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Intraday Trade in Dealership Markets

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Abstract

We develop and test a structural asymmetric information transaction model to characterize the price impact of information on the NYSE. Unlike previous literature, we allow for mixed entry strategies on the part of informed traders and obtain an equilibrium where trades are temporally separated. In addition, when it is costly to transact, informed agents will not trade small quantities. Estimation of the structural parameters is performed using a maximum likelihood procedure. The price impact of information and the average informational innovation are found to be positive and significant. However, when the overidentifying restrictions are tested, the model is rejected with probability one.

Introduction

This paper develops and tests an asymmetric information market transaction model to explain the price and volume moments that characterize intraday transaction-by-transaction trade on the NYSE. All trades are completed individually.[1] We allow for the possibility of mixed entry strategies by the potentially informed speculators and solve for the equilibrium. The resulting pricing function for intraday trades is the same as that obtained when a single (competitive) specialist pools simultaneous trades (if they exist) and meets the net order flow. Since markets are thin in the intraday period, the (adverse selection component of the) pricing function is *invariant* to fluctuations in arrival rates of traders (the depth of the market), in contrast to previous literature (Admati and Pfleiderer, etc.). Consequently, the theory is consistent with the U-shaped volume pattern found in previous empirical research (e.g. Wood, McInish and Ord [1985]). Because spreads can also arise due to transaction costs, we consider the possibility that the specialist or dealer incurs both fixed and variable costs when completing transactions.

In contrast to Kyle [1985], Admati and Pfleiderer [1988] or Foster and Viswanathan [1988a], characterization of the equilibrium yields explicit implications for *transaction-by-transaction* behavior as a function of the model's structural parameters during the trading day. We then use the Fitch database which contains transaction-by-transaction NYSE price and volume data to test the model. In contrast to the previous literature, rather than concetrate on the reduced form, we estimate the structural parameters of the model. We exploit the nonlinear restrictions provided by the model to estimate not only the probability that a particular trade is consummated at the bid or ask,[2] but also the probability that the specialist faces an informed trader, the amount and quality of inside information, and the cost of information acquisition.

Mixed entry strategies on the part of the potentially informed imply that price changes and traded quantities are conditionally but not unconditionally normally distributed. Thus, unlike Foster and Viswanathan [1989] or Hasbrouk [1989], neither ordinary nor generalized least squares are efficient estimation procedures *even when bid-ask indicators are observable.* The mixed unconditional distribution for volume helps identify the probability a particular trader is informed. That is, informed orders may be larger or smaller than uninformed orders. We therefore maximize the joint likelihood function for price and volume.

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[1]We allow for the possibility that simultaneous trades are completed *independently* by different dealers.

[2]As in Glosten and Harris [1989].

The empirical results indicate that the adverse selection component to the bid-ask spread is generally positive and significant. While this adverse selection component is economically unimportant for small orders, for a 10,000 share order the tariff can exceed $2000. The adverse selection component varies throughout the day, and it does not necessarily follow a U-shaped curve. Variations appear to be caused not so much by intradaily fluctuations in the amount of inside information, as by fluctuations in the quality of information available to potential insiders. The noise in the signal available to insiders is much larger in the middle of the day than it is near either open or close. This may explain why the cost of information is lower in the middle of the day. In addition, insiders are found to trade far greater quantities than uninformed traders, up to 20 times larger. In light of this, it is perhaps surprising that we fail to reject the linear pricing rule against a quadratic alternative.

When transaction costs are integrated into the model, identification of some structural parameters becomes infeasible. However, both a per share transaction cost and a quantity cutoff below which adverse selection will not occur can be estimated. Unlike the linear pricing rule estimated by Glosten and Harris [1989], the resulting equilibrium pricing function is kinked. For our sample, the magnitude of estimated cutoff cannot be explained merely by a per-share transaction cost. We strongly reject the linear pricing rule in favor of the kinked rule.

In an important recent paper, Admati and Pfleiderer [1988] investigate some of the ramifications of informational asymmetries in a dealership market. Their fundamental contributions are to endogenize both information acquisition by the potentially informed and strategic behavior by discretionary liquidity traders who can choose when to time their trades. Their principal finding is that when their equilibrium exists, in order to minimize the cost to transacting, all discretionary liquidity traders concentrate their trades in the same period which, in turn, leads many of the informed to follow suit.

Unfortunately, in models such as those of Admati and Pfleiderer, Kyle [1985], or Foster and Viswanathan [1988a], which restrict potentially informed agents to pure strategies, an equilibrium with insider trading only exists when informed traders transact each period. Consequently, the market must *always* be sufficiently thick that each period the (necessarily) multiple transactions of informed and uninformed traders can be pooled together to obtain one net transaction price. Terry

[1986], shows that during the day there are often long intervals when no transaction occurs[3], and that consecutive trades are generally sufficiently separated temporally. Bronfman [1990] notes that trades are accepted independently, even when volume is very heavy - e.g. 5000 trades per day:[4] intraday transactions are almost never pooled on the NYSE.

Bernhardt and Hughson [1990] extend Admati and Pfleiderer [1988] by demonstrating the existence of mixed strategy equilibria in an environment where multiple orders in a single period are pooled (if they exist). Equilibrium exists even when the market is too thin to support pure entry strategy equilibria.[5]

On the NYSE the specialist holds a formal auction in which bids are crossed and the specialist meets the net transaction only at the open, and perhaps at close[6]. Indeed, Stoll and Whaley [1990] assert that due to the different trading process, the specialist may have more monopoly power, particularly at the open. The paper presented here consequently examines *only* intraday trading - trade between open and close[7]. We assume that the specialist's price schedule is updated after each trade. Equivalently, one can assume that market orders within a period are processed independently by the specialist and various brokers who use the same price schedule, one which is set and honored throughout the period.[8]

During each intraday trading period, in equilibrium, potentially informed traders randomize their decisions to acquire information and trade. The specialist and brokers then incorporate the new

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[3]He finds that the average time between trades for stocks on the Dow 30 and for all NYSE stocks are 2.7 and 15.2 minutes respectively on December 31, 1989. Further, since these averages are (total trading minutes)/(total number of firms), the weight to the heavily traded stocks is proportionally greater. Terry also provides an equally weighted average for the Dow 30 - an average of 4.3 minutes between trades.

[4]USX volume on 23 October 1989.

[5]Whenever the market is sufficiently thick, discretionary liquidity traders concentrate their trade only at open and close. They do so because, as in Admati and Pfleiderer [1988], concentration lowers the degree of adverse selection in the specialist's pricing function, and hence the costs of trading. Unlike Admati and Pfleiderer [1988] the timing of the informed trades in pinned down by the arrival process of the potentially informed and the equilibrium refinement that no coalition of potentially informed can be made better off by switching their trading time (e.g. to noon). In equilibrium, the potentially informed, who learn of their need to trade after the open, simply trade at the close. Those who learn of their need to trade before the open choose between trading in the midst of the mass of pent-up overnight demand, (i.e. at the open) and trading at the close. Consequently, under weak regularity conditions, both trading volume and price variance are higher at open and close.

[6]Conversations with specialists suggest that they have an informal auction at close.

[7]This is precisely the period in which the equilibria described by Foster and Viswanathan [1988a] and Admati and Pfleiderer [1988] do not exist.

[8]This assumption is similar to that made in Admati and Pfleiderer [1989].

information and can choose to set new bid-ask spreads after each separated intraday transaction. Market makers expect to earn zero profits on a trade by trade basis. Unlike Kyle [1985], etc., where orders are pooled to determine the price, we view each trader as arriving at the trading floor, asking for price quotes, and *then* deciding whether to trade.

Intraday fluctuations in the amount and quality of information lead to corresponding fluctuations in the specialist's pricing function. However, the specialist's resulting linear pricing function is invariant to intraday variations in arrival rates; varying concentrations of liquidity traders do not affect the cost of trading. In equilibrium, a potentially informed agent who adopts a randomized strategy must be indifferent about whether or not to acquire information: he must expect zero profits net of information costs. To effect this outcome, the sampling probabilities of the informed must be perfectly correlated with the arrival rate of liquidity traders. In contrast, at open and close (as in Admati and Pfleiderer [1988]) the market is thick enough that potential insiders compete, and variations in liquidity trade directly affect the specialist's equilibrium pricing function.

Because each transaction is met individually, we can consider the possibility that the market makers incur both a fixed fee and a per-share cost to completing each individual transaction.[9] We distinguish this transaction cost component from the informational asymmetry components of the equilibrium bid-ask spread. The fixed transaction cost leads the adverse selection component to vanish for small trading quantities. A cutoff quantity such that no informed agents trade a smaller quantity is found by equating the informed agent's trading costs with his expected profits from trading the critical quantity. Insiders do not trade smaller quantities because the transaction costs exceed the expected profits derived from their information. For larger transactions, an adverse selection component to the specialist's pricing function exists, but is smaller than it would be in the absence of these costs. The greater the fraction of the expected trading costs (information acquisition and fixed transacting costs) incurred by the informed which are transaction costs, the flatter is the specialist's pricing function, the greater the transaction volume level below which informed agents do not transact, and the less likely an agent is to seek inside information. The transaction costs also induce negative serial correlation in the price series. The transaction cost rationale for the absence of an adverse selection component for small transactions differs from that of Easley and O'Hara [1987]

[9]If there are transaction costs, a pooling model is inappropriate because buyers and sellers may be charged different prices. In addition, the number of trades affects transaction costs. Two purchases of 100 shares is not equivalent to one purchase of 200 shares due to the additional fixed fee.

where an adverse selection component may not exist due to the interaction of a discrete stochastic process and the discrete quantities in which agents are constrained to trade.

In an extension of the model, we consider the case where innovations to the asset's value need not occur every period: Potential insiders may seek inside information but be unable to find it. The probability of an informational innovation in a period with trade is therefore greater. *Ceteris paribus*, the thicker the market, the greater the probability the specialist faces an informed trader, and the greater the price variance *per unit time*.

2. The Model.

Consider a market run by a risk neutral specialist and or floor traders. During a day of T periods the market is open from period T_0 until period T, $1 < T_0 < T$. A new day starts immediately after the previous day's close. We are interested in characterizing trade in the intraday trading periods T_0+1 to T-1: At open and close, different market clearing mechanisms which involve pooling of transactions operate, so that an assumption that simultaneous transactions are handled independently is inappropriate. Admati and Pfleiderer [1988] consider a model with discretionary uninformed liquidity traders who trade a fixed quantity but can select which period in which to trade. Under weak regularity conditions (see Bernhardt and Hughson [1990]), discretionary uninformed traders transact only at open and close. Since in this paper here, we are not concerned with trade at these dates, discretionary traders are excluded, and there are two types of risk neutral traders: uninformed liquidity traders who trade a fixed (stochastic) quantity in a fixed period, and potentially informed traders who can pay a cost to acquire inside information.

As in Admati and Pfleiderer [1988], agents trade claims to a risk free asset with single period gross return r and a single risky asset whose value in period τ is given by:

$$F_\tau = r^\tau F_0 + \sum_{t=1}^{\tau} r^{\tau-t}\delta_t H_t ,$$

where δ_t, the informational innovation is a normal independently distributed random variable with zero mean and variance v_t, and H_t is an indicator function which is equal to one if there is an innovation in period t and is equal to zero otherwise. While H_t is not used in the subsequent empirical work, it might prove important particularly when there are long periods when there is no informational event. In that scenario, trades would cluster around informational events, since the informed would only trade then.[10] The probability of an innovation in any given period is given by γ. F_0 is the liquidation value of the asset at the end of the previous trading day, and includes (discounted) any future deterministic component of the firm. F_T is the liquidation value after the current trading day's close. Both F_0 and F_T can be interpreted as common perceptions of the value of the risky asset at those moments. Since the impact of the risk free rate on the intraday evolution of price moments is shown to be small in Bernhardt and Hughson [1990],[11] we set r = 1 to reduce the notational burden.

On the NYSE, the actual receipt of securities is invariant to the intraday timing of trades. At each moment in a day agents trade claims to the risky asset whose value is given by its worth at close. The value in period τ of these claims is given by

$$V_\tau = E_\tau[V_T] = F_0 + \sum_{t=1}^{\tau} \delta_t H_t.$$

In period t, F_{t-1} is assumed to be common knowledge, so that uncertainty about the risky asset's time t value only concerns whether there was an innovation in period t, and if so, its value, δ_t. Thus, as in Admati and Pleiderer [1988], information is only good for one period.

Liquidity traders must trade as soon as they learn of their need. Each period there is a (very) large number of *potential* liquidity traders, M, each of whom may need to trade in period t with probability η_t/M. The parameter η_t is a measure of (perhaps time varying) market depth - when η_t is small, there is little background trading noise in which an informed agent can hide, so little trade of any kind occurs. Liquidity trader k's arrival is not observed by other agents, nor can he distinguish himself as an uninformed trader. He has inelastic demand, z_{kt}, which is the realization of an independently and identically distributed normal random variable with zero mean and variance σ.

Each period t there are N (very large) potentially informed traders. The potentially informed take the specialist's pricing function for that period as given and select a probability π_{it} of acquiring inside information. Potentially informed trader i can pay a cost c to attempt to find inside information about an innovation in t. If an innovation occurs in t, he receives a private signal about its value, $\delta_t + \varepsilon_{it}$,

[10] This might explain why the time between trades is not an important predictor of price variance in Glosten and Harris [1989].

[11] This follows because the entire risk-free return is earned overnight.

where ε_{it} is an independently and identically distributed normal random variable with zero mean and variance ϕ. Since, in period t+1, δ_t becomes common knowledge, this information is valuable to trader i only in period t. If there is no innovation, the investment is wasted - and the indicator function is $H_t = 0$.

Agent i selects a demand, $x^*_{it}(\delta_t + \varepsilon_{it})$, to maximize expected profits:

$$MAX_{x_{it}}\ E\,[x_{it}[F_t - p^*_t(x_{it})] \mid \delta_t + \varepsilon_{it}]$$

conditional on his signal $\delta_t + \varepsilon_{it}$ about the increment δ_t to the firm's worth. The probability a potentially informed agent chooses to become informed, π^*_{it}, then maximizes his expected profits within that period taking into account his subsequent profit-maximizing demand if informed:

$$MAX_{\pi_{it}}\ E\,[\ \pi_{it}[MAX_{x_{it}}[E[H_t x^*_{it}[F_t - p^*_t(x^*_{it})] \mid \delta_t{+}\varepsilon_{it}, H_t] - c_i]]].$$

The market is made by a specialist and many floor traders. In accord with practice on the NYSE, we assume that orders received by the market makers during the intraday trading period are handled independently:[12] during the trading day on the NYSE, the price schedule is updated after nearly every trade. This contrasts with the practice at open and close on the NYSE where larger pooling auctions are held (and contrasts with Admati and Pfleiderer [1988], Kyle [1985], or Foster and Viswanathan [1988]), where the specialist sets his price conditional on the net order flow. We consequently focus our attention on the intraday trading periods, T_0+1 to T-1.

In the intraday trading periods, market makers are essentially indistinguishable.[13] A market maker selects his pricing function taking into account the subsequent strategic response of the potentially informed traders. Each period, he first selects a linear pricing function which details for each possible transaction level, the price at which the market maker is willing to transact. Agents then look at the

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[12]We think of each period lasting only a couple of minutes, so that multiple orders in the same period are unlikely. In the unlikely event of multiple orders, the market makers do not condition their pricing schedule on the number of traders in the market.

[13]The specialist may have access to more information (his book of limit orders) than other floor brokers, but he only handles a small fraction of the trades. Since in this model, limit orders are necessarily placed by the uninformed, we will not distinguish among market makers.

price schedule and decide whether to become informed and/or trade. Risk neutrality[14] and competition together imply that the identity of the agent who takes the opposite side of each order is irrelevant.

Market makers are not privy to any private information about the risky asset. Further a market maker does not know the identity of the agent with whom he transacts (informed or uninformed). Therefore, the common price schedule offered by the dealers is such that they earn zero expected profits:

$$E_{H_t,\delta_t}\,[\omega_t^*[F_t - p_t^*(\omega_t^*)]\,] = 0 \;\text{ for all t,}$$

where ω_t^* is x_t^* if the trader is informed and z_t otherwise.

3. Equilibrium.

An equilibrium is a pricing function $p_t^*(\omega_t)$; a set of sampling probabilities $\{\pi_{it}^*\}$ $i = 1,\ldots, N_t$ $\pi_{it}^* \in [0,1]$; an associated set of demands for the risky asset by the potentially informed agents $\{x_{it}^*(\delta_t + \varepsilon_{it})\}$; for all t such that given $p_t^*(\omega_t)$,

1. If agent i is informed, he selects his demand, $x_{it}^*(\delta_t + \varepsilon_{it})$, to maximize expected profits:

$$MAX_{x_{it}}\, E\,[x_{it}[F_t - p_t^*(x_{it})] \mid \delta_t + \varepsilon_{it}].$$

2. The probability a potentially informed agent chooses to become informed, π_{it}^*, maximizes expected profits given his subsequent behavior if informed,

$$MAX_{\pi_{it}}\, E\,[\,\pi_{it}[MAX_{x_{it}}[E[H_t x_{it}^*[F_t - p_t^*(x_{it}^*)] \mid \delta_t{+}\varepsilon_{it}, H_t] - c_i]]].$$

3. The informed traders earn zero expected profits in equilibrium.

$$E\,[\,\pi_{it}[MAX_{x_{it}}[E[H_t x_{it}^*[F_t - p_t^*(x_{it}^*)] \mid \delta_t{+}\varepsilon_{it}, H_t] - c_i]]] \;=\; 0.$$

4. The market maker accepting the order earns zero expected profits

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[14]The risk neutrality assumption renders this model inappropriate for estimating an inventory cost component to the spread.

For securities where the specialist handles only a small fraction of trades in a stock, inventory cost models may be difficult to estimate. Since a market maker is relatively free to pick and choose which transactions to take, it is straightforward for him to keep a balanced portfolio. In equilibrium, the market maker who takes a transaction is likely to be the one who benefits most from the resulting rebalancing - and hence offers the most attractive price. One might even expect the inventory cost component to the bid-ask spread to be negative on occasion. For securities where the specialist completes most trades, an inventory cost model might be estimable, although our competitivity assumption would be violated.

$$E\,[\omega_t^*[F_t - p_t^*(\omega_t^*)]] = 0 \text{ for all t.}$$

Given each agent's time t information set and the optimal strategies of all other agents, each agent maximizes his expected profits, and the dealer earns zero expected profits each period.

3.1 The period problem.

The approach to solving the static one period problem is similar to that used in Kyle [1984], or Admati and Pfleiderer [1988]. For any given expected population of traders, we postulate a linear equilibrium pricing rule, $p_t = F_{t-1} + \lambda_t\omega_t$, for the risky asset which yields the broker zero expected profits conditional on the order he meets. Given these pricing rules, we then determine the equilibrium sampling and trading probabilities for the traders, and the equilibrium linear demands of the potentially informed, $x_{it} = \beta_{it}[\delta_t + \varepsilon_{it}]$ if they have inside information. We suppress the time subscripts because of the static nature of the single period problem.

An agent who has paid a cost c to become informed then solves:

$$MAX_{x_i}\, E[x_i[\delta - \lambda\omega]|\, \delta + \varepsilon_i]. \tag{1}$$

Since intraday transactions are accepted independently, the informed agent knows that volume, ω, consists only of his own demand, x_i, and substitutes for ω in (1). Differentiating with respect to x_i and applying the projection theorem[15] yields:

$$x_i = \frac{v(\delta + \varepsilon_i)}{2\lambda(v + \phi)} = \beta[\delta + \varepsilon_i]. \tag{2}$$

where

$$\beta = \frac{v}{2\lambda(v + \phi)}. \tag{3}$$

Since π is the probability a trader attempts to acquire information and γ is the probability of an informational event, let $\theta = \frac{\pi\gamma}{\eta + \pi\gamma}$ be the probability from the broker's perspective that a given trader

[15]The projection theorem can be used because δ and ε are normally distributed.

is informed.[16] The zero expected profit condition for the broker requires that the broker equate expected losses to the informed to the expected gains from trading with the uninformed:

$$\theta E\,[\,\beta[\delta + \varepsilon_i][\delta - \lambda\beta[\delta + \varepsilon_i]]\,] = [1 - \theta]E(z\lambda z). \tag{4}$$

Taking the unconditional expectation of both sides and solving for λ yields:

$$\lambda = \frac{\theta\beta\nu}{\theta\beta^2\,(\nu + \phi) + (1 - \theta)\sigma} \tag{5}$$

Substituting for β and solving for λ yields:

$$\lambda = \frac{\nu\sqrt{\theta}}{2\sqrt{(\nu + \phi)(1 - \theta)\sigma}}. \tag{6}$$

This implies that:

$$\beta = \frac{\sqrt{(1 - \theta)\sigma}}{\sqrt{\theta(\nu + \phi)}}. \tag{7}$$

The expected number of informed can be found by equating the expected profits of the informed with the acquisition cost, c. The market makers' zero profit condition implies that his expected losses to the informed, $\theta c/\gamma$, must equal his expected profits from the noise traders, $(1 - \theta)\lambda\sigma$:

$$(1 - \theta)\lambda\sigma = \frac{\theta c}{\gamma}, \qquad or \qquad \eta\lambda\sigma = \pi c,$$

when we substitute for π and η. Substituting for λ and solving yields:

Proposition 1: The probability the broker trades with an informed agent in period $t \in (T_0, T)$, given there is a transaction is given by:

$$\theta_t = \frac{\gamma^2\sigma\nu^2}{\gamma^2\sigma\nu^2 + 4c^2(\nu + \phi)}. \tag{8}$$

[16]A consequence of mixed strategies is that order size not a normal random variable. Hence expected price change given order size is not linear in the order size, and the market makers do not expect zero profits on a trade by trade basis. The specialist will make money on some trades, and lose on others. Large (small) trades subsidize small (large) ones whenever the variance of uninformed (informed) trade exceeds that of informed (uninformed trade).

The linear coefficients of the broker's pricing function and an informed agent's demand function are given by

$$\lambda = \frac{\gamma v^2}{4c(v+\phi)}; \quad \beta = \frac{2c}{v\gamma}. \ [] \tag{9}$$

Observe that when sampling is virtually costless, the conditional probability the broker faces an informed trader approaches one. As sampling costs become arbitrarily large, or the probability of an event becomes arbitrarily small, the probability of meeting an informed trader goes to zero. If an informational event is certain, λ, β, and θ are equivalent to those obtained in Bernhardt and Hughson [1990] when there is but a single potential insider and the market is not thick enough to support his certain entry.

The expected number of potential traders that become informed is given by:

$$\pi = \frac{\gamma\sigma v^2 \eta}{4c^2(v+\phi)}. \tag{10}$$

The equilibrium pricing function is invariant to the number of potential intermediaries when intraperiod trades are accepted individually. Indeed, the pricing function is the same for intraday periods whether the intraperiod market orders are pooled by the specialist or met independently by a variety of brokers. One consequently feels more comfortable about the ability of the "pooling" models (e.g. Kyle [1985], Admati and Pfleiderer [1988]) to capture pricing behavior on the NYSE even though they may not capture the actual trade process well.

There are multiple equilibria in the following sense: Only the expected number of informed agents is identified by the broker's zero profit conditions. Because of the linearity of the price schedule and its invariance to the actual number of arrivals within a period, the broker's expected profits are unaffected by how expected entry is divided among the potentially informed. Variation in the number of potentially informed who sample with positive probability does not affect the specialist's pricing function. For reasons given in Bernhardt and Hughson [1990], we concentrate on the equilibrium where there is a single potentially informed trader so that π becomes the probability of informed trade.

The pricing function, λ in (9) is *independent* of the aggregate variance of liquidity trade, but is not independent of variations in the quantity or quality of private information. This result differs markedly from that in Admati and Pfleiderer, where their equilibrium λ_t is decreasing in σ_t. Notice that π is linear in σ. The market maker's profits from the informed are also linear in σ. Thus the two effects *exactly offset each other*. This invariance occurs in markets which are too thin for Admati and Pfleiderer's equilibrium to exist.[17] However, if either the quantity of private information as reflected by the variance in the innovations, v_t, or the quality of information, $(\phi_t)^{-1}$, changes, the broker's pricing function will fluctuate during the day.

3.2. Transaction Costs and Brokerage Fees

Suppose agents now face an additional tariff to execute their trades, $c_1|\omega| + c_2$, where c_1 is a per-share transaction cost paid to the market maker and c_2 is a deadweight cost incurred if the trade is executed. We consider this alternative specification to bring the model into rough conformity with the data. If c_1 is positive, the transaction price series will exhibit the negative serial correlation that is observed on the NYSE. One can think of c_2 as the (unmodelled) opportunity cost of the potentially informed agent's time. Thus, c_1 is incorporated directly in the transaction price recorded on the Fitch tapes, while c_2, the deadweight cost, is not.[18] Hence, c_1 is essentially half the bid-ask spread (see e.g. Roll [1984]).

In addition to determining whether to obtain information, the potentially informed must also determine whether it is optimal to trade given the signal obtained. The ex-ante expected fixed costs to entry faced by a potentially informed trader (which include c, the cost of information acquisition) become:

$$c^* = c + 2c_2[1 - \Phi(\delta + \varepsilon)^*],$$

where $\Phi(\delta + \varepsilon)^*$ is the cumulative distribution function for a normal random variable with zero mean and variance $v + \phi$, and $2[1 - \Phi(\delta + \varepsilon)^*]$ is the probability an informed trader receives a signal sufficiently 'large' that he expects profits from trade which exceed the transactions costs. This then is the probability that he actually trades given that he is informed. The broker's conjectured pricing function is:

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[17]The result is due to the thinness of the markets, not to the mixed entry strategies of the informed.

[18]This specification is chosen in part for tractability. It also builds on Glosten and Harris [1989] who do not find a fixed transaction cost to be significantly different from zero.

$$
\begin{aligned}
&P_t = F_{t-1} + \lambda_t\omega_t + c_1 && \text{if } \omega_t \geq \omega(\delta+\varepsilon)^*,\\
&P_t = F_{t-1} + c_1 && \text{if } 0 \leq \omega_t < \omega(\delta+\varepsilon)^*,\\
&P_t = F_{t-1} - c_1 && \text{if } -\omega(\delta+\varepsilon)^* \leq \omega_t < 0,\\
&P_t = F_{t-1} - \lambda_t\omega_t - c_1 && \text{if } \omega_t < -\omega(\delta+\varepsilon)^*,
\end{aligned}
$$

where $\omega(\delta+\varepsilon)^* \equiv \omega^*$ is the critical trading quantity and $|(\delta+\varepsilon)^*|$ is the associated maximum signal below which no adverse selection problem exists. For small trading quantities, the profits an informed agent expects from his information are exceeded by his transaction (and/or brokerage) fees. For signals which would lead to such trades, it is not profitable for the informed to exploit his limited private information.

The broker expects zero profits net of transaction costs for any trade. For small trades, no adverse selection problem emerges since the informed do not trade such small quantities. Glosten and Harris [1989] attempt to distinguish econometrically the transaction cost components but their formulation does not recognize that when information is costly to obtain, the adverse selection problem vanishes for small transaction quantities. This may lead them to underestimate the size of the adverse selection component to the pricing function. Here, although closed-form solutions for λ cannot be obtained, we can characterize it explicitly and solve numerically for the appropriate parameters. Glosten [1987] speculates that "Since statistical properties of transaction prices are typically a function of both the spread and the composition of the spread, there is no obvious candidate for a simple spread-estimation procedure based on the moments of transactions returns...Furthermore, the results suggest than any attempt to estimate the spread from transaction prices should estimate two components." We agree with his sentiment that it is important to capture the two components, and find that here, the solution takes an estimable form.

A solution to the more general problem is not more difficult, but to ease notation, we restrict attention to the case where $\gamma = 1$. Let $I_t = 1{*}\mathrm{sgn}[\omega_t]$. Dropping the time subscripts for convenience:

Lemma 2: The demand function for an informed agent is:

$$x_i(\delta+\varepsilon) = MAX\left\{\left[\frac{\nu(\delta+\varepsilon)}{(\nu+\phi)} - c_1 I\right]\frac{1}{2\lambda}, 0\right\}$$

if $\delta + \varepsilon > 0$ and

$$x_i(\delta+\varepsilon) = MIN \left\{ \left[\frac{\nu(\delta+\varepsilon)}{(\nu+\phi)} - c_1 I\right]\frac{1}{2\lambda}, 0 \right\}$$

if $\delta + \varepsilon \leq 0$.

Proof: See appendix 2.

To solve the equilibrium problem we break it down into three steps:

1. Solve for the critical signal $(\delta + \varepsilon)^*$ below which no informed traders will trade.
2. >From the zero profit condition for the broker, we solve for the broker's pricing function, λ, for an arbitrary entry probability, π.
3. >From the condition that the ex-ante expected profits of an informed agent must be πc^*, we solve for the entry probability, π.

Recall that $\beta = \frac{\nu}{2\lambda(\nu+\phi)}$. Substituting β, expected profits from trading equal:

$$E\Pi = E\left[\left(\beta(\delta+\varepsilon) - \frac{c_1 I}{2\lambda}\right)\left(\delta - \lambda\left(\beta(\delta+\varepsilon) - \frac{c_1 I}{2\lambda}\right) - c_1 I\right) \mid (\delta+\varepsilon)\right].$$

In order to find the informed agent's expected profits conditional on his signal, we must find $E(\delta^2 \mid \delta+\varepsilon)$ and $E(\delta\varepsilon \mid \delta+\varepsilon)$.

Lemma 2a:

$$E(\delta^2 \mid \delta+\varepsilon) = \frac{\nu^2(\delta+\varepsilon)^2}{(\nu+\phi)^2} + \frac{\nu\phi}{(\nu+\phi)}, E(\delta\varepsilon \mid \delta+\varepsilon) = \frac{\nu\phi(\delta+\varepsilon)^2}{(\nu+\phi)^2} - \frac{\nu\phi}{(\nu+\phi)}.$$

Proof: See appendix 2. []

Given these relations, and given his signal, the expected profits of an informed trader to trading can be rewritten as:

$$E\Pi = \frac{\nu^2(\delta+\varepsilon)^2}{4\lambda(\nu+\phi)^2} - \frac{c_1 I\nu(\delta+\varepsilon)}{2\lambda(\nu+\phi)} + \frac{c_1^2}{4\lambda}.$$

Substituting β for $\frac{\nu}{2\lambda(\nu+\phi)}$ yields:

$$E\Pi = \frac{\beta v(\delta + \varepsilon)^2}{2(v + \phi)} - c_1 I \beta(\delta + \varepsilon) + \frac{\beta c_1^2 (v + \phi)}{2v}.$$

This value must be greater than zero for non-zero trading quantities. Note that even absent fixed costs, there remains a critical value below which informed traders will not transact,

$$|(\delta + \varepsilon)^*| = \frac{c_1}{2\beta\lambda} = \frac{c_1(v + \phi)}{v}.$$

When c_2 is added, to determine the critical signal $(\delta + \varepsilon)^{**}$ below which an informed agent does not trade, we must first find the signal at which the expected profits from transacting equal c_2, his deadweight trading cost. This yields $(\delta + \varepsilon)^{**}$, where there is a discrete jump in the specialist's pricing function (see figure 2). But $(\delta + \varepsilon)^{**}$ cannot be the critical value. If it were, an informed agent could profit by trading the quantity ω^{**} minus some very small quantity, face no adverse selection tariff, and make positive profits. To find ω^*, we must determine the slope of the specialist's pricing function and find where it crosses the horizontal line, $p_t = f_{t-1} + c_1$, where c_1 reflects the specialist's per-share cost of doing business. This intersection determines ω^*. A consequence is that between ω^{**} and ω^*, no equilibrium pricing function exists where both the specialist and informed traders earn zero expected profits. If the pricing function is flat between ω^* and ω^{**} the informed trader profits, and if the function is linear (with slope λ) then the market maker profits. We disregard this problem in the later empirical sections.

Equating $E\Pi$ with c_2 yields the following quadratic equation:

$$\frac{\beta v(\delta + \varepsilon)^{**\,2}}{2(v + \phi)} - c_1 I(\delta + \varepsilon)^* + \frac{\beta c_1^2 (v + \phi)}{2v} - c_2 = 0.$$

If $c_1 = 0$,

$$|(\delta + \varepsilon)^{**}| = \left(\frac{2(v + \phi)c_2}{\beta v}\right)^{1/2}.$$

Otherwise:

$$|(\delta + \varepsilon)^{**}| = \frac{c_1 I(v + \phi)}{v} + \frac{2(v + \phi)\sqrt{\lambda c_2}}{v}.$$

Equation (4) no longer details the zero expected profit condition for the broker because the conditional probability the broker faces an informed trader is altered - an informed agent trades only if the absolute value of his signal exceeds $(\delta + \varepsilon)^{**}$. For smaller signals it is not profitable for an informed agent to transact. Conditional on the volume exceeding ω^{**}, the broker recognizes that he may be trading with an informed trader. The broker's zero expected profit condition then requires that expected profits net of transaction costs from trading with uninformed liquidity traders equal his losses from trading with the informed:

$$\pi \int_{-\infty}^{\infty} \int_{-\delta+(\delta+\varepsilon)^{**}}^{\infty} \Big[E\Pi \Big] f_\delta(\delta) f_\varepsilon(\varepsilon) d\varepsilon d\delta = \lambda \eta \int_{\omega^{**}}^{\infty} z^2 f_z(z) dz, \tag{11}$$

where $f_\delta(d), f_\varepsilon(e)$, and $f_z(z)$ are the normal probability density functions for δ, ε, and z respectively. Solving implicitly for λ, we obtain:

$$\lambda = \Bigg[\frac{\pi}{\eta} \frac{1}{4(v+\phi)^2} \int_{-\infty}^{\infty} \int_{-\delta+(\delta+\varepsilon)^{**}}^{\infty} [v^2(\delta+\varepsilon)^2 - 2c_1 I(v+\phi)(\delta+\varepsilon) + c_1^2 (v+\phi)^2] f_\delta(\delta) f_\varepsilon(\varepsilon) d\varepsilon d\delta \, / \tag{12}$$

$$\int_{\beta(\delta+\varepsilon)^{**}}^{\infty} z^2 f_z(z) dz \Bigg]^{1/2}.$$

The right hand side of the equation is continuously decreasing in λ, approaching infinity as λ goes to zero and zero as λ approaches infinity.[19] Hence for a given π, there is a unique solution for λ. Observe that only the ratio $\frac{\pi}{\eta}$ enters so that once again λ is invariant to fluctuations in η, although now, λ is a complicated function of σ, the variance of uninformed trade.

To solve for the equilibrium entry probability for the informed, π, we equate expected profits from sampling to the equilibrium expected sampling costs. Equivalently, the expected revenues from being informed are equated to the expected costs of becoming informed (including the possibility of trading):

$$\frac{1}{4(v+\phi)^2} \int_{-\infty}^{\infty} \int_{-\delta+(\delta+\varepsilon)^{**}}^{\infty} \frac{1}{\lambda(\pi)} [v^2(\delta+\varepsilon)^2 - 2c_1 I(v+\phi)(\delta+\varepsilon) + c_1^2 (v+\phi)^2] f_\Delta(\delta) f_\varepsilon(\varepsilon) d\varepsilon d\delta \tag{13}$$

$$= c^*(\lambda(\pi)).$$

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[19]As λ approaches zero, insider profits become infinite, since the risk neutral insiders will trade very large amounts. Conversely, when λ becomes large, insider trading vanishes, as do expected profits.

Here we write λ explicitly as a function of π. Observe that both sides of the equation are continuous and that the left hand side is monotone decreasing approaching infinity as π goes to zero and zero as π approaches 1. The right hand side is monotone decreasing, approaching $c + c_2$ as π goes to zero, and approaching c as π approaches 1. Hence a solution exists, and one can show that the equilibrium sampling probability is unique.

One can integrate and explicitly write equations (12) and (13) as functions of normal distributions. For instance, if δ, ε and z are each standard normal random variables, and c_1 is set to zero, these equations are shown in appendix 3 to reduce to

$$\lambda = \frac{\sqrt{2c_2\lambda/pi}\, e^{-2c_2\lambda} + 1 - \Phi(2\sqrt{c_2\lambda})}{4(c + 2c_2(1 - \Phi(2\sqrt{c_2\lambda}))}$$

$$(\delta + \varepsilon)^{**} = \sqrt{8c_2\lambda}$$

$$\frac{\pi}{\eta} = \frac{2\lambda(1 - \Phi(c_2/2\lambda) + \sqrt{c_2/4\lambda pi}\, e^{-c_2/4\lambda})}{c + 2c_2(1 - \Phi(2\sqrt{c_2\lambda}))},$$

where we write "pi" to distinguish the number from the sampling probability. Note that λ can be solved for directly (numerically) and then resubstituted to obtain $(\delta + \varepsilon)^{**}$, and π/η. In figures 3 - 6, the relationship between transaction costs and the endogenous variables are illustrated. Observe how quickly the probability that a potentially informed agent actually seeks to acquire inside information falls as transaction costs rise. For example, if c = .5, and brokerage and transaction fees, c_2, are even one tenth the cost of information acquisition, c, the probability of acquiring inside information falls by 15%, if total expected costs are held constant. For c = 1, the reduction is even greater - 20%. In response to the decreased likelihood of trading with an insider, the broker pares the adverse selection component, λ, of his pricing function by 8%. That is, small transaction costs can have significant effects on the broker's pricing function. Note also how the minimum trade size such that an informed trader is willing to trade increases with brokerage fees. To induce him to transact as these fees increase, the informed trader requires an ever more promising signal that he possesses significant private information. Consequently, there are more signals for which the informed trader will walk away from the broker's desk. As brokerage fees rise, the probability an informed agent trades falls accordingly.

It is possible to further characterize the equilibrium solution when $c_1 = 0$. Define the locus of combinations of c and c_2 which yield the same equilibrium expected cost, c^*: $c^* \equiv c + 2[1 - \Phi(\delta + \varepsilon)^*]c_2$. Then, writing the equilibrium λ as a function of the fraction of costs which are due to information acquisition, c/c^*, $\lambda(c/c^*)$ must be increasing in its argument, c/c^*. For suppose, for instance, that it were constant. Then for any given signal the informed would demand the same quantity for all c, conditional on actually trading. But then the informed agent's (positive) expected profits in those trading states are the same for different sampling costs, c. However, the greater is c then the lower is c_2, so the more signals at which the informed profitably trade. Since expected costs are equal to c^* in each scenario, it must be that net expected profits are greater the larger the fraction c represents of expected costs. But net expected profits must be zero independent of the composition of costs, a contradiction. Consequently, as the costs of becoming informed consume a smaller proportion of c^*, the adverse selection component of the bid-ask spread falls, and the trade volume below which no adverse selection component to the bid-ask spread exists rises. A corollary is that the equilibrium probability that an informed samples must fall as c comprises a smaller fraction of c^*, in order for the broker to earn zero expected profits. These observations are illustrated in figures 7 and 8 where the relationship between c/c^* and the endogenous variables are detailed for expected total trading costs for the informed of $c^* = .5$.

Observe too that at least some liquidity traders benefit from larger transaction costs - they may actually be helped by a tax on transactions. If transaction costs increase marginally, that increase may be sufficient that it ceases to be in the interest of informed traders to trade the same quantity as some liquidity trader. The liquidity trader incurs a marginal increase in costs due to the transaction cost, but receives a lump sum benefit because the adverse selection component to price no longer exists. Similarly, because λ falls as transaction costs rise, discretionary traders who trade sufficiently large quantities also benefit. In contrast, liquidity traders who trade very small quantities lose as transaction costs increase because the degree of adverse selection the broker faces for such transaction volumes is similarly small. See figure 9. Ex ante, however, liquidity traders do not gain from an increase in transaction costs.

If $c_1 > 0$, there is negative serial correlation in the price series which provides additional information about the sign of ω_i. The entire trading history is now relevant because the fixed transaction costs imply prices no longer follow a martingale. The estimate of c_1 here (and in Glosten and Harris [1989]) provides a lower bound on the cost of trading since the total trading cost presumably includes brokerage fees. A more relevant statistic may be an estimate of ω^*, the smallest trade insiders are willing to make. This value is unaffected by the division of transaction costs into observed (price) and unobserved (brokerage fee) components. From ω^*, the estimate of brokerage fees can then be unraveled.

4. Testable implications.

In the absence of transaction costs, not only can λ, the slope of the specialist's pricing function, be estimated, but we can obtain predictions about the nature of the relationships between volume, adverse selection and transaction costs. This is because the theory provides non-linear restrictions which identify the structural parameters, v, the variance of information, ϕ, the noise in the signal, c, the cost of information, and σ, the variance of uninformed trade. These structural parameters in turn provide information about other quantities of interest: the probability the specialist faces an informed trader, and the average quantities traded by both informed and uninformed agents.

Equation (9) implies that λ_t, does not vary with intraday variations in trading volume (due to variations in non-discretionary liquidity trade). Estimation intervals must then not include opening and closing transactions, because the specialist's pricing function is generally different then. By estimating the pricing function over different intervals throughout the day, on different days of the week, or after different events, one can test whether there is more inside information at different times. We estimate the pricing function, λ_t over different intraday intervals using transaction price differences, accounting for the unobservability of both the bid/ask indicators and the identity of the traders (informed or uninformed).

We then test to see if λ_t is positive (i.e. if there exists an adverse selection problem). Assuming information acquisition costs do not change, evidence that λ_t varies over the day is evidence of changing quantities of private information, v_t, or of private signals, ϕ_t. Rises in v_t and falls in ϕ_t both worsen the degree of adverse selection and lead to a larger λ_t. Estimation of v_t and ϕ_t allows us to distinguish between these two explanations. We then test the linear pricing rule of the market maker against a quadratic alternative. Finally, we test the nonlinear overidentifying restrictions which identify the structural parameters in the model.[20]

When transaction costs are integrated into the model, identification of some structural parameters

[20]There are six reduced form parameters, four structural parameters, and two nonlinear restrictions which map the reduced form parameters to the structural parameters. The alternative simply does not impose the restrictions.

becomes infeasible. In addition, λ is no longer independent of σ (see section 3). However, we can still estimate the probability a particular trader is informed as well as the average quantities traded by informed and uninformed traders. In addition, both a per-share transaction cost, c_1, and a quantity cutoff, ω^{**} below which there is no adverse selection are estimated. Unlike the linear pricing rule estimated by Glosten and Harris [1989], the resulting pricing function is kinked. We therefore test the linear pricing rule against the kinked alternative.

5. Estimation.

5.0. The data.

The Francis Emory Fitch transaction database is used to test the model. The data are all transactions for all stocks on the NYSE between January 2, 1980 and January 6, 1981. We remove securities which experience stock splits or receive distributions other than normal cash dividends. Regular dividends are paid overnight, and their effects are felt only at open. For the remaining stocks, we examine the entire price series, forming price differences between adjacent trades. Since prices on open and close may be determined by call auctions or *de facto* call auctions, the first trade of the day, the last trade of the day, and all other trades processed at these times are eliminated. All price differences which contain these trades are also removed. The same sort of criterion removes trades around trading halts, trades which appear out of order on the tape, trades which are marked with a correction code or certain condition codes, trades at negative prices (known errors), and trades with price changes of more than 20%. For frequently traded securities, we restrict attention to the first 5000 remaining price differences. Finally, when comparing parameters across different time periods, we eliminate price changes which cross hourly boundaries.

5.1. Estimation procedure.

We assume that all trades are processed sequentially as they appear on the tape, and an informational innovation occurs each period - $\gamma = 1$.

Estimation without transaction costs when the sign of ω is unobservable.

Let there be n transactions and let prices, volumes, and indicator functions be indexed by time, not by transaction. Estimation is accomplished by taking price differences from successive trades. Suppose adjacent transactions occur at times s-t and s. Recall:

$$p_s = F_{s-1} + \lambda_s\omega_s; \qquad p_{s-t} = F_{s-t-1} + \lambda_{s-t}\omega_{s-t}.$$

We can rewrite p_s as:

$$p_s = F_{s-t-1} + \sum_{\tau=s-t}^{s-1} \delta_\tau + \lambda_s \omega_s.$$

Then,

$$\Delta p_s = p_s - p_{s-t} = [\delta_{s-t} - \lambda_{s-t}\omega_{s-t}] + \sum_{\tau=s-t+1}^{s-1} \delta_\tau + \lambda_s \omega_s. \tag{14}$$

$[\delta_{s-t} - \lambda_{s-t}\omega_{s-t}]$ is the projection error due to the information released about δ_{s-t} immediately following the trade at s-t. $\sum_{\tau=s-t}^{s-1} \delta_\tau$ is the balance of the information released between the two trades. Thus [14] can be writen as:

$$\Delta p_s = \lambda_s \omega_s + e_s. \tag{15}$$

Although quantities traded by both informed and uninformed traders are normal random variables with zero mean and variances $\beta^2(v + \phi)$ and σ, and δ is itself normally distributed, e_s and ω_s are mixtures of normal distributions (see e.g. Amemiya [1985] p 120). Were ω_s observable, ordinary least squares would provide consistent but inefficient parameter estimates.[21] Assuming that price increases (decreases) indicate buy (sell) orders clearly biases the results upward. The correct procedure for estimating λ when the sign of ω is unobservable is given in appendix 5.

In Table 4 we compare the correct procedure with estimates of λ obtained using OLS, assuming that price increases (decreases) indicate buy (sell) orders, and lack of price movement indicates that the sign of the bid-ask indicator is the same as that of the previous transaction.[22]

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[21]The inefficiency is due to the unobservability of the identities of the traders (informed or uninformed).

[22]While the sign of ω_i is not included by Fitch, the new ISSM data tapes provides bid-ask quotes in addition to trading prices. To the extent that the fact that a trade at the bid indicates a sell, and a trade at the ask indicates a buy, the problem of identifying which trades are buy orders and which are sell orders is eased. Any residual uncertainty will, however, necessitate the use of mixtures of distributions as in Glosten and Harris [1989], or this paper here. In addition, even when it is known when agents are buying or selling, in the context of our model, it is still not possible to use OLS or GLS to estimate. Table 4 shows how using OLS biases estimates of λ upward. However, it would be no longer necessary to integrate conditional likelihoods for Δp over the unconditional marginal distribution for I, the sign of ω.

Foster and Viswanathan [1988b] estimate λ for a different model, where entry and trade by agents is deterministic, which implies that both ω and ε are normally distributed. They estimate this model using Hasbrouk's [1989] procedure on the ISSM data tapes. If the transaction price is closer to the ask (bid), the transaction is a buy (sell). Transactions which are equidistant from the bid and ask are omitted. This still overestimates λ. Suppose that there is perfect correlation between the bids and asks and the quotes. Even then, omission of trades midway between the bid and ask reduces the slope coefficient λ. For example, if $\Delta p = \lambda\omega + \varepsilon$, $\lambda = \Sigma\omega\Delta p/\Sigma\omega^2$. Omitting those trades reduces the denominator without affecting the numerator. At the same time, the procedure may reduce much of the noise associated with the unobservability of buys and sells.

Mixtures of distributions are due not only to the unobservability of the sign of ω but also because informed and uninformed agents (who trade quantities drawn from different normal distributions) cannot be distinguished from each other. These additional mixtures identify the structural parameters of the model. Our estimation procedure, which maximizes the *unconditional* likelihood of jointly observing the sequence of price changes $\{\Delta p\} = \{\Delta p_1, \dots, \Delta p_n\}$, and the volume sequence $\{|\omega|\} = \{|\omega_1|, \dots, |\omega_n|\}$ is given in appendix 4.

5.2. Results.

5.2.1. Estimation of λ when the sign of ω is unobservable.

We first estimate the slope of the specialist's pricing function, λ (\$/share/1000 shares) when the sign of ω is not observable (see appendix 5). Since, in this section we are mainly interested in comparing these λ with the λ obtained using the OLS procedure detailed in 5.1, we momentarily ignore our theory and restrict var(e_i) to be a constant. Note, however, that our theory implies that var(e_i) is *ceteris paribus* linear in the time between trades. We justify this simplifying assumption because, when we allowed var(e_i) to be linear in the time between transactions, (i.e. $\text{var}(e)_i = k_0 + k_1 t$, where t is the time between trades), estimates of k_1 were found to be insignificantly different from zero in most cases. In addition, the economic impact of including time between trades on var(e_i) was small. Finally, estimating k_1 sometimes caused serious problems with convergence of the minimization algorithm.

Var(e_i) is also a function of the identity of the trader (informed or uninformed) at both time s and time s - t. This more complicated relationship identifies the structural parameters of the model. Those results are presented in section 5.2.2.

Parameter estimates obtained for nine securities are presented in Table 1. SYM is the stock symbol on the Fitch tape. The next column is the total number of trades for the year 1980. The following column is the number of trades included for estimation. λ is the estimate of the slope of the specialist's pricing function in dollar price change per 1000 share trade, taking into account the unobservability of the sign of ω. Therefore, if the price rises, the $\text{pr}(\omega > 0 \mid \Delta p > 0)$ will not be one. It will, however, be increasing in the magnitude of the price movement. Except for ABC, there appears to be a significantly positive adverse selection component to the bid-ask spread.[23] λ appears

[23]When λ approaches zero, the algorithm GQOPT ceases to converge - there are no longer four independent first order conditions at the limit.

to decrease with trading volume. Although the model places no restrictions on the relationship between firm size and adverse selection, this result would be consistent with Foster and Viswanathan's [1988b] observation that less private information is available about large, frequently traded, securities.

Var(e) is the estimate of the variance of public information release between trades. Recall that surprisingly, the variance of price changes does not appear to be affected by the time between trades.[24] One explanation for this may be that an informational innovation does not occur every period. When γ, the probability of an innovation, is small, trades tend to clump around informational events because insiders trade only then.

When the sample is divided into six trading hours, (see Table 2), we reject the hypothesis that λ and var(e) are constant across the trading day ($X^2(10) = 24.13$),[25] even after we omit transactions (e.g. opening and closing trades) which fail our editing criteria. Theory also predicts that the adverse selection component is smaller at open and close if there is discretionary liquidity trade. Unfortunately, the specialist pools orders on open and close, and we do not have a formal procedure for extracting net order flow from price and volume data. However, we informally test the hypothesis that the adverse selection component is smaller on open and close by estimating the model during the first and last 15 minutes of the day assuming no pooling and including opening and closing trades and find that λ is higher there than at any other time. λ is high during hours 1, 2, and 6, but it is also high during hour 4. The lack of systematic correlation between λ and price variance is even more pronounced for AAA. Here, we *cannot* reject the null hypothesis that λ is constant across the day ($X^2(10) = 5.75$).

5.2.2. Structural parameter estimation.

Here we estimate the structural parameters for the model in the absence of transaction costs. The likelihood function, which takes into account the unobservability of the sign of ω and the unobserved identities of the traders (informed or uninformed) is derived in appendix 4. Due to the shape of the likelihood function, gradient methods of estimation did not converge. Consequently, the maxima were found using a grid search. Standard errors are found numerically as well.

[24]Hausman, Lo, and MacKinlay [1991] find that when they account for discreteness, the time between trades is indeed a determinant of the variance of public information release. Their finding is consistent with the model presented here.

[25]In the restricted model, we estimate two parameters, λ, and var(e), rather than 12.

Estimates are provided for Alcoa Aluminum in Table 5. The structural parameters are all found to be statistically significant. It is comforting to find that the estimates of the slope parameter, λ, are similar in magnitude to those in Table 4.

Previous literature attributes changes in λ to changes in the "amount" of information about the stock. Our estimation procedure distinguishes between the "amount" of information and its "quality". We find that v, the variance of inside information, is nearly constant throughout the day. In Table 5, we see small changes in v accompanied by large changes in λ (see hours 2 and 3). However, ϕ, the noise in the signal is much higher in the middle of the day. Thus, changes in λ appear to be almost completely determined by changes in ϕ.

Over the day, the average informational innovation (informed traders are not concerned about the sign) is slightly greater than an eighth, \$.1353, and the signal to noise ratio is about .5. Not surprisingly, insiders trade much larger quantities than uninformed traders, averaging 7002 and 458 shares respectively. See figure 10 for the distribution of traded quantities for the 5000 Alcoa orders. Only about 300 trades exceed 7000 shares. The large majority of orders are less than 1000 shares. While the estimated probability a given trader is informed is .43%, it is close to zero for small orders, (.03% for hundred share orders, .3% for 1000 share orders), rises precipitously between 1000 and 2000 shares (see figure 11), and is about 100% for trades above 3000 shares. The small fraction of informed traders is implausible. Examination of the specialist's zero profit condition (equation (4)) shows that the expected profits of the informed on a 10000 share order are extremely high - almost \$100,000, almost \$10 per share. When the nonlinear restrictions are relaxed (see Table 6), the probability of an informed trader rises to over 9%, and the expected profits of the informed drop to a more reasonable \$4200.

For Alcoa (AA), a 1000 share purchase (approximately \$60,000) results in a tariff of \$4.19 due to adverse selection. While this seems small, the tariff rises to \$419 for a 10,000 share trade. This latter number is perhaps more relevant, since insiders trade 7002 shares on average. Indeed, for some securities, the adverse selection tariff rises to \$2000 (see Table 7).

In Table 6, the model is tested against two alternatives. First, the two nonlinear restrictions implied by the model are relaxed: we just estimate the reduced form parameters of the model. This exercise is, in some sense, equivalent to that in Glosten and Harris, who estimate an econometric specification rather than a structural model. When the mapping from the reduced form to the structural parameters is removed, some structural parameters are now unidentifiable, but the fit improves dramatically. We strongly reject the nonlinear restrictions imposed by the model ($X^2(2) = 612$). Symptoms of the bad fit include the low probability of informed trade and the unreasonably large profits of the insiders when they trade large quantities. While the slope of the pricing function and the quantities traded by market participants are unaltered, the probably of informed trade increases to over 9%. Second, the model is tested against a nonlinear rule. The immediate concern is whether the specialist is also inferring whether large trades are due to informed agents. Since the linearity is a consequence of normal distributions, there is in general little reason to expect that the pricing function is linear. As a simple diagnostic, we test the model against a quadratic pricing rule. Define d as a quadratic term in the pricing function. Surprisingly, we are unable to reject linearity, since d is neither economically nor statistically significant. We conjecture that d may be negative only because transaction costs are excluded from the model here. If the price increases by C whenever shares are bought, regardless of the size of the order, a first order quadratic approximation without transaction costs would appear to be concave.

In Table 7, parameter estimates are given for other securities. Note that for the smaller stocks (not ABC), both informed and uninformed agents tend to trade smaller quantities, and λ appears to be greater. Note also that λ_{ABC} is quite low, the adverse selection tariff is \$1.90 for a thousand share order. This is consistent with the (lack of an) estimate for λ_{ABC} in Table 1.

5.2.3. The Effect of Transaction Costs.

When transaction costs are included, structural parameter estimation becomes infeasible. The structure of the model looks like the unrestricted alternative from Table 6 with the addition of transaction costs. When investors face a fixed cost to execute their trades, below a critical value $(\delta + \varepsilon)^*$, there is no adverse selection problem. Figure 3 shows that ignoring the consequences of c_2 biases estimates of λ downward. The additional per-share cost, c_1, paid to the specialist, introduces serial correlation, so the past transaction history is relevant. The likelihood function becomes complicated, because it is now necessary to integrate over all possible transaction paths to form the likelihood function. The procedure is similar to that used to integrate over all possible series of informed/uninformed indicators. See appendix 6 for details. Because the overnight non-trading period is long, and there are presumably many informational innovations, we assume the probability that the opening trade is a buy is 1/2. This significantly reduces the relevant number of potential transaction paths because the likelihood of Δp_s depends only on trades which take place in the same day. Estimation of the critical level of trade ω^* below which no adverse selection problem exists involves including ω as an explanatory variable above the critical level, and omitting it otherwise [Judge 1985 pp 803-806].

Unfortunately, we were unable to obtain convergence for the full structural model when the minimum cutoff for transaction sizes of informed traders is included. Instead we estimate λ, the critical cutoff, and the variances of both informed and uninformed trade. Results for Alcoa Aluminum (AA) are given in Table 8. They indicate that the introduction of transaction costs to the econometric modele reduces λ to .0025. Transaction costs are estimated to be about \$.032 per share and the critical cutoff is estimated to be 1300 shares ($X^2(1) = 248.32$ (> .99)), which means that informed traders do not place orders for Alcoa of less than about \$65,000. As one might have expected, the estimated average trade sizes of both the informed and uninformed are unaffected by the introduction of transaction costs. The reason that this estimate does not change is that even without transaction costs, the estimated probability that an individual trading a quantity less than 1400 shares is informed is close to nil.

6. Extensions and Comments.

6.1. Probabilistic informational innovations and estimation when simultaneous transactions are accepted independently.

Here, price differences are not formed between two essentially simultaneous trades. If for example, trades A and B have the same time stamp, and trade C occurred two minutes prior, the price differences created are $p_A - p_C$ and $p_B - p_C$.

In addition, we relax the restriction that γ, the probability of an informational event, equals one. Thus, informational events do not necessarily occur each period. Informed agents will not trade in the absence of an informational event. Thus j, the number of trades is a given interval, contains information about the probability of an informational event. We first find the probability of an event conditional on j, and then use this probability to calculate the variance of price changes. While we do not estimate this version of the model, it can potentially explain why the time between trades does not significantly affect the variance of price changes. Below, the time subscripts are suppressed to ease notation.

Proposition 3: Given j trades in a period:

$$pr(no\ event \mid j\ trades) = \frac{(1-\gamma)\eta^j}{(1-\gamma)\eta^j + (\pi+\eta)\gamma e^{-\pi}},$$

and:

$$pr(event \mid j\ trades) = \frac{(\pi+\eta)^j \gamma e^{-\pi}}{(1-\gamma)\eta^j + (\pi+\eta)^j \gamma e^{-\pi}}.$$

Proof: See appendix 3. []

Now Δp_s can be written as:

$$\Delta p_s = [\delta_{s-t} H_{s-t} - \lambda_{s-t}\omega_{s-t}] + \sum_{\tau=s-t+1}^{s-1} \delta_\tau H_\tau + \lambda_s \omega_s, \tag{16}$$

where H_τ is an indicator which equals one if there is an informational innovation in period τ and zero otherwise. The above equation can still be written as:

$$\Delta p_s = \lambda_s \omega_s + e_s. \tag{17}$$

The unconditional distribution of e_s is now complicated by the need to mix over the marginal distribution for η conditional on the number of trades which occur in each period. Fortunately, this allows one to estimate the probability of an informational innovation given j trades, and test the restriction that the probability of an informational innovation is increasing in the number of trades. In addition, we obtain the restrictions (not tested) that as j_s - j_{s-t} increases, so does the variance of price changes, and as j_{s-t} increases, the variance of e_s declines.

6.2. Event Studies.

The methodology outlined in sections 5.1 and 5.2 can be used to examine whether increased price variance around major events such as earnings, dividend, and merger announcements can be explained by an increase in the amount of inside information near the announcement dates.

6.3. Cross-sectional tests.

Informational events may affect more than a single security. The analysis presented in this paper can be extended to the case where there is contemporaneous correlation in informational innovations across securities, i.e. to where the informational innovations contain both a common market-wide component and a security-specific idiosyncratic component. However, there is the additional problem of non-synchronous trading across securities. There will be only some overlap of price change intervals for different securities. Fortunately, the model implies the correlation between the regression errors of two securities is linear in the time overlap, and it is not difficult to apply a modified "seemingly unrelated" regressions technique to estimate the model. Given the amount of cross-sectional correlation in security returns, it is not unreasonable to expect an efficiency gain from this approach. Details are given in Bernhardt and Hughson [1990b].

6.3. Discreteness in Prices and Time Between Trades.

Harris [1986], and Glosten and Harris [1989] recognize that transaction prices come from a discrete price grid with increments of an eighth which is about three times the estimated per-share adverse selection component for a 1000 share order for Alcoa. They assume that the "true value" is a continuous variable and that agents round the price to the nearest eighth. They note that rounding is fundamentally ad-hoc. Unfortunately, it is reasonable to believe that the magnitude of the error introduced by discreteness may be large relative to information effects. Our results suggest that not only does discreteness play an important role in determining the price series, but it may also play an important role in determining the strategies of the potentially informed agents, and hence, the specialist. These issues are examined in Bernhardt and Hughson [1991a], [1991b].

7. Conclusion.

We have created a model of insider trading on the NYSE which is consistent with many of the stylized facts found in the empirical literature. Equilibrium exists in our environment during the trading day. An important result is that the adverse selection component to the bid-ask spread need not be related to the quantities traded by uninformed traders. Since transactions are processed independently, we can incorporate transaction cost components into the model. Then the adverse selection component to the specialist's pricing function vanishes for small transaction sizes, and the price series is negatively serially correlated. The theory imposes testable restrictions on transaction-by-transaction data. These econometric tests are performed using maximum likelihood estimation which takes into account the unobservability of the side of the trade taken by the broker. The theory provides additional non-linear restrictions which allow identification of the structural parameters in the economy: the variance of information, the noise in the signal, the cost of information, and the variance of uninformed trade. While the linear pricing function is not rejected when tested against a quadratic alternative, the non-linear restrictions which identify the structural parameters are tested and decisively rejected.

When fixed and variable transaction costs are introduced, estimation of the structural parameters is infeasible. However, a transaction cost component and a critical value below which no adverse selection exists emerges can be estimated. The critical cutoff is found to be large - insiders do not trade small quantities. We strongly reject the linear pricing rule when it is tested against the kinked alternative.

We note that the magnitude of the error introduced by discreteness may be large relative to information effects. Our results suggest that not only does discreteness play an important role in determining the price series, but it may also play an important role in determining the strategies of the potentially informed agents, and hence, the specialist.

Finally, we note that the price impact of information of a single trade appears to be small economically. This may be in part due to the structure of the NYSE itself, in particular, the existence of limit orders and price continuity requirements for the specialist. This may cause the price impact of information to be spread over several trades. It is also plausible that in the presence of a linear pricing rule, and long-lived information, informed agents will attempt to disguise their actions by adopting dynamic trading strategies where they split their orders over time. This may also minimize the price impact of information of a single trade, and suggests a direction for future research.

Appendix 1. Tables.

TABLE 1

PARAMETER ESTIMATES FOR 13 SECURITIES**

SYMBOL TOTAL TRADES	FIRM TRADES INCLUDED	λ	var(e)	μ_0 trend	μ_1 components
AA	Alcoa Aluminum				
13002	5000	.00420	.01790	.00131	-.00038
		(.00074)	(.00046)	(.00219)	(.00032)
AAA	American Savings and Loan of Florida				
2153	847	.05113	.02122	.00089	-.00061
		(.01303)	(.00100)	(.00651)	(.00041)
AAE	Amerace Corp.				
1903	709	.03138	.02232	.00790	-.00011
		(.00987)	(.00097)	(.00769)	(.00055)
ABC	American Broadcasting Co.				
10434	5000	0*			
ABF	Airborne Flight Corp.				
2323	945	.01117	.02471	.00397	-.00038
		(.00224)	(.00157)	(.00670)	(.00045)
ABT	Abbot Corp.				
10057	5000	.00580	.01521	.00379	-.00037
		(.00090)	(.00033)	(.00230)	(.00026)
ABY	Albany International				
3289	1788	.01498	.02838	.00103	.00062
		(.00838)	(.00085)	(.00494)	(.00039)
ABZ	Arkansas Best Corp.				
1438	365	.03190	.00962	-.00176	.00051
		(.00840)	(.00066)	(.00763)	(.00041)
ACA	Arcata Corp.				
3150	1519	.00259	.02092	.00959	-.00056
		(.00205)	(.00074)	(.00050)	(.00033)

[26,27,28]

[26]* Estimation procedure did not converge - all parameters are not separately identified when λ is in the neighborhood of zero.

[27]** These preliminary results are derived assuming that the errors have constant variance. That is, it is not affected by identity of the trader (informed or uninformed).

[28]Standard errors are given in parentheses directly below the point estimates.

TABLE 2

HOURLY PARAMETER ESTIMATES FOR ALCOA ALUMINUM (AA) AND AMERICAN SAVINGS AND LOAN OF FLORIDA (AAA)

SYMBOL HR	TRADES	λ	var(e)	μ_0 trend	μ_1 components
AA				.00168	-.00033
				(.00219)	(.00006)
1	1068	.00502	.01834		
		(.00199)	(.00084)		
2	1035	.00661	.01690		
		(.00238)	(.00092)		
3	703	.00264	.01687		
		(.00060)	(.00116)		
4	575	.00474	.01485		
		(.00194)	(.00098)		
5	750	.00367	.01874		
		(.00104)	(.00095)		
6	857	.00447	.02048		
		(.00218)	(.00164)		
AAA				.00781	-.00057
				(.00675)	(.00327)
1	203	.05678	.02299		
		(.01193)	(.00204)		
2	249	.02617	.02399		
		(.03658)	(.00192)		
3	112	.08025	.01841		
		(.03130)	(.00274)		
4	84	-.00735	.01797		
		(.27496)	(.00278)		
5	114	.00000	.02229		
		(.00466)	(.00231)		
6	85	.07251	.01957		
		(.02552	(.00327)		

[29,30]

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[29] AA is Alcoa Aluminum, AAA is American Savings and Loan of Florida

[30] Standard errors are given in parentheses directly below the point estimates.

TABLE 3

ESTIMATES OF PRICE AND VOLUME

SYM	HR	VAR(price)	$E[\omega^2]$* per 1000	$E[\|\omega\|]$ thousands	SAMPLE SIZE
AA	ALL	0.01813	12.692	1.266	5000
	1	0.01862	10.569	1.354	1068
	2	0.01724	8.281	1.182	1035
	3	0.01698	17.461	1.211	703
	4	0.01485	15.585	1.373	575
	5	0.01899	18.228	1.319	750
	6	0.02050	9.888	1.185	857
AAA	ALL	0.02245	0.443	0.399	847
	1	0.02503	0.650	0.483	203
	2	0.02277	0.446	0.362	249
	3	0.01996	0.323	0.363	112
	4	0.01734	0.285	0.364	84
	5	0.02246	0.297	0.396	114
	6	0.02207	0.451	0.392	85

* Note that E(vol squared) is a measure of var(ω), because the theory implies E(ω) = 0.

TABLE 4

OLS PARAMETER ESTIMATES FOR 13 SECURITIES

All estimates were highly significant

SYM	λ (OLS)	var(e) (OLS)	λ table 1	var(e) table 1
AA	.009	.017	.004	.018
AAA	.113	.017	.051	.021
AAE	.098	.018	.031	.022
ABC	.005	.010	*	*
ABF	.017	.023	.011	.025
ABT	.013	.014	.006	.015
ABY	.037	.026	.015	.028
ABZ	.065	.007	.032	.010
ACA	.010	.020	.002	.021

[31]

[31] * Estimation procedure did not converge - all parameters are not separately identified when λ is in the neighborhood of zero. See appendix 4 for details.

TABLE 5

HOURLY PARAMETER ESTIMATES FOR ALCOA ALUMINUM (AA)

HOUR	ALL	1	2	3	4	5	6
TRADES	5000	1068	1035	703	575	750	857
STRUCTURAL PARAMETERS							
v	variance of inside information						
	.01829	.01888	.01736	.01704	.01536	.01921	.02096
	(.00037)	(.00084)	(.00077)	(.00091)	(.00924)	(.00101)	(.00103)
ϕ	noise in the signal						
	.08212	.07315	.11480	.15360	.05075	.07211	.05401
	(.02404)	(.04194)	(.08388)	(.16174)	(.03239)	(.04988)	(.04988)
σ	variance of uninformed trade						
	.20940	.22395	.20938	.20777	.14253	.18319	.23239
	(.00334)	(.00759)	(.00888)	(.00765)	(.00691)	(.00769)	(.00904)
c	cost of information $						
	20.213	18.933	13.826	17.293	21.709	27.199	24.231
	(2.553)	(4.614)	(4.532)	(8.406)	(5.784)	(7.927)	(5.747)
SHALLOW PARAMETERS							
λ	slope of specialist's pricing function						
	.00412	.00512	.00412	.00246	.00411	.003714	.00604
	Adverse selection tariff for a 1,000 share order, $\$\lambda\omega^2$.						
	4.12	5.12	4.12	2.46	4.11	3.71	6.04
	Adverse selection tariff for a 10,000 share order, $\$\lambda\omega^2$.						
	412.00	512.00	412.00	246.00	411.00	371.00	604.00
θ	ex-ante probability a trader is informed						
	.00425	.00602	.00621	.00295	.00269	.00250	.00584
$\sigma^{1/2}$	average size of an uninformed trade (shares)						
	458	473	458	456	378	428	485
	average size of an informed trade (shares)						
	7002	6083	5790	8384	7267	8558	6332
$v^{1/2}$	average size of price shock $						
	.1353	.1374	.1318	.1305	.1240	.1386	.1448
$\phi^{1/2}$	average amount of noise $						
	.2866	.2705	.3388	.3919	.2253	.2685	.2324

[32,33]

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[32]Standard errors are given in parentheses directly below the point estimates.

[33]Test of whether hourly parameter estimates significantly differ from each other: ($X^2(20)$): 152.002 (> .995)

TABLE 6

TEST OF NONLINEAR RESTRICTIONS IMPLIED BY THE MODEL TEST OF LINEARITY OF PRICING FUNCTION FOR ALCOA ALUMINUM (* = UNIDENTIFIED)[34,35,36]

		THE MODEL	UNRESTRICTED ALTERNATIVE	NONLINEAR RULE
STRUCTURAL PARAMETERS				
v	variance of inside information			
		.01829	*	.01836
		(.00037)	*	(.00038)
ϕ	noise in the signal			
		.08212	*	.06900
		(.02404)	*	(.01472)
σ	variance of uninformed trade			
		.20940	.20843	.20948
		(.00334)	(.00448)	(.00334)
c	cost of information $			
		20.213	*	21.728
		(2.553)	*	(1.972)
SHALLOW PARAMETERS				
λ	slope of specialist's pricing function			
		.00412	.00419	.00444
		*	(.00070)	*
	Adverse selection tariff for a 1,000 share order, $\$\lambda\omega^2$.			
		4.12	4.19	4.44
	Adverse selection tariff for a 10,000 share order, $\$\lambda\omega^2$.			
		412.00	419.00	444.00
d	quadratic pricing function term			
		*	*	-.0000140
		*	*	(.0000141)
θ	ex-ante probability a trader is informed			
		.00425	.09048	.00426
		*	(.00767)	*
$\sigma^{1/2}$	average size of an uninformed trade (shares)			
		458	457	458
	average size of an informed trade (shares)			
		7002	8049	6995
		*	(369.395)	*
$v^{1/2}$	average size of price shock $			
		.1353	*	.1355

[34] *: The nonlinear restrictions identify some, but not all structural parameters.

[35] Standard errors are given in parentheses directly below the point estimate s.

[36] Test against unrestricted alternative: $X^2(2) = 612.059!$ (> .999).

$\phi^{1/2}$	average amount of noise \$		
	.2866	*	.2627

TABLE 7

PARAMETER ESTIMATES FOR OTHER NYSE SECURITIES

SYMBOL	AAA	AAE	ABC	ABF	ABT
TRADES	1380	1187	2000	945	2000
STRUCTURAL PARAMETERS					
v variance of inside information	.02392	.02380	.01150	.02571	.01563
	(.00092)	(.00092)	(.00037)	(.00118)	(.00048)
ϕ noise in the signal	.05379	.81912	.05457	.04121	.16636
	(.02044)	(.57903)	(.01604)	(.02035)	(.14401)
σ variance of uninformed trade	.0566	.0606	.4332	.1394	.2323
	(.02056)	(.00230)	(.01881)	(.01259)	(.00584)
c cost of information $	9.114	2.025	26.343	33.824	11.171
	(1.3236)	(.70085)	(3.4378)	(6.0060)	(4.5367)
SHALLOW PARAMETERS					
λ slope of specialist's pricing function	.0202	.0083	.0019	.0073	.0031
Adverse selection tariff for a 1,000 share order, $\$\lambda\omega^2$.	20.20	8.30	1.90	7.30	3.10
Adverse selection tariff for a 10,000 share order, $\$\lambda\omega^2$.	2020.00	830.00	190.00	730.00	310.00
θ ex-ante probability a trader is informed	.01239	.02424	.00311	.00300	.00630
$\sigma^{1/2}$ average size of an uninformed trade (shares)	238	246	658	373	482
average size of an informed trade (shares)	2124	1561	11775	6807	6052
$v^{1/2}$ average size of price shock $	.1547	.1543	.1073	.1603	.1250
$\phi^{1/2}$ average amount of noise $	.2319	.9051	.2336	.2030	.4046

[37]

[37] Standard errors are given in parentheses directly below the point estimates.

TABLE 8
TRANSACTION COST PARAMETER ESTIMATION FOR ALCOA ALUMINUM (* = UNIDENTIFIED)

PARAMETERS	NO TC	PER SHARE TC	PER SHARE TC + CRITICAL CUTOFF
sigma	variance of uninformed trade		
	.2084	.2085	.2085
	(.00448)	(.00449)	
c1	variable transaction cost		
	*	.03210	.03210
	*	(.00347)	
critical cutoff	shares		
	*	*	1400
λ	slope of specialist's pricing function		
	.00419	.00250	.00250
	(.00070)	(.00069)	
θ	ex-ante probability a trader is informed		
	.09048	.09050	.09050
	(.00767)	(.00767)	
$\sigma^{1/2}$	average size of an uninformed trade (shares)		
	457	457	457
	average size of an informed trade (shares)		
	8049	8062	8049
	(369.395)	(369.399)	

[38,39,40]

[38]*: The nonlinear restrictions identify some, but not all structural parameters.

[39]Standard errors are given in parentheses directly below the point estimates.

[40]We are unable to test formally whether the cutoff point is statistically different from zero, since we are unable to obtain convergence. Instead we estimate λ, 2g(s), c_1 and the cutoff, fixing the other parameters. Then, we perform a likelihood ratio test to see if the cutoff is significantly different from zero. The resulting chi-square value is: $X^2(1) = 248.32$ (> .99).

Appendix 2. Integration of transaction costs

Proof to lemma 2:

An agent who has paid a cost c to become informed now solves:

$$MAX_{x_i} E[x_i(\delta - \lambda\omega - c_1 I) - c_2].$$

Differentiating with respect to x_i and applying the projection theorem yields:

$$x_i = \left[\frac{v(\delta + \varepsilon)}{(v + \phi)} - c_1 I\right]\frac{1}{2\lambda}.$$

However an informed agent will not choose to sell (buy) if he receives a signal greater (less) than zero. []

Proof to lemma 2a:

Recall that δ can be written as:

$$\delta = E(\delta \mid \delta + \varepsilon) + \xi.$$

Applying the projection theorem,

$$\delta = \frac{v}{v + \phi}(\delta + \varepsilon) + \xi. \tag{18}$$

To calculate the variance of δ given $\delta + \varepsilon$, first compute the variances of both sides of (18). This yields:

$$v = \frac{v^2}{v + \phi} + var(\xi).$$

Since $E(\delta \mid \delta + \varepsilon)$ is not random, $var(\xi) = var(\delta \mid \delta+\varepsilon)$. Thus, $var(\xi) = \frac{v\phi}{v+\phi}$. To obtain $E((\delta + \varepsilon)^2 \mid \delta + \varepsilon)$ just recall that $var(\delta \mid \delta+\varepsilon) = E(\delta^2 \mid \delta+\varepsilon) - (E(\delta \mid \delta+\varepsilon))^2$.

Then to obtain $E(\delta\varepsilon \mid \delta + \varepsilon)$, observe that $E((\delta + \varepsilon)^2 \mid \delta + \varepsilon) = E(\delta^2 \mid \delta + \varepsilon) + E(2\delta\varepsilon \mid \delta + \varepsilon) + E(\varepsilon^2 \mid \delta + \varepsilon) = (\delta + \varepsilon)^2$. Solving for $E(\varepsilon^2 \mid \delta + \varepsilon)$ as above and then solving for $E(2\delta\varepsilon \mid \delta + \varepsilon)$ yields the result. []

Simplifying assumptions used to produce figures 2-9.

Under the assumptions that δ, ε, and z are standard normal random variables,

$$(\delta + \varepsilon)^* = \sqrt{8c_2\lambda},$$

so that (14) reduces to:

$$\int_{-\infty}^{\infty}\int_{\sqrt{8c_2\lambda}-\delta}^{\infty}(3\delta^2 + 2\varepsilon\delta - \varepsilon^2)f_\delta\delta f_\varepsilon\varepsilon d\varepsilon d\delta = 8\lambda[c + 2c_2(1 - \Phi(2\sqrt{c_2\lambda}))].$$

Substituting w = δ + ε, and switching the order of integration, the left hand side becomes:

$$\frac{1}{2\pi}\int_{\sqrt{8c_2\lambda}}^{\infty}\int_{-\infty}^{\infty}[4w\delta - w^2]e^{-\delta^2/2}e^{-(w-\delta)^2/2}d\delta dw$$

$$= \frac{1}{2\pi}\int_{\sqrt{8c_2\lambda}}^{\infty}4we^{w^2/4}\int_{-\infty}^{\infty}\delta e^{-(\delta-w/2)^2}d\delta dw - \frac{1}{2\pi}\int_{\sqrt{8c_2\lambda}}^{\infty}w^2e^{w^2/4}\int_{-\infty}^{\infty}e^{-(\delta-w/2)^2}d\delta dw.$$

Substituting $x = \sqrt{2}\delta - w/\sqrt{2}$, we obtain:

$$\int_{\sqrt{8c_2\lambda}}^{\infty}\int_{-\infty}^{\infty}\frac{e^{-w^2/4}}{\pi}w(x + \frac{w}{\sqrt{2}})e^{-x^2/2}\,dxdw - \int_{\sqrt{8c_2\lambda}}^{\infty}\frac{1}{2\sqrt{\pi}}w^2e^{-w^2/4}dw.$$

Noting that:

$$\int_{-\infty}^{\infty}xe^{-x^2/2}dx = 0,$$

this reduces to

$$\int_{\sqrt{8c_2\lambda}}^{\infty}\frac{1}{2\sqrt{\pi}}w^2e^{-w^2/4}dw.$$

Integrating by parts, letting u=w and dv=$we^{-w^2/4}$, and solving, the left hand side equals:

$$\left[\frac{8c_2\lambda}{\pi}\right]^{1/2}e^{-2c_2\lambda} + 2(1 - \Phi(2\sqrt{c_2\lambda})).$$

Solving implicitly for λ yields the result. The same integration techniques are adopted to solve the general problem with arbitrary variances, v, ϕ and σ.

Appendix 3. Proof to proposition 3.

Proof to Proposition 3:

Given the respective arrival rates of the informed and uninformed, π/N, and η/M, we have

$$pr(j\ trades\ by\ uninformed) = \lim_{M \to \infty} \left[\binom{M}{j} \left(\frac{\eta}{M}\right)^{j} \left(1 - \frac{\eta}{M}\right)^{M-j} \right].$$

Note first that:

$$\lim_{M \to \infty} \left[\frac{M!}{(M-j)!M^j} \right] = 1,$$

and

$$\lim_{M \to \infty} \left(1 - \frac{\eta}{M}\right)^{M-j} = e^{-\eta},$$

a Poisson distribution. Therefore,

$$pr(j\ trades\ by\ uninformed) = \frac{\eta^j e^{-\eta}}{j!}.$$

Similarly,

$$pr(j\ samples\ by\ informed) = \frac{\pi^j e^{-\pi}}{j!},$$

and the resulting probability of j trades by the uninformed is given by:

$$pr(j\ trades\ by\ informed) = \frac{\gamma\pi^j e^{-\pi}}{j!}.$$

Now,

$$pr(event \mid no\ trade) = \frac{pr(event \cap no\ sample \cap no\ trade)}{pr(event \cap no\ sample \cap no\ trade) + pr(no\ event \cap no\ trade)}$$

$$= \frac{\gamma e^{-\pi} e^{-\eta}}{\gamma e^{-\pi} e^{-\eta} + (1-\gamma)e^{-\eta}},$$

$$= \frac{\gamma e^{-\pi}}{\gamma e^{-\pi} + (1-\gamma)}.$$

Note that pr(event | j trades) = 1 - pr(no event | j trades) and:

$$pr(no\ event \mid j\ trades) = \frac{pr(no\ event \mid j\ liquidity\ trades)}{pr(j\ trades)}$$

$$= \frac{(1-\gamma)\eta^j e^{-\eta}}{j!} \left(\frac{(1-\gamma)\eta^j e^{-\eta}}{j!} + \gamma \sum_{i=0}^{j} \frac{\pi^i e^{-\pi} \eta^{j-i} e^{-\eta}}{i!(j-i)!} \right)^{-1},$$

which reduces to:

$$pr(no\ event \mid j\ trades) = \frac{(1-\gamma)\eta^j}{(1-\gamma)\eta^j + \gamma e^{-\pi} \sum_{i=0}^{j} \binom{j}{i} \pi^i \eta^{j-i}}.$$

Define $\theta = \frac{\pi}{\pi+\eta}$ so $1-\theta = \frac{\eta}{\pi+\eta}$. Substituting, we obtain:

$$pr(noevent \mid j\ trades) = \frac{(1-\gamma)\eta^j}{(1-\gamma)\eta^j + (\pi+\eta)\gamma e^{-\pi} \sum_{i=0}^{j} \binom{j}{i} \theta^i (1-\theta)^{j-i}},$$

$$= \frac{(1-\gamma)\eta^j}{(1-\gamma)\eta^j + (\pi+\eta)\gamma e^{-\pi}}.$$

Therefore,

$$pr(event \mid j\ trades) = \frac{(\pi+\eta)^j \gamma e^{-\pi}}{(1-\gamma)\eta^j + (\pi+\eta)^j \gamma e^{-\pi}}.[]$$

Appendix 4.

Derivation of the unconditional joint density of Δp_s and $|\omega_s|$ in the absence of transaction costs.

Under weak regularity conditions (Amemiya [1985]), maximum likelihood maximizers {B]} are consistent efficient estimators of $\{\beta_0\}$ for mixtures of normal distributions. Let L(B) = L({Δp,$|\omega|$,B}) be unconditional likelihood function for the price series for Δp and $|\omega|$. The error, e_s is conditionally normally distributed with mean zero and variance σ_j, where j = 1 if the specialist faced an informed trader *at time s - t* (probability θ) and j = 2 if the specialist faced an uninformed trader then (probability 1 - θ). Let I_s, the bid-ask indicator, equal one if a trader is buying and minus one if he is selling. Finally, ω is conditionally normally distributed with mean zero and variance σ_w, where w = 3 if the specialist faces an informed agent *at time s* (probability θ) and w = 4 if that agent is uninformed (probability 1 - θ).

Given our assumptions, the conditional density of Δp_s given past prices, bid-ask indicator, I_s, variance indicators, j and w, is given by:

$$f_s(\Delta p,\mathrm{B} \mid I,j,\bullet) = \frac{1}{\sqrt{2\pi\sigma_j}} e^{\frac{-(\Delta p_s - \lambda|\omega_s|I_s)^2}{2\sigma_j}}. \tag{19}$$

Integrating over the unconditional marginal distribution for the bid-ask indicator I_s[41] yields:

$$f_s(\Delta p,\mathrm{B} \mid j,\bullet) = \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma_j}} e^{\frac{-(\Delta p_s - \lambda|\omega_s|)^2}{2\sigma_j}} + \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma_j}} e^{\frac{-(\Delta p_s + \lambda|\omega_s|)^2}{2\sigma_j}}. \tag{20}$$

Glosten and Harris [1989] would write this as the *unconditional* density for Δp_s in the absence of

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[41] In this model, buying and selling is equally likely. However, this procedure can accommodate more complicated forms for this distribution, for example, serial correlation. Serial correlation might arise due to portfolio insurance strategies.

transaction costs.[42] Here we must still integrate over the unconditional marginal density for j. In general, the probability a particular agent is informed is a function of the trade size. Define θ'_s as the pr(informed | $|\omega_s|$). Using Bayes' rule yields:

$$\theta'_s = \left(\theta\sqrt{\sigma_4} e^{\frac{-\omega^2}{2\sigma_3}} \right) \left(\theta\sqrt{\sigma_4} e^{\frac{-\omega^2}{2\sigma_3}} + (1-\theta)\sqrt{\sigma_3} e^{\frac{-\omega^2}{2\sigma_4}} \right)^{-1} . \tag{21}$$

Now:

$$f_s(\Delta p,\mathrm{B} \mid w,\bullet) = \theta'_{s-t}\left[\frac{1}{2}\frac{1}{\sqrt{2\pi\sigma_1}}e^{\frac{-(\Delta p_s-\lambda|\omega_s|)^2}{2\sigma_1}} + \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma_1}}e^{\frac{-(\Delta p_s+\lambda|\omega_s|)^2}{2\sigma_1}}\right] + \tag{22}$$

$$(1-\theta'_{s-t})\left[\frac{1}{2}\frac{1}{\sqrt{2\pi\sigma_2}}e^{\frac{-(\Delta p_s-\lambda|\omega_s|)^2}{2\sigma_2}} + \frac{1}{2}\frac{1}{\sqrt{2\pi\sigma_2}}e^{\frac{-(\Delta p_s+\lambda|\omega_s|)^2}{2\sigma_2}}\right].$$

Using only information contained in the price series, the unconditional likelihood function can be written as:

$$L(\mathrm{B}) = \prod_{s=1}^{n} f_s(\Delta p \mid \bullet).$$

However, the volume series provides additional information. Integrating over the unconditional marginal distribution for w, we find the unconditional density for ω_s:

$$g_s(\omega|\bullet) = \frac{\theta'_s}{\sqrt{2\pi\sigma_3}}e^{\frac{-\omega^2}{2\sigma_3}} + \frac{(1-\theta'_s)}{\sqrt{2\pi\sigma_4}}e^{\frac{-\omega^2}{2\sigma_4}}. \tag{23}$$

Unfortunately, it is not appropriate to write some $h_s = f_s g_s$ and then write the unconditional likelihood function as $\prod_{s=1}^{n} h_s(\Delta p,|\omega| \mid \bullet)$. The reason is that lagged ω's determine σ_j and current ω's determine the distribution for σ_w. The unconditional likelihood function L has a recursive structure.

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[42]However, they estimate a model with transaction costs. In addition, their model places no structure on the probability the specialist faces an informed trader. Instead, they assume that traded quantities are normally distributed. Therefore, their unconditional likelihood function is an equally weighted average over 2^n unobserved bid-ask paths $(I_1,\ldots,I_n)$ of the conditional densities detailed above. That is:

$$L(g(D)p,\mathrm{B}) = \sum_{1}^{2^n} \frac{1}{2^n} \prod_{s=1}^{n} f_s(\Delta p,\mathrm{B} \mid I,\bullet).$$

This can be computed recursively, as detailed in Glosten and Harris [1989].

Define f_s^{**} as the conditional joint density of Δp_s and ω_s, where the first superscript indicates whether the trader at time s-t was informed and the second superscript indicates whether the trader at time s is informed. Let * = i if a trader is informed and u otherwise. Thus:

$$f_s^{ii} = \frac{\theta'_{s-t}}{2\sqrt{2\pi\sigma_1}} e^{\frac{-(\Delta p_s - \lambda|\omega_s|)^2}{2\sigma_1}} \frac{\theta'_s}{\sqrt{2\pi\sigma_3}} e^{\frac{-\omega_s^2}{2\sigma_3}} + \frac{\theta'_{s-t}}{2\sqrt{2\pi\sigma_1}} e^{\frac{-(\Delta p_s + \lambda|\omega_s|)^2}{2\sigma_1}} \frac{\theta'_s}{\sqrt{2\pi\sigma_3}} e^{\frac{-\omega_s^2}{2\sigma_3}},$$

$$f_s^{ui} = \frac{(1-\theta'_{s-t})}{2\sqrt{2\pi\sigma_2}} e^{\frac{-(\Delta p_s - \lambda|\omega_s|)^2}{2\sigma_2}} \frac{\theta'_s}{\sqrt{2\pi\sigma_3}} e^{\frac{-\omega_s^2}{2\sigma_3}} + \frac{(1-\theta'_{s-t})}{2\sqrt{2\pi\sigma_2}} e^{\frac{-(\Delta p_s + \lambda|\omega_s|)^2}{2\sigma_2}} \frac{\theta'_s}{\sqrt{2\pi\sigma_3}} e^{\frac{-\omega_s^2}{2\sigma_3}},$$

$$f_s^{iu} = \frac{\theta'_{s-t}}{2\sqrt{2\pi\sigma_1}} e^{\frac{-(\Delta p_s - \lambda|\omega_s|)^2}{2\sigma_1}} \frac{(1-\theta'_s)}{\sqrt{2\pi\sigma_4}} e^{\frac{-\omega_s^2}{2\sigma_4}} + \frac{\theta'_{s-t}}{2\sqrt{2\pi\sigma_1}} e^{\frac{-(\Delta p_s + \lambda|\omega_s|)^2}{2\sigma_1}} \frac{(1-\theta'_s)}{\sqrt{2\pi\sigma_4}} e^{\frac{-\omega_s^2}{2\sigma_4}},$$

$$f_s^{uu} = \frac{(1-\theta'_{s-t})}{2\sqrt{2\pi\sigma_2}} e^{\frac{-(\Delta p_s - \lambda|\omega_s|)^2}{2\sigma_2}} \frac{(1-\theta'_s)}{\sqrt{2\pi\sigma_4}} e^{\frac{-\omega_s^2}{2\sigma_4}} + \frac{(1-\theta'_{s-t})}{2\sqrt{2\pi\sigma_2}} e^{\frac{-(\Delta p_s + \lambda|\omega_s|)^2}{2\sigma_2}} \frac{(1-\theta'_s)}{\sqrt{2\pi\sigma_4}} e^{\frac{-\omega_s^2}{2\sigma_4}}.$$

Now define $f_s^i = f_s^{ii} + f_s^{ui}$ and $f_s^u = f_s^{iu} + f_s^{uu}$. Also, let $L_1 = f_1^i + f_1^u$. Now:

$$L_2 = f_1^i(f_2^{ii} + f_2^{iu}) + f_1^u(f_2^{ui} + f_2^{uu}).$$

In general:

$$L_s = f_{s-t}^i(f_s^{ii} + f_s^{iu}) + f_{s-t}^u(f_s^{ui} + f_s^{uu}),$$

where:

$$f_s^i = f_{s-t}^i f_s^{ii} + f_{s-t}^u f_s^{ui},$$

and,

$$f_s^u = f_{s-t}^i f_s^{iu} + f_{s-t}^u f_s^{uu}.$$

Indexing by transaction number rather than by transaction time yields:

$$L_n = f_{n-1}^i(f_n^{ii} + f_n^{iu}) + f_{n-1}^u(f_n^{ui} + f_n^{uu}),$$

where $L_n = L(\Delta p, |\omega| \mid \bullet)$. If there is a break in the transaction price series for any reason, e.g. the end of the day, we restart the iterative process. That is, θ'_{s-t} is reset to θ. Thus, the log of the likelihood function becomes the sum of the log likelihoods over the regions where the iteration occurs. Were there M breaks, the log of the likelihood, LL, would be represented by:

$$LL = \sum_{m=1}^{M} log[L_{n_m}]$$

This assumption makes the most sense overnight, where there is a long interval without trade: Information release continues overnight. When a break occurs in the middle of the day, we prefer to throw away information rather than pollute the time series by introducing observations which may be generated by a different forcing process.

In general, the likelihood function depends on the parameters λ, σ_1, σ_2, σ_3, σ_4, and θ. However, these are functions of structural parameters, v ϕ, c, σ, and k, a scaling constant which is needed because time periods are not necessarily scaled in minutes. In the estimation, we finesse this issue by assuming that there is a single innovation between trades.

Recall:

$$\lambda = \frac{v^2}{4c(v+\phi)}; \quad \beta = \frac{2c}{v}; \quad \theta = \frac{\sigma v^2}{\sigma v^2 + 4c^2(v+\phi)}.$$

In addition, $\sigma_1 = E[(\delta - \lambda\omega)^2 \mid \text{informed trader}]$ and $\sigma_2 = E[(\delta - \lambda\omega)^2 \mid \text{uninformed trader}]$. By substituting for λ and recalling that $\sigma_3 = \beta^2(v + \phi)$ and $\sigma_4 = \sigma$, we obtain:

$$\sigma_1 = kvt - \frac{3v^2}{4(v+\phi)}$$
$$\sigma_2 = kvt - \frac{v^4\sigma}{16c^2(v+\phi)^2}.$$

Appendix 5. Derivation of the likelihood function used to generate table 1.

This likelihood function is used to generate the results in table 1. Variable transaction costs are not considered ($c_1 = 0$). Also, e_s has constant variance. As a result, *this* likelihood function need not be derived recursively. Recall that $J_t = 1$ only if ω exceeds ω^*. As a diagnostic, we subtract $\mu_0 + \mu_1 t$ to check for deterministic components to price changes. Therefore, -2log[l({p})] - (constant) can be written as:

$$-2logL = \sum_{t=1}^{n} \left\{ log(var(e_t)) - 2log\left[e^{\frac{(\Delta p - \lambda|\omega|J_t - \mu_0 - \mu_1 t)^2}{2var(e)}} + e^{\frac{(\Delta p + \lambda|\omega|J_t - \mu_0 - \mu_1 t)^2}{2var(e)}} \right] \right\}.$$

Define the expression in square brackets as T1 + T2, where T1 is the first exponential, and T2 the second. Further, let T1 = $e^{-E1/2var(e)}$ and T2 = $e^{-E2/2var(e)}$. Taking the derivatives with respect to var(e), μ_0, μ_1, and λ, we obtain:

$$\sum_{t=1}^{n} \frac{-2\partial l_t}{\partial \lambda} = \sum_{t=1}^{n} \frac{-2|\omega U_t}{(T1 + T2)var(e)}(E1 \times T1 - E2 \times T2) = 0, \tag{24}$$

$$\sum_{t=1}^{n} \frac{-2\partial l_t}{\partial \mu_0} = \sum_{t=1}^{n} \frac{-2}{(T1 + T2)var(e)}(E1 \times T1 + E2 \times T2) = 0, \tag{25}$$

$$\sum_{t=1}^{n} \frac{-2\partial l_t}{\partial var(e)} = \sum_{t=1}^{n} \left[1 - \frac{-1}{(T1 + T2)(var(e))^2}(E1^2 T1 + E2^2 \times T2)\right] = 0. \tag{26}$$

All 4 parameters are theoretically identified, except when λ approaches zero. Then, T1 approaches T2, and (24) collapses to zero, leaving the first derivative matrix short of full rank.

Appendix 6

Here, we introduce both fixed and per-share transaction costs to the model.

Recall that $J_s = 1$ if $\omega > \omega^*$. Let $K_s = 1$ if $I_s = 1$ and $I_{s-t} = -1$, $K_s = -1$ if $I_s = -1$ and $I_{s-t} = 1$, and $K_s = 0$ otherwise. That is, there is no bid-ask "bounce" if I_{s-t} and I_s have the same sign.

We begin by writing the conditional density for Δp_s:

$$f_s^{\bullet\bullet} = f_s(\Delta p, B \mid I_s, I_{s-t} j_s, \bullet) = \frac{1}{\sqrt{2\pi\sigma_j}} e^{\frac{(\Delta p_s - \lambda|\omega_s|I_s J_s - 2c_1 K_s)^2}{2\sigma_j}}, \tag{27}$$

where $\bullet$ = b if I = 1 and s otherwise. Glosten and Harris [1989] have $J_s = 1$, and do not distinguish between informed and uninformed traders. They need only weight the 2^n possible bid-ask paths to construct the likelihood function for Δp.

Here, estimation is complicated by the dependence of σ on past realizations of volume, ω(s-t), due to the unobserved identity (informed or uninformed) of the trader in both at time t and at time t-1. Thus, we also weight over all possible sequences of identities. We write the likelihood function in pieces in order to highlight its recursive construction. Define

$$f_n^{\bullet\bullet\bullet\bullet} = \frac{pr(\bullet\bullet\bullet\bullet)}{8\pi\sqrt{\sigma_{j,n-1}\sigma_{j,n}}} e^{\frac{-\omega_s^2}{2\sigma_j}} e^{\frac{-(\Delta p_s - \lambda|\omega_s|I_s J_s - 2c_1 K_s)^2}{2\sigma_j}}, \tag{28}$$

where the four arguments in the superscript of $f_n^{\bullet\bullet\bullet\bullet}$ are: $\{\{i,u\}_{n-1}, \{i,u\}_n, \{b,s\}_{n-1}, \{b,s\}_n\}$. That is, the first argument is i if an informed agent traded in period n-1 and u otherwise; the second argument is i if an informed agent traded in period n and u otherwise; the third argument is b if an agent bought period n - 1 and s otherwise; and the fourth argument is b if an agent bought period n and s otherwise. $pr(\bullet\bullet\bullet\bullet)$ is the probability of a particular state occurring. Note that when ω is below the critical cutoff, the probability of an informed trader must be zero. The recursive structure is similar to that given in appendix 4, but there are now more terms:

$$L_n = f^{ib}_{n-1}[f^{iibb}_n + f^{iibs}_n + f^{iubb}_n + f^{iubs}_n] + f^{is}_{n-1}[f^{iisb}_n + f^{iiss}_n + f^{iusb}_n + f^{iuss}_n] +$$
$$f^{ib}_{n-1}[f^{uibb}_n + f^{uibs}_n + f^{uubb}_n + f^{uubs}_n] + f^{ib}_{n-1}[f^{uisb}_n + f^{uiss}_n + f^{uusb}_n + f^{uuss}_n]$$

where:

$$f^{ib}_n = f^{ib}_{n-1} f^{iibb}_n + f^{is}_{n-1} f^{iisb}_n + f^{ub}_{n-1} f^{uibb}_n + f^{us}_{n-1} f^{uisb}_n,$$

$$f^{is}_n = f^{ib}_{n-1} f^{iibs}_n + f^{is}_{n-1} f^{iiss}_n + f^{ub}_{n-1} f^{uibs}_n + f^{us}_{n-1} f^{uiss}_n,$$

$$f^{ub}_n = f^{ib}_{n-1} f^{iubb}_n + f^{is}_{n-1} f^{iusb}_n + f^{ub}_{n-1} f^{uubb}_n + f^{us}_{n-1} f^{uusb}_n,$$

$$f^{us}_n = f^{ib}_{n-1} f^{iubs}_n + f^{is}_{n-1} f^{iuss}_n + f^{ub}_{n-1} f^{uubs}_n + f^{us}_{n-1} f^{uuss}_n.$$

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Figure 1: Intraday Volume and Variance

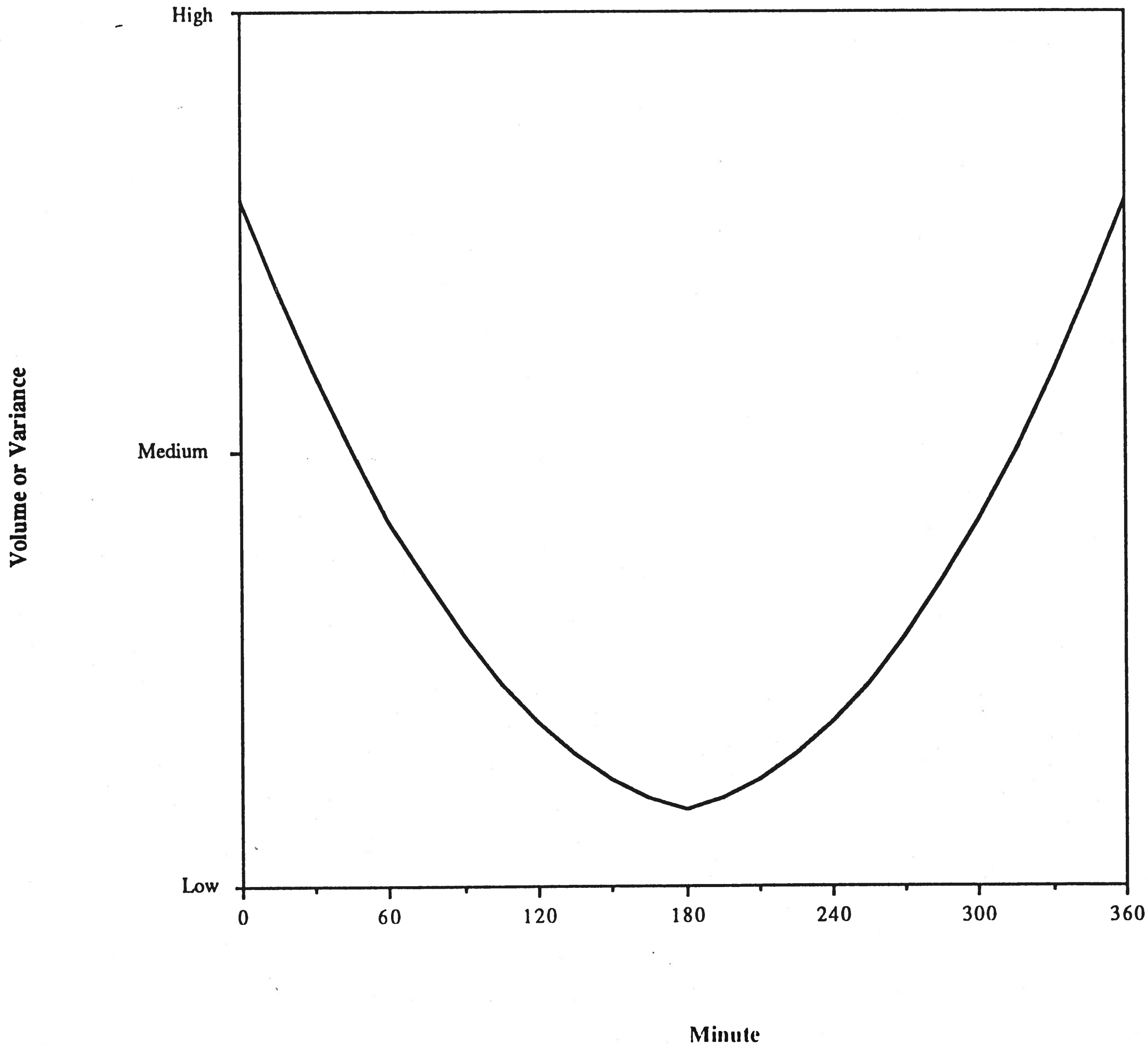


Figure 2: Share Transaction Price with Transaction Cost

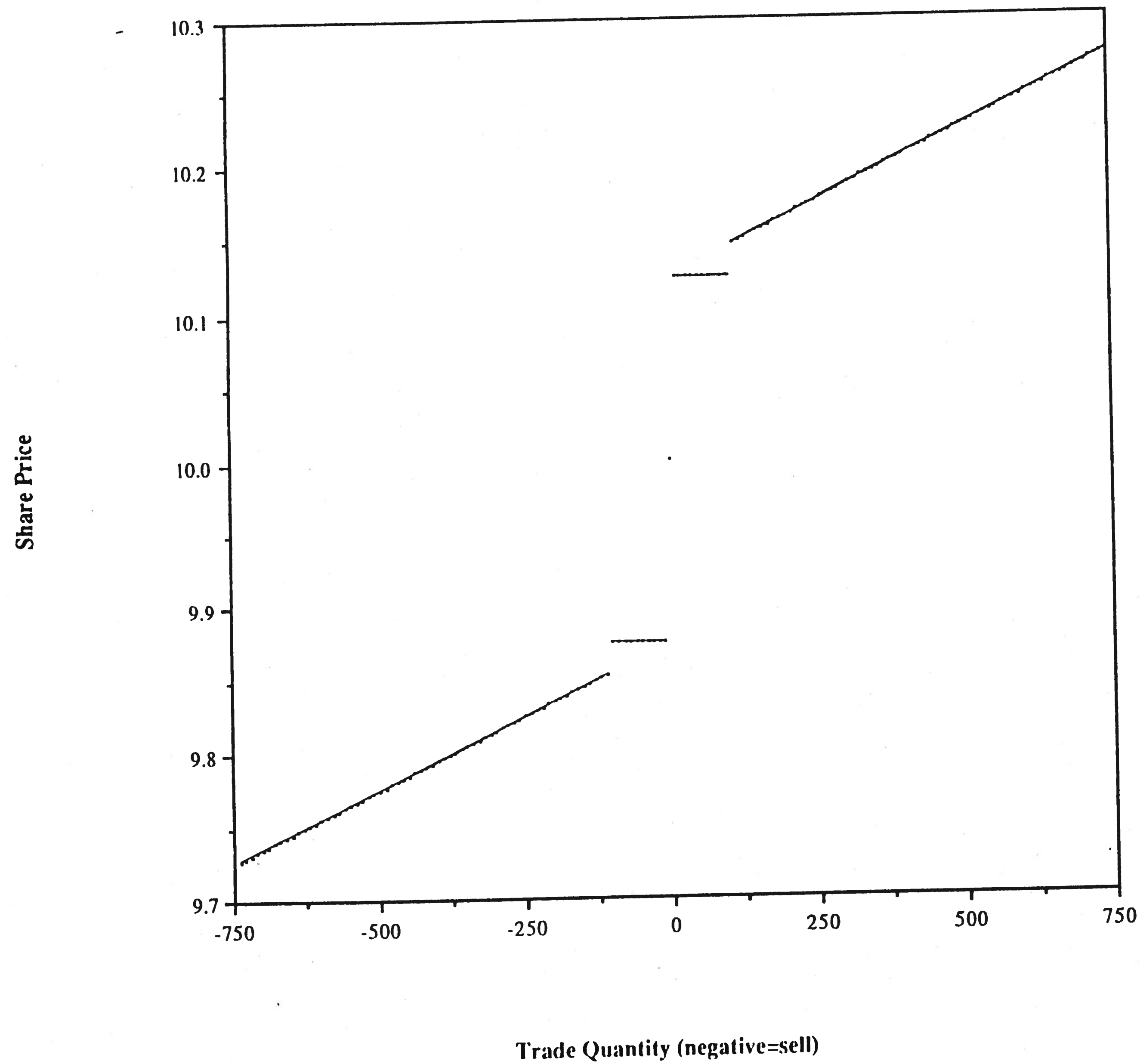


Figure 3: Lambda, Pi/Eta vs. Trading Costs, c=.5

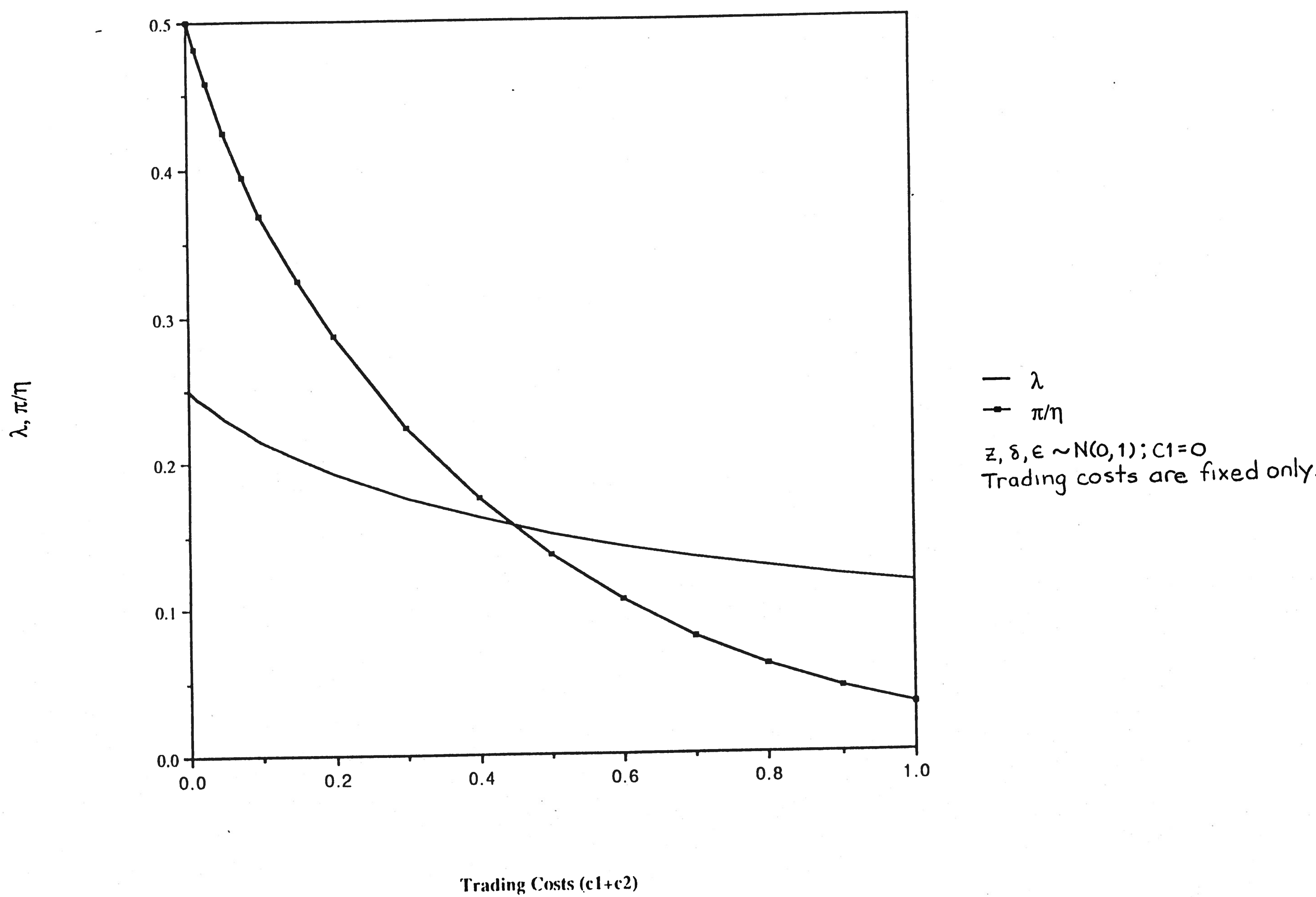


Figure 4: Lambda, Pi/Eta vs. Trading Costs, c=1.

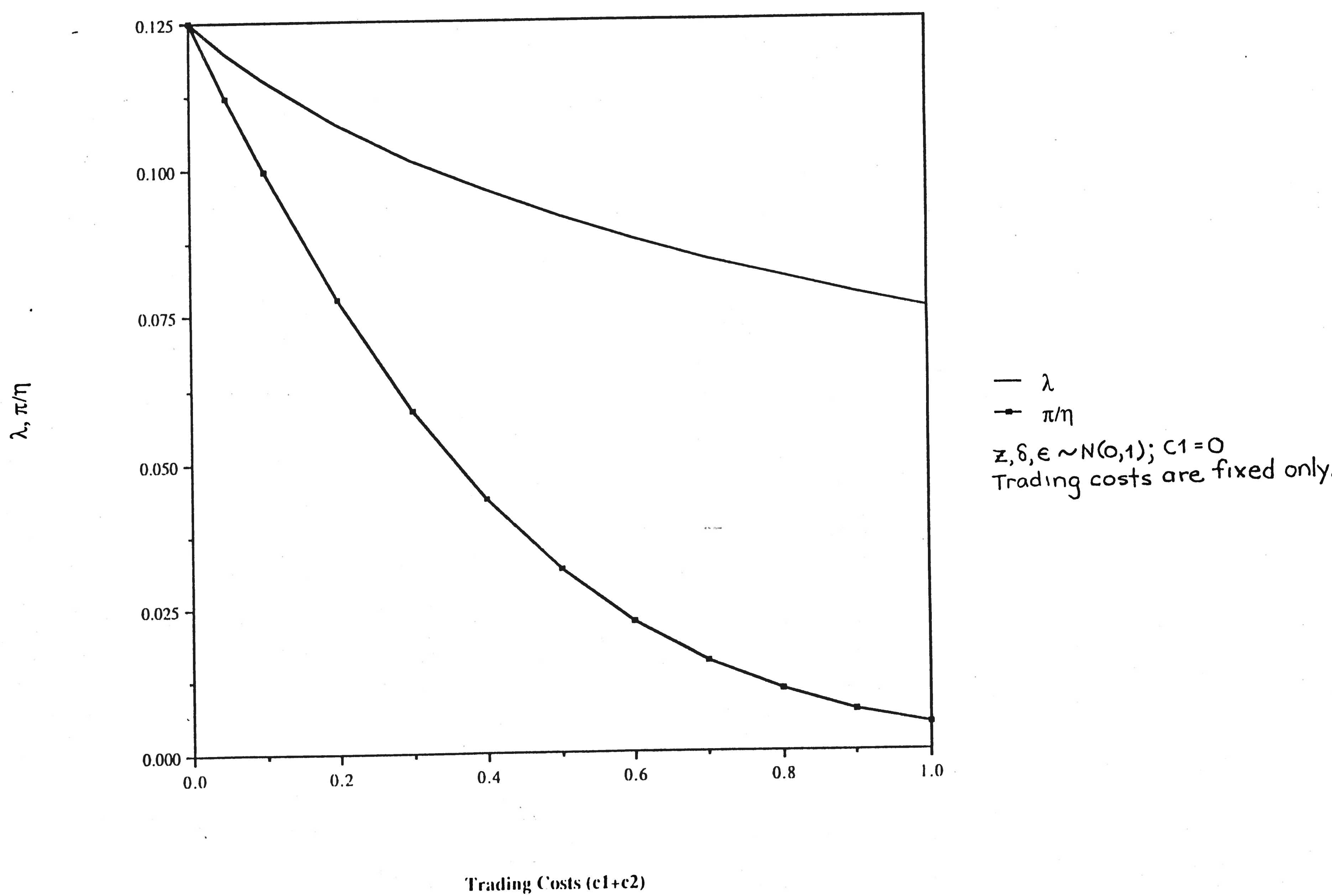
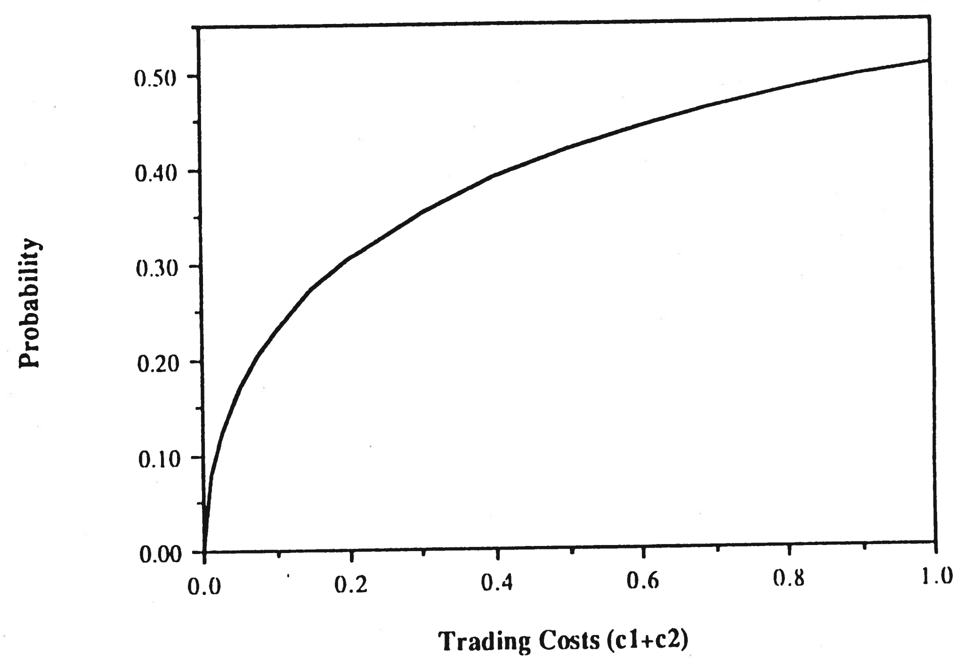


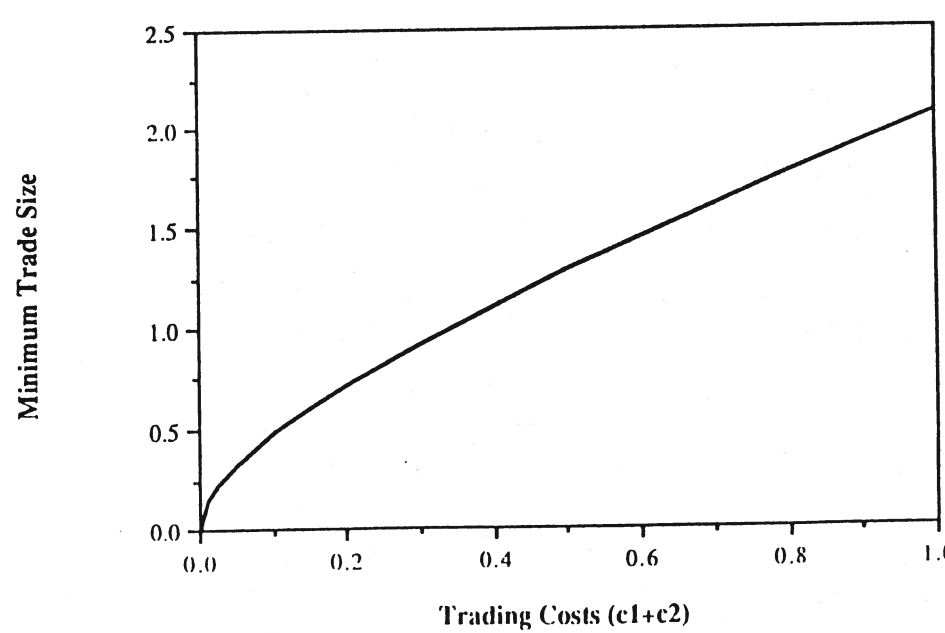
Figure 5a: Prob(No Trade|Informed) vs. Trading Costs, c=.5



Probability signal is small enough that informed cannot recoup trading costs by trading.

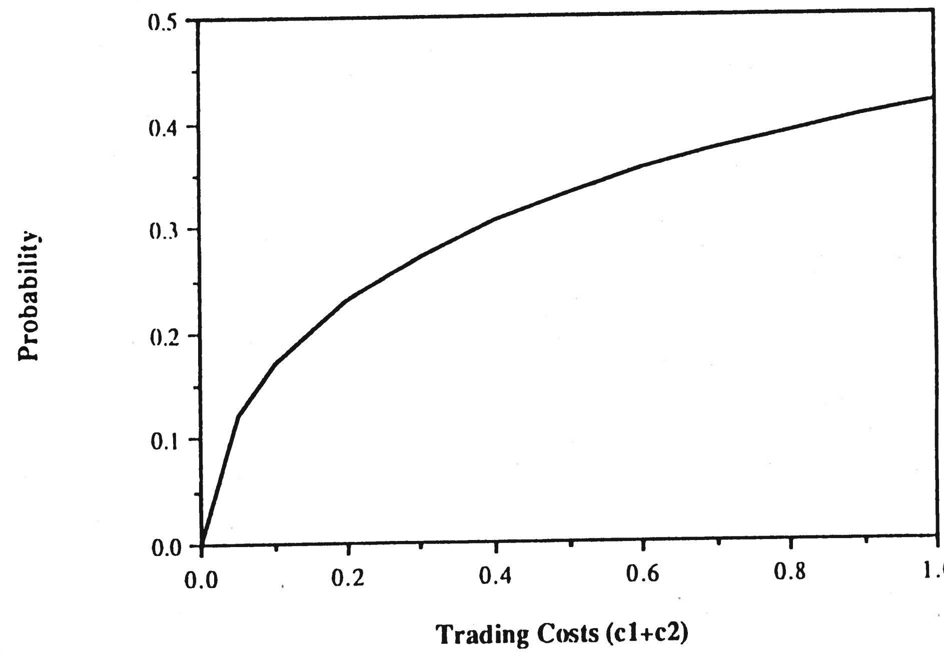
$z, \delta, \epsilon \sim N(0,1)$; $C1 = 0$
Trading costs are fixed only.

Figure 5b: Minimum Informed Trade Size vs. Trading Costs, c=.5



Quantity below which trade by informed is unprofitable.

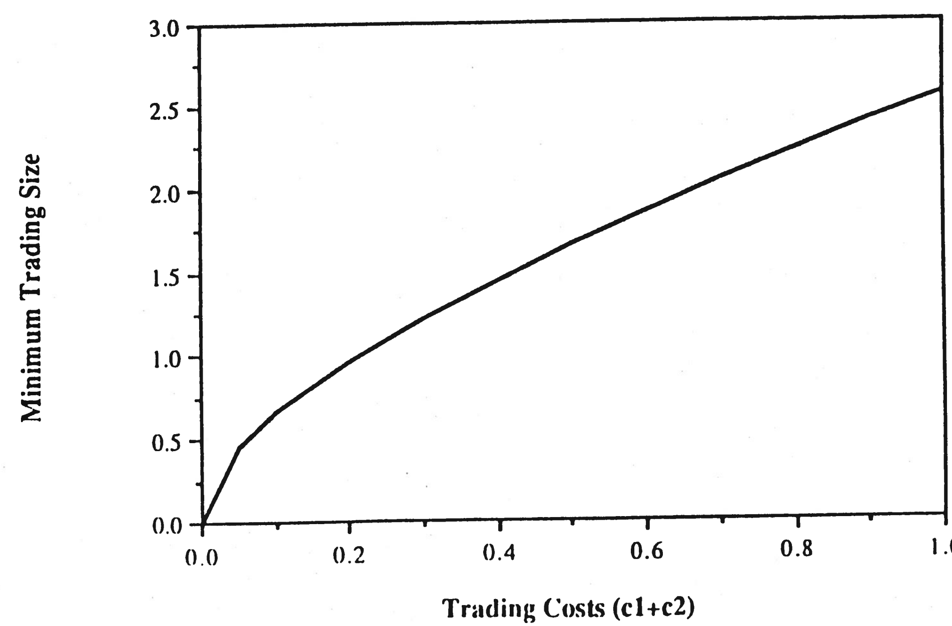
Figure 6a: Prob(No Trade|Informed) vs. Trading Costs, c=1.0



Probability signal is small enough that informed cannot recoup fixed costs by trading.

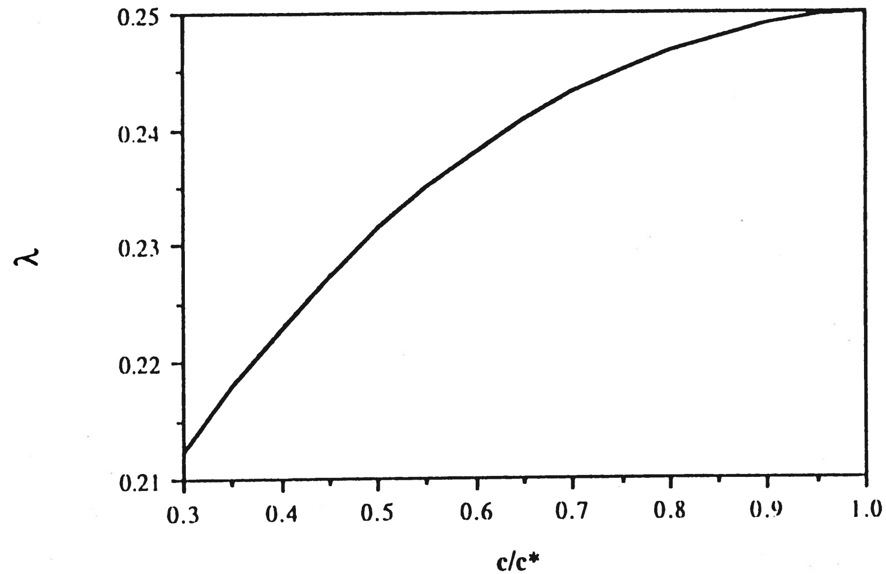
$z, \delta, \epsilon \sim N(0,1)$; $C1 = 0$
Trading costs are fixed only.

Figure 6b: Minimum Informed Trade Size vs. Trading Costs, c=1.0



Quantity below which trade by informed is unprofitable.

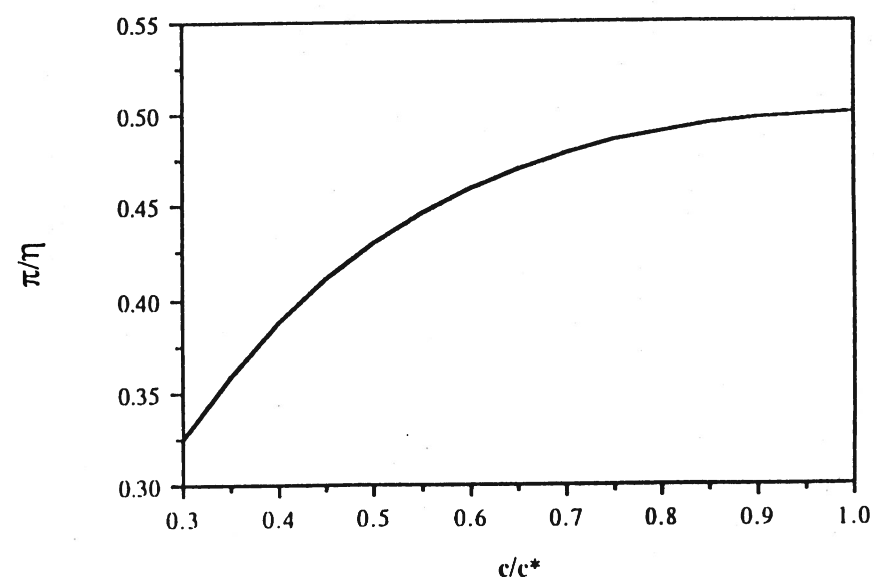
Figure 7a: Lambda vs c/c*



Lambda rises as information acquisition cost comprises a larger fraction of fixed costs.

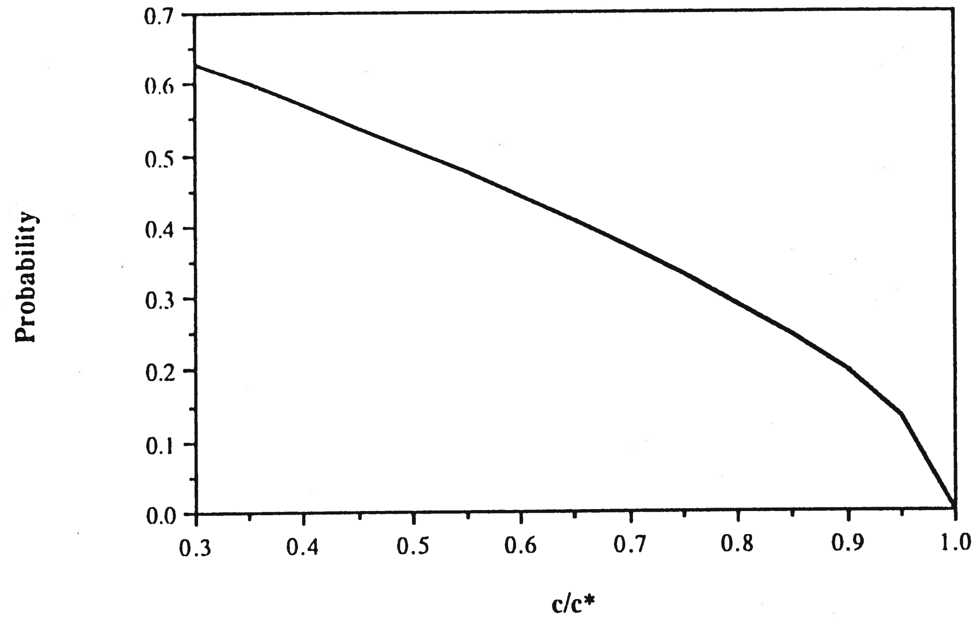
$z, \delta, \epsilon \sim N(0,1)$; $C1 = 0$
Trading costs are fixed only.

Figure 7b: Pi/Eta vs. c/c*



Probability specialist faces an informed trader also rises as acquisition cost comprises a larger fraction of fixed costs.

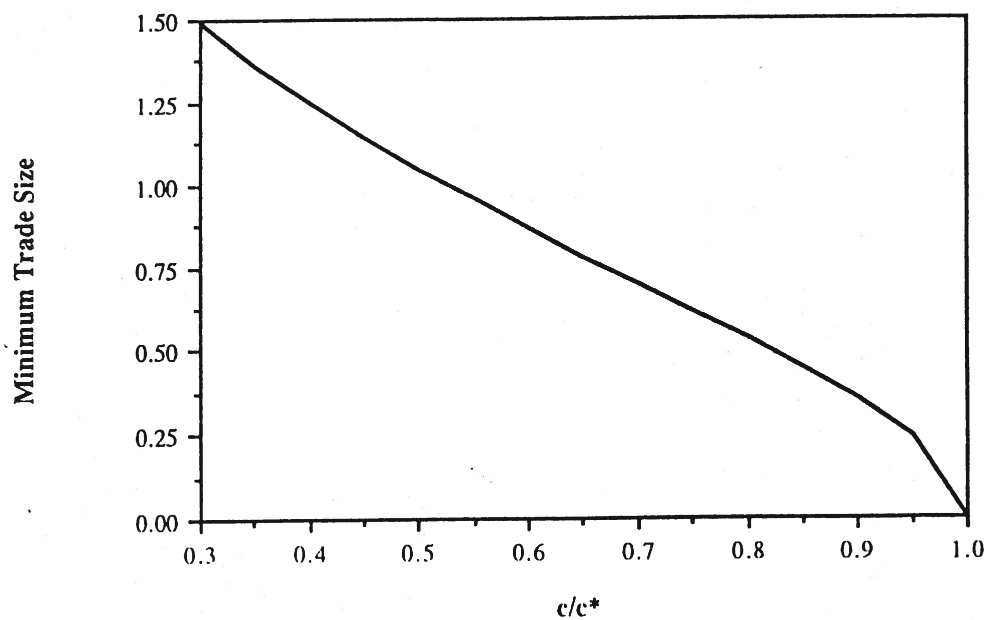
Figure 8a: Prob(No Trade|Informed) vs. c/c*



The probability an informed trader will walk away without trading decreases as trading costs form a smaller part of fixed costs.

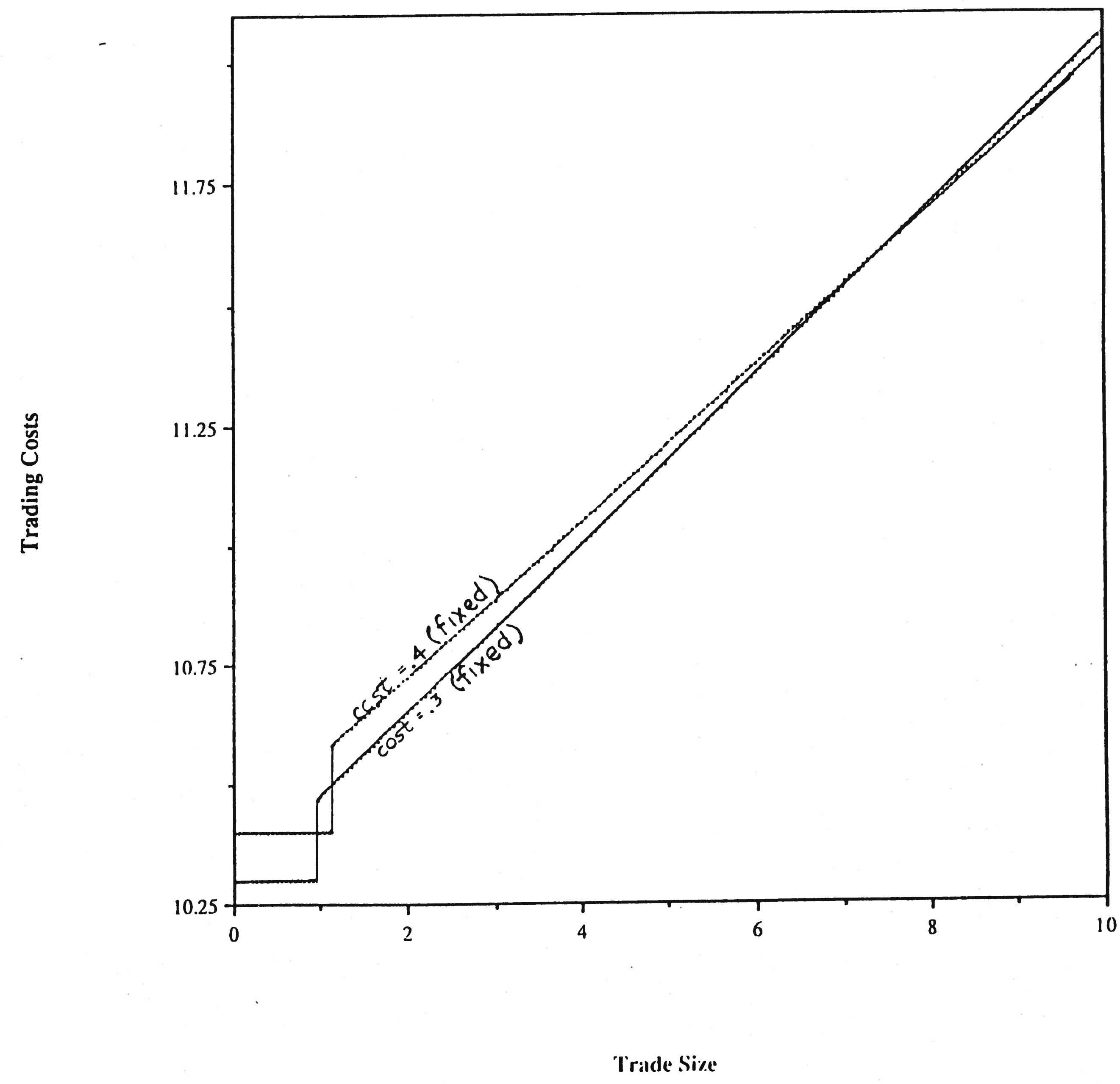
$z, \delta, \epsilon \sim N(0,1)$; $C1 = 0$
Trading costs are fixed only.

Figure 8b: Minimum Informed Trade Size vs. c/c*



As trading costs diminish relative to acquisition costs, $w^* \longrightarrow 0$.

Figure 9: Trading Cost vs. Trade Size, Different Txn Costs



Note that the relationship between cost $(C+C_2)$ and trade size can cross for different values of fixed costs.

Figure 10a: Alcoa Trading Frequency (volume < 1000)

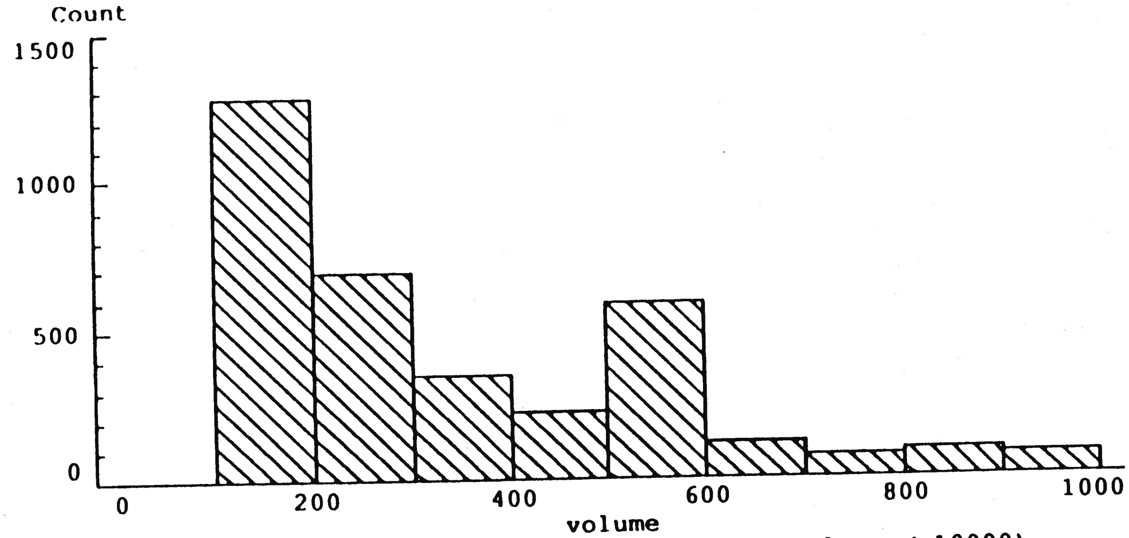


Figure 10b: Alcoa Trading Frequency (volume < 10000)

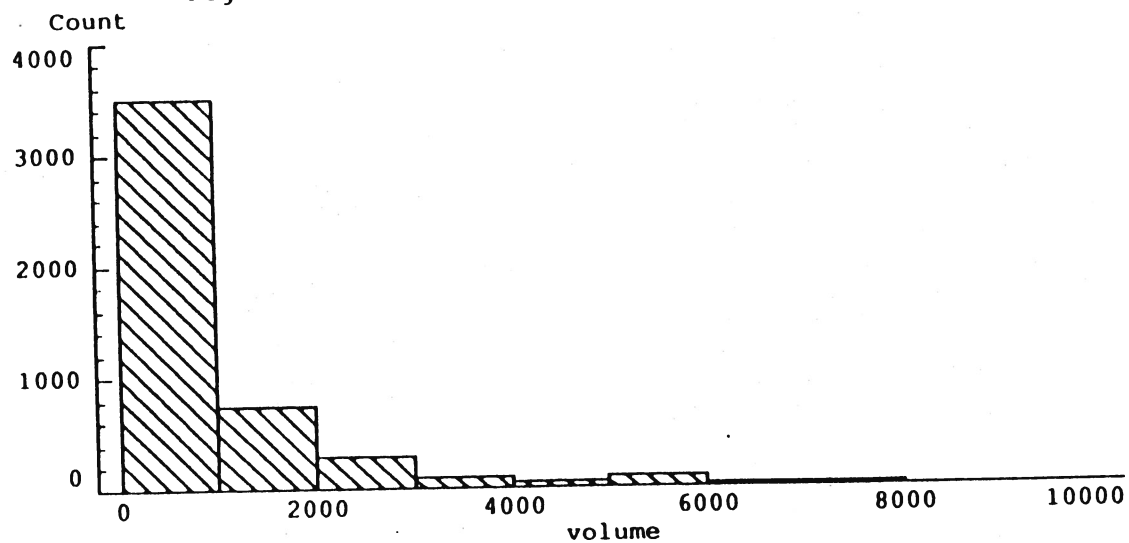


Figure 11: Probability of Informed Trade as a Function of Trade Size for Alcoa Aluminum