both counter- and procyclical movements in markups through the interaction of opposing factors is considered. This framework can account for observed variations in the cyclical behavior of markups across industries. Technology shocks and endogenous labor supply are introduced into a model with implicit collusion and periodic reversion to non-collusive behavior. Within either a collusive or non-collusive regime, markups are positively correlated with output. Switches between regimes, however, result in opposing movements in markups and output, reducing the overall correlation of the two series. Our findings imply that weak cyclicalities of markups is not inconsistent with a large role for changes in market power in accounting for cyclical fluctuations. Offsetting effects may make the overall correlation of the markup with output low, while still allowing for the instability of the cartel to have important cyclical implications.

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## I. Introduction

The divergence of prices from marginal costs, or “markups,” generally interpreted as a measure of market power, has been the focus of a large body of empirical work. Evidence on cyclical fluctuations in markups has been somewhat contradictory, owing in part to the sensitivity of results to the assumptions employed in the measurement of costs. Most empirical studies, however, document substantial variation in the cyclical pattern of markups across industries. Explaining these differences has received little attention in the theoretical literature. This paper presents an environment with oligopolistic firms which is capable of generating both pro- and countercyclical markups, depending on industry characteristics. The model is also consistent with empirical evidence on the links between markups and technological change, changes in competitive conditions, and industry concentration.

We begin by briefly summarizing the empirical evidence on the cyclicity of markups. In a study that emphasizes the importance of the premium paid for overtime labor, Bils (1987) finds evidence that markups are countercyclical in U.S. manufacturing. Using different methods, Rotemberg and Woodford (1991) also find countercyclical markups in U.S. manufacturing. Their findings are, however, qualified by Ramey (1991) who illustrates that the Rotemberg and Woodford estimates are sensitive to assumptions regarding the average markup. She argues that the average markups they use are unrealistically high and demonstrates that if lower average markups are used, much of the countercyclicity that Rotemberg and Woodford report disappears and several industries in their sample actually exhibit procyclical markups.

In contrast, several studies have found evidence of procyclical fluctuations in markups. In a series of papers, Domowitz, Hubbard, and Peterson (1988,1986) study U.S. manufacturing industries at the two- and four-digit (SIC) level. They find an overall procyclical tendency in markups, with some industries exhibiting this tendency more strongly than others, and a minority exhibiting markups that are countercyclical. They also find that industry concentration tends to strengthen the procyclicity of markups, although markups in highly concentrated non-durable goods industries tend to be countercyclical. Chirinko and Fazzari (1994) consider firm level data in eleven four-digit U.S. manufacturing industries and also find both a general tendency toward procyclicity and a substantial degree

of dispersion across industries.

Morrison (1994,1992) also finds an overall tendency for weakly procyclical variations in markups in U.S., Canadian, and Japanese manufacturing. Again, cyclical behavior is found to vary across industries. She approaches the issue of cyclical behavior by considering the effects on markups in manufacturing industries of various factors, which are considered in themselves to be either pro- or countercyclical. The evidence suggests that these factors have differing, in some cases counteracting, effects that combine to determine the overall cyclical behavior of markups. Changes in technology, inputs, and input prices tend to provoke procyclical movements in markups. Changes in certain aggregate demand variables, such as aggregate consumer expenditure, also affect Canadian markups procyclically, but have weakly countercyclical effects in the U.S. and Japan. In contrast, changes in both scale economies and import prices (the latter of which she interprets as proxies for changes in competitiveness) tend to produce countercyclical movements.

In summary, the empirical literature documents substantial dispersion in the cyclical properties of markups across industries and countries and suggests that market power is affected by a variety of factors over the cycle. Also, the evidence suggests that cyclical fluctuations in markups are actually quite weak in some industries. This evidence may be seen as damaging to theories of economic fluctuations which depend heavily on markup fluctuations, but do not allow different types of cyclical phenomena to affect markups differently. In dynamic models of economic fluctuations with imperfect competition, market structure (in particular the cyclical behavior of markups) is often crucial for generating particular cyclical comovements among prices and quantities. For example, Rotemberg and Woodford (1992) employ the cartel structure of Rotemberg and Saloner (1986). In this environment, fluctuations in aggregate demand alone drive markups. A positive demand shock weakens the cartel, causing market power and markups to be strongly countercyclical. In the absence of this strong countercyclical behavior of market power, the propagation mechanism embodied in this cartel strategy is unlikely to be a significant force in aggregate fluctuations.

In this paper, we consider a theoretical model capable of generating both counter- and procyclical movements in markups through the interaction of opposing factors. Stochastic technology and endogenous labor supply are introduced into the Green and Porter

(1984) model of implicit collusion. In the economy, an oligopolistic manufacturing industry alternates between episodes of colluding to restrict output below the non-collusive (Cournot) level and “reversionary” episodes in which output is not restricted. The periodic reversions are necessary to support episodes of collusive behavior in the presence of unobservable market demand. This environment is used to study the co-movements among markups, output, wages, and employment in response to observable technology shocks and unobservable demand shocks.

In our model, *ceteris paribus*, markups are high during collusive periods and low during non-collusive periods. Hence, as the economy cycles between collusive and non-collusive “regimes,” output and markups tend to move in opposite directions. This introduces a countercyclical force in markup fluctuations which is consistent with Morrison’s findings with regard to the effects of changes in “competitiveness.” In contrast, within either regime, output and markups will respond in the same direction to a technology shock. This produces a procyclical force in markups which is consistent with Morrison’s results regarding the effects of technological change. Combining these two effects provides an environment in which the relationship between output and markups depends on technology through variation in productivity and on market structure through the frequency of reversion to non-collusive behavior. The model is also consistent with Morrison’s finding that strong aggregate demand, which increases the probability of collusion, may be associated with high markups. Furthermore, the model generates a negative relationship between wages and markups which is consistent with Morrison’s finding that the elasticity between input prices and markups is negative.

The model can account for differences in the cyclical properties of markups across industries in two ways. First, differences across industries may be due to sampling variability in both technology and demand shocks. That is, even if all industries are symmetric, over a given time period, a particular industry may have experienced more shifts between collusive and non-collusive behavior than other industries because of industry specific shocks. Such an industry will exhibit a greater tendency toward countercyclical movements in markups. In a numerical experiment, we illustrate that for time horizons similar to those considered by most empirical studies, sampling variability with regard to the correlation

between the markup and output can be substantial.

Another way in which the model can account for differences across industries involves industry characteristics. For example, we find that, *ceteris paribus*, more highly concentrated industries tend to exhibit more procyclical markups. This finding is consistent with evidence in Domowitz, Hubbard, and Peterson (1988) and Rotemberg and Woodford (1991). Also, differences across industries with regard to the parameters of the collusive agreement, the demand for industry output, the supply of industry labor, and the distribution of demand shocks can all affect the overall cyclicity of markups. All of these factors affect the cyclicity of the markup only by changing the importance of markup fluctuations within a regime relative to those associated with switching regimes. None of them affect the basic character of the responses of markups to either demand or technology shocks. In particular our results show that neither weak overall cyclicity of markups nor variations in the cyclical behavior of markups across industries is necessarily inconsistent with a large role for changes in market power in the business cycle. In our model, reversions to non-collusive behavior provoke increases in both output and employment that can be as large or larger than those associated with technology shocks. Offsetting effects may make the overall correlation of the markup with output low, while still allowing the instability of the cartel to have important cyclical implications for output, wages, and employment.

The remainder of the paper is organized as follows. Section II describes the model and its equilibrium and characterizes the relationships among output, markups, wages, and employment within and across regimes. Section III provides a numerical example to illustrate the model's ability to generate differences in the cyclical properties of markups across industries. Section IV concludes.

## II. The Model

We consider a model which incorporates stochastic technology shocks and endogenous labor supply into the Green and Porter (1984) model of implicit collusion in the presence of unobservable demand. There is a single manufactured good which is produced by a homogeneous oligopoly with a stochastic technology that uses only labor. The industry is comprised of a fixed number,  $N$ , of risk neutral firms each of which produces a homogeneous



good. The output of firm  $i$  at time  $t$  is denoted  $q_{it}$  for  $i = 1, 2, \dots, N$ . Industry output is denoted  $Q_t$ , where  $Q_t = \sum_{i=1}^N q_{it}$ . Firms observe market price,  $\hat{p}_t$ , given by:

$$\hat{p}_t = \theta_t p(Q_t). \quad (1)$$

Here  $p'(\cdot) < 0$ , and  $\theta$  is an *i.i.d.* demand shock with mean  $\bar{\theta}$  and cumulative distribution  $F$ .  $F$  and its derivative,  $f$ , are continuously differentiable with  $F(0) = 0$  and  $F(\infty) = 1$ . Firms do not observe the demand shock directly, nor do they observe either industry output or total employment. Thus they can in no way infer the realized value of  $\theta$ .

The industry operates as in Green and Porter.<sup>1</sup> That is, firms collude to restrict output under threat of reversion to Cournot levels whenever the observed price,  $\hat{p}_t$ , falls below a predetermined trigger price level,  $\tilde{p}$ . Reversion implies Cournot output levels for a finite number of periods,  $T - 1$ . Periods are classified as “collusive” (or “cooperative”) and “non-collusive” (or “noncooperative”) as by Porter (1983a):

- (a)  $t = 0$  is a collusive period;
- (b) if  $t$  is a collusive period and  $\hat{p}_t \geq \tilde{p}$  then  $t + 1$  is a collusive period;
- (c) if  $t$  is a collusive period and  $\hat{p}_t < \tilde{p}$ , then  $t + 1, \dots, t + T - 1$  are non-collusive periods, and  $t + T$  is a collusive period.

We refer to the two classes of time periods as collusive and non-collusive “regimes”.

The technology for producing goods in the manufacturing sector is identical across all firms and is assumed to be linear in labor input:

$$q_{it} = \gamma_t l_{it}, \quad (2)$$

where  $\gamma_t \in \{\gamma^1, \gamma^2\}$  is a stochastic technology parameter and  $l_{it}$  is labor employment by firm  $i$  at time  $t$ .<sup>2</sup> We assume that  $\gamma^1 > \gamma^2$ . The technology shock evolves as a two-state Markov process with transition matrix  $\Pi$ . We use  $\pi(\gamma_{t+1} = \gamma^j | \gamma_t)$  for  $j = 1, 2$  to denote

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<sup>1</sup> The cartel equilibrium that we study is not an optimal equilibrium of the type studied by Abreu, Pearce, and Stacchetti (1990, 1986). The technological uncertainty and endogeneity of labor supply present in this model complicate the characterization of an optimal equilibrium analogous to theirs, although such an equilibrium may exist.

<sup>2</sup> The results presented in this paper generalize to an environment with a finite number of technology parameters.

the probability that the technology shock at time  $t + 1$  equals  $\gamma^j$  given that the current state is  $\gamma_t$ . The industry faces a labor supply curve,  $L^s(w_t)$ , where  $w_t$  denotes the wage at time  $t$ . It is assumed that firms take the wage as given, and that  $L^{s'}(\cdot) > 0$ .

Let  $d_t \equiv w_t/\gamma_t$  denote the unit cost of production at time  $t$  and let  $Q_{it} = Q_t - q_{it}$  be the output of all firms other than firm  $i$  at time  $t$ . We can then write the single period *expected* (i.e.  $\theta_t = \bar{\theta}$ ) profits for firm  $i$  at time  $t$  contingent on the observed technology shock as

$$R_i(\mathbf{q}_t, \gamma_t) \equiv [\bar{\theta}p(Q_{it} + q_{it}) - d_t]q_{it}, \quad (3)$$

where  $\mathbf{q}_t = (q_{1t}, q_{2t}, \dots, q_{Nt})$  denotes the vector of outputs at time  $t$ . The single period expected profit function is assumed to be strictly concave in a firm's own output,  $q_{it}$ .

We consider the firm's profit maximization problem in each regime separately. In the non-collusive regime, firms choose quantities in Cournot competition. Thus they take all other firms' outputs as given and choose output to maximize (3) contingent upon the realized technology shock. Denote the non-collusive profit maximizing quantities in a symmetric equilibrium by  $q^n(\gamma_t)$  (firm subscripts have been dropped because of symmetry) and let industry output be given by  $Q^n(\gamma_t) \equiv Nq^n(\gamma_t)$ . Given these quantities, labor market clearing determines equilibrium wages as a function of the technology shock in non-collusive periods,  $w^n(\gamma_t)$ , as follows:

$$Q^n(\gamma_t) = \gamma_t L^s(w^n(\gamma_t)). \quad (4)$$

Similarly, unit costs in non-collusive periods are denoted  $d^n(\gamma_t)$ . Finally, expected non-collusive profits for each firm as a function of the technology shock are given by

$$R^n(\gamma_t) \equiv \left[ \bar{\theta}p(Q^n(\gamma_t)) - d^n(\gamma_t) \right] q^n(\gamma_t). \quad (5)$$

We restrict the inverse demand function and the technology shocks so that firms earn positive profits in the non-collusive equilibrium.

We define the markup as the ratio of price to marginal cost. The markup is a function of the realized demand shock and the technology shock, and in the non-collusive regime can be written

$$m^n(\theta_t, \gamma_t) \equiv \frac{\theta_t p(Q^n(\gamma_t))}{d^n(\gamma_t)} = \left( \frac{\varepsilon^n(\gamma_t)}{\varepsilon^n(\gamma_t) - 1} \right) \left( \frac{\theta_t}{\bar{\theta}} \right), \quad (6)$$

where  $\varepsilon^n(\gamma_t) > 0$  is the elasticity of demand facing a single firm when each firm produces  $q^n(\gamma_t)$ , i.e.  $\varepsilon^n(\gamma_t) \equiv \frac{-p(Q^n(\gamma_t))}{p'(Q^n(\gamma_t))q^n(\gamma_t)}$ . We restrict attention to inverse demand functions for which elasticity is greater than one, and for which  $p' + p''q \leq 0$ . These restrictions guarantee the existence and uniqueness of the Cournot equilibrium in the non-collusive regime. We then have the following proposition which characterizes the effect of technology shocks on equilibrium costs, quantities, and markups in the non-collusive regime. Proofs of all propositions are contained in the appendix.

*Proposition 1:*

Assume that the elasticity of demand is decreasing in output. Then the non-collusive equilibrium is characterized by:

- i.  $d^n(\gamma^1) < d^n(\gamma^2)$
- ii.  $q^n(\gamma^1) > q^n(\gamma^2)$
- iii.  $m^n(\theta, \gamma^1) > m^n(\theta, \gamma^2) \quad \forall \theta$ .

This proposition tells us that within a non-collusive regime, when costs are low, output will be high and markups will also tend to be high. Note, however, that it does not tell us the effect of technology shocks on either wages or employment. These effects depend on the nature of the labor supply function, which has been restricted only in that it is assumed to be increasing.

In the collusive regime, firms face a dynamic maximization problem with a common discount factor,  $0 < \beta < 1$ . Dynamics arise as a firm's discounted future profits depend on the future regime, and the probability of triggering a reversion depends in part on the firm's choice of output. Firm  $i$ 's maximization problem in a collusive period can be written as a dynamic programming problem with the current technology shock as the state:

$$\begin{aligned}
 V_i(\gamma_t) = \max_{q_{it}} & R_i(\mathbf{q}_t, \gamma_t) + [1 - F(\tilde{p}/p(Q_t))] \left[ \beta \sum_{j=1,2} \pi(\gamma_{t+1} = \gamma^j | \gamma_t) V_i(\gamma^j) \right] \\
 & + F(\tilde{p}/p(Q_t)) \left[ \sum_{\tau=1}^{T-1} \beta^\tau \sum_{j=1,2} \pi^{(\tau)}(\gamma_{t+\tau} = \gamma^j | \gamma_t) R^n(\gamma^j) \right. \\
 & \left. + \beta^T \sum_{j=1,2} \pi^{(T)}(\gamma_{t+T} = \gamma^j | \gamma_t) V_i(\gamma^j) \right], \tag{7}
 \end{aligned}$$

where  $F(\tilde{p}/p(Q_t))$  is the probability that the realized equilibrium price this period is below the trigger level,  $\tilde{p}$ , and  $\pi^{(s)}(\cdot | \cdot)$  denotes the  $s > 0$  step ahead probability distribution over

$\gamma$ . The first term on the right-hand side of (7) is the one-period profit from production. The second term is expected future returns from collusion weighted by the probability that the realized price will be above the trigger price and firms will continue to collude. The final term is expected returns from reverting to the non-collusive equilibrium for  $T - 1$  periods and then returning to the collusive regime weighted by the probability that the realized price will be below the trigger price. Note that the probability of triggering is affected by the firm's choice of output.

A symmetric equilibrium in collusive periods is characterized by output levels,  $q^c(\gamma)$ ,  $Q^c(\gamma) \equiv Nq^c(\gamma)$ , expected discounted returns,  $V^c(\gamma)$ , wages  $w^c(\gamma)$ , and employment levels,  $L^c(\gamma)$ , all of which depend on the technology shock. The equilibrium is also characterized by output prices,  $p^c(\theta, \gamma) \equiv \theta p(Q^c(\gamma))$  which depend on the technology shock and the demand shock. Quantities, expected discounted returns, and employment levels are all the same across firms. Let  $z^c(\gamma) \equiv \tilde{p}/p(Q^c(\gamma))$ . Then, in equilibrium the following first-order conditions must be satisfied:

$$\begin{aligned} \frac{\partial R_i(\mathbf{q}^c(\gamma_t), \gamma_t)}{\partial q_{it}} + f(z^c(\gamma_t)) \left[ \frac{\tilde{p}'(Q^c(\gamma_t))}{p(Q^c(\gamma_t))^2} \right] & \left[ \beta \sum_{j=1,2} \pi(\gamma_{t+1} = \gamma^j | \gamma_t) V_i(\gamma^j) \right. \\ & - \sum_{\tau=1}^{T-1} \beta^\tau \sum_{j=1,2} \pi^{(\tau)}(\gamma_{t+\tau} = \gamma^j | \gamma_t) R^n(\gamma^j) \\ & \left. - \beta^T \sum_{j=1,2} \pi^{(T)}(\gamma_{t+T} = \gamma^j | \gamma_t) V_i(\gamma^j) \right] = 0. \end{aligned} \quad (8)$$

In addition, the collusive equilibrium wages must clear the labor market in each state:

$$Q^c(\gamma_t) = \gamma_t L^s(w^c(\gamma_t)). \quad (9)$$

We now examine a special case in which the technology shocks are *i.i.d.*. In this case we can derive analytical relationships among output and markup levels within the collusive regime and across regimes. We consider the effects of persistence in the technology shocks numerically in the next section. There we show that the relationships demonstrated here

continue to hold in the presence of persistent shocks. Let the transition matrix be restricted so that shocks are *i.i.d.*,

$$\Pi = \begin{bmatrix} \pi & 1 - \pi \\ \pi & 1 - \pi \end{bmatrix}, \quad (10)$$

where  $0 < \pi < 1$ . In this case, the functional equations can be written,

$$\begin{aligned} V^c(\gamma_t) = & R_i(\mathbf{q}^c(\gamma_t), \gamma_t) + [1 - F(\tilde{p}/p(Q_t))] \left[ \beta [\pi V^c(\gamma^1) + (1 - \pi) V^c(\gamma^2)] \right] \\ & + F(\tilde{p}/p(Q_t)) \left[ \lambda (\pi R^n(\gamma^1) + (1 - \pi) R^n(\gamma^2)) \right. \\ & \left. + \beta^T [\pi V^c(\gamma^1) + (1 - \pi) V^c(\gamma^2)] \right], \end{aligned} \quad (11)$$

and the first order conditions are given by

$$\begin{aligned} \frac{\partial R_i(\mathbf{q}^c(\gamma_t), \gamma_t)}{\partial q_{it}} + f(z^c(\gamma_t)) \left[ \frac{\tilde{p}p'(Q^c(\gamma_t))}{p(Q^c(\gamma_t))^2} \right] \left[ (\beta - \beta^T)(\pi V^c(\gamma^1) + (1 - \pi) V^c(\gamma^2)) \right. \\ \left. - \lambda (\pi R^n(\gamma^1) + (1 - \pi) R^n(\gamma^2)) \right] = 0, \end{aligned} \quad (12)$$

where  $\lambda \equiv (\beta - \beta^T)/(1 - \beta)$ .

Let  $R^c(\gamma)$  denote equilibrium expected single period profits in a collusive period as a function of the state. Solving the functional equations, (11), for  $V^c(\gamma)$  and substituting into the first order conditions, (12), allows us to write the following:

$$\begin{aligned} \frac{\partial R_i(\mathbf{q}^c(\gamma_t), \gamma_t)}{\partial q_{it}} + f(z^c(\gamma_t)) \left[ \frac{\tilde{p}p'(Q^c(\gamma_t))}{p(Q^c(\gamma_t))^2} \right] \left[ \Psi(\beta - \beta^T) \right] \times \\ \left[ \pi (R^c(\gamma^1) - R^n(\gamma^1)) + (1 - \pi)(R^c(\gamma^2) - R^n(\gamma^2)) \right] = 0, \end{aligned} \quad (13)$$

where  $\Psi \equiv (1 - \beta) + (\beta - \beta^T)(\pi F(z^c(\gamma^1)) + (1 - \pi)F(z^c(\gamma^2)))$ . This equation makes it clear that, as in Green and Porter (1984), existence of an equilibrium is not an issue. Equation (13) is satisfied by the non-collusive (Cournot) output levels, and the degenerate equilibrium in which firms produce their one-period Cournot profit maximizing quantities in each period satisfies our definition of a “collusive” equilibrium. We are interested, however, in cases in which there is a solution to (13) such that output is collusively restricted below

the non-collusive levels. Whether such an equilibrium exists depends on the parameters of the collusive agreement,  $\tilde{p}$  and  $T$ , on the particular form of the inverse demand and labor supply functions and on the distribution of the demand shocks.

Let the unit cost in a collusive period be given by  $d^c(\gamma_t) \equiv \frac{w^c(\gamma_t)}{\gamma_t}$  and let the markup in a collusive period be denoted  $m^c(\theta_t, \gamma_t) \equiv \frac{\theta_t p(Q^c(\gamma_t))}{d^c(\gamma_t)}$ . We then have the following proposition characterizing the effect of technology shocks on costs, quantities, and markups within a collusive regime.

*Proposition 2:*

Let the inverse demand function satisfy the following restriction:<sup>3</sup>  $p'' \leq \frac{(p')^2}{p}$ . Let the cumulative density function for demand shocks,  $F(\theta)$ , be convex. Then the collusive regime in equilibrium is characterized by:

- i.  $d^c(\gamma^1) < d^c(\gamma^2)$
- ii.  $q^c(\gamma^1) > q^c(\gamma^2)$
- iii.  $m^c(\theta, \gamma^1) > m^c(\theta, \gamma^2) \quad \forall \theta$ .

Analogous to Proposition 1, Proposition 2 establishes that within the *collusive* regime, when costs are low, output is high and markups also tend to be high. Again, the effects of technology shocks on wages and employment are ambiguous. Finally, we have the following proposition that compares costs, output levels, and markups across regimes.

*Proposition 3:*

In comparing unit costs, quantities, and markups across regimes, we state the following for

$\gamma^j \in \{\gamma^1, \gamma^2\}$ :

- i.  $d^n(\gamma^j) \geq d^c(\gamma^j)$
- ii.  $q^n(\gamma^j) \geq q^c(\gamma^j)$
- iii.  $m^n(\theta, \gamma^j) \leq m^c(\theta, \gamma^j) \quad \forall \theta$ .

As immediate corollaries to Proposition 3, we have  $w^n(\gamma^j) \geq w^c(\gamma^j)$  and  $L^n(\gamma^j) \geq L^c(\gamma^j)$ .

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<sup>3</sup> This is similar to but more restrictive than the sufficient condition for existence and uniqueness of an equilibrium in non-collusive periods given by  $p' + p''q \leq 0$ . The restriction in this proposition can be written as  $p' + p''\varepsilon q \leq 0$ . Both conditions are satisfied if the inverse demand function is concave.

Taken together, Propositions 1-3 establish the following relationships between quantities and markups across states and regimes, under the given restrictions on  $\Pi$ ,  $p(\cdot)$ , and  $F(\cdot)$ :

- i.  $q^n(\gamma^1) > q^n(\gamma^2)$   
 $m^n(\theta, \gamma^1) > m^n(\theta, \gamma^2) \quad \forall \theta$
- ii.  $q^c(\gamma^1) > q^c(\gamma^2)$   
 $m^c(\theta, \gamma^1) > m^c(\theta, \gamma^2) \quad \forall \theta$
- iii.  $\pi q^n(\gamma^1) + (1 - \pi)q^n(\gamma^2) \geq \pi q^c(\gamma^1) + (1 - \pi)q^c(\gamma^2)$   
 $\pi m^n(\theta, \gamma^1) + (1 - \pi)m^n(\theta, \gamma^2) \leq \pi m^c(\theta, \gamma^1) + (1 - \pi)m^c(\theta, \gamma^2) \quad \forall \theta$

Hence, within a regime, markups vary positively with output, owing to the fact that the elasticity of demand is decreasing in output. Comparing across regimes, however, when average quantity is low, average markups are high. The contemporaneous correlation of the markup with output is, therefore, ambiguous, because of offsetting effects. Within either regime, a higher technology shock will increase both output and the markup. This is consistent with Morrison's (1994) finding of a positive elasticity between technological change and markups. If, however, the industry switches from a collusive period to a non-collusive one (i.e. moves from less competitive output levels to more competitive ones), output will rise and the markup will fall. This is broadly consistent with Morrison's (1994) finding of a negative relationship between "competitiveness" (measured in her study by the inverse of import prices) and markups.

Overall, if regime switches are frequent enough, and if they cause large enough changes in average markups and output, the contemporaneous correlation between industry output and markups will be negative. In the absence of cost shocks, output and markups are likely to be negatively correlated (although demand shocks within a regime will produce fluctuations in markups without affecting output). As the variability of costs increases, the fluctuations *within* a regime become increasingly important and if costs (and, therefore, output) are sufficiently volatile, output and markups may be positively correlated. Note that regardless of the overall correlation between output and markups, changes in market structure will contribute to cyclical output fluctuations, as reversions to non-collusive

behavior lead to increases in both output and employment which may be as large or larger than those caused by technology shocks.

With regard to wages and employment, as noted above, technology shocks have ambiguous effects within a regime. Comparing across regimes, however, expected wages and employment are both unambiguously higher in the non-collusive regime than in the collusive regime. This suggests that there are forces in the model which can generate a negative relationship between wages and markups, in accordance with Morrison (1994). On the other hand, in this case, the model would also generate a negative relationship between employment and the markup which is inconsistent with Morrison's findings. Also, it is possible that wages will exhibit only weak correlations with output, and for these correlations to be either positive or negative. In the next section, we examine the relationships between movements in output and markups in a parametric example and consider the effects of economy parameters on this relationship.

### III. A Parametric Example

We now turn to a parametric example which is illustrative of the type of relationships between markups and other economic variables that the model can generate. Suppose the inverse demand function is linear:

$$p(Q) = a - bQ, \quad (14)$$

where  $b > 0$  and  $a > 0$  is chosen so that non-zero production is optimal in all states. This demand function satisfies the restriction specified in Proposition 2 and exhibits an elasticity that is decreasing in  $Q$ . Let the labor supply function be given by

$$L^s = w^\rho \quad (15)$$

where the labor supply elasticity is given by  $\rho > 0$ . Let the cumulative density for the demand shock,  $\theta$ , be given by

$$F(\theta) = \left[ \frac{\alpha\theta}{1+\alpha} \right]^\alpha, \quad (16)$$

for  $0 \leq \theta \leq (1+\alpha)/\alpha$  and  $\alpha > 1$ . This density is taken from Porter (1983a) and as he demonstrates

$$\bar{\theta} = 1 \quad \sigma_\theta^2 = \frac{1}{\alpha(2+\alpha)}. \quad (17)$$



With  $\alpha > 1$ ,  $F(\cdot)$  is convex. Technology shocks are no longer be restricted to be *i.i.d.*, but instead follow a covariance stationary two-state Markov process with time invariant transition matrix  $\Pi$ .

Our goal is to investigate the overall cyclical behavior of the markup measured by its correlation with industry output,  $Corr(m, Q)$ , given that different factors cause the two variables to co-vary in different directions. We begin by choosing a set of benchmark parameters, which is reported in Table 1. These parameters are not meant to be interpreted as a calibration. Rather, they are just one of many collections of parameters that will deliver a population correlation between output and the markup of zero.<sup>4</sup> In our benchmark parameterization we also maintain the assumption of *i.i.d.* technology shocks, and set the probability of each technology parameter to .5. From this benchmark, we can then vary the parameters and examine the direction of the effect that they have on  $Corr(m, Q)$ .

**Table 1: Benchmark Parameters**

<i>Oligopoly Parameters</i>	<i>Consumer Parameters</i>	<i>Technology Shocks</i>	<i>Demand Shocks</i>
$T = 3$	$a = 5$	$\bar{\gamma} = 3.7$	$\alpha = 6$
$\tilde{p} = 1.2$	$b = 1$	$\sigma_{\gamma} = .06$	
$N = 4$	$\rho = .5$		
$\beta = .99$			

We first use this example to clarify the analytical relationships derived in the previous section. Consider Figure 1 which contains what may be thought of as a typical realization of the equilibrium stochastic processes for markups and output. In the figure, markups and output have been scaled for illustrative purposes; in fact, for these parameters markups are more volatile than output. The shaded areas in the diagram are periods of reversion to the non-collusive regime. Note that *within* a regime, output and markups tend to move together in response to technology shocks (markups are also varying in response to demand shocks, which leave output unchanged). In contrast, in the transition periods

<sup>4</sup> The population correlation is approximated by calculating a single realization of the economy's equilibrium stochastic process for 5,000 periods and computing the correlation. Repetition of this experiment indicated that this time horizon was sufficient to virtually eliminate sampling variability with regard to this correlation.

*between* regimes, output and markups tend to move in opposite directions; i.e. output rises and the markup generally falls when entering a reversionary period while output falls and the markup generally rises when the reversion ends. The correlation between the markup and output then depends on the relative importance of these two effects.

One goal of this paper is to describe an environment which is capable of generating both pro- and countercyclical markups as suggested by previous empirical studies. This model can account for differing properties of markups and output across industries in two ways: sampling variability and industry specific parameters. We first explore sampling variability. Imagine that there are several industries operating as suggested by our model. Even if all of these industries are entirely symmetric, over a given time horizon they will experience different realizations of cost and demand shocks and may experience different numbers of switches between collusive and non-collusive regimes. Even though the actual stochastic relationship between markups and output is the same in all industries, over finite time horizons we would expect to see a certain amount of variability in the sample correlation coefficients.

Consider the case of the benchmark parameter values. Figure 2 contains a histogram of the correlations between markups and output generated in 1000 simulations, each of which is 26 periods in length.<sup>5</sup> Over such a time horizon and with the parameters given in Table 1, the model predicts a correlation ranging anywhere from roughly -.6 to .6. This range encompasses most of the correlations either directly estimated or implied by the empirical studies discussed above. Of course, by changing the parameters it is possible to move the mean of this distribution into either the positive or negative range and also to change the size of its support. Only under extreme assumptions, however, will the model be consistent with only a very narrow range of correlations.

Substantial sampling variability should not, however, be interpreted as a prediction that “anything can happen” in this model. As stated above, the sampling variability arises from differences in the relative importance of technology shocks and regime shifts across

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<sup>5</sup> The number of periods was chosen to be consistent with the length of the series used by Morrison (1990) in her study of the cyclical properties of markups. Samples of similar length are also studied by Domowitz, Hubbard, and Peterson (1988,1987,1986) and Rotemberg and Woodford (1991).

realizations. Throughout the distribution, the correlation between output and the markup is systematically related to other economic predictions of the model. For example, the model predicts a positive monotonic relationship between the output-markup correlation and the volatility of output. The volatility of output in the model emanates primarily from the effects of technology shocks within a regime. The more variable the technology shocks, the larger are these fluctuations and the associated procyclical movements in markups relative to the countercyclical movements that occur during regime switches. Thus, more variable output is typically associated with a more procyclical markup in this model. This relationship is depicted in Figure 3 which plots  $Corr(m, Q)$  against the standard deviation of output for the same sample as depicted in the histogram in Figure 2. The upward sloping line represents the estimate of an OLS regression. That the positive relationship between these two moments is strongly statistically significant is apparent from the Spearman rank correlation coefficient for this sample. The Spearman coefficient here is .39, indicating a positive relationship between  $Corr(m, Q)$  and the standard deviation of output that is significant at the .01% level.

Some empirical evidence for the existence of a positive relationship between the volatility of output and its correlation with the markup can be found using estimates of annual markups taken from Morrison (1990). Morrison provides estimates of markups from 1960-1986 for seventeen U.S. manufacturing industries at the two-digit level and for manufacturing as a whole.<sup>6</sup> We use these markup estimates and three measures of output for the corresponding industries to examine the prediction of the model. The three measures of output considered are (1) the index of industrial production, (2) capacity utilization by industry, and (3) gross production by industry. Since neither the markups nor the measures of industry output are stationary over this time horizon, all data are in first differences of logarithms. The Spearman coefficients between the output-markup correlation and the standard deviation of output for each output measure are as follows: (1) .57, (2) .47, and (3) .22. The coefficients associated with the index of industrial production and with capacity utilization are positive and significant at the 5% level. This provides some evidence

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<sup>6</sup> These are taken from Table 2A in Morrison (1990).

that the variance of industry output is positively related to its cyclical correlation with industry markups.

Another way that the model can account for variations in the cyclical properties of markups across industries hinges on industry specific characteristics. To the extent that key parameters differ across industries, those industries would be expected to exhibit different cyclical properties of markups even in the long run, once we have accounted for sampling variability. We consider the effect of each parameter on the *population* moment,  $Corr(m, Q)$ , holding all other parameters constant at their values in Table 1. Population correlations are approximated by computing the equilibrium stochastic process for a single realization (in each case) 5000 periods in length. Recall that the benchmark parameterization was chosen so that it produced a population correlation of zero.

Parameters affect the cyclicity of the markup by changing the importance of fluctuations within a regime, which produce procyclical movements in the markup, relative to switches between regime, which produce countercyclical movements. With regard to the parameters of the oligopoly, we find that the discount factor,  $\beta$ , has a small negative effect on  $Corr(m, Q)$ . The lower the discount factor, the smaller the collusive restriction of output that can be sustained by the threat of Cournot reversion. This tends to increase the importance of markup fluctuations within a regime relative to those associated with regime switching, and to make markups more procyclical. The trigger price,  $\tilde{p}$ , has a negative effect on the  $Corr(m, Q)$  as increases in the trigger price raise the probability of reversion and the relative importance of regime switches. The length of a reversionary episode,  $T$ , has no significant effect on the correlation. Finally, the number of firms is negatively related to  $Corr(m, Q)$ . A given collusive agreement more readily breaks down when there are more firms in the industry, increasing the relative importance of regime switches. This characteristic of the model is consistent with the empirical finding of Domowitz, Hubbard, and Peterson (1988), that more highly concentrated industries tend to exhibit more strongly procyclical markups.

Turning to the parameters of the inverse demand function,  $a$  and  $b$  have a positive and a negative effect on  $Corr(m, Q)$ , respectively. Increases in the intercept parameter,  $a$ , reduce the probability of a reversion for a given distribution of  $\theta$  and thereby increase the

relative importance of within regime fluctuations. Increases in the slope parameter  $b$ , in contrast, increase the effect on price of given fluctuations in  $\theta$ , increasing the probability of reversions and thereby the relative importance of switches between regimes. The elasticity of labor supply, given by  $\rho$ , has virtually no effect on  $Corr(m, Q)$ . In this environment, more elastic labor supply tends to dampen wage fluctuations, without affecting the relative importance of output and markup fluctuations within and across regimes. If, however, the labor elasticity is driven to zero, then there is no possibility of a distinct collusive regime (in this case, the labor supply and the technology shock alone determine output). In this case output fluctuates only due to technology shocks, and the markup will be positively correlated with output.

Regarding the technology and demand shocks, increases in the mean of the technology shocks decrease the output-markup correlation. This occurs because an increase in this mean, increases average quantities and decreases average prices. This increases the probability of realizing a price below the trigger price and, increasing the probability of reversion, and tending to lead to countercyclical markups. Increases in the variance of the technology shocks increase the importance of markup fluctuations within a regime, leading to more procyclical markups. Changes in the persistence of those shocks have no significant effect on the cyclicity of markups. Finally, increases in the parameter  $\alpha$  in the demand shock process increase the probability of reversion and tend to lead to countercyclical markups.

#### **IV. Conclusion**

This paper considers a theoretical model capable of generating both counter- and procyclical movements in markups through the interaction of opposing factors. Within either a collusive or non-collusive regime, markups are positively correlated with output. Switches between regimes, however, result in opposing movements in markups and output, reducing the overall correlation of the two series. This framework can account for variations in the cyclical behavior of markups across industries either due to sampling variability, or because of industry specific characteristics. In particular, both the variance of technology shocks and the concentration of the oligopolistic industry tend to contribute to the procyclicality of markups. The former is shown to be consistent with the properties of markups in U.S. manufacturing measured by Morrison (1990). The latter relationship

is consistent with empirical evidence in Domowitz, Hubbard, and Peterson (1988) and Rotemberg and Woodford (1991).

One implication of the results is that weak cyclicalities of markups is not inconsistent with a large role for changes in market power in the business cycle. In the model studied here, reversions to non-collusive behavior provoke increases in both output and employment that can be as large or larger than those associated with technology shocks. Offsetting effects may make the overall correlation of the markup with output low, while still allowing for the instability of the cartel to have important cyclical implications.

Several avenues for future research seem apparent. First, the results are inconsistent with the findings of Morrison (1994) with regard to the effects of changes in factor employment on markups. An extension of the model in which labor supply may shift either due to sectoral reallocations or income effects could be used to address this issue. Also, a formal test of the basic regime switching framework applied to markup fluctuations would be useful. Several researchers have considered the empirical plausibility of models of cartel instability, typically focusing on fluctuations in prices. For example, Ellison (1994) finds evidence in favor of the Green and Porter (1984) model in a study of the pricing behavior of the Joint Executive Committee (JEC). This accords (and somewhat strengthens) evidence in favor of this model found by Porter (1983b) and Lee and Porter (1984) in earlier studies of the JEC. In another recent study, Suslow (1994), using pre-World War II data, finds that the strength of international cartels varies over the business cycle. Studying fluctuations in markups is somewhat more difficult than studying fluctuations in prices, since calculation of the markup requires measurement of costs. Also, those studies which have considered markups (e.g. Morrison (1994,1992) and Domowitz, Hubbard, and Peterson (1986,1988)) have also typically considered annual data. Data at annual frequencies is unlikely to be sufficient for the study of reversions to non-collusive behavior as these reversions are likely to be relatively short-lived. For example, Porter (1983b) estimates that the price wars among railroads in the JEC period lasted on average only 10 weeks. In further research, monthly data on Canadian manufacturing industries at the four-digit SIC level is being collected so that tests of the regime switching framework can be conducted on markups.

## Appendix

*Proof of Proposition 1:*

(i.) and (ii.) The first order condition for a solution to the non-collusive profit maximization problem can be written as

$$\bar{\theta}p(Q^n(\gamma_t)) \left[ \frac{\varepsilon^n(\gamma_t) - 1}{\varepsilon^n(\gamma_t)} \right] = d^n(\gamma_t). \quad (A1)$$

Dividing the first order conditions for each state gives

$$\left[ \frac{p(Q^n(\gamma^1))}{p(Q^n(\gamma^2))} \right] \left[ \frac{\varepsilon^n(\gamma^1) - 1}{\varepsilon^n(\gamma^1)} \right] \left[ \frac{\varepsilon^n(\gamma^2)}{\varepsilon^n(\gamma^2) - 1} \right] = \left[ \frac{d^n(\gamma^1)}{d^n(\gamma^2)} \right]. \quad (A2)$$

- (a.) Suppose  $d^n(\gamma^1) \geq d^n(\gamma^2)$  and  $q^n(\gamma^1) > q^n(\gamma^2)$ . Since prices and elasticities are decreasing in output, the left hand side of (A2) is less than one but the right hand side is greater than or equal to one. This implies a contradiction.
- (b.) Suppose  $d^n(\gamma^1) < d^n(\gamma^2)$  and  $q^n(\gamma^1) \leq q^n(\gamma^2)$ . Since prices and elasticities are decreasing in output, the left hand side of (A2) is greater than or equal to one but the right hand side is greater than one. This implies a contradiction.
- (c.) Suppose  $d^n(\gamma^1) \geq d^n(\gamma^2)$  and  $q^n(\gamma^1) \leq q^n(\gamma^2)$ . The relationship between unit costs implies that  $w^n(\gamma^1) > w^n(\gamma^2)$  (since  $\gamma^1 > \gamma^2$ ). Labor market clearing, then, implies the following:

$$Nq^n(\gamma^1) = \gamma^1 L^s(w^n(\gamma^1)) > \gamma^2 L^s(w^n(\gamma^2)) = Nq^n(\gamma^2).$$

This implies a contradiction.

(iii.) Dividing the markup equations given by equation (6) gives

$$\left[ \frac{m^n(\gamma^1)}{m^n(\gamma^2)} \right] = \left[ \frac{\varepsilon^n(\gamma^1)}{\varepsilon^n(\gamma^1) - 1} \right] \left[ \frac{\varepsilon^n(\gamma^2) - 1}{\varepsilon^n(\gamma^2)} \right] \quad (A3).$$

Since  $q^n(\gamma^1) > q^n(\gamma^2)$  and since elasticities are decreasing in output, we have the result.

*QED*

*Proof of Proposition 2:*

(i.) and (ii.) Combining the first order conditions for a collusive equilibrium given by (13) gives

$$\left[ \frac{\frac{\partial R_i(\mathbf{q}^c(\gamma^1), \gamma^1)}{\partial q_i}}{\frac{\partial R_i(\mathbf{q}^c(\gamma^2), \gamma^2)}{\partial q_i}} \right] = \left[ \frac{f(z^c(\gamma^1))}{f(z^c(\gamma^2))} \right] \left[ \frac{p(Q^c(\gamma^2))}{p(Q^c(\gamma^1))} \right] \left[ \frac{\varepsilon^c(\gamma^2)q^c(\gamma^2)}{\varepsilon^c(\gamma^1)q^c(\gamma^1)} \right] \quad (A4)$$

where  $\varepsilon^c(\gamma^j) \equiv \frac{-p(Q^c(\gamma^j))}{p'(Q^c(\gamma^j))q^c(\gamma^j)}$ .

- (a.) Suppose  $d^c(\gamma^1) < d^c(\gamma^2)$  and  $q^c(\gamma^1) \leq q^c(\gamma^2)$ . Concavity of the single period profit function in own output implies that the left hand side of this equation is greater than one. Since price is decreasing in output, we have  $p(Q^c(\gamma^1)) \geq p(Q^c(\gamma^2))$  and  $z^c(\gamma^1) \leq z^c(\gamma^2)$ . Convexity of  $F(\theta)$  implies  $f(z^c(\gamma^1)) \leq f(z^c(\gamma^2))$ . The restriction on the inverse demand function implies that  $\varepsilon q$  is non-increasing in  $q$  and, therefore, that the product of the final two terms is less than or equal to one. Hence, the right-hand side is less than one, implying a contradiction.
- (b.) Suppose  $d^c(\gamma^1) \geq d^c(\gamma^2)$  and  $q^c(\gamma^1) > q^c(\gamma^2)$ . Concavity of the single period profit function in own output implies that the left hand side of this equation is less than one. Since price is decreasing in output, we have  $p(Q^c(\gamma^1)) < p(Q^c(\gamma^2))$  and  $z^c(\gamma^1) > z^c(\gamma^2)$ . Convexity of  $F(\theta)$  implies  $f(z^c(\gamma^1)) > f(z^c(\gamma^2))$ . The restriction on the inverse demand function that the product of the final two terms is greater than one. Hence, the right-hand side is greater than one, implying a contradiction.
- (c.) Suppose  $d^c(\gamma^1) \geq d^c(\gamma^2)$  and  $q^c(\gamma^1) \leq q^c(\gamma^2)$ . The relationship between unit costs implies that  $w^c(\gamma^1) > w^c(\gamma^2)$  (since  $\gamma^1 > \gamma^2$ ). Labor market clearing, then, implies the following:

$$Nq^c(\gamma^1) = \gamma^1 L^s(w^c(\gamma^1)) > \gamma^2 L^s(w^c(\gamma^2)) = Nq^c(\gamma^2).$$

This implies a contradiction.

(iii.) Part (ii.) implies that the right hand side of equation (A4) is greater than one. Now the left hand side of that equation can be written as follows:

$$\left[ \frac{\frac{\partial R_i(\mathbf{q}^c(\gamma^1), \gamma^1)}{\partial q_i}}{\frac{\partial R_i(\mathbf{q}^c(\gamma^2), \gamma^2)}{\partial q_i}} \right] = \frac{\left( \frac{p(Q^c(\gamma^1))}{p(Q^c(\gamma^2))} \right) \left[ \frac{\varepsilon^c(\gamma^1)-1}{\varepsilon^c(\gamma^1)} - \left( \frac{1}{m^c(\gamma^1)} \right) \left( \frac{\theta}{\bar{\theta}} \right) \right]}{\left[ \frac{\varepsilon^c(\gamma^2)-1}{\varepsilon^c(\gamma^2)} - \left( \frac{1}{m^c(\gamma^2)} \right) \left( \frac{\theta}{\bar{\theta}} \right) \right]}. \quad (A5)$$



Since the elasticity is decreasing in output, we have

$$\left[ \frac{\frac{\partial R_i(\mathbf{q}^c(\gamma^1), \gamma^1)}{\partial q_i}}{\frac{\partial R_i(\mathbf{q}^c(\gamma^2), \gamma^2)}{\partial q_i}} \right] > \frac{\left( \frac{p(Q^c(\gamma^1))}{p(Q^c(\gamma^2))} \right) \left[ \frac{\varepsilon^c(\gamma^1)-1}{\varepsilon^c(\gamma^1)} - \left( \frac{1}{m^c(\gamma^1)} \right) \left( \frac{\theta}{\bar{\theta}} \right) \right]}{\left[ \frac{\varepsilon^c(\gamma^1)-1}{\varepsilon^c(\gamma^1)} - \left( \frac{1}{m^c(\gamma^2)} \right) \left( \frac{\theta}{\bar{\theta}} \right) \right]}. \quad (A6)$$

Suppose that  $m^c(\gamma^1) \leq m^c(\gamma^2)$ , then we have

$$\left[ \frac{\frac{\partial R_i(\mathbf{q}^c(\gamma^1), \gamma^1)}{\partial q_i}}{\frac{\partial R_i(\mathbf{q}^c(\gamma^2), \gamma^2)}{\partial q_i}} \right] > \frac{\left( \frac{p(Q^c(\gamma^1))}{p(Q^c(\gamma^2))} \right) \left[ \frac{\varepsilon^c(\gamma^1)-1}{\varepsilon^c(\gamma^1)} - \left( \frac{1}{m^c(\gamma^2)} \right) \left( \frac{\theta}{\bar{\theta}} \right) \right]}{\left[ \frac{\varepsilon^c(\gamma^1)-1}{\varepsilon^c(\gamma^1)} - \left( \frac{1}{m^c(\gamma^2)} \right) \left( \frac{\theta}{\bar{\theta}} \right) \right]} = \left( \frac{p(Q^c(\gamma^1))}{p(Q^c(\gamma^2))} \right). \quad (A7)$$

But since  $p(Q^c(\gamma^1)) < p(Q^c(\gamma^2))$ , this expression is less than one, implying a contradiction.

*QED*

*Proof of Proposition 3:*

(i.) and (ii.)

(a.) Suppose  $d^n(\gamma^j) < d^c(\gamma^j)$  and  $q^n(\gamma^j) \geq q^c(\gamma^j)$  for  $j = 1, 2$ . The relationship between costs implies that  $w^n(\gamma^j) < w^c(\gamma^j)$  for  $j = 1, 2$ . Labor market clearing, then, implies the following:

$$Nq^n(\gamma^j) = \gamma^j L^s(w^n(\gamma^j)) < \gamma^j L^s(w^c(\gamma^j)) = Nq^c(\gamma^j).$$

This implies a contradiction.

(b.) Suppose  $d^n(\gamma^j) \geq d^c(\gamma^j)$  and  $q^n(\gamma^j) < q^c(\gamma^j)$  for  $j = 1, 2$ . The relationship between costs implies that  $w^n(\gamma^j) \geq w^c(\gamma^j)$  for  $j = 1, 2$ . Labor market clearing, then, implies the following:

$$Nq^n(\gamma^j) = \gamma^j L^s(w^n(\gamma^j)) \geq \gamma^j L^s(w^c(\gamma^j)) = Nq^c(\gamma^j).$$

This implies a contradiction.

(c.) Suppose  $d^n(\gamma^j) < d^c(\gamma^j)$  and  $q^n(\gamma^j) < q^c(\gamma^j)$  for  $j = 1, 2$ . Recall the first order condition given by equation (13),

$$\frac{\partial R_i(\mathbf{q}^c(\gamma_t), \gamma_t)}{\partial q_{it}} + f(z^c(\gamma_t)) \left[ \frac{\tilde{p}'(Q^c(\gamma_t))}{p(Q^c(\gamma_t))^2} \right] \left[ \Psi(\beta - \beta^T) \right] \times \quad (13)$$

$$\left[ \pi(R^c(\gamma^1) - R^n(\gamma^1)) + (1 - \pi)(R^c(\gamma^2) - R^n(\gamma^2)) \right] = 0.$$

The last term in brackets in this expression is the difference in expected discounted returns between colluding forever and not colluding forever. For the collusive equilibrium to be incentive compatible, this term must be non-negative. Therefore, since  $\beta < 1$  and  $p'(\cdot) < 0$ , the second term in the sum in equation (16) must be non-positive. Hence, the first order condition implies that the collusive equilibrium quantities must satisfy  $\frac{\partial R_i(q^c(\gamma^j), \gamma^j)}{\partial q_i} \geq 0$  for  $j = 1, 2$ . Since the non-collusive output levels satisfy these with equality, strict concavity of the single period profit function implies that if unit costs of production are identical across regimes (i.e.  $d^n(\gamma^j) = d^c(\gamma^j)$ ), then output would be higher in the non-collusive regime. Therefore, since output is decreasing in unit costs and since  $d^n(\gamma^j) < d^c(\gamma^j)$ , we must have  $q^n(\gamma^j) > q^c(\gamma^j)$ , implying a contradiction.

(iii.) Result (ii.) implies that  $p(Q^n(\gamma^j)) \leq p(Q^c(\gamma^j))$ . Hence from (i.) and (ii.) and the definition of markups we have

$$\frac{m^n(\gamma^j)}{m^c(\gamma^j)} = \left[ \frac{p(Q^n(\gamma^j))}{p(Q^c(\gamma^j))} \right] \left[ \frac{d^c(\gamma^j)}{d^n(\gamma^j)} \right] \leq 1.$$

*QED*

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# Quantities & Markups (Deviations)





