Queen's Economics Department Working Paper No. 902

Measuring World Business Cycles

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6-1994

Discussion Paper #902

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June 1994

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Keywords: G7, Kalman filter, scoring

JEL Classification: C32, E32

Abstract

Using Kalman filtering and dynamic factor analysis, we decompose fluctuations in real aggregate output, consumption, and investment for the G7 countries into factors that are (i) common across all countries and aggregates, (ii) common across aggregates within a country, and (iii) specific to each data series. In quarterly data for the period 1970-1993, fluctuations in all of the aggregates contain world and country-specific common components which are significant both statistically and economically. Over this period all seven countries experience business cycle episodes primarily attributable to the world cycle and other episodes driven primarily by the country-specific factor. The share of the variance of aggregate output accounted for by the world business cycle in our estimates ranges from 13% for the U.K. to 67% for France. Also, the world common component in growth rates is more strongly serially correlated than is output growth in any of the seven countries.

We thank David Backus, James Nason, and Gregor Smith for helpful comments and Christian Sigouin and David Watt for research assistance. Financial support from the Social Sciences and Humanities Research Council of Canada and Fonds FCAR is gratefully acknowledged.

1. Introduction

It has long been observed that many countries experience similar fluctuations in macroeconomic aggregates, and that these fluctuations, or business cycles, exhibit substantial synchronization across countries. These observations date back at least to the work of the National Bureau of Economic Research in the early 1920's on constructing business cycle chronologies for several nations. Moore and Zarnowitz (1986) survey some of this research. Many recent studies have also documented business cycle similarities and international co-movements (particularly positive cross-country correlations of output and consumption) among developed countries (Backus and Kehoe (1992), Baxter and Crucini (1993), Fiorito and Kollintzas (1992), and many others). Canova and Dellas (1993) and Head (1994) also document similarities in fluctuations for samples including developing and less developed countries.

Similarities in aggregate fluctuations in different countries have been interpreted as a challenge to economic theory, suggesting the development of business cycle theories which focus on the functioning of market economies in a general sense, rather than on institutions particular to individual countries (Lucas (1977)). In our view the similarity of cycles across countries, and their positive co-movements also call for further measurement. To what extent are these fluctuations all driven by a single "world business cycle"? We use Kalman filtering techniques and dynamic factor analysis in a study of business cycles in Canada, the U.S., Japan, France, Italy, Germany, and the U.K. (i.e. the "G7" countries). Specifically, with quarterly data for the period 1970-93 we decompose fluctuations in real aggregate output, consumption, and investment for these countries into distinct factors that are: i) common across all countries and aggregates (a world common factor); ii) common across aggregates within a country (seven country-specific common factors); and iii) specific to each data series. Hence our approach involves measuring a total of eight latent common factors in international data. Except for the number of factors, the methodology follows closely that used to estimate the single-factor coincident indicator models of Stock and Watson (1991) and Watson (1994) (see also Camen (1987) and Diebold and Rudebusch (1994)). We interpret our estimated world-wide common component as a measure of a "world business cycle".

Our goal is to consider a general enough framework to permit the data to tell us where the similarities and co-movements in aggregate fluctuations lie. Overall, our findings suggest that the dynamic factor model provides a useful decomposition of international business cycle phenomena. Fluctuations in all of the aggregates contain world and country-specific common components which are significant both statistically and economically. Fluctuations in the world common component are very persistent, in most cases more so than fluctuations in the country-specific components. Quantitatively, cycles in the G7 are to a significant extent driven by the world business cycle (i.e. fluctuations in the world common component), and yet are also significantly driven by country-specific factors. The importance of both common and country-specific factors accords with the results of Stockman (1988) who found that both world-wide sectoral and national aggregate disturbances were important for explaining fluctuations in industrial output for seven European countries, and Costello (1993) who obtained results similar to Stockman's in a study of productivity growth in a group of six countries.

The influence of the world business cycle on aggregate fluctuations in each of the G7 countries is estimated by computing the shares of the variances of their aggregates for which it accounts. These shares range widely across countries, but in general the world cycle accounts for a relatively large share of output fluctuations in Japan, France, Germany, and Italy, and a smaller share in Canada, the U.S., and the U.K. In addition, we find that the world cycle accounts for a greater share of fluctuations in the seven countries' outputs than it does in their consumption fluctuations. This accords with earlier findings that cross-country correlations of output typically exceed those of consumption (Backus, Kehoe, and Kydland (1992) and Stockman and Tesar (1991)). Our findings thus tend to reinforce the evidence that the empirical properties of international co-movements do not support the predictions of theoretical models which posit a high degree of international risk sharing, as these generally predict that cross-country correlations of consumption will exceed those of output. We also find that the world cycle accounts for a larger share of fluctuations in investment than in consumption. Co-movements in investment have not received the same degree of attention as those in consumption and output. The evidence suggests that perhaps they are worthy of a closer look, both theoretically and empirically.

Our methodology also enables us to characterize the timing of the world business cycle. The world cycle measured by the dynamic factor model largely accords with our knowledge of historical events. For example, our estimated world common component experienced a sharp downturn in the mid-1970's, a steady expansion throughout the mid and late 1980's, and another sharp downturn in the early 1990's. During the time period studied, all countries experienced certain business cycle episodes primarily attributable to the world cycle and other episodes that were primarily country-specific. For example, certain U.S. business cycle peaks and troughs (as reported by the NBER dating committee) correspond almost exactly to peaks and troughs in our measure of the world business cycle, while others appear to be primarily U.S. phenomena. The U.S. trough of 1975 appears to be closely associated with a world-wide recession, very likely due to the oil shock. In contrast, the U.S. recession of 1982 appears to be much more heavily influenced by the country-specific factor. This country-specific effect could be closely related to the tight monetary policy that we know was in effect in the U.S. at that time.

The remainder of the paper is organized as follows. Section 2 describes the data and estimation procedure, with technical details contained in an appendix. Sections 3 and 4 present the empirical findings. Section 5 discusses the implications of these findings for theoretical research, and also suggests several directions for refinements and further measurement.

2. A Multiple Dynamic Factor Model

Let Y_{jt}, C_{jt}, and I_{jt} denote respectively the logarithms of output, consumption and investment for country j, where $j = 1, ..., 7$. All series are taken from the *International Financial Statistics* of the International Monetary Fund. Y_t is real gross national product at 1985 prices, C_t and I_t are consumption and gross capital formation in current dollars respectively, deflated using the implicit deflator, with 1985=100. The data span the period 1970:1-1993:1 at quarterly frequencies and are seasonally adjusted. Since we are interested in relatively short term fluctuations, it is necessary at the outset to isolate high frequency movements in the data. Here we take the two standard approaches. In one approach, we decompose each data series into trend and cyclical components using the Hodrick-Prescott

(HP) (1980) filter with smoothing parameter 1600. This has become a standard method of detrending data in the business cycle literature and thus has the advantage of producing results that can be compared to those of a large number of previous studies (e.g. Backus and Kehoe (1992)). In another approach, we first difference the data in logarithms. This approach may be thought of as analyzing the cycle in growth rates. The two approaches lead to measures of the world business cycle different in some respects, but similar in many others. For the countries and series considered, fluctuations in log first differences differ from those in deviations from HP trend principally in their serial correlation properties. Since the approaches lead to somewhat different interpretations of the world business cycle, we present results from both as two complementary measures. For all series, the sample mean is removed and the variance is standardized to one.

Let W_t be an index of world economic activity (unobservable) common to each of the variables of G7 countries and N_{jt} be a component common to each of the variables in country j (also unobservable). We will refer to these as the world common and country-specific common factors, and index them by $f = w, n$ respectively. Each series, Y_{jt}, C_{jt}, and I_{jt}, $j = 1 \dots 7$, is decomposed into three separate components:

$$\begin{aligned} Y_{jt} &= \gamma_{wj}^{Y} W_t + \gamma_{nj}^{Y} N_{jt} + \eta_{jt}^{Y} \\ C_{jt} &= \gamma_{wj}^{C} W_t + \gamma_{nj}^{C} N_{jt} + \eta_{jt}^{C} \\ I_{jt} &= \gamma_{wj}^{I} W_t + \gamma_{nj}^{I} N_{jt} + \eta_{jt}^{I}. \end{aligned} \tag{2.1}$$

Here η_{jt}^{k} represents a component specific to series k, where $k = Y, C, I$ indexes the aggregates for each country. γ_{fj}^{k} is the impact coefficient on factor f for aggregate k in country j. In the language of Kalman filtering, (2.1) is a system of 21 measurement equations. Following Stock and Watson (1991), we assume that each of the three components follows a first-order stationary univariate autoregressive process:

$$\begin{aligned} W_t &= \rho_w W_{t-1} + \epsilon_{wt} \\ N_{jt} &= \rho_{nj} N_{jt-1} + \epsilon_{njt} \\ \eta_{jt}^{k} &= \rho_j^{k} \eta_{jt-1}^{k} + \epsilon_{jt}^{k}. \end{aligned} \tag{2.2}$$

These 29 equations comprise the transition equations. To estimate the parameters of the model, we assume for identification that all the errors are uncorrelated both contemporaneously and autoregressively, and normalize the variances of ϵ_w and ϵ_{nj}, $j = 1, \ldots, 7$ to unity. The assumption that errors are contemporaneously uncorrelated allows no explicit channel for co-movement among the common factors. Nevertheless, we do find some interesting correlations among the estimated latent variables, and we return to this issue in our final remarks.

In the appendix, the model is cast in state-space/measurement equation form and the Gaussian log-likelihood is developed. It should be noted that the model as specified (even with the restriction that the errors are uncorrelated) has a very large number of parameters. Due to the size of the model, the maximum likelihood estimation is computationally difficult. There are 92 parameters to estimate with eight common factors in 21 measurement equations. The dimension of the model prevented us from using the EM algorithm to get initial estimates and the scoring algorithm failed to converge for arbitrary starting values. Therefore, initial estimates were obtained in a two-step procedure. We first estimated a model with a single world common component and no country-specific common components. Estimation of this smaller model (65 parameters and only one common factor) provided a first-round estimate of a world common component, $\hat{W}_t$. We then estimated country by country single country-specific common components for the three variables Y_{jt}, C_{jt}, and I_{jt}, using the $\hat{W}_t$ from the first round as an exogenous regressor in the measurement equations. This second round yields 91 estimated parameters (13 for each country), which together with the first-round estimate of the autocorrelation for the world component (ρ_w) are used as the 92 initial estimates for the entire multiple factor model.

In maximizing the likelihood, we found on occasion that one or two of the estimated variances of the series-specific components (ϵ_{jt}^k) moved into the non-positive region. Evidently, the observed variables were completely described by the world (W_t) and country-specific (N_{jt}) common factors. For these cases, we eliminated the series-specific components from the estimating equations. Thus, in the HP filtered data, Japanese output and Italian investment are functions of the world and country-specific components only, leaving

88 parameters to be estimated. In the first-differenced data, only Japanese output proved problematic. With its series-specific component omitted, 90 parameters are estimated.

Even with good starting values, the estimation was computationally intensive. All of the calculations were conducted on the IBM RS/6000 Model 355 workstation and a Gauss-Newton scoring algorithm with analytical derivatives was used to maximize the likelihood. The program used is described in detail in Raynauld, Simonato and Sigouin (1993). The code was written in GAUSS as implemented for a UNIX platform and required 50 megabytes of memory. Each iteration required about eight hours and the estimation of the HP filter and log first-differenced models presented here took approximately four months of continuous estimation time.

3. Findings: Deviations from Hodrick-Prescott Trend

We begin by considering the general properties of our estimated world common factor. Figure 1 plots this factor, (W), together with the the estimated U.S. country-specific common component (N), and detrended U.S. output (Y) over the sample period. In the figure the common and U.S. component have been weighted by their impact coefficients so that the sum of these components with the series-specific component in U.S. output (not shown) is equivalent to the U.S. output series. The vertical bars emanating from the 0-axis indicate peaks (+1) and troughs (-1) of the U.S. business cycle over this period, as reported by the NBER dating committee. This picture permits comparison of our measured "world" business cycle, represented here by fluctuations in W, with fluctuations in actual U.S. output (and NBER peaks and troughs), and with an indicator of the U.S.-specific business cycle, represented by N. Consider first the timing of the fluctuations in W. Given our knowledge of economic events over the past 25 years, this measure of a world business cycle appears to be a sensible one. By our measure, the world experienced significant downturns in the mid-1970's, early 1980's and early 1990's; experienced strong upturns in the early and late 1970's, and went through a relatively long, slow expansion throughout the mid and late 1980's. The fact that fluctuations in the world component can be easily related to observed events, like the oil shock of 1973 and the severe U.S. recession of 1982, is encouraging.

Now consider the relationships among the world business cycle, our measured indicator of the U.S.-specific business cycle, N, and fluctuations in U.S. output. In some cases the peaks and troughs of the world cycle correspond quite closely with those of the U.S. output (and with those of the NBER) but not always. For example, the NBER peaks of 1973.4 and 1980.1, and the NBER trough of 1975.1 correspond to world business cycle peaks and troughs almost exactly. In contrast, the NBER trough and peak in 1980.3 and 1981.3 respectively appear to be principally U.S.-specific phenomena. Similarly, the NBER trough of 1991.1, appears to occur at least two years prior to a trough in the world business cycle, which evidently had not occurred as of 1993.1. There also appear to be other episodes in which fluctuations in U.S. output lead the world business cycle. For example, the booms of the early and late 1970's also exhibit this pattern. In other episodes, however, the U.S. does not lead the world cycle.

In addition to some differences in the timing of U.S. and world business cycles, it appears that the world and U.S. common components are of different relative quantitative importance for U.S. output fluctuations at different times. Roughly speaking, it appears that in the 1970's the world cycle played a larger role in determining the amplitude of fluctuations in U.S. output than it did in the 1980's. The effect of the world common factor in the early 1970's boom and mid-1970's recession was roughly equal to that of the country-specific component. In contrast, the U.S. country-specific component was quantitatively much more important in the U.S. recession of the early 1980's. These differences in quantitative impacts also accord roughly with intuition. The mid-1970's recession is widely attributed to effects of the first major oil shock, which is most appropriately thought of as a global phenomenon. The U.S. recession of 1982, in contrast, is often attributed to a tight monetary policy, perhaps more appropriately thought of as a country-specific factor. Overall, Figure 1 suggests that cycles in U.S. output are closely related to movements in the world business cycle, but that there are important episodes in the U.S. business cycle that are primarily country-specific.

Over the time period considered, not only the U.S., but all of the G7 countries experienced distinct business cycle episodes in which the principle factor was the world business cycle, and other episodes that were principally country-specific. For example, Figure 2 plots the world common factor, (W), the country-specific common factor for France, (N), and French output, (Y). This diagram suggests that fluctuations in French output were principally driven by the world cycle in the mid-1970's, late 1980's and early 1990's, but were driven primarily by country-specific factors in the early 1970's and early 1980's. The pattern that output fluctuations were strongly influenced by the world cycle in the mid-1970's and early 1990's, but largely country-specific throughout the 1980's, seems apparent to some degree in the fluctuations of all seven countries. We will return to this observation in our final remarks. At this point, however, it is useful to consider the estimates in somewhat closer detail.

Tables 1 and 2 contain parameter estimates for the dynamic factor model using HP filtered data. Table 1 contains the estimated impact coefficients and their standard errors for all countries and variables on both the world and country-specific common factors, and Table 2 contains the estimates of the first order autoregressive coefficients for the world common, country-specific common, and series-specific factors. Throughout the tables, aggregates are indexed by k, where $k = Y, C, I$ denotes aggregate output, consumption, and investment respectively. The world common and country-specific common factors are indexed by $f = w, n$ respectively. Since all equations are symmetric for each of the seven countries, the country subscript, j, is suppressed where possible.

Considering first the estimated impact coefficients (Table 1), note that nearly all of estimates are statistically significant at the 5% level. In addition, the following broad regularities apply. In all cases, the estimated impact coefficient on the world component is larger for output than for either consumption or investment (i.e. $\hat{\gamma}_w^Y > \hat{\gamma}_w^C$ and $\hat{\gamma}_w^Y > \hat{\gamma}_w^I$). Also, in all but one case (Italian consumption) the impact coefficients on the country-specific factors are larger than those on the world factor for both consumption and investment. These patterns in the impact coefficients suggest that the world cycle is most strongly evident in output fluctuations. With regard to the estimated autoregressive coefficients (Table 2), the coefficients for the world common and country-specific common factors are all positive and statistically significant. This is also true of nearly all of the series-specific factors. The strong autocorrelations suggest a great deal of persistence in both world-wide and country-specific fluctuations. We will not focus on this persistence here, however, because it is to be expected to some extent in HP filtered data. The persistence left in deviations from an HP trend is related to the choice of smoothing parameter (here we use 1600, as is standard). Presumably, varying this factor would have substantial effect on the estimated autoregressive coefficients. For this reason, we postpone a discussion of the persistence of world fluctuations to the next section, in which we consider data in log first-differences.

Our estimates enable us to measure the quantitative influence of variations in the different common factors on fluctuations in aggregate output, consumption, and investment. Let R_f^k denote the share of the variance of aggregate k accounted for by variation in the factor f. This measure can be used to give a quantitative economic interpretation to the magnitudes of the parameter estimates in Tables 1 and 2. Under the assumption that the world common, country-specific common, and series-specific factors are orthogonal, the variance of each series can be decomposed into three terms:

$$\sigma_k^2 = \gamma_w^{k^2} \sigma_w^2 + \gamma_N^{k^2} \sigma_n^2 + \sigma_{\eta^k}^2 \qquad k = Y, C, I. \tag{3.1}$$

With the variances of the innovations to the world and country-specific common components normalized to unity, we can compute estimates of R_f^k as follows,

$$\hat{R}_f^k = \frac{\frac{\hat{\gamma}_f^{k^2}}{1-\hat{\rho}_f^{k^2}}}{\frac{\hat{\gamma}_w^{k^2}}{1-\hat{\rho}_w^2} + \frac{\hat{\gamma}_n^{k^2}}{1-\hat{\rho}_n^2} + \frac{\hat{\sigma}_{\epsilon^k}^2}{1-\hat{\rho}^{k^2}}}, \qquad k = Y, C, I \qquad f = w, n. \tag{3.2}$$

where $\hat{\sigma}_{\epsilon^k}^2$ denotes the estimated variance of the innovation to the series-specific component in aggregate k. Standard errors of the $\hat{R}_f^k$ are computed using the delta method.

The estimated variance shares (Table 3) for the most part reflect the estimates of the impact coefficients given in Table 1. In all cases, the share of the variance in output accounted for by fluctuations in the world common factor is greater than the comparable share for either consumption or investment. It is also generally the case (Italy is the sole exception) that in consumption, the share of variance accounted for by the country-specific factor is greater than that accounted for by the world factor. The greater influence of the world cycle on output fluctuations than on consumption fluctuations conflicts with an interpretation of international co-movements centered on international risk sharing. This interpretation suggests that co-movements of consumption should be strongest, as countries use intertemporal trade to diversify country-specific income risk. It has, however, been widely noted (e.g. by both Backus, Kehoe, and Kydland (1992) and Stockman and Tesar (1991)) that international output correlations are typically larger than those of consumption. Our finding that consumption fluctuations are driven by country-specific factors to a greater extent than are output fluctuations accords with this evidence. With regard to investment, although the share of variance accounted for by the world factor is lower than the corresponding share in output fluctuations for all countries, the size of $\hat{R}_w^I$ varies widely across countries and is greater than .4 in three countries. International co-movements in investment have not been highlighted in the recent literature on international business cycles; the results here suggest that perhaps that the implications of theories for these correlations (typically models of international risk sharing predict negative correlations) should be given more attention.

In terms of the shares of variances accounted for by the common factor, the quantitative importance of the world business cycle varies substantially across countries. In four countries: Japan, France, Germany, and Italy the world common factor accounts for more than 50% of the variance of output, with Germany having the largest share (66.7%) attributable to this factor. In contrast, the world business cycle has substantially lesser effect on output fluctuations in Canada, the U.S., and the U.K.. It is also interesting that for all countries, fluctuations in the world and country-specific common factors together account for more than 80% of the variance of output, leaving less than 20% to be accounted for by series-specific fluctuations.

Overall, the estimation of the dynamic factor model suggests that there is a statistically significant common cycle in output, consumption, and investment for all of the G7 countries distinct from the country-specific cycle that jointly affects the three aggregates within a country. Measured in this way, the "world business cycle" is quantitatively important, accounting for between 14.5% and 66.7% of the variance of output in these seven countries. In addition, all countries experience certain episodes (in particular the mid-1970's and early 1990's recessions) in which the world cycle appears to be the primary factor driving aggregate fluctuations.

4. Findings: Log First-Differences

We now consider the dynamic factor model applied to aggregate data in the first-differences of logarithms, or growth rates. For the present purposes, the principle difference between these data and the deviations from HP trend discussed in the previous sections lies in their serial correlation properties. In considering the persistence of fluctuations in the world and country-specific components, the serial correlations of log first differences have a clear interpretation in terms of growth rates. This contrasts somewhat with the HP filtered data, the serial correlation properties of which depend on the smoothing parameter chosen. For considerations other than serial correlation properties, the findings in log first-differences are remarkably similar to those in HP filtered data.

Tables 4 and 6 contain the estimates of impact coefficients and of the shares of variances accounted for by the common factors respectively. Note first that many of the same patterns observed in Table 1 are present in Table 4. In all cases $\hat{\gamma}_w^Y > \hat{\gamma}_w^C$ and for all countries except Italy (as in Table 1), $\hat{\gamma}_n^C > \hat{\gamma}_w^C$. Table 6 also bears considerable similarity to Table 3. While the shares of the variances accounted for by the world factor are almost all lower for log first-differences than for HP filtered data, the same overall patterns are evident. For all countries, the world common component accounts for a larger share of the variance of output than of the variance of consumption, reinforcing the observation that the world business cycle seems inconsistent with explanations based on international risk sharing. In this case, the country-specific nature of consumption fluctuations is perhaps even more striking; only for Italy does the world factor account for more than 20% of the variance of consumption. Again, investment fluctuations appear to be more strongly affected by the world cycle. Finally, note that while the ranking of countries by $\hat{R}_w^y$ differs somewhat depending on whether we consider HP filtered data or log first-differences, France, Germany, and Italy still have a relatively large share of their output variance accounted for by fluctuations in the world common component, and Canada, the U.S., and the U.K. still occupy the low end in this regard.

Turning to the estimates of the first order autoregressive coefficients (Table 5), a particularly interesting result emerges concerning the relative serial correlations of the dynamic factors. The world common component is strongly positively autocorrelated, as are the country-specific common components for Canada, the U.S., France, and Italy. The other three countries' country-specific components, however, are either negatively autocorrelated (Japan and Germany), or serially uncorrelated (the U.K.). The autoregressive coefficients on the series specific components are for the most part negative (the exception is Italian investment) and many are statistically insignificant. The series-specific components in output are negatively autocorrelated for all countries except Italy, and Japan, for which no series-specific component in output was estimated, has a negatively autocorrelated country-specific common component.

These estimated autoregressive parameters for the output equations can be contrasted with the estimated autocorrelations of the actual output series (the last row of Table 7). Except for Italy, all of the series autocorrelation coefficients are much lower than that of the estimated world common factor. Of particular note are Japan, Germany, and the U.K., whose output series appear to be serially uncorrelated. Nevertheless, despite the lack of autocorrelation in the individual series, each is strongly related to a persistent world factor. Overall the parameter estimates suggest that the world cycle in growth rates is strongly autocorrelated, while country and series-specific factors serve to reduce the serial correlations of the growth rates of aggregates. This pattern is evident to a certain extent for the U.S. in Figure 3, which contains the world common component (W), the U.S. country-specific common component (N), and deviation of the U.S. output growth rate from its mean (Y). In this picture, the world common component is noticeably smoother than the first differences of the logs of U.S. output, which is not surprising given the strong negative serial correlation in the series-specific component for U.S. output (-.436).

Table 7 also contains the contemporaneous correlations of each country's output growth rate with those of the other countries, and with the world common component. With the exception of Canada (which has a high pair-wise output correlation with the U.S.) all countries exhibit much higher correlations of output with the world common component than with the output of any of the other countries. The strong correlations of output growth in the individual countries with the world common factor suggest that the world cycle may account in part for similarities in the serial correlation of output growth across countries. Considering that there are substantial differences in the serial correlations of the country-specific common factors (the second column of Table 5), it appears that certain countries might have negatively autocorrelated output growth if not for the effect of the world business cycle.

Overall, our findings indicate that there is a statistically significant world cycle in the growth rates of output, consumption, and investment for the G7 countries. Measured in this way, the world business cycle is strongly persistent, much more so than might be expected considering the autocorrelations and cross-correlations of the individual series. In contrast, country-specific cycles in Japan, Germany, and the U.K. are negatively autocorrelated or serially uncorrelated, and the series-specific fluctuations in output are negatively serially correlated for all countries. Except with regard to serial correlation, the findings do not differ markedly from those reported for HP filtered data, although the shares of variances accounted for by the world business cycle are nearly all lower when the cycle is measured in log first-differences. Finally, cycles in output growth for all seven countries display distinct periods in which each of the factors appears to be driving the cycle. An example of this can be seen by returning to Figure 3. Again, cycles in U.S. growth rates experience episodes in which the world common factor appears dominant (e.g. 1972-75) and other episodes in which the country-specific factor appears most important (e.g. 1982-85).

5. Final Remarks

Our findings suggest that dynamic factor analysis provides a useful framework for analyzing international business cycle phenomena. The statistical and quantitative significance of the world common factor in both HP filtered data and log first-differences provide a quantitative answer to our original question: To what extent are the different countries' business cycles influenced by a single common cycle? Evidently both world-wide and country-specific factors play major roles in determining the properties of a given country's aggregate fluctuations, and either may be dominant during a particular episode. With the cycle measured using growth rates, it appears that not only the co-movements, but also certain similarities in the time series properties of output fluctuations across countries may be principally due to the influence of a world cycle.

This analysis also suggests some extensions to refine our measurements of the world business cycle. In this study we began with a relatively general framework, allowing for factors specific to each country and series, as well as a world factor. Our assumption that there be no covariance among these factors seemed natural as a starting point. In addition, due to the large number of parameters inherent in the specification and the difficulty in obtaining estimates, ruling out covariances and additional factors (perhaps affecting sub-groups of the seven countries) was also an assumption of considerable convenience. Analysis of the findings, however, suggests that certain of these restrictions may not be appropriate. Table 8 contains the contemporaneous correlations of the world and country-specific common factors. Generally speaking, the estimated country-specific factors do not exhibit strong correlations with the world factor, and so an assumption of orthogonality may not be overly restrictive. With regard to the country-specific factors, however, one anomaly stands out in particular. In both HP filtered data and log first-differences there is a strong positive correlation between the country factors for the U.S. and Canada. This point is illustrated vividly in Figure 4, which contains these two factors in HP filtered data. Clearly, the co-movement is very strong, suggesting that the two country-specific factors could be combined into a single, "North American", factor. This is just one example of a refinement that could be investigated, and there are no doubt others to be considered.

This evidence of a world cycle also calls for theoretical work to identify phenomena that account for it. Important policy issues may hinge on what theory is most convincing. If aggregate fluctuations are largely due to global phenomena producing common movements, then the business cycle may not be as responsive to domestic policies as previously thought. If, however, particular episodes in the world cycle are caused by propagation of disturbances in a leading country (which could change from episode to episode), then domestic policies may be appropriate.

Our measurements do not address the issue of whether co-movements in aggregate fluctuations are due to common shocks or international propagation of country-specific disturbances and may be consistent with theories based on either possibility, or a combination of both. In the literature on international business cycles, co-movements in total factor productivity, usually measured by the Solow residual, have been taken as indicators of the degree to which technology shocks are common across countries (see e.g. Backus, Kehoe, and Kydland (1992), Costello (1993), and Stockman and Tesar (1991)). As a first pass at considering these issues, we estimated a dynamic factor model using Solow residuals for each of the G7 countries. Since there is only one variable per country, there is a single world common factor. Table 9 contains the estimates for both log first-differences and HP filtered data; the notes to the table contain a description of the Solow residual and the data used to construct measures for the seven countries. The estimates in the table suggest that there is a common component in Solow residuals that is both statistically and economically significant, and thus to the extent that Solow residuals may be indicators of technology shocks, this may provide a measure of the extent to which these shocks occur simultaneously throughout the G7. Table 9 also suggests a relatively parsimonious method for measuring a common component in disturbances (which could be applied to measures of other, perhaps non-technological shocks) that could be used in the calibration of dynamic general equilibrium models of the interaction of multiple countries.

Other areas for further empirical research include identification of factors that cause, or at least predict, movements in the world business cycle. Preliminary research in this area has suggested that oil prices may be a significant factor, but there is much more to be done. Another area for investigation is whether one country appears to "lead" the world cycle over certain episodes. This does not appear to be the case for any one country over the entire 1970-93 time horizon, but it may be the case for certain sub-periods. As noted earlier, there are three episodes in which fluctuations in U.S. output appear to lead the world cycle. Finally, another extension could address the issue of whether the influence of the world business cycle on fluctuations in individual countries changes over time. As noted in section 3, inspection of the estimated common factors in HP filtered data suggests a sense in which the influence of the world cycle differed from the 1970's to the mid-1980's. It would be interesting to incorporate a notion of "regime switching", perhaps developed from that proposed by Diebold and Rudebusch (1994), in their case aimed at the analysis of business cycle asymmetries. At present, the main impediments in all of these areas of proposed extensions are the availability of comparable data for groups of several countries, and the sheer computational intensity of the estimation.

Appendix: State-Space Representation and Log Likelihood

In this appendix we cast the dynamic factor model in state-space form and develop the Gaussian log likelihood. As the methods used to estimate the model are standard the presentation is terse; interested readers are referred to Stock and Watson (1991) and Harvey (1989) for a more complete description. Dynamic factor models have also been studied extensively in the macroeconometric literature (e.g. by Geweke (1977), Geweke and Singleton (1980), Sargent and Sims (1977), Watson (1994), and Watson and Engle (1983)). The state-space equations give a representation on the evolution of W_t, N_{jt} and η^k_{jt} and their lags and the measurement equations link the observed variables to the elements of the state vector. Collect $(Y_{1t},...,Y_{7t},C_{1t},...,C_{7t},I_{1t},...,I_{7t})$ into a (21×1) vector, y_t. It is straightforward to express the system given by (2.1) and (2.2) in the companion form:

$$\alpha_t = T\alpha_{t-1} + \omega_t, \qquad y_t = Z\alpha_t,$$

with $\alpha_t = (W_t, N_{jt}, \eta^k_{jt})$, $\omega_t = (\epsilon_{wt}, \epsilon_{njt}, \epsilon^k_{jt})'$, and with T and Z the appropriately defined coefficient matrices. Denote the variance-covariance matrix of ω_t by Σ.

The Kalman filter consists of the prediction and updating equations. Let $\alpha_{t|\tau}$ be the estimate of α_t based upon information $(y_1,...,y_\tau)$ and $P_{t|\tau} = E\left((\alpha_{t|\tau} - \alpha_t)(\alpha_{t|\tau} - \alpha_t)^T \right)$. The prediction equations are:

$$\alpha_{t|t-1} = T\,\alpha_{t-1|t-1}$$

$$P_{t|t-1} = T\,P_{t-1|t-1}T' + \Sigma.$$

The updating equations are:

$$\alpha_{t|t} = \alpha_{t|t-1} + P_{t|t-1}Z'F_t^{-1}\nu_t,$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}Z'F_t^{-1}\nu_t,$$

where $F_t = E(\nu_t\nu_t^T) = ZP_{t|t-1}Z^T$ and $\nu_t = y_t - y_{t|t-1}$. Given initial estimates of T, Σ, Z and starting values $\alpha_{0|0} = 0$ and $vec(P_{0|0}) = (I - T \otimes T)^{-1}vec(\Sigma)$ recursive calculation gives the prediction state vector $\alpha_{t|t-1}$ and covariance $P_{t|t-1}$. The Gaussian log likelihood (excluding the constant) is:

$$L = \frac{-1}{2}\sum_{t=1}^{T} \nu_t^T F_t^{-1}\nu_t \frac{-1}{2}\sum_{t=1}^{T} ln \det F_t. \tag{A.1}$$

The problem is to maximize (A.1) with respect to the parameters of the model. In this study, we maximize this function using a Gauss-Newton scoring algorithm with analytical derivatives developed by Raynauld, Simonato, and Sigouin (1993).

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Table 1: Impact Coefficients

Deviations from Hodrick-Prescott Trend

	World Factor			Country Factors		
	$\hat{\gamma}_w^Y$	$\hat{\gamma}_w^C$	$\hat{\gamma}_w^I$	$\hat{\gamma}_n^Y$	$\hat{\gamma}_n^C$	$\hat{\gamma}_n^I$
Canada	.150	.058	.091	.478	.382	.292
	(.061)	(.066)	(.056)	(.059)	(.063)	(.053)
U.S.	.197	.167	.165	.381	.355	.329
	(.052)	(.056)	(.047)	(.039)	(.045)	(.036)
Japan	.305	.205	.256	.516	.409	.285
	(.056)	(.067)	(.050)	(.042)	(.069)	(.041)
France	.356	.135	.289	.255	.283	.316
	(.053)	(.071)	(.057)	(.060)	(.087)	(.069)
Italy	.324	.249	.204	.227	.210	.450
	(.051)	(.056)	(.057)	(.043)	(.052)	(.036)
Germany	.379	.205	.316	.365	.393	.369
	(.058)	(.064)	(.066)	(.060)	(.078)	(.073)
U.K.	.208	.172	.186	.400	.476	.228
	(.058)	(.061)	(.059)	(.052)	(.054)	(.059)

Notes:

The following equations are estimated:

$$Y_j = \gamma_{wj}^Y W_t + \gamma_{nj}^Y N_{jt} + \eta_{jt}^Y \qquad W_t = \rho_w W_{t-1} + \epsilon_{wt}$$
$$C_j = \gamma_{wj}^C W_t + \gamma_{nj}^C N_{jt} + \eta_{jt}^C \qquad N_{jt} = \rho_{nj} N_{jt-1} + \epsilon_{njt}$$
$$I_j = \gamma_{wj}^I W_t + \gamma_{nj}^I N_{jt} + \eta_{jt}^I \qquad \eta_{jt}^k = \rho_j^k \eta_{jt-1}^k + \epsilon_{jt}^k$$

The seven countries are indexed by $j = 1, ..., 7$. Aggregate output, consumption, and investment are indexed by $k = Y, C, I$, respectively. Standard errors are in parentheses.

Table 2: Autoregressive Coefficients

Deviations from Hodrick-Prescott Trend

	World Factor $\hat{\rho}_w$	Country Factors $\hat{\rho}_n$	Series-Specific Factors $\hat{\rho}^Y$	$\hat{\rho}^C$	$\hat{\rho}^I$
World	.904 (.049)				
Canada		.810 (.064)	.454 (.528)	.716 (.076)	.846 (.056)
U.S.		.869 (.053)	.153 (.247)	.627 (.085)	.815 (.074)
Japan*		.610 (.094)	—	.587 (.084)	.849 (.055)
France		.845 (.076)	.672 (.132)	.571 (.096)	.645 (.146)
Italy*		.856 (.056)	.647 (.094)	.590 (.088)	—
Germany		.761 (.091)	.142 (.318)	.234 (.121)	.571 (.103)
U.K.		.789 (.070)	.520 (.110)	-.046 (.312)	.727 (.073)

Notes:

The following equations are estimated:

$$Y_j = \gamma_{wj}^Y W_t + \gamma_{nj}^Y N_{jt} + \eta_{jt}^Y \qquad W_t = \rho_w W_{t-1} + \epsilon_{wt}$$

$$C_j = \gamma_{wj}^C W_t + \gamma_{nj}^C N_{jt} + \eta_{jt}^C \qquad N_{jt} = \rho_{nj} N_{jt-1} + \epsilon_{njt}$$

$$I_j = \gamma_{wj}^I W_t + \gamma_{nj}^I N_{jt} + \eta_{jt}^I \qquad \eta_{jt}^k = \rho_j^k \eta_{jt-1}^k + \epsilon_{jt}^k$$

The seven countries are indexed by $j = 1, ..., 7$. Aggregate output, consumption, and investment are indexed by $k = Y, C, I$, respectively. Standard errors are in parentheses.

* The series-specific components on Japanese output and Italian investment were omitted; see text.

Table 3: Shares of Variance Accounted for by Common Factors

Deviations from Hodrick-Prescott Trend

	World Factor			Country Factors		
	$\hat{R}_w^Y$	$\hat{R}_w^C$	$\hat{R}_w^I$	$\hat{R}_n^Y$	$\hat{R}_n^C$	$\hat{R}_n^I$
Canada	.145	.021	.057	.783	.481	.312
	(.124)	(.048)	(.072)	(.159)	(.146)	(.132)
U.S.	.250	.178	.211	.696	.597	.624
	(.156)	(.133)	(.139)	(.158)	(.148)	(.150)
Japan	.545	.236	.441	.455	.272	.160
	(.159)	(.147)	(.163)	(.159)	(.102)	(.068)
France	.642	.099	.459	.213	.280	.354
	(.152)	(.106)	(.176)	(.134)	(.162)	(.174)
Italy	.599	.387	.230	.202	.189	.770
	(.156)	(.165)	(.147)	(.110)	(.109)	(.147)
Germany	.667	.218	.458	.268	.348	.271
	(.142)	(.139)	(.168)	(.131)	(.128)	(.131)
U.K.	.287	.195	.238	.510	.719	.173
	(.162)	(.145)	(.151)	(.155)	(.148)	(.097)

Notes:

R_f^k measures the share of the variance in aggregate k (where $k = Y, C, I$) accounted for by variation in the common factor f. Here $f = w, n$ denote the world and country factors respectively. R_f^k is defined as the ratio of the variance of the common factor weighted by the appropriate impact coefficient, to the sum of the variances of the common and country factors (weighted by their impact coefficients) and the variance of the series-specific component η_j^k, where $j = 1, \ldots, 7$. That is,

$$\hat{R}_f^k = \frac{\frac{\hat{\gamma}_f^{k^2}}{1-\hat{\rho}_f^{k^2}}}{\frac{\hat{\gamma}_w^{k^2}}{1-\hat{\rho}_w^2} + \frac{\hat{\gamma}_n^{k^2}}{1-\hat{\rho}_n^2} + \frac{\hat{\sigma}_{\epsilon^k}^2}{1-\hat{\rho}^{k^2}}}, \qquad k = Y, C, I \qquad f = w, n$$

where $\hat{\sigma}_{\epsilon^k}^2$ is the estimated variance of the innovation to the series specific component in aggregate k. Standard errors, computed by the Delta method, are in parentheses.

Table 4: Impact Coefficients

Log First-Differences

	World Factor			*Country Factors*		
	$\hat{\gamma}_w^Y$	$\hat{\gamma}_w^C$	$\hat{\gamma}_w^I$	$\hat{\gamma}_n^Y$	$\hat{\gamma}_n^C$	$\hat{\gamma}_n^I$
Canada	.247	.101	.219	.750	.587	.542
	(.090)	(.083)	(.090)	(.094)	(.091)	(.096)
U.S.	.326	.270	.329	.637	.544	.679
	(.086)	(.082)	(.094)	(.076)	(.079)	(.077)
Japan	.333	.218	.336	.863	.522	.574
	(.071)	(.072)	(.080)	(.069)	(.093)	(.079)
France	.484	.206	.404	.411	.322	.337
	(.082)	(.075)	(.076)	(.115)	(.110)	(.097)
Italy	.423	.338	.329	.399	.171	.582
	(.087)	(.081)	(.091)	(.098)	(.099)	(.137)
Germany	.366	.127	.221	.624	.374	.629
	(.069)	(.063)	(.067)	(.130)	(.117)	(.138)
U.K.	.239	.165	.236	.736	.723	.466
	(.075)	(.072)	(.073)	(.105)	(.105)	(.103)

Notes:

The following equations are estimated:

$$Y_j = \gamma_{wj}^Y W_t + \gamma_{nj}^Y N_{jt} + \eta_{jt}^Y \qquad W_t = \rho_w W_{t-1} + \epsilon_{wt}$$
$$C_j = \gamma_{wj}^C W_t + \gamma_{nj}^C N_{jt} + \eta_{jt}^C \qquad N_{jt} = \rho_{nj} N_{jt-1} + \epsilon_{njt}$$
$$I_j = \gamma_{wj}^I W_t + \gamma_{nj}^I N_{jt} + \eta_{jt}^I \qquad \eta_{jt}^k = \rho_j^k \eta_{jt-1}^k + \epsilon_{jt}^k$$

The seven countries are indexed by $j = 1, ..., 7$. Aggregate output, consumption, and investment are indexed by $k = Y, C, I$, respectively. Standard errors are in parentheses.

Table 5: Autoregressive Coefficients

Log First-Differences

	World Factor	*Country Factors*	*Series-Specific Factors*		
	$\hat{\rho}_w$	$\hat{\rho}_n$	$\hat{\rho}^Y$	$\hat{\rho}^C$	$\hat{\rho}^I$
World	.742				
	(.090)				
Canada		.386	-.351	-.127	.243
		(.122)	(.260)	(.121)	(.115)
U.S.		.458	-.436	-.179	.594
		(.110)	(.129)	(.114)	(.153)
Japan*		-.192	—	-.080	.182
		(.108)		(.105)	(.105)
France		.511	-.348	-.196	-.297
		(.194)	(.183)	(.110)	(.122)
Italy		.536	.176	.036	-.221
		(.157)	(.131)	(.110)	(.284)
Germany		-.236	-.293	-.260	-.185
		(.157)	(.191)	(.107)	(.147)
U.K.		.071	-.298	-.369	-.154
		(.144)	(.182)	(.159)	(.112)

Notes:

The following equations are estimated:

$$Y_j = \gamma_{wj}^Y W_t + \gamma_{nj}^Y N_{jt} + \eta_{jt}^Y \qquad W_t = \rho_w W_{t-1} + \epsilon_{wt}$$
$$C_j = \gamma_{wj}^C W_t + \gamma_{nj}^C N_{jt} + \eta_{jt}^C \qquad N_{jt} = \rho_{nj} N_{jt-1} + \epsilon_{njt}$$
$$I_j = \gamma_{wj}^I W_t + \gamma_{nj}^I N_{jt} + \eta_{jt}^I \qquad \eta_{jt}^k = \rho_j^k \eta_{jt-1}^k + \epsilon_{jt}^k$$

The seven countries are indexed by $j = 1, ..., 7$. Aggregate output, consumption, and investment are indexed by $k = Y, C, I$, respectively. Standard errors are in parentheses.

* The series-specific component on Japanese output was omitted; see text.

Table 6: Shares of Variance Accounted for by Common Factors

Log First-Differences

	World Factor			Country Factors		
	$\hat{R}_w^Y$	$\hat{R}_w^C$	$\hat{R}_w^I$	$\hat{R}_n^Y$	$\hat{R}_n^C$	$\hat{R}_n^I$
Canada	.136	.023	.111	.666	.419	.359
	(.093)	(.038)	(.087)	(.123)	(.098)	(.107)
U.S.	.219	.162	.238	.477	.374	.575
	(.105)	(.093)	(.122)	(.098)	(.091)	(.124)
Japan	.242	.106	.252	.758	.283	.343
	(.090)	(.067)	(.101)	(.090)	(.081)	(.081)
France	.511	.095	.362	.224	.141	.153
	(.117)	(.079)	(.111)	(.107)	(.068)	(.079)
Italy	.403	.254	.245	.227	.041	.482
	(.126)	(.105)	(.118)	(.114)	(.047)	(.170)
Germany	.300	.037	.111	.414	.152	.427
	(.095)	(.036)	(.064)	(.163)	(.088)	(.167)
U.K.	.128	.062	.123	.548	.541	.216
	(.075)	(.125)	(.072)	(.128)	(.125)	(.085)

Notes:

R_f^k measures the share of the variance in aggregate k (where $k = Y, C, I$) accounted for by variation in the common factor f. Here $f = w, n$ denote the world and country factors respectively. R_f^k is defined as the ratio of the variance of the common factor weighted by the appropriate impact coefficient, to the sum of the variances of the common and country factors (weighted by their impact coefficients) and the variance of the series-specific component η_j^k, where $j = 1, \ldots, 7$. That is,

$$\hat{R}_f^k = \frac{\frac{\hat{\gamma}_f^{k2}}{1-\hat{\rho}_f^{k2}}}{\frac{\hat{\gamma}_w^{k2}}{1-\hat{\rho}_w^2} + \frac{\hat{\gamma}_n^{k2}}{1-\hat{\rho}_n^2} + \frac{\hat{\sigma}_{\epsilon^k}^2}{1-\hat{\rho}^{k2}}}, \qquad k = Y, C, I \qquad f = w, n$$

where $\hat{\sigma}_{\epsilon^k}^2$ is the estimated variance of the innovation to the series specific component in aggregate k. Standard errors, computed by the Delta method, are in parentheses.

Table 7: Cross-Country Correlations and First Order Autocorrelations

Log First-Differences

	World	*Canada*	*U.S.*	*Japan*	*France*	*Italy*	*Germany*	*U.K.*
$\rho(Y_i, Y_j)$								
Canada	.412							
U.S.	.444	.530						
Japan	.506	.140	.145					
France	.779	.206	.248	.240				
Italy	.715	.168	.199	.179	.534			
Germany	.590	.080	.192	.320	.420	.380		
U.K.	.345	.230	.276	.104	.241	.096	.305	
$\rho(Y_{jt}, Y_{j,t-1})$								
	.710	.349	.279	.070	.287	.507	.006	.072

Notes:

The contemporaneous correlation of the first-difference of the logarithm of output in country i with the first-difference of the log of output in country j is denoted $\rho(Y_i, Y_j)$. When "country j" refers to "*World*" the correlation is that of output in country i with the world common factor. The first-order autocorrelation of output in country j is denoted $\rho(Y_{jt}, Y_{j,t-1})$. Again, the first column contains the analogous statistic for the world common factor.

Table 8: Correlations of World and Country-Specific Common Factors

Log First Differences \ Deviations from Hodrick-Prescott Trend

	World	*Canada*	*U.S.*	*Japan*	*France*	*Italy*	*Germany*	*U.K.*
World	—	.278	.292	.199	.044	.150	-.056	.329
Canada	.102	—	.648	-.219	-.273	.069	-.204	.258
U.S.	.048	.457	—	-.089	-.494	-.432	.055	.356
Japan	.070	-.044	-.167	—	.021	-.077	.262	-.096
France	.176	-.141	-.219	-.250	—	.091	-.241	.008
Italy	.170	-.089	-.231	-.169	-.200	—	-.328	-.250
Germany	.125	-.266	-.054	.013	-.129	-.160	—	-.499
U.K.	.053	.129	.172	-.018	-.042	-.353	.078	—

Notes:

The table contains contemporaneous correlations of the estimated world and country-specific common factors. Numbers above the diagonal pertain to the common factors in Hodrick-Prescott filtered data, those below the diagonal refer to common factors in log first-differences.

Table 9: Estimated Dynamic Factor Model, Solow Residuals

	World	Canada	U.S.	Japan	France	Italy	Germany	U.K.
Deviations from Hodrick-Prescott Trend								
$\hat{\gamma}_w$		.324	.406	.355	.334	.305	.365	.222
		(.075)	(.068)	(.073)	(.085)	(.076)	(.064)	(.073)
$\hat{\rho}_w$	.838							
	(.065)							
$\hat{\rho}^{SR}$		.455	.367	.353	.639	.735	-.041	.222
		(.105)	(.126)	(.120)	(.097)	(.082)	(.132)	(.112)
$\hat{R}_w^{SR}$		.190	.605	.446	.443	.406	.284	.176
		(.081)	(.122)	(.135)	(.165)	(.167)	(.115)	(.106)
Log First-Differences								
$\hat{\gamma}_w$		.253	.312	.309	.522	.571	.318	.204
		(.115)	(.104)	(.102)	(.118)	(.125)	(.088)	(.095)
$\hat{\rho}_w$	.410							
	(.162)							
$\hat{\rho}^{SR}$		-.107	-.263	-.314	-.129	.008	-.502	-.358
		(.115)	(.112)	(.113)	(.139)	(.140)	(.104)	(.106)
$\hat{R}_w^{SR}$		.078	.118	.116	.335	.397	.123	.051
		(.067)	(.074)	(.070)	(.121)	(.133)	(.063)	(.046)

Notes to Table 9:

Solow residuals were computed as both in log first differences:

$$\Delta \ln SR = \Delta \ln Y - \alpha \Delta \ln H,$$

and as deviations from Hodrick-Prescott trend:

$$\ln SR = \ln Y - \alpha \ln H.$$

Here Y and H refer to aggregate output and total employment respectively. For the deviations from Hodrick-Prescott trend, we compute $\ln SR$ as above and then apply the HP filter with smoothing parameter 1600. All data were taken from the *International Financial Statistics* of the IMF, and all are quarterly, covering the period 1970.1-1989.4.

Measures of the share of labor compensation in aggregate output, α, are taken from Stockman and Tesar (1991). These measures are:

CAN	GER	JPN	UK	FRA	ITA	USA
.650	.593	.530	.645	.570	.500	.631

The following equations are estimated:

$$SR_{jt} = \gamma_{wj} W_t + \eta_{jt}^{SR} \qquad \begin{aligned} W_t &= \rho_w W_{t-1} + \epsilon_{wt} \\ \eta_{jt}^{SR} &= \rho_j^{SR} \eta_{jt-1}^{SR} + \epsilon_{jt}^{SR} \end{aligned}$$

The seven countries are indexed by $j = 1, ..., 7$. Standard errors are in parentheses.

Figure 1: U.S. HP-Output
World and Country-Specific Components

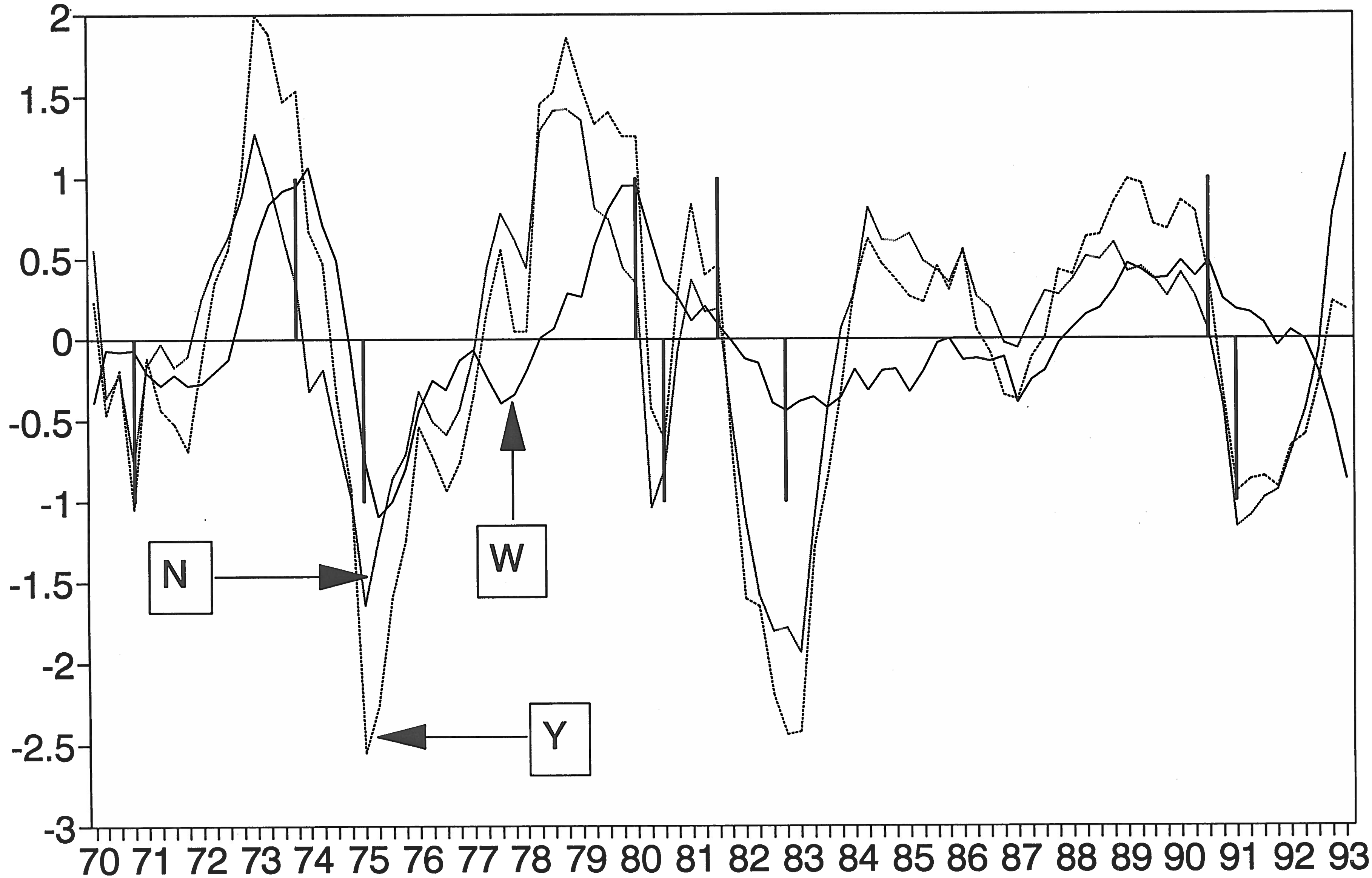

Figure 2: France: HP Output
World and Country-Specific Components

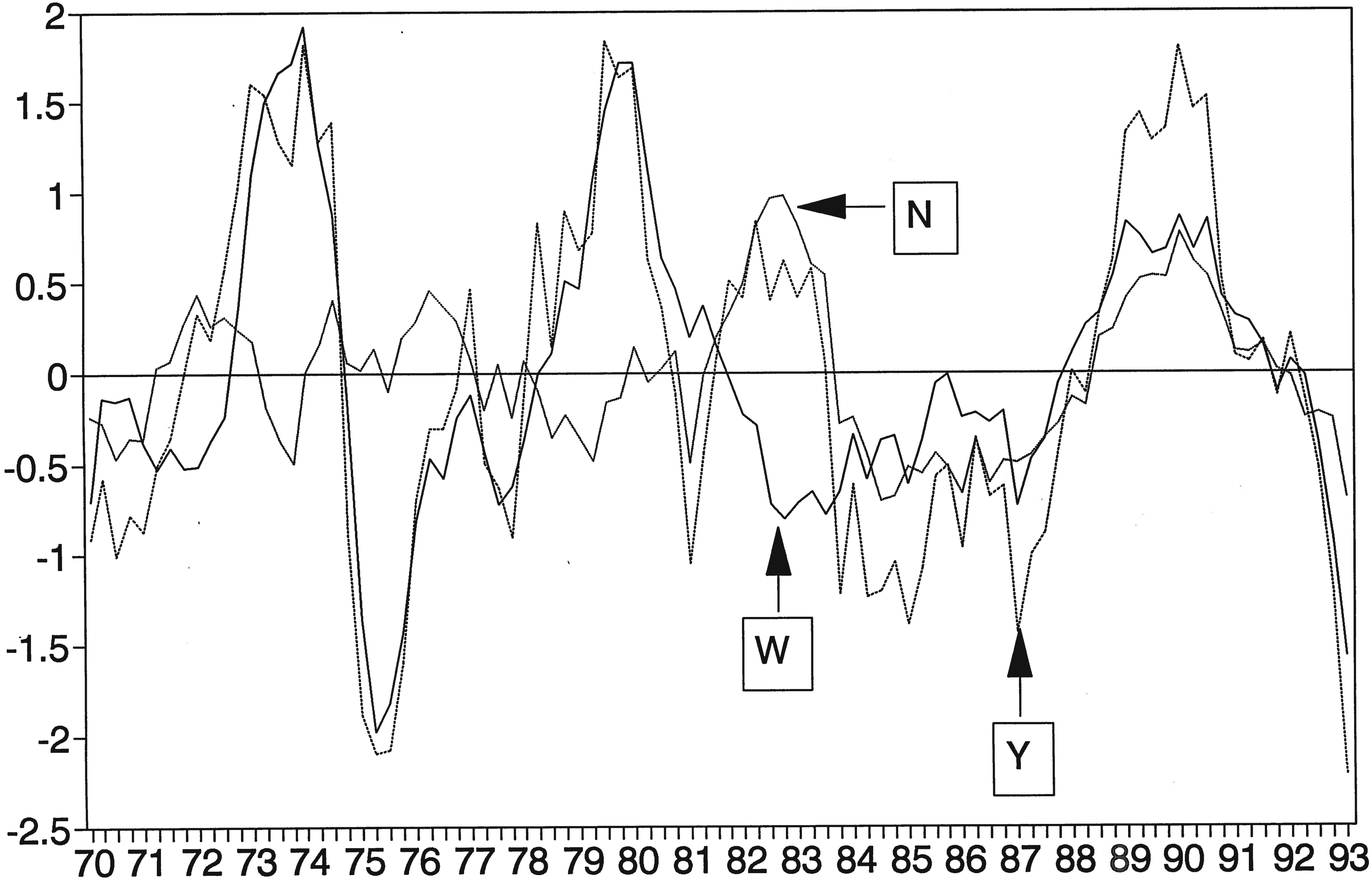

Figure 3: US Log Difference Output

World and Country-Specific Components

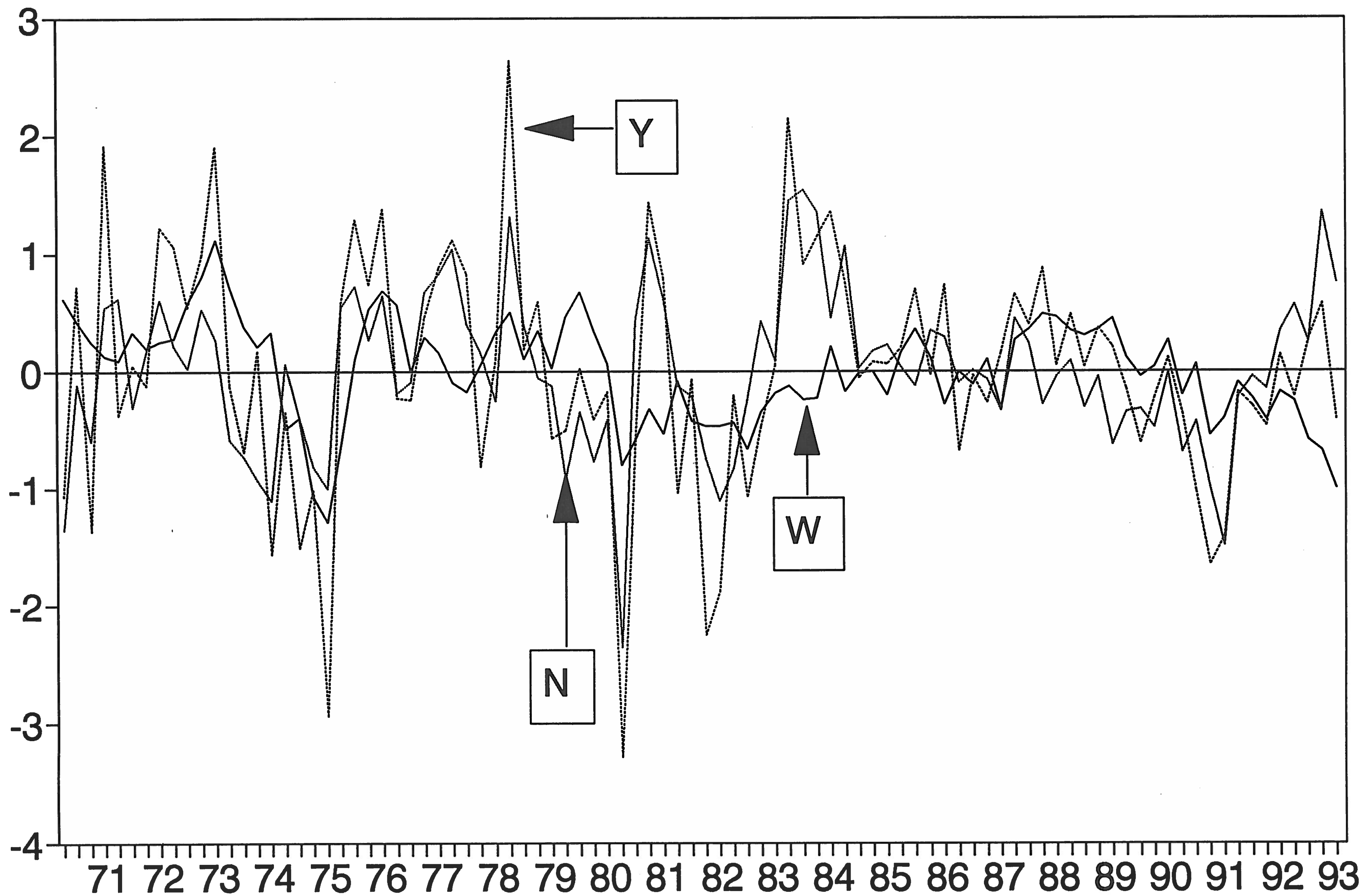

Figure 4: US and Canada-HP

Comparing Country-Specific Components