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Queen's Economics Department Working Paper No. 860

# Investment in Education and the Time Inconsistency of Redistributive Tax Policy

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10-1992

Discussion Paper #860

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\* We would like to thank Sam Bucovetsky and Benoît Delage for helpful comments on an earlier draft. Financial support of the Social Sciences and Humanities Council of Canada is gratefully acknowledged.

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**ABSTRACT**

Time inconsistency of tax policy is shown to arise in a setting in which households differ in their ability to accumulate wealth and the government has redistributive objectives. The government can levy non-distorting taxes but is precluded from redistributing optimally by a self-selection constraint. The analysis is done for the case in which all wealth is human capital, and education is a private good. An argument can be made for public intervention in the provision of education.

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## 1. Introduction

The purpose of this paper is to investigate the problem of redistributive tax policy in a setting where individual incomes are partly determined by education, and where governments cannot commit to a future tax policy before the education decision is taken. If the government could precommit to a structure of redistribution in the future, it would take into consideration the effect that redistribution would have on the decision of households to invest in human capital. However, such a policy will be time-inconsistent if the government is able to change its tax policy after household education decisions have been taken. This is because the human capital accumulated will then effectively be fixed in size. We might expect that in a time-consistent equilibrium where households rationally anticipate future government policies, non-optimal levels of education will be chosen, and the level of welfare attained will be non-optimal. A case can then be made for considering other policies which either correct for the implied distortion of the education decision or precommit the government to some policies in advance. Examples include provisions for mandatory education, public education or the subsidization of education, all of which are observed in practice.

Since the application of the notion of time inconsistency to redistributive tax policy is somewhat different than in the instances of time inconsistency previously analyzed, it is worth indicating at the outset how our problem compares with other cases found in the literature. The problem of the time inconsistency of economic policy is, of course, a well-known one. It can occur when agents must take decisions whose outcomes depend on policies that can be chosen after the agents have acted. Strotz (1958) showed how time inconsistency in dynamic problems can arise as a result of preferences changing over time. In a decentralized economy, it can also occur when the planner uses as indicators of individual welfare in the society's objective function something other than the individuals' own utility functions. (See Fischer (1980) and Chari, Kehoe and Prescott (1989)). We are, however, more interested in cases in which the planner's objective function fully respects individual preferences. In this context, as argued by Hillier and Malcomson (1984) and by Calvo and Obstfeld (1988), a requirement for time inconsistency of dynamic policy is that the planner does not have enough instruments at his disposal to implement the first-best optimum.

There are two general sorts of reasons why first-best optimal policies are not attainable. One is that the planner is restricted to using distortionary policy instruments, such as taxes. In a dynamic setting, second-best tax policies will generally be time-inconsistent so that the second-best allocation cannot, in fact, be implemented. Much of the analysis of dynamic inconsistency in public finance has been in this context. The seminal paper is that of Fischer (1980) who showed how time inconsistency can arise in an economy consisting of a representative individual with a

two-period life cycle. The individual takes a saving-consumption decision in the first period, and finances second period consumption from capital income and variable labour earnings. The government can tax labour and capital incomes in the second period to finance variable public expenditures. From an *ex ante* point of view, both capital and labour income taxes are distortionary, and the second-best optimal tax mix will generally include a mix of both types. However, once the household has taken its saving decision, capital income becomes fixed, and the government entering the second period will have an *ex post* incentive to generate as much of its tax revenue as possible from a capital income tax. In the time-consistent equilibrium, the capital income tax will be too high and the capital stock too low with the result that utility is lower than in the second-best optimal tax equilibrium. This analysis has been extended to slightly more general settings and tax instruments by Rogers (1987), Bruce (1990), and Persson and Tabellini (1990).

The Fischer example involves time-inconsistency resulting from the use of distortionary taxes. As he argues, if lump-sum taxes were available in these settings so that the first-best dynamic path could be achieved, the dynamic policy would be fully time-consistent. These applications typically involve issues of efficiency related to taxation. The case we consider in this paper involves time inconsistency arising from a second source of deviation from first-best allocations. The first-best social welfare-optimizing allocation may not be available because of the existence of imperfect information between the planner and the households. The planner cannot implement the optimal first-best redistributive policy because this requires information on household abilities that the planner does not have. The first-best lump-sum tax optimum would require households to reveal information about themselves that it is not in their interest to reveal. The planner is thus faced with a self-selection constraint which prevents the first-best from being reached even given non-distorting taxes. As we show, this can result in time inconsistency of redistributive tax policy. The purpose of this paper is to analyze the consequences of this in a particular setting.<sup>1</sup>

For illustrative purposes, we consider a very simple stylized case intended to

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<sup>1</sup> The issue of time inconsistent taxes and redistribution has been raised by Rogers (1986) and Persson and Tabellini (1990). In these analyses, Fischer (1980)'s model is enriched by differentiating workers from capital owners. They conclude that redistributive goals reduce the incentives of the planner to renege on its announced policies because fully redistributing income from capital owners would have adverse redistributive consequences. In both contributions, the source of the inconsistency is the absence of non-distortionary taxes in the first period, and the planner's instruments are not explicitly redistributive. This is very different from the current analysis where the planner's redistributive goals themselves are the source of the time inconsistency. To emphasise this point, we retain the assumption of lump-sum redistributive taxes throughout.

capture the bare essentials of the argument. There are two ability types of agents whose only decision is the amount of education to undertake. The cost of education is forgone current earnings. The high-ability type is more productive at converting education into future wages. The government cannot observe ability types or amounts of education, but can observe the consequences of the education (i.e., earnings) next period. The government is interested in redistributing in the second period, and uses lump-sum taxes for the purpose. In the optimum with commitment, the government would announce a redistributive tax policy in the first period to be implemented in the second, and the individuals would take their education decisions accordingly. However, such a policy would not be time-consistent.

Our analysis proceeds by first analyzing the second-best (full-commitment, open-loop) policy and showing that it is, in fact, time-inconsistent. Next the time-consistent (no-commitment) equilibrium is analyzed and compared with the previous one. Finally, policy instruments for partially offsetting the adverse effects of the time-consistent policy are considered. The restriction of our analysis to lump-sum taxes is for simplicity of analysis. In the time-inconsistent case where a self-selection constraint applies, the planner can do better by using non-linear redistributive taxes (see Nichols and Zeckhauser (1982)). However, our qualitative results can be illustrated much more easily by using lump-sum taxes, so we restrict the planner to them for pedagogical purposes. Those results will continue to apply in the non-linear tax case.

## 2. The Model

The model we use, designed primarily with simplicity of analysis in mind, abstracts from a number of features of the real world. Unlike the case of Fischer (1980), whose focus is purely on the efficient raising of tax revenues in an intertemporal setting, our analysis is based on a redistributive objective. Thus, we consider a two-person economy with some features borrowed from the two-person optimal non-linear income tax model of Stiglitz (1982). However, we simplify the model to avoid some of the complications that arise in the latter by restricting both the sorts of decisions made by households and the instruments available to the planner. At the same time, the model is made somewhat more complicated by introducing an element of dynamics into it.

There are two types of households in the economy — type 1 (low ability) and type 2 (high ability). There are  $N_i$  identical households of type  $i$ , and they differ from the other type only in ability. Each household lives for two periods and obtains utility solely from consumption in each period. A household's utility function can be written  $u(c_i) + v(d_i)$ , where  $c_i$  and  $d_i$  are first and second period consumption levels of a household of type  $i$ , and per period utility functions are strictly concave. Each

household is endowed with an amount of time  $k$  in the first period which can be divided between working at a wage rate  $\bar{w}$  and investing in education. The amount of time devoted to education is  $s_i$ , and the forgone earnings are the sole cost of education. Thus,  $c_i = \bar{w} \cdot (k - s_i)$ . In the second period, the household works a fixed amount of hours and earns labour income given by  $w_i(s_i)$  where  $w'_i > 0$ ,  $w''_i < 0$ ,  $w_1(0) = w_2(0) = \bar{w}$ , and  $w_2(s) \geq w_1(s) \forall s$ . Therefore, the only thing which distinguishes the two types is their second period wage function, i.e., their return to education. Household 2 is the higher ability person in the sense that he can convert a given amount of time into a higher future wage than person 1 can. Household  $i$  also pays a lump-sum tax in the second period of  $T_i$  so that  $d_i = w_i(s_i) - T_i$ .

It is assumed that all household saving takes the form of accumulation of human capital; that is, there is no borrowing or lending on capital markets. Although this is assumed mainly for simplicity, it also has a reasonable justification in our context. To the extent that education is productive, it generates income in the future. The household's stream of consumption is financed solely by labour income in the two periods. Under reasonable assumptions about education productivity, the household would prefer to be a net borrower in the first period. We adopt the common assumption that the household cannot borrow solely against future earnings.<sup>2</sup> A type- $i$  household solves the following problem:

$$(1) \quad \max_{\{s_i\}} u[\bar{w} \cdot (k - s_i)] + v[w_i(s_i) - T_i].$$

The first-order condition is:

$$(2) \quad -u'(c_i)\bar{w} + v'(d_i)w'_i = 0.$$

Equation (2) defines a demand-for-education function  $s_i = s_i(T_i)$  which can be shown to satisfy  $s'_i > 0$ . Substituting this function back into the maximand in (1) gives an indirect utility function  $V_i(T_i)$ . Then, from the envelope theorem:

$$(3) \quad \frac{\partial V_i}{\partial T_i} = -v'(d_i) < 0.$$

Two useful benchmark allocations are the *laissez faire* equilibrium and the *first-best* optimum. In the former,  $T_i = 0$ . In this case, clearly utility is higher for the

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<sup>2</sup> Liquidity constraints of this sort are common in the literature. See, for example, Becker and Tomes (1976).



high-ability person, or  $V_2^L > V_1^L$ , where the superscript  $L$  refers to the laissez-faire equilibrium. In this equilibrium, it is not possible to say whether  $s_2(0) > s_1(0)$ , or not. That is, given his higher education productivity, the high-ability person can obtain more consumption in the second period than the low-ability person while investing less in education. Whether he chooses to do so or not will depend jointly upon the elasticity of the wage function and the elasticity of the per period utility functions. In what follows, we shall assume for illustrative purposes that  $s_2(0) > s_1(0)$ , that is,  $s_2^L > s_1^L$ .

To obtain the planning optimum for this model, we assume that the planner uses a utilitarian social welfare function, though any quasi-concave social welfare function would do as well. Indeed, we could also have followed Stiglitz (1982) and set the problem up as a Pareto-optimizing one. The notion of first-best used here is somewhat restrictive in the sense that the planner is limited to using lump-sum taxes for redistributive purposes in the second period and is not able to overcome the capital market imperfection facing the consumer. Later on in the paper we consider widening the number of instruments available to the planner. The first-best optimum so defined is the solution to the following simple problem:

$$(4) \quad \max_{\{T_1, T_2\}} N_1 V(T_1) + N_2 V(T_2)$$

subject to

$$N_1 T_1 + N_2 T_2 = 0,$$

where the latter is the government budget constraint. The solution to this problem is simply to equate the marginal utilities of the two types of persons in the second period:  $v'(d_2) = v'(d_1)$ , that is, to set  $w_1 - T_1 = w_2 - T_2$ .

We will denote the solution to this problem by the superscript  $O$ . Clearly, it will be the case that redistribution will go from the high-ability to the low-ability person so that  $T_2^O > 0 > T_1^O$ . Given that  $s_i' > 0$  and recalling that  $s_2^L > s_1^L$ , it will also be the case that  $s_2^O > s_2^L > s_1^L > s_1^O$ . Furthermore, since consumption is the same in the second period for both, but the high-ability person takes more education, it will be the case that  $V_2^O < V_1^O$ . This is reminiscent of the Mirrlees (1974) result in the optimal income tax literature, as also reported in Stiglitz (1987), and analyzed in Dasgupta and Hammond (1980). The high-ability person is made worse off because, while his second-period consumption is the same as that of the low-ability person, he is required to invest more in education in the first period because of his higher ability to convert present time into future output. This implies that the first-best solution is not incentive-compatible. The high-ability person would not willingly reveal his type to the planner. We must, therefore, turn to the second-best problem of the planner.

### 3. The Second-Best Planning Problem

In the second-best setting, the policy options of the planner are limited by a self-selection constraint. As mentioned, we also assume that the planner uses only lump-sum taxes, that is, taxes with a zero marginal tax rate. This is done solely for simplicity since the use of non-linear taxes complicates the analysis enough to obscure the main message being delivered. The self-selection constraint rules out person 2 mimicking person 1 by their choice of first-period education.<sup>3</sup> The behaviour of the mimicker is straightforward. A mimicker of type 2 (denoted by  $\hat{2}$ ) will set  $\hat{s}_2$  such that  $w_2(\hat{s}_2) = w_1(s_1)$  so that second period income is the same as the low-ability person. Given that  $s_1$  is increasing in  $T_1$ , an education function is defined such that  $\hat{s}_2 = \hat{s}_2(T_1)$  with  $\hat{s}_2' > 0$ . The indirect utility function of a mimicker is given by:

$$(5) \quad \hat{V}_2(T_1) = u[w \cdot (k - \hat{s}_2(T_1))] + v[w_2(\hat{s}_2(T_1)) - T_1].$$

The effect of an increase in  $T_1$  on the utility of a type  $\hat{2}$  person is given by:

$$(6) \quad \frac{\partial \hat{V}_2}{\partial T_1} = [-u'(\hat{c}_2)w + v'(\hat{d}_2)\hat{w}_2']\hat{s}_2' - v'(\hat{d}_2).$$

The second-best planning problem is the same as the first-best one given by (4) with the addition of a self-selection constraint. This extra constraint precludes the high-ability person from being better off by mimicking the low-ability person, that is,  $\hat{V}_2(T_1) \leq V_2(T_2)$ . In our context, the self-selection constraint will be binding. Therefore, since there are two constraints and two policy variables, the solution can be inferred directly from solving the constraints. It will be denoted using  $S$  (for second best) as the superscript. In this equilibrium, utility in the second period is the same for the mimicker as for person 1, and this entails  $\hat{s}_2^S < s_1^S$ . Furthermore, the mimicker takes less education than the low-ability person in the first period since their second-period incomes are the same. This implies that:

$$(7) \quad V_2^S = \hat{V}_2^S > V_1^S.$$

This in turn implies  $T_2^S > 0 > T_1^S$ .

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<sup>3</sup> In the Pareto-optimizing framework of Stiglitz (1982), the possibility that the self-selection constraint applies to the low-ability person cannot be ruled out. Given the social welfare function that we are adopting, this possibility will not arise.

To summarize, the relationship between the first-best and the second-best optima is as follows:

$$T_2^O > T_2^S > 0 > T_1^S > T_1^O$$

and, assuming that  $s_2^L > s_1^L$  in the *laissez-faire* equilibrium,

$$s_2^O > s_2^S > s_2^L > s_1^L > s_1^S > s_1^O.$$

Furthermore,

$$V_2^L > V_1^L,$$

$$V_2^S = \hat{V}_2^S > V_1^S,$$

and

$$V_2^O < V_1^O.$$

Some redistribution is carried out in the second-best equilibrium, but not as much as in the first-best because of the self-selection constraint. This, in turn, implies that the high-ability persons obtain less education in the second-best policy optimum than in the first-best (and even less than in the *laissez faire*), while the low-ability persons obtain more.

It is clear that the second-best optimum will not be time-consistent, however. To see this, notice that in the second-best equilibrium, second-period consumption levels will differ between the two types of persons. In particular, since utility is lower in the first period for person 2, while lifetime utility is higher, second-period consumption must be higher. Thus, if the planner has an opportunity to remaximize, he will redistribute further away from the high-ability person, after the latter has already committed to an educational decision. The outcome will be an equilibrium in which persons are fooled, and as a result social welfare is actually higher than in the second-best equilibrium.<sup>4</sup> However, if persons know that the government is able to remaximize, they will anticipate it and change their behaviour accordingly. The result is the no-commitment equilibrium, to which we now turn.

#### 4. The No-Commitment Equilibrium

The characterizing feature of the no-commitment equilibrium, and the one which differentiates it from the second-best case considered above, concerns the timing of decisions by the various agents in the economy. In the second-best, or open-loop, case, the planner can be thought of as the first mover. He commits to a policy

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<sup>4</sup> This was pointed out by Fischer (1980).

before the households take their only decision, which is to invest in education. Of course, the planner anticipates the behaviour of the households when deciding on a policy. In the no-commitment case, the order is reversed. Households must take their education decisions before the planner chooses a redistributive policy. In this case, it is the households who must anticipate the behaviour of the planner. We assume that they do so correctly, that is, with rational expectations.

Given this change in the order of decision-making, we adopt the usual procedure of solving the agents' problems in reverse order. We begin with the planner's problem in period 2, given the behaviour of households in the previous period. Then we look at household behaviour in the first period, anticipating the planner's behaviour. Since there are many households of the two types all taking decisions simultaneously, we must specify an equilibrium concept to be used in analyzing the first period. We assume a Nash equilibrium in period 1.

The basic model we use is the same as that used in the previous sections. The planner is solely involved with redistributing income in the second period, which is when information concerning incomes becomes available. We begin by characterizing the no-commitment equilibrium for that case, and compare it with the second-best one discussed above. We then look at alternative policy instruments for use in the first period, again given the absence of information to the planner. We show how social welfare can be improved in the no-commitment case by various measures which impinge on the education decision in the first period.

### *Period 2: The Planner's Problem*

At time 2, the households have already taken the only decision they have to take, which is the amount of investment in education. Thus, their income levels are determined, and observable to the planner. Given these income levels, the planner selects a redistributive tax policy to maximize second-period aggregate utility. Since the planner can observe all relevant information, there is no self-selection constraint to worry about. The Lagrangian expression for the planner's second-period problem may be written as follows:

$$(8) \quad \max_{\{T_1, T_2\}} N_1 v(w_1 - T_1) + N_2 v(w_2 - T_2) + \mu(N_1 T_1 + N_2 T_2)$$

where pre-tax incomes  $w_1$  and  $w_2$  are given.

The straightforward solution to this problem is to equate the marginal utilities (and thus, given identical utility functions, the utilities):

$$(9) \quad v'(d_1) = v'(d_2),$$

where  $d_1 = w_1 - T_1 = w_2 - T_2 = d_2$ . The solution to this problem gives the tax rates  $T_i$  as a function of the incomes  $w_i$ . Since incomes depend upon education taken in the first period, we can equivalently say that the second-stage problem determines tax rates  $T_1$  and  $T_2$  as a function of education levels  $s_1$  and  $s_2$ .

All individuals know that the planner is going to behave in this way and take their actions accordingly. It is useful to rewrite the tax functions of the planner from the perspective of each individual household's decisions. Since we are invoking the Nash assumption about individual behaviour, each individual makes educational choice given the behaviour of the others. We can therefore write the tax function facing an individual as the planner's tax function to the above problem, given the behaviour of the other households. Let us denote this tax function for household  $i$  as  $T_i(s_i, \tilde{s}_i, \tilde{s}_j)$ , where the "tilde" denotes all other households. Thus, for a household of type  $i$ ,  $\tilde{s}_i$  denotes the vector of education levels undertaken by the other  $N_i - 1$  individuals of his type, while  $\tilde{s}_j$  refers to the education levels of the  $N_j$  individuals of the other type. In the equilibrium, the levels chosen by all persons of a given type are the same. However, in describing individual behaviour, that cannot be presumed. Given that the solution to the planner's second-period problem is to equalize after-tax incomes, an individual's tax will be the difference between his pre-tax income and the average income for the economy as a whole. Thus, the tax function facing individual  $i$  is:

$$(10) \quad T_i(s_i, \tilde{s}_i, \tilde{s}_j) = w_i(s_i) - \frac{w_i(s_i) + \sum_{\tilde{N}_i} w_i(\tilde{s}_i) + \sum_{\tilde{N}_j} w_j(\tilde{s}_j)}{N},$$

where  $N = N_1 + N_2$ . This expression will be used in solving the problem of households in the first period. It is also useful for future reference to define  $R$  as the average income in the economy, or, equivalently, the after-tax income of *all* individuals whatever their type. Thus, we can write the tax on household  $i$  as

$$(10'). \quad T_i = w_i - R$$

### *Period 1: Household Decisions*

As mentioned, each of the  $N$  households is assumed to behave with rational expectations regarding the government's second-period behaviour, and with Nash beliefs about the behaviour of the other  $N - 1$  households.<sup>5</sup> These two features are captured in the tax rule (10). Consider an individual of type  $i$ . He will solve:

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<sup>5</sup> Fischer (1980) considers some of the consequences for time inconsistency of allowing house-

$$(11) \quad \max_{\{s_i\}} u[\bar{w}(k - s_i)] + v[w_i(s_i) - T_i(s_i, \tilde{s}_i, \tilde{s}_j)],$$

where  $T_i(\cdot)$  is the function given by (10) and is correctly anticipated by the individuals. Substituting for the tax function from equation (10), the first-order condition for this problem may be written:

$$(12) \quad -\bar{w}u'(c_i) + \frac{w'_i v'(d_i)}{N} \leq 0; \quad s_i \left( -\bar{w}u'(c_i) + \frac{w'_i v'(d_i)}{N} \right) = 0,$$

where, as before,  $c_i = \bar{w}(k - s_i)$ , and  $d_i = w_i(s_i) - T_i$ . Comparing (12) with (2) we can see that the household decision rule for investment in education is quite different in the no-commitment case than in the second-best (full-commitment) case. In the latter, the household chooses the amount of education such that, at the margin, the utility value of the additional earnings generated in the second period just equals the utility value of the forgone earnings in the first period. That is, the household captures the entire marginal return on investment in education. In the no-commitment case, the household only obtains a fraction  $1/N$  of the additional earnings generated in the second period. The remainder is spread among all other households in the economy.  $s_i$  is declining in  $N$ ; if  $N$  is very large, there will be very little incentive to invest in education. If  $w'_i(0) < \infty$ ,  $s_i$  will be zero for a finite size of  $N$ . Unless otherwise stated, in what follows we assume (12) is satisfied with equality.

The solution to (12) yields the demand for education  $s_i$  by the household as a function of the exogenous variables facing the household, which, given our assumptions, are the education levels of the other  $N - 1$  households. We can write this solution as  $s_i = s_i(\tilde{s}_i, \tilde{s}_j)$  and interpret it as a reaction function for household  $i$ . The no-commitment equilibrium will then just be the Nash Equilibrium in which all household demands are consistent. Since all households of a given type are identical, we concentrate on an equilibrium which is symmetric, that is, in which all households of a given type have the same value of  $s_i$ . A no-commitment equilibrium is then a vector  $[s_1^* \cdots s_1^*; s_2^* \cdots s_2^*]$  such that  $s_i^* = s_i(s_i^*, s_j^*)$ ,  $\forall i, j$ .

Since, in the symmetric equilibrium, all households of a given type demand the same level of education, we can recast the equilibrium solution in terms of the

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holds to behave cooperatively. In his context, the time-inconsistency problem can be circumvented completely with fully cooperative behaviour among the identical households in his economy. In our two-class economy, we would have to specify a bargaining equilibrium between the two household types to analyze the outcome in which households of each type cooperate.

demand for education by the representative household of each type. In a symmetric equilibrium, (10) and (12) will both be satisfied for the common values of  $s_1^*$  and  $s_2^*$  for the two types of households. Explicitly, a symmetric equilibrium will be the solution of the following two equations for  $s_1^*$  and  $s_2^*$ :

$$(12') \quad -\bar{w}u'(\bar{w}(k - s_i^*)) + \frac{w'_i(s_i^*)v'(R(s_1^*, s_2^*))}{N} = 0, \quad i = 1, 2$$

where  $R$  is average income, which is received by all persons in period 2:

$$R(s_1^*, s_2^*) = \frac{N_1 w_1(s_1^*) + N_2 w_2(s_2^*)}{N}$$

It is helpful to characterize the solution to the above equations in terms of reaction functions of the representative household of each type, taking into account the identical behaviour within each group. Denote the reaction function for the representative household of type 1 as  $s_1(s_2)$ . It is the solution to (12') for individual 1 using the above definition of  $R(s_1^*, s_2^*)$ . The slope of the reaction function is obtained by differentiating (12') to yield:

$$(13) \quad \begin{aligned} \frac{ds_1}{ds_2} &= - \left[ \frac{N_2 w'_1 w'_2 v''(d_1)}{N^2} \right] \left[ \bar{w}^2 u''(c_1) + \frac{w'_1 v'(d_1)}{N} + \frac{N_1 w_1'^2 v''(d_1)}{N^2} \right]^{-1} \\ &= - \left[ \frac{N_2 w'_1 w'_2 v''(d_1)}{N^2} \right] D^{-1} < 0. \end{aligned}$$

A similar reaction function applies symmetrically for household 2. These reaction functions will be used in the analysis in the following section in which we introduce other government policies. The no-commitment equilibrium will be the simultaneous solution of the two reaction functions  $s_1(s_2)$  and  $s_2(s_1)$ . This is shown in Figure 1. It immediately follows from (13) and the fact that  $v''(d_1) = v''(d_2)$  that

$$\left| \frac{\partial s_1}{\partial s_2} \right| < \left| \frac{\partial s_2}{\partial s_1} \right|^{-1}.$$

Therefore, the equilibrium, assuming it exists, will be unique and stable as depicted in the diagram.

— FIGURE 1 HERE —

### *Properties of the No-Commitment Equilibrium*

It is now useful to characterize some relevant properties of the no-commitment equilibrium. The following properties can be seen to hold in the no-commitment equilibrium where a superscript  $N$  is used to denote equilibrium variables.

First, assuming that a positive amount of education is demanded, high-ability persons will invest more in education than low-ability persons ( $s_2^N > s_1^N$ ). To see this, note that equations (9) and (12) imply that

$$(14) \quad \frac{u'(c_1^N)}{w_1(s_1^N)} = \frac{u'(c_2^N)}{w_2(s_2^N)}.$$

We can show by contradiction that  $s_2^N \leq s_1^N$  cannot apply. Suppose that it did. Then, since  $\bar{w}(k - s_1^N) \leq \bar{w}(k - s_2^N)$ , we have that  $u'(c_1^N) \geq u'(c_2^N)$  from  $u'' < 0$ . Moreover,  $s_1^N \geq s_2^N$  implies that  $w_1'(s_1^N) < w_2'(s_2^N)$ . Therefore,  $s_1^N \geq s_2^N$  contradicts equation (14). Therefore,  $s_2^N > s_1^N$ . This result occurs simply because the rate of return on education is higher for the high-ability than for the low-ability households.

The fact that high-ability households spend more time in education implies that they work less, so they consume less in the first period ( $c_1^N > c_2^N$ ). Since we know from (9) that both types consume the same amounts in the second period ( $d_1^N = d_2^N$ ), this implies that lifetime utility must be lower for the high-ability types ( $V_2^N < V_1^N$ ). This might be contrasted with the second-best optimum above in which the high-ability persons are better off than the low-ability persons.

Since high-ability persons obtain more education than the low-ability persons, they also earn more income before tax ( $w_2(s_2^N) > w_1(s_1^N)$ ). Since  $w_2(s_2^N) - T_2^N = d_2^N = d_1^N = w_1(s_1^N) - T_1^N$ , it must be the case that  $T_2^N > 0 > T_1^N$ . That is, redistribution goes from the high-ability persons to the low-ability persons, as expected.

We can summarize these various results for the no-commitment equilibrium with positive education as follows:

$$\begin{aligned} s_2^N &> s_1^N \\ T_2^N &> 0 > T_1^N \\ V_2^N &< V_1^N. \end{aligned}$$

Of course, if (12) holds with inequality for both persons so no education is taken, income for both is identical in both periods, there are no taxes, and utilities are equal. This no-education no-commitment equilibrium will serve as a useful benchmark in the next section.



It would be nice to analyze whether social welfare is higher or lower in the no-commitment equilibrium compared with the second-best one, and also whether the two types of persons are individually better off or worse off. Unfortunately, the comparison is not easy to make. Furthermore, it is not clear that the result would be unambiguous. In the Fischer (1980) case with only one household type, the no-commitment equilibrium is unambiguously worse than the full-commitment one. The reason is simply that, in the full-commitment case, the planner could have committed to the no-commitment policy for the second period at the outset, but chose not to. In our case, the matter is not quite so simple. It is true that in the no-commitment case the incentive to invest in education is dulled considerably. At the same time, the no-commitment equilibrium involves a policy which would violate the self-selection constraint. It is therefore not a feasible policy for the full-commitment equilibrium. Thus, the no-commitment equilibrium has an advantage in this regard. However, if there are a large number of persons, the incentive to invest in education in the no-commitment equilibrium will be so small that, presumably, social welfare should be lower as well, assuming that such investment is at all productive.

The feature of the no-commitment equilibrium that is most striking is the reduction in the incentive to invest in education which results from the government not being able to commit to a future redistribution policy. This is clearly seen by comparing the household's optimality condition for determining  $s_i$  in the no-commitment case given by (12) with that for the full-commitment case given by (2). This suggests that if we could find policies which either provide a direct incentive to households to invest in education or allow the planner to take actions in the first period which have implications for the second-period outcome, social welfare might improve. In the following section we analyze an example of such a policy.

## 5. Education Policy in the No-Commitment Equilibrium

In this section, we analyze one sort of policy which can be used to counteract the significant disincentive to investment in education that results from the inability of government to commit to future redistributive policies. The example we use is that of imposing mandatory education requirements on all households. In our setting, this involves imposing a minimum amount of time that must be devoted to education by all persons. This is only one of many alternative policies that could have been used, and is intended only to be illustrative. It is, however, a policy which has its counterpart in the real world. We also consider supplementing mandatory education by a uniform lump-sum subsidy in the first period financed by second-period tax revenues. It must be uniform because we assume that persons are identical from the perspective of the planner in the first period. Although a first-period subsidy may have no direct value as a redistributive device, it can be a useful policy instrument in our model where households face a liquidity constraint which prevents them

from borrowing against future earnings. One might well consider other reasonable policy instruments such as public education or a subsidy on educational investment. However, the ones we have chosen illustrate the principles involved.

Mandatory education involves specifying a minimum amount of education that must be taken by all households. Obviously, the planner must be able to monitor this amount of education, and it could be thought of as taking place in public institutions. Time spent at education over and above that cannot be monitored in the first period; only its rewards in the second period can. Denote by  $\bar{s}$  the level of mandatory education imposed by the government.<sup>6</sup>

We will consider the imposition of mandatory education starting from the no-commitment equilibrium described above. Obviously, if  $\bar{s} \leq s_1^N < s_2^N$ , mandatory education has no impact whatsoever. As  $\bar{s}$  is increased above  $s_1^N$ , type 1's become constrained. Then, as  $\bar{s}$  is further increased, a point will come where the type 2's will also become constrained. For convenience, denote by  $\bar{s}_1$  and  $\bar{s}_2$  the levels of education at which individuals of type 1 and 2 become constrained. Obviously,  $\bar{s}_1 = s_1^N$ . However, the education level chosen by individuals of type 2 will be affected by the level of education of individuals of type 1 through the reaction function  $s_2(s_1)$ . Therefore, as  $\bar{s}$  increases and the latter are forced to increase their education levels beyond  $s_1^N$ , the level chosen by the high-ability persons will change. This implies that it will not be the case that  $\bar{s}_2 = s_2^N$ . In fact, as we have seen in (13),  $s_2$  will fall as  $s_1$  rises, so that  $\bar{s}_2 < s_2^N$ .

Our analysis will involve evaluating the social welfare effects of small changes in the level of mandatory education taken at different points. It will always be assumed that the no-commitment equilibrium is in effect. To simplify notation, we will drop the superscript  $N$  in what follows except where superscripts are necessary for clarification. We begin by deriving a general expression for the change in social welfare from a change in  $\bar{s}$  for  $\bar{s}_1 \leq \bar{s} \leq \bar{s}_2$ . In this range,  $s_1 = \bar{s}$ . However, high income persons will not be constrained as long as  $\bar{s} < \bar{s}_2$ . They can choose their level of education freely, but, given the Nash assumption, the level they choose will be affected by that imposed on persons of type 1. Using the slope of the reaction function for persons of type 2 given by (13), the response of persons of type 2 to changes in the level of education imposed on persons of type 1 will be given by:

$$(13') \quad \frac{ds_2}{d\bar{s}} = - \left[ \frac{N_1 w'_1 w'_2 v''(d_2)}{N^2} \right] D^{-1} < 0.$$

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<sup>6</sup> This specification of mandatory education could also be interpreted as the provision of uniform public education to all persons financed by an equal per capita tax. Persons are allowed to supplement mandatory or public education with additional investments.

That is, persons of type 2 unambiguously decrease their education level as the mandatory level imposed on persons of type 1 is increased, assuming the latter to be binding.

Next, consider the effect on social welfare of changes in  $\bar{s}$ . We denote social welfare by  $\Omega(\bar{s})$ . It will be simply the sum of utilities:

$$(15) \quad \Omega(\bar{s}) = N_1 [u(\bar{w}(k - \bar{s})) + v(R(\bar{s}))] + N_2 [u(\bar{w}(k - s_2(\bar{s}))) + v(R(\bar{s}))].$$

where  $s_2(\bar{s})$  is the reaction function for the representative person of type 2 whose first derivative is given by (13'), and  $R(\bar{s})$  is the average income for the economy as a whole:

$$(16) \quad R(\bar{s}) = \frac{N_1 w_1(\bar{s}) + N_2 w_2(s_2(\bar{s}))}{N}.$$

Differentiating (16) with respect to  $\bar{s}$  yields:

$$(17) \quad \frac{dR}{d\bar{s}} = \frac{N_1 w'_1}{N} + \frac{N_2 w'_2}{N} \frac{ds_2}{d\bar{s}}.$$

Using (13') it can be readily shown that  $dR/d\bar{s} > 0$  despite the fact that  $ds_2/d\bar{s} < 0$ :

$$\frac{dR}{d\bar{s}} = \frac{N_1 w'_1}{N} \left[ \bar{w}^2 u''(c_2) + \frac{w''_2 v'(d_2)}{N} \right] D^{-1} > 0.$$

The change in social welfare  $\Omega(\bar{s})$  from a change in  $\bar{s}$  is obtained by differentiating (15) with respect to  $\bar{s}$  and using (17) to give:

$$(18) \quad \frac{d\Omega}{d\bar{s}} = N_1 [-u'(c_1)\bar{w} + v'w'_1] + N_2 [-u'(c_2)\bar{w} + v'w'_2] \frac{ds_2}{d\bar{s}}.$$

This expression has an intuitive interpretation. The terms in square brackets are the *social* values of an additional unit of education to the two types of persons. For type-2 persons, for whom (12) holds, this is unambiguously positive because of the policy-induced disincentive to invest in education. For type-1 persons, it may be negative if  $\bar{s}$  exceeds  $\bar{s}_1$  by enough since, in that case, these persons are forced to consume more than they otherwise would. If  $N$  is large, we might expect this term to be positive. Let us consider the values of (18) for various values of  $\bar{s}$ .

*Case 1:  $d\bar{s}$  evaluated at  $\bar{s} = \bar{s}_1$*

In this case, neither type-1 nor type-2 persons are constrained, so (12') applies to each. Substituting (12') into (18) and using (13') for  $ds_2/d\bar{s}$  yield:

$$\left. \frac{d\Omega}{d\bar{s}} \right|_{\bar{s}=\bar{s}_1} = \frac{N-1}{N} v' D^{-1} \left[ N_1 w'_1 (\bar{w}^2 u''(c_2) + \frac{w''_2 v'}{N}) \right] > 0.$$

This is summarized in the following proposition:

**Proposition 1:** Imposing mandatory education at a level marginally above the one where the low-ability individuals become constrained is welfare-improving.

*Case 2:  $d\bar{s}$  evaluated at  $\bar{s} = \bar{s}_2$*

Next we consider an increase in mandatory education at the point where the individuals of type 2 become constrained, i.e.,  $\bar{s}_2$ . In this case, both individuals are constrained to take the same amount of education, so  $u'(c_1) = u'(c_2)$ . Furthermore, type-2's can no longer choose  $s_2$  freely so  $ds_2/d\bar{s} = 1$ . Type-1 persons are "out of equilibrium" in the sense that they are forced to take more education than they would prefer. However, at the point  $\bar{s} = \bar{s}_2$ , conditions (12') still applies for the type-2's. Substituting these conditions into (18) yields:

$$\left. \frac{d\Omega}{d\bar{s}} \right|_{\bar{s}=\bar{s}_2} = N_1 w'_1 v' + (N_2 - 1) w'_2 v' > 0.$$

Thus, we have:

**Proposition 2:** Increasing mandatory education marginally from the level where the high-ability individuals become constrained is welfare-improving.

*Case 3:  $d\bar{s}$  evaluated at  $\bar{s} > \bar{s}_2$*

Beyond  $\bar{s}_2$ , both types of persons are constrained to invest in more education than they would choose so (12') applies with equality to neither. Given that  $ds_2/d\bar{s} = 1$ , the expression for welfare change (18) becomes:

$$\left. \frac{d\Omega}{d\bar{s}} \right|_{\bar{s} > \bar{s}_2} = N_1 [-u'(c_1) \bar{w} + v' w'_1] + N_2 [-u'(c_2) \bar{w} + v' w'_2].$$

As  $\bar{s}$  increases above  $\bar{s}_2$ , this expression will be initially positive, but its value will gradually fall as  $\bar{s}$  rises and eventually will become negative. Thus, there will be a local optimum at some  $\bar{s} > \bar{s}_2$ .

*Case 4:  $d\bar{s}$  evaluated at  $\bar{s}_1 < \bar{s} < \bar{s}_2$*

In this case, equation (18) applies with  $ds_2/d\bar{s}$  given by (13'). In general, its sign will be ambiguous, given that the first term can be positive or negative, the term in the second square brackets is positive by (12') and  $ds_2/d\bar{s} < 0$ . However, there are two sorts of circumstances in which increases in  $\bar{s}$  will yield continuous welfare gains up to  $\bar{s}_2$ .

The first case occurs when  $N_1$ ,  $N_2$  and  $N$  are "large". By substituting (13') into (18) and rearranging, we can obtain:

$$\begin{aligned} \left. \frac{d\Omega}{d\bar{s}} \right|_{\bar{s}_1 < \bar{s} < \bar{s}_2} &= -D^{-1}N_1\bar{w} \left[ \left( u''(c_2)\bar{w}^2 + \frac{v'w_2''}{N} \right) \left( u'(c_1) - N \frac{w_1' u'(c_2)}{w_2'} \right) \right. \\ &\quad \left. + \frac{w_2' N_2 v''}{N^2} (-u'(c_1)w_2' - u'(c_2)w_1') \right]. \end{aligned}$$

As  $N_1$ ,  $N_2$  and  $N$  rise, the second term in the square bracket vanishes while the first term is dominated by the  $N$  in the second braces. Thus, for large populations, social welfare will increase monotonically with  $\bar{s}$  between  $\bar{s}_1$  and  $\bar{s}_2$ .

The second case is that in which the no-commitment equilibrium involves no education. This will be the case if  $N$  is large enough and  $w_i'(0)$  is finite. In this case,  $\bar{s}_1 = \bar{s}_2 = 0$  and  $ds_2/d\bar{s} = 1$ . The change in welfare is given by:

$$\left. \frac{d\Omega}{d\bar{s}} \right|_{\bar{s}_1 < \bar{s} < \bar{s}_2} = N_1[-u'(c_1)\bar{w} + v'w_1'] + N_2[-u'(c_2)\bar{w} + v'w_2'].$$

In this instance, starting at  $s_1 = s_2 = 0$ , we might expect that an increment of investment in education would be socially profitable. (Otherwise, the study of education would be uninteresting.)

Also note that when the number of individuals is large enough so that  $s_1 = s_2 = 0$ , it is possible to compare the no-commitment equilibrium with the full-commitment second-best equilibrium. This is so because this no-commitment equilibrium has both types of individuals achieving the same welfare level. The self-selection constraint is therefore not binding as is usually the case in the no-commitment equilibrium. Since this no-commitment equilibrium could have been achieved with full commitment when appropriate tax rates are chosen, and since the second-best does not entail those tax rates, it has to be the case that the second-best equilibrium dominates the no-commitment equilibrium.<sup>7</sup>

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<sup>7</sup> Any comparison of equilibria using this type of argument requires that the self-selection constraint is not binding. This is precisely the reason why a comparison of the no-commitment and the second-best equilibria is possible in this case.

## 6. Supplementing Education with Debt Policy

In our model, non-optimality arises for two sorts of reasons. The first is that, because of the inability of the government to commit to a future redistribution policy, the households only capture  $1/N$  of the social returns on their investment in human capital. The other is that households are constrained from borrowing against future earnings. In this section, we address mainly the second issue by allowing the government to pay households a lump-sum transfer in the first period financed by debt.<sup>8</sup> This is effectively a way for the government to borrow on behalf of the households.

Our analysis involves considering an incremental first-period subsidy in addition to mandatory education.<sup>9</sup> The timing of decision-making in the no-commitment equilibrium is as follows. In the first period, knowing the behaviour of the households and given  $\bar{s}$ , the planner provides a lump-sum transfer of  $\sigma$ . Since the planner cannot distinguish one type of person from the other, the same subsidy is provided to all persons. Households take  $\sigma$  (and  $\bar{s}$ ) as given and select their education levels. In the second period, given the behaviour of households in the first period, the planner sets tax rates  $T_1$  and  $T_2$  both for redistributive purposes and to repay its first-period debt. Our procedure is to assume that taxes in the second period will be set optimally and investigate changes in the first period tax (subsidy) rate. We need first to specify the second-period policy outcome since it will be slightly different here than before.

At time 2, incomes  $w_i(s_i)$  and the subsidy  $\sigma$  will already have been determined. The planner's choice of second period taxes will need to satisfy the following budget constraint:

$$(19) \quad N_1 T_1 + N_2 T_2 = (N_1 + N_2)(1 + r)\sigma.$$

where  $r$  is the market interest rate, assumed to be exogenous to the economy. As in (8) above, the planner chooses second-period taxes to maximize the sum of utilities subject to its budget constraint. The Lagrangian expression is:

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<sup>8</sup> The results of last section are not sensitive to the assumption that households are liquidity constrained. If they were not constrained, they would still under-invest in education in the no-commitment equilibrium as they would still be deprived of most of the return on their investment. Things are clearly worse for the households if, as in last section, they can not borrow. This is why we introduce a debt policy. But although this debt policy helps, the commitment problem remains.

<sup>9</sup> If we follow the interpretation suggested earlier of treating  $\bar{s}$  as public education, this approach would be analogous to financing part of the public education by borrowing.

$$(20) \max_{\{T_1, T_2\}} N_1 v(w_1 - T_1) + N_2 v(w_2 - T_2) + \mu [N_1 T_1 + N_2 T_2 - (N_1 + N_2)(1 + r)\sigma].$$

The first-order condition for this problem is, as before, given by equation (9).

At time 1, households rationally anticipate the government's second-period policy choice. Given the government budget constraint (19), household  $i$  expects a tax liability given by  $T_i = w_i - R + (1 + r)\sigma$ , where  $R$  is average income as above. The household's problem then is

$$(21) \max_{\{s_i\}} u[\bar{w}(k - s_i) + \sigma] + v[R - (1 + r)\sigma]$$

where the values for  $s_i$  of the other households as well as  $\sigma$  are taken as given. The first-order condition is the same as (12) above.

In the no-commitment Nash equilibrium, these first-order conditions will be solved simultaneously for the education levels of all households. As before, in the symmetric equilibrium, the no-commitment equilibrium will be the simultaneous solution to the following two equations:

$$(12'') \quad \bar{w}u'[\bar{w}(k - s_i^*) + \sigma] + \frac{w'_i(s_i^*)v'[R(s_1^*, s_2^*) + (1 + r)\sigma]}{N} = 0, \quad i = 1, 2.$$

From the equation for, say, type 1, we can obtain the reaction function  $s_1(s_2, \sigma)$ . It can easily be shown as above that (13) holds, so  $ds_1/ds_2 < 0$ , and the reaction curve slopes downward. Also, differentiating (12'') with respect to  $s_1$  and  $\sigma$  yields:

$$(22) \quad \frac{\partial s_1}{\partial \sigma} = D^{-1} \left( \bar{w}u''(c_1) + \frac{w'_1}{N}v''(1 + r) \right) > 0$$

where  $D$  is defined as earlier in (13). A similar result holds for person 2. Therefore, increasing the subsidy in period 1 to each person will cause both reaction functions to shift outward. This is intuitive since the subsidy indirectly works to loosen the liquidity constraint facing the households. We cannot be sure that the consequence of both reaction curves shifting outward will be to cause the equilibrium levels of both  $s_1$  and  $s_2$  to rise. Of course, if either household is constrained by mandatory education, their level of education  $\bar{s}$  will be unaffected by  $\sigma$ .

The general expression for social welfare given mandatory education plus the subsidy is:

$$\begin{aligned}\Omega(\bar{s}, \sigma) = & N_1 [u(\bar{w}(k - \bar{s}) + \sigma) + v(R - (1 + r)\sigma)] \\ & + N_2 [u(\bar{w}(k - s_2(\bar{s})) + \sigma) + v(R - (1 + r)\sigma)]\end{aligned}$$

where  $s_1$  and  $s_2$  would equal  $\bar{s}$  if the latter were binding. The change in social welfare from a change in  $\sigma$  is given by differentiating this expression:

$$(23) \quad \begin{aligned}\frac{d\Omega(\bar{s}, \sigma)}{d\sigma} = & N_1 \left[ u'(c_1) \left( -\bar{w} \frac{ds_1}{d\sigma} + 1 \right) - v' \left( (1 + r) - \frac{N_1}{N} w'_1 \frac{ds_1}{d\sigma} - \frac{N_2}{N} w'_2 \frac{ds_2}{d\sigma} \right) \right] \\ & + N_2 \left[ u'(c_2) \left( -\bar{w} \frac{ds_2}{d\sigma} + 1 \right) - v' \left( (1 + r) - \frac{N_1}{N} w'_1 \frac{ds_1}{d\sigma} - \frac{N_2}{N} w'_2 \frac{ds_2}{d\sigma} \right) \right]\end{aligned}$$

where  $ds_i/d\sigma$  is the change in the Nash equilibrium value of  $s_i$ . This welfare change is to be evaluated at alternative levels of  $\bar{s}$ . Only two cases need to be considered.

*Case 1:  $d\sigma$  evaluated at  $\bar{s} \geq \bar{s}_1$*

In this case,  $s_1 = \bar{s}$  so  $ds_1/d\sigma = 0$ . As well, if the high ability person can choose  $s_2$  freely, first-order condition (12') applies with equality; if not,  $ds_2/d\sigma = 0$ . The combination of these implies that  $(-u'(c_2)\bar{w} + \frac{v'w'_2}{N}) ds_2/d\sigma = 0$ . Furthermore, if type-2 is unconstrained,  $ds_2/d\sigma > 0$  since his reaction curve shifts outward and  $s_1$  is fixed at  $\bar{s}$ . Equation (23) can be rewritten as:

$$(24) \quad \begin{aligned}\left. \frac{d\Omega(\bar{s}, \sigma)}{d\sigma} \right|_{\bar{s} \geq \bar{s}_1} = & N_1 (u'(c_1) - v'(1 + r)) + N_2 (u'(c_2) - v'(1 + r)) \\ & + \frac{N_1(N - 1)}{N} v' w'_2 \frac{ds_2}{d\sigma}.\end{aligned}$$

The third term will be positive for  $\bar{s} < \bar{s}_2$  and zero for values above that. The first two terms represent the net benefits to the households of the government borrowing on behalf of them. If household  $i$  is liquidity-constrained,  $u'(c_i) - v'(1 + r) > 0$ . Thus, we have the following result:

**Proposition 3.** For  $\bar{s} \geq \bar{s}_1$ , increasing the lump-sum subsidy will be welfare-improving if both types of individuals are liquidity-constrained.

As  $\sigma$  is increased, eventually one and then both households become unconstrained. Social welfare will eventually begin to fall.



*Case 2:  $d\sigma$  evaluated at  $\bar{s} < \bar{s}_1$*

Here the mandatory education is not effective so both type-1 and type-2 persons are unconstrained in their investment in education. Using the first-order conditions (12''), equation (23) can be written:

$$(25) \quad \left. \frac{d\Omega(\bar{s}, \sigma)}{d\sigma} \right|_{\bar{s} < \bar{s}_1} = N_1(u'(c_1) - v'(1+r)) + N_2(u'(c_2) - v'(1+r)) \\ + \frac{N_1(N-1)}{N} v' w'_1 \frac{ds_1}{d\sigma} + \frac{N_2(N-1)}{N} v' w'_2 \frac{ds_2}{d\sigma}.$$

From (25) the following proposition is immediately apparent:

**Proposition 4.** For  $\bar{s} < \bar{s}_1$ , an increase in  $\sigma$  will be welfare-improving if both types of household are liquidity-constrained and if the levels of education of both types rise with  $\sigma$  ( $ds_i/d\sigma > 0$  for  $i = 1, 2$ ).

We would expect this to be satisfied at  $\sigma = 0$ . The reaction curves for both  $s_1$  and  $s_2$  shift outward and we would expect the equilibrium values of both  $s_1$  and  $s_2$  to rise. However, we cannot be certain of it since one curve could shift out much further than the other causing one of the  $s_i$ 's to fall.

## 7. Concluding Remarks

In this paper, we have constructed a very simple and abstract model to illustrate an important issue. It is that when governments have as an objective to redistribute from the well-off to the less well-off, and when a person's utility depends in part on the amount of education they undertake, governments which are unable to commit to redistributive policies until after persons have taken their human capital investment decisions will treat the human capital accumulated as a quasi-fixed factor. This implies that they will redistribute more *ex post* than they would like to commit to *ex ante*. The resultant no-commitment equilibrium will imply that the incentive to invest in human capital has been considerably blunted by the expectation by households of future redistributive taxes.

This inability of governments to precommit to a future redistributive policy is a different form of time-inconsistency problem than one usually finds in the literature. Existing analyses in the taxation literature have stressed time inconsistency arising from inefficiencies of the tax system. In our setting, it arises from the fact that a self-selection constraint precludes the government from pursuing full redistribution. To emphasize this, and to simplify the analysis, we have restricted our policy instruments to lump-sum taxes, but restricted by asymmetry of information. It would

have been possible to allow the government to use non-linear income taxes. This would have complicated the analysis considerably without changing the qualitative nature of the results. Given the inability to commit to a future redistributive policy, we then investigated as an example an instrument that could be used to counteract the problem, given the limited information then available to the government. We showed that mandatory education policies, which are commonly used in the real world, would be unambiguously welfare-improving, especially if supplemented with government borrowing.

The model we have used is a very simple one. There are various ways in which it could be extended to make it more realistic. For example, other capital assets could be added besides human capital. Also the model could be made explicitly into an overlapping generations one. These would, however, add considerable complexity.

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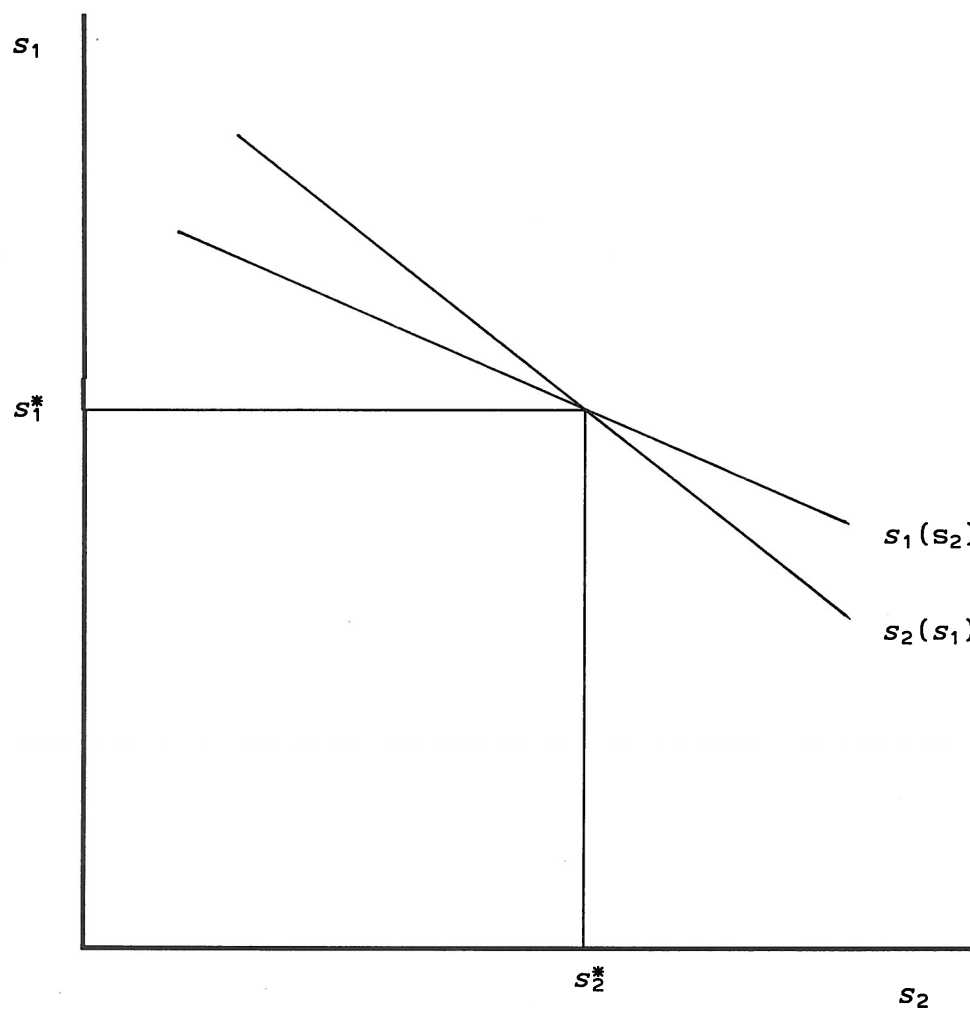


Figure 1