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Education and Poverty Trap

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EDUCATION AND THE POVERTY TRAP

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Abstract

An overlapping generations model is constructed in which individual wealth is related to educational attainment, and in which liquidity constraints may induce children to invest in a sub-optimal level of education given their ability. Borrowing for educational attainment is obtained from within the family. Abilities differ among children and may be related to parental ability. Stationary state equilibria are found to exist in which children of poorer families are caught in a poverty trap because of an inability to finance their education. The role of redistributive policy is studied in this context.

Education and the Poverty Trap

I. Introduction

It is becoming widely accepted that human capital is a key determinant of economic progress alongside physical capital and the generation of new knowledge through research and development. Human capital accumulation is unique among these forms of investment in that it is largely undertaken by households,[1] but households differ with respect to their capacity to accumulate human capital. For one thing, some households may be more capable than others, thereby making a given amount of investment more productive. For another, households may differ in their ability to finance education, and may not be able to use capital markets to make up the difference. The human capital investment process will thus affect not only the rate of growth of the economy but also the distribution of utility among households.

Our focus in this paper is largely on the distributional consequences of different households having unequal opportunity to acquire human capital. We construct a simple overlapping generations model of a stationary economy in which households take only two types of decision: an educational investment decision and a savings decision. Education yields a rate of return in the form of higher earnings, while savings earn interest. The opportunity cost of education consists of two components — foregone earnings and a financial cost. However, we assume that there are in addition non-pecuniary benefits to education, either because it is more agreeable to spend time being educated rather than working during youth, and/or because it is pleasanter to work as an educated rather than an uneducated person. An important feature of our model is that households cannot borrow in capital markets against future earnings, but only from their parents.[2] Although parents are not altruistic, they are willing to lend money to their children for education since they know whether or not their children will succeed and reimburse their loan.[3]

The two fundamental components of our model are the characterization of household behaviour and the technology of education. In both cases, we work with very simple representations designed to capture more complex underlying structures. We suppose that households live for three distinct periods. During the first period, i.e., their youth, they choose to work or to acquire education and, if they attend school, what quality of schooling to acquire. If they get educated, they must borrow from their parents to pay for tuition fees and childhood consumption. During the second period, i.e., their adulthood, they work a fixed amount of time, receiving a wage which depends on their education. They repay their parents and undertake additional savings for retirement consumption;

[1] This is not entirely true; some human capital investment takes place on the job and is part of the activities of the firm.

[2] This assumption is not uncommon in the literature (e.g., see Becker and Tomes (1976)), and we discuss it further below.

[3] Altruism could be added to our model at the expense of some simplicity. Its existence should not change the qualitative nature of the results as long as the altruism is kept within the family.

some of this saving can take the form of lending to their children. They are retired from work in the third period, and finance consumption by decumulating assets.

Our analysis concentrates on the distribution of incomes achieved in the stationary state under the above assumptions. In particular, we show how the existence of a liquidity constraint can condemn some persons to a "poverty trap" in which they remain uneducated despite the social profitability of being educated. We also investigate optimal policy in this setting; though our model is highly stylized, we think it may be suggestive of one of the reasons for which the public sector is so heavily involved in the provision of education.

In our model, expenditure on tuition determines the quality of education purchased — higher quality, which results in a higher wage during adulthood, has a higher variable cost. It is natural to think of an educational facility (i.e., a school) as being like a club good. Several pupils share the same classroom, and the larger is the size of the class, the lower is the share of the cost borne by each student, but the lower is the quality of the education acquired. This latter effect can thus be interpreted as the congestion effect in club theory.

II. The Three-Period Life-Cycle Model

Households live for three periods of equal length — childhood (period 0), adulthood (period 1) and retirement (period 2).[4] Lifetime utility is assumed to take the simple form:

$$U \;=\; \log(c_0) + \log(c_1) + \log(c_2) + \delta H \tag{1}$$

where c_t denotes consumption in period t, H denotes the utility premium from being educated, and δ takes the value one if educated and zero if uneducated. As mentioned, H can be interpreted as the difference in utility between working or becoming educated early in life, or alternatively may be viewed as the non-pecuniary benefit of working as an educated person in the next period of life.

During childhood, agents choose between working for a given wage, w, or full-time school attendence. If they attend school, they incur a cost e, the level of which is under their control. It can be interpreted as the cost of obtaining different qualities of education. For example, if we think of a school as a club good as discussed above, with a cost of operating the school denoted by K and the number of pupils by N, the variable cost per pupil e is then K/N. Agents can be thought of as choosing a classroom size N. Decreasing N increases e and improves the quality of education. During adulthood, an educated person receives a wage of $\omega(e)$, where the earnings

[4]The term childhood is used for convenience. It covers the first period of an individual's life, that is, until age 25. We assume that children are in fact taken care of by their parents until they turn 16. They then become autonomous and must choose between getting an (advanced) education or beginning their working life.

function $\omega(e)$ is increasing in educational expenditure and is strictly concave. The uneducated worker again receives wage w. Hours of work are fixed and labour units are normalised so that each person supplies one unit of labour per working period. During retirement, no work is supplied; agents consume the assets they have saved in earlier periods.

Capital markets are rather special in this model. Agents can acquire assets which yield a return r, and will do so during adulthood and, for the uneducated, during childhood. However, agents cannot borrow in capital markets against future earnings to finance their education. Instead, they must borrow from their parents. The latter, being non-altruistic and being able to acquire financial assets with a rate of return r, will lend to their children at the same interest rate r.[5] Since this liquidity constraint is a key ingredient in our model, it is worth spending some time justifying it.

In the simplest version of the model, in which agents are equally capable and there is no uncertainty with respect to the return to education, it is difficult to justify the absence of a credit market for education expenses: banks would surely be willing to lend against future income, and all borrowers would be capable of repayment. However, by introducing asymmetric information and borrower heterogeneity,[6] a persuasive story can be told which is essentially a variant on Akerlof's classic 'lemons' model.[7] Thus, consider the simple case in which if an individual is of low ability investment in education has no effect on earnings during adulthood, whereas education is socially profitable for high ability individuals. Suppose further that agents cannot be forced to make loan repayments above an amount that reduces their net income below that of an uneducated worker; consequently, if low ability individuals obtain educational loans, and use this money to increase their consumption during youth, they cannot be punished when they subsequently default on their loan repayment. Nonetheless, in this economy competitive banks will be willing to lend to *all* students at an interest rate $\tilde{r}$ such that $\pi_H(1+\tilde{r}) = 1+r$, where π_H denotes the proportion of high ability agents, and r is the risk-free interest rate. However, if parents know whether or not their children are of high or low ability, they will be willing to lend their savings to their *high* ability children at interest rate $r < \tilde{r}$; thus, no child whose parents can lend them the desired level of educational expenditure will apply to the bank for a loan, and the remaining high ability

[5] When households are liquidity-constrained by a shortage of parental saving, they would be willing to pay more than r to borrow from their parents. It would seem natural in this case to allow parents to lend to their children at a higher interest rate until an equilibrium has been achieved within each household. We have not pursued this line of reasoning further since it complicates the analysis considerably. We would not expect it to have a qualitative effect on the results.

[6] The same approach to borrower heterogeneity is taken subsequently in this paper when we analyse the issue of the steady-state distribution of wealth.

[7] Jaffee & Russell (1976) and Mankiw (1986) differentiate borrowers according to their likelihood of repayment. In Mankiw's article, which is particularly pertinent for our analysis, it may be the case that *despite the social profitability of education*, there is no interest rate at which educational loans are made. Using Mankiw's results directly, however, is difficult in our model, since if parents can obtain a certain rate of return r by depositing their savings at the bank, they will require a higher rate of return from their children than would be consistent with inducing an efficient level of investment in education. This is why we prefer to use the lemons model and limited liability to explain the absence of the credit market.

children will only wish to borrow the difference between the desired amount and their parents' savings. This withdrawal from the borrowing pool of high ability individuals therefore increases the riskiness of the remaining pool of borrowers, and the interest rate at which banks are willing to make educational loans must therefore rise also. If the relative proportion of bad risks increases fast enough to always drive out the marginal good quality borrower, then the credit market will collapse. In the following, we therefore assume that financing educational expenses through the capital market is impossible.

There are three sorts of persons in our model. The first are those persons who choose to remain uneducated. The second are those who invest in education and who can borrow as much as desired from their parents. The third are the persons who invest in education, but whose parents' savings are insufficient to allow them to choose their most-preferred quality of schooling. We refer to these types of persons respectively as the uneducated (U), the unconstrained educated (E) and the liquidity-constrained educated (C). All have the same preferences, but may differ in ability and/or parental resources. We consider the behaviour of each in turn. Where necessary, we use the superscripts U, C or E to distinguish between them. Where the meaning is clear, the superscripts are suppressed.

The uneducated (U)

Let s_0 denote savings during childhood, and s_1 denote savings during adulthood. The problem for an uneducated individual can be written as:[8]

$$\max_{s_0, s_1} \quad \log(w - s_0) + \log(w + [1+r]s_0 - s_1) + \log(s_1[1+r]). \tag{2}$$

The solution to this problem is:[9]

$$s_0 = \frac{w[1+2r]}{3[1+r]} \quad \text{and} \quad s_1 = \frac{w[2+r]}{3} \tag{3}$$

so that lifetime utility is:

$$V^U = 3\log\left(\frac{w[2+r]}{3}\right) = 3\log s_1. \tag{4}$$

Note that the fact that lifetime utility can be expressed in terms of period 1 savings only will be used often in what follows.

The unconstrained educated (E)

Let b be the amount the unconstrained individual desires to borrow from his/her parents. The problem for the unconstrained educated person becomes:

$$\max_{b,e,s} \quad \log(b - e) + \log(\omega(e) - b[1+r] - s_1) + \log(s_1[1+r]) + H. \tag{5}$$

[8] Remark that brackets are used to indicate multiplication, whereas arguments of functions are enclosed in parentheses.

[9] These results are readily obtained by realising that with the Cobb-Douglas utility function with equal weights, $c_0 = \frac{c_1}{1+r} = \frac{c_2}{[1+r]^2}$, and $s_0 = w - \frac{W_0}{3}$; $s_1 = \frac{W_1}{2}$ where W_0 is lifetime wealth ($w + \frac{w}{1+r}$) while W_1 is wealth in period 1 ($w + s_0[1+r]$).

Education expenditure is determined by:

$$\omega'(e) = 1 + r \tag{6}$$

which simply implies that the marginal rate of return on education must be equal to that on financial assets. Note that since working during youth and becoming educated are mutually exclusive activities, there is also a foregone earnings component, w.

The solutions for b and s_1 are given by:

$$b = \frac{\omega(e) + 2e[1+r]}{3[1+r]} \quad \text{and} \quad s_1 = \frac{\omega(e) - e[1+r]}{3} \tag{7}$$

and lifetime utility is:

$$V^E = 3\log\left(\frac{\omega(e) - e[1+r]}{3}\right) + H = 3\log\, s_1 + H. \tag{8}$$

Two further things should be remarked about the unconstrained solution. First, this outcome can only be maintained from generation to generation if $s_1 \geq b$ at the optimum, that is, if unconstrained educated persons save enough to lend their own children the desired amount b. Furthermore, to choose to become educated requires $V^E \geq V^U$. In the special case in which $H = 0$, so that there are no non-pecuniary benefits to becoming educated, this is equivalent to $s_1^E \geq s_1^U$, which can be written as:

$$\frac{\omega(e)}{1+r} \geq e + w + \frac{w}{1+r} \tag{9}$$

where e is defined by (6). This says that the present value of earnings if educated must equal the sum of the cost of education plus the present value of the stream of earnings that would have been obtained if uneducated. More generally, if $H > 0$, $V^E \geq V^U$ implies $3\log(s_1^E) + H > 3\log s_1^U$. Thus, it is possible for s_1^E to be less than s_1^U if $H > 0$.

The constrained educated (C)

Children whose parents savings are less than b are constrained in their education decision and solve:

$$\max_{e,s_1} \quad \log\left(\bar{b} - e\right) + \log\left(\omega(e) - \bar{b}[1+r] - s_1\right) + \log\left(s_1[1+r]\right) + H \tag{10}$$

where $\bar{b}$ denotes the amount they can borrow from their parents. The level of education chosen is the solution to:

$$\omega'(e) = \frac{\omega(e) - \bar{b}[1+r]}{2\left[\bar{b} - e\right]} \tag{11}$$

and first-period saving is given by:

$$s_1 = \frac{\omega(e) - \bar{b}[1+r]}{2}. \tag{12}$$

To interpret this, remark that (11) can be rewritten as:

$$\frac{\omega'(e)}{1+r} = \frac{s_1}{[1+r]c_0} = \frac{c_2}{[1+r]^2 c_0}. \tag{13}$$

In the fully unconstrained case, $\omega'(e) = 1 + r$ and $c_0 = c_2/[1+r]^2$. So (13) is also satisfied in this case. Note, however, that because of the borrowing constraint $\omega'(e) > 1 + r$, and therefore $c_0 < c_2/[1+r]^2$. In other words, constrained households reduce both educational spending and childhood consumption relative to unconstrained ones. This also implies, in counterpart, that consumption rises during the last two periods of life.

Lifetime utility can now be written:

$$\begin{aligned} V^C &= \log(c_0) + \log(s_1) + \log(s_1[1+r]) + H \\ &= \log(c_0[1+r]) + 2\log(s_1) + H. \end{aligned} \tag{14}$$

This expression for V^C is in fact directly comparable with (4) for V^U and (8) for V^E (since then $\omega'(e) = 1+r$). Both of the latter can be written in the form of (14) (less the H term for V^U) since $c_0[1+r] = c_2/1+r = s_1$. However, (14) cannot be written in the form of (4) or (8) since, due to the liquidity constraint, $c_0[1+r] \neq s_1$.

Again, for the constrained educated person to choose to become educated, it must be the case that $V^C > V^U$, or:

$$\log\left(c_0^C[1+r]\right) + 2\log(s_1^C) + H \geq \log(c_0^U[1+r]) + 2\log(s_1^U). \tag{15}$$

To interpret (15), note that the utility of constrained educated persons, V^C, is increasing in $\bar{b}$. Denote by $\underline{s}$ the level of parental savings for which (15) is satisfied as an equality (assuming $\underline{s}$ exists). If $H = 0$, we can infer that $c_0^C < c_0^U$, and so $s_1^C > s_1^U$.[10] This fact is important for it implies that with $H = 0$, the children of parents who invest in education are less liquidity constrained than their parents were.

In contrast, as H increases above zero, the values of c_0^U and c_1^U which satisfy (15) with equality rise relative to c_0^C and c_1^C. Beyond some threshold value of H, $c_1^U > c_1^C$ and so $s_1^U > s_1^C$. When this is true, a child who can just borrow enough to make investing in education worthwhile ($\bar{b} = \underline{s}$) will save less than his/her parent, and so his/her own child will not choose to become educated. This corresponds below to the case in which the savings curve lies below the 45° line for low levels of parental savings.

This completes our characterisation of household behaviour. We now turn to a description of equilibria. We first analyse the sorts of equilibria that can exist in an economy of identical

[10] In this case, (15) can be written, $\log(c_0^C[1+r]) + 2\log(c_1^C) = \log(c_0^U[1+r]) + 2\log(c_1^U)$. Since $c_0^C[1+r] < c_1^C$, while $c_0^U[1+r] = c_1^U$, it must be the case that $c_0^C < c_0^U$ and $c_1^C > c_1^U$ or $s_1^C > s_1^U$.

individuals. Depending on initial family income, which for now we take as historically given, different equilibria can occur. We restrict ourselves to the case in which population is constant — there is one parent per child; this was already implicit in the above analysis.

III. Stationary State Equilibria in the Homogeneous Household Case

To identify those stationary states towards which households with different histories will tend, we must characterise the dynamics of inter-generational behaviour. To proceed, we note that a person's lifetime utility is weakly monotonic in his parent's utility. In particular, for the uneducated, utility is independent of parental saving. As parental saving increases, a point may exist at which it is just worthwhile for a child to become constrained educated ($\underline{s}$). As parental saving increases further, the offspring's utility increases monotonically until parental saving is enough to eliminate the constraint. Beyond this level, childrens' utility is independent of parental saving.

In more analytic terms, the dynamic analysis involves investigating how saving evolves from generation to generation. Consider a constrained educated household of generation t. Let us rewrite equations (11) and (12) as:

$$\omega' = \frac{\omega(e) - s_{t-1}[1+r]}{2[s_{t-1} - e]} \tag{11'}$$

$$s_t = \frac{\omega(e) - s_{t-1}[1+r]}{2} \tag{12'}$$

where s_t *refers to period* 1 *(adulthood) savings of generation* t. Differentiating (12′) we obtain:

$$\frac{ds_t}{ds_{t-1}} = \frac{\omega'(e)\frac{de}{ds_{t-1}} - [1+r]}{2} \tag{16}$$

where, from (11′),

$$\frac{de}{ds_{t-1}} = \frac{3\omega'(e) - [\omega'(e) - [1+r]]}{3\omega'(e) - 2[s_{t-1} - e]\omega''(e)}. \tag{17}$$

From (16) and (17) we can infer how family savings evolve as long as successive generations remain constrained.

Note first that since $\omega'(e) > 1 + r$ for the constrained person, $0 < de/ds_{t-1} < 1$ if $\omega(e)$ is strictly concave. This implies that additional amounts of borrowing are partly devoted to educational spending and partly to increased childhood consumption. However, we cannot infer the sign of $\frac{ds_t}{ds_{t-1}}$ from (16) which may be positive or negative even when $s_{t-1} = \underline{s}$. We can, however, be certain that $\frac{ds_t}{ds_{t-1}}$ eventually turns negative, for when the household is unconstrained $\omega'(e) = 1 + r$ and therefore $ds_t/ds_{t-1} < 0$.

Given this, a number of different shapes for the curve of s_t versus s_{t-1} are possible, each one giving rise to different families of equilibria. For expositional purposes, we restrict ourselves to the case in which $s_t(s_{t-1})$ is initially positively sloped at $s_{t-1} = \underline{s}$ and is strictly concave.[11] The following analyses the equilibria that may exist. In each situation considered, a *poverty trap* may or may not exist. We shall say a poverty trap exists if it is socially desirable for a child to become educated, but insufficient parental saving precludes this. It is socially efficient to be educated if some b exists which satisfies (15), in other words, if the household would become educated if borrowing were freely available. Given that education is socially profitable, a poverty trap exists if $\frac{w[2+r]}{3} < \underline{s}$, so that a child with an uneducated parent will not be able to become educated (recall that $w[2+r]/3$ is the savings of adult uneducated workers).

Three separate cases are considered — that in which education is efficient and no poverty trap exists, that in which a poverty trap exists, and that in which education is socially unprofitable. For each, stable and/or unstable equilibria may exist. In what follows, we denote by e^* the level of education chosen by the unconstrained household (i.e., $w(e^*) = 1 + r$).

Case 1: No poverty trap and education socially profitable

This case is illustrated in Figures 1-5. These figures depict the curve of s_t versus s_{t-1} for the constrained household over the relevant range. Each curve begins at $s_{t-1} = \underline{s}$, which is the minimum level of saving required to satisfy (15); recall that because no poverty trap exists, $\underline{s} \leq w[2+r]/3$. It then extends to $s_t = e^* + c_0^E$, at which point the household becomes unconstrained educated. As one moves to the right along these curves, welfare of generation t is strictly increasing. The curves differ with respect to the location of the points $s_t(\underline{s})$ and $s_t(e^* + c_0^E)$ vis-à-vis the 45° line. Recall that, depending on the value of H, $s_t(\underline{s}) \gtrless \underline{s}$, and that the higher is the value of H, the lower will be the value of $s_t(\underline{s})$, so that for sufficiently large H this point lies below the 45° line. The curves show five different situations in which education is socially profitable and there is no poverty trap.

[11] A sufficient (but far stronger than necessary) condition for this to be true is that $\omega''' \leq 0$.

(a.) One stable unconstrained equilibrium

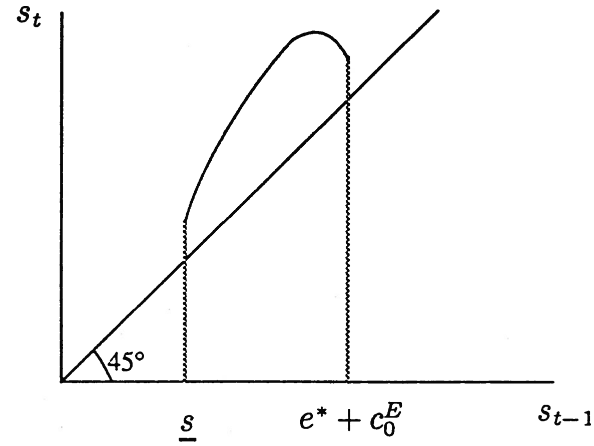

Figure 1

Figure 1 represents the situation in which the curve lies everywhere above the 45° line. Furthermore, since there is no poverty trap, the children of uneducated workers will choose to invest in education, and will in turn save more than $w[2+r]/3$, and so on until the family becomes unconstrained. Thus, every family will converge in finite time to the unconstrained educated equilibrium. From then on, it will be unconstrained educated, and successive generations will continue to borrow $e^* + c_0^E$ and to save $s(e^* + c_0^E)$. Furthermore, if, for whatever reason, parents' savings were larger than $s(e^* + c_0^E)$, children would not choose to borrow more than $e^* + c_0^E$, and would themselves save only $s(e^* + c_0^E)$.

(b.) One stable constrained equilibrium

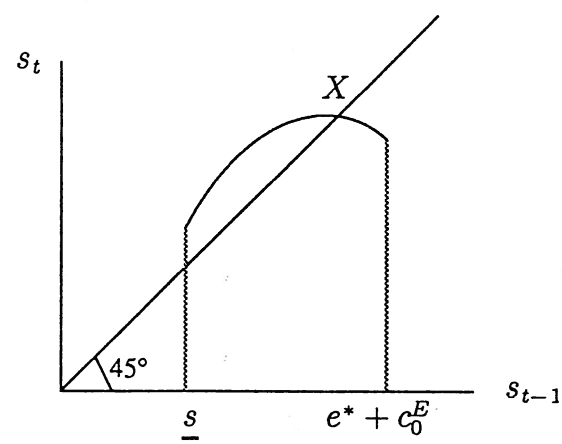

Figure 2

Figure 2 shows the case in which a person who is unconstrained saves less than their parent

$(s_t(e^* + c_0^E) < e^* + c_0^E)$. This person's offspring will therefore be constrained. In fact, it is readily apparent that a stable equilibrium will exist at the point X where the household is constrained at a sub-optimal level of education. If no liquidity constraint existed, this person would borrow $e^* + c_0^E$, thereby attaining a higher level of utility.

(c.) One unstable constrained equilibrium and one stable unconstrained equilibrium

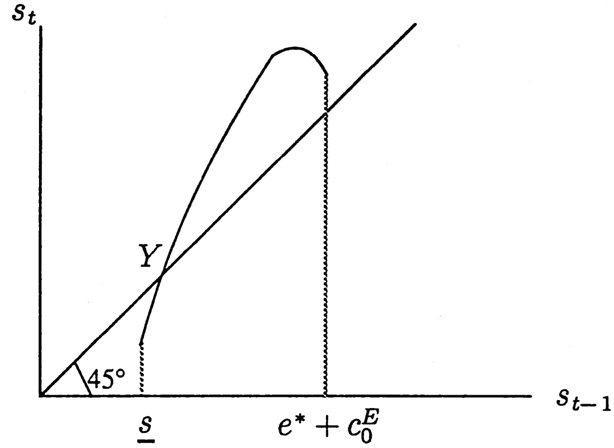

Figure 3

Figure 3 depicts this case. The curve $s_t(s_{t-1})$ begins below the 45° line, reflecting a high value of H; $\frac{w[2+r]}{3}$ the savings of the uneducated, exceeds $\underline{s}$ but is less than the level required to achieve Y.[12] There are therefore two equilibria . Consider persons whose parents are uneducated. They borrow $\frac{w[2+r]}{3}$ from their parents, become educated and generate $s_t < \frac{w[2+r]}{3}$. Their offspring in turn may become educated (if $s_t \geq \underline{s}$), but they in turn save still less. At any rate, after a finite number of generations, an educated person's child will not choose to invest in education, and the cycle will begin again. This cycle of an uneducated generation followed by a finite sequence of constrained educated ones would continue indefinitely. The instability of the constrained equilibrium (Y) implies that a family with an uneducated person in some generation will never attain the unconstrained educated equilibrium, whereas if there were no liquidity constraint he would choose this solution immediately. In contrast, if a family starts with parental saving to the right of Y, after a finite number of generations it will become unconstrained, and will remain so thereafter.

[12] If $\frac{w[2+r]}{3}$ exceeded that required to go to Y, this case would be essentially identical to case a above.

(d.) One unstable constrained equilibrium and one stable constrained equilibrium

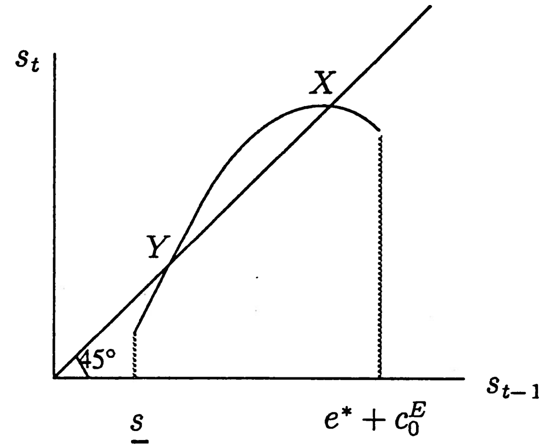

Figure 4

This is depicted in Figure 4. It combines the stable constrained equilibrium X of case b above with the unstable constrained equilibrium of case c. The same explanations apply. Again, in the absence of liquidity constraints, all persons would borrow enough to attain the unconstrained equilibrium.

(e.) No stable equilbrium

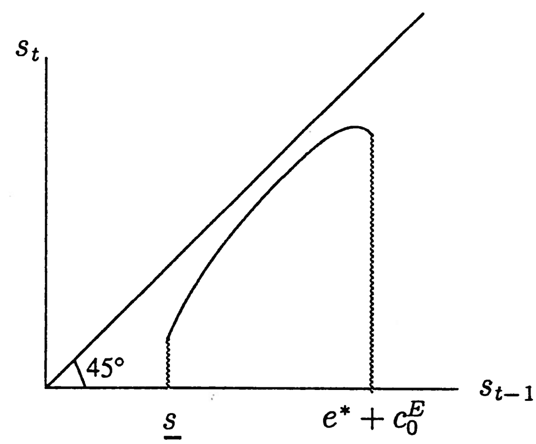

Figure 5

Figure 5 illustrates the case in which no stable equilibrium exists. As in the unstable constrained case of Figure 3, families will cycle repeatedly: an uneducated generation will be followed by a finite number of constrained and educated ones, with successively less saving. Note that this case cannot exist if $H = 0$.

Case 2: Poverty trap exists but education socially profitable

The same diagrams, Figures 1–5, can be used to illustrate this case. The essential difference is that now $s_1^U = \frac{w[2+r]}{3} < \underline{s}$, so a poverty trap exists. Children of uneducated parents cannot borrow enough to make investment in education worthwhile. They will therefore remain uneducated, as will all their descendents. If, however, parents are educated, different types of equilibrium are possible depending upon the initial level of parental saving and the location of the $s_t(s_{t-1})$ curve. If parental saving exceeds Y at some point in time, the family will remain educated in perpetuity; whether or not it becomes unconstrained depends upon which of $a - - - d$ apply. In contrast, families which start out with a level of wealth below Y will always eventually be caught in the poverty trap when parental saving starts below $\underline{s}$. Once the poverty trap is reached (in a finite number of generations) the family will remain in this state permanently. If Figure 5 applies, all households end up in the poverty trap in a finite number of periods, and remain their forever, regardless of the initial level of savings. Notice also that, for all the examples in Case 2, if liquidity constraints were absent, the household would borrow enough to become educated at the unconstrained level (e^*).

Case 3: Education not socially profitable

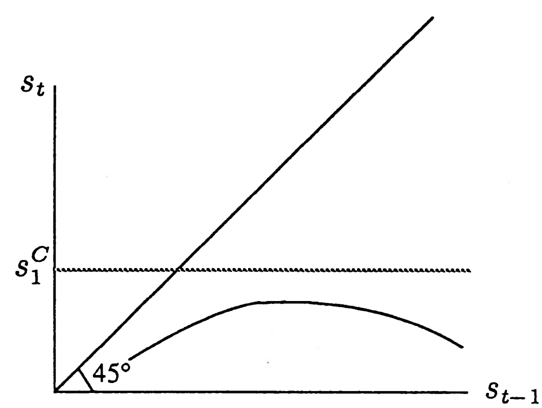

Figure 6

Finally, Figure 6 illustrates the case in which education is not socially profitable, and thus *regardless* of the level of parental savings that can be borrowed, no child will ever choose to invest in education — the return is too low. In other words, $\underline{s}$ does not exist, because the level of constrained adult saving which is required to satisfy equation (15), denoted s_1^C, is always greater than what an educated worker saves.

This completes our description of household behaviour and of the sorts of equilibria when individuals are equally able but may start out with different levels of resources. We now wish to study the stationary state equilibria which occur in economies of heterogeneous individuals. We first consider a fairly simple special case, and then a more general one.

IV. Stationary Equilibria in a Simple Two-Ability Model

We consider an economy with individuals of two ability levels, high and low, and with a very simple structure. A person's lifetime utility will be determined jointly by ability and parental savings; the latter will depend upon the family's history of ability levels. The distribution of wealth and welfare in the stationary state will depend upon the location of the $s_t(s_{t-1})$ curves for the persons of high and low ability. In this section we suppose that low ability agents, denoted L, are unable to benefit from education, and so remain uneducated regardless of the level of parental savings (i.e., they are characterised by Case 3 above). In contrast, high ability individuals, denoted H, are described by Case $1a$ above — no poverty trap exists ($\frac{w[2+r]}{3} > \underline{s}$), and Figure 1 applies. Whether or not they are constrained in their educational investment decision depends upon the level of their parent's savings.

Ability is determined stochastically at birth. Suppose that, conditional on parents being of low ability, the probability of being born with high ability is π_L. For simplicity, we assume this to be independent of the previous family history; in particular, it does not depend on how many consecutive generations were of type L.[13]

For high ability persons, family history is of importance, because it determines parental saving. Denote by R the number of generations that have passed since a member of the family was last of type L ($R = 1, 2, ...$). Since uneducated workers save $w[2 + r]/3$ during adulthood, it is straightforward to determine the level of education and savings chosen by each successive child of high ability. Consequently, the resources available to a given child of type H are fully characterized by the index R. In particular, since Figure 1 applies, after a finite number of periods, denoted by R^*, high ability children are no longer constrained. For $R < R^*$, household utility is increasing in parental saving, whereas it is constant after the family has been of high ability for at least R^* generations. Let π_R be the probability that a child is of high ability, conditional on his parents being of high ability and of type R; note that π_R may depend upon R. However, once a low ability person is born, the probability of having a child of high ability reverts to π_L.

In an economy consisting of a very large number of families, a stationary regime will eventually be reached with a given distribution of population amongst states L and R, where $R = 1, 2, ...$ We denote the proportions of the populations in these various types by the vector:

$$P = (P_L, P_1, P_2, ...). \tag{18}$$

To determine an explicit expression for P_L and P_R ($R = 1, 2, \ldots$) in terms of the underlying probabilities π_L and π_R ($R = 1, 2, \ldots$), we note that the matrix of transition probabilities Π of the underlying Markov chain can be written:

[13] This could be relaxed with little complication.

$$\Pi = \begin{bmatrix} 1-\pi_L & \pi_L & 0 & 0 & 0 & \cdot & \cdot & \cdot \\ 1-\pi_1 & 0 & \pi_1 & 0 & 0 & \cdot & \cdot & \cdot \\ 1-\pi_2 & 0 & 0 & \pi_2 & 0 & \cdot & \cdot & \cdot \\ 1-\pi_3 & 0 & 0 & 0 & \pi_3 & \cdot & \cdot & \cdot \\ 1-\pi_4 & 0 & 0 & 0 & 0 & \pi_4 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \tag{19}$$

where π_{ij} (i.e. the element of this matrix at the intersection of row i and column j) is the probability that a parent in state i will have a child in state j. We know that the vector of stationary probabilities (i.e., the steady-state proportions of the population in the various states) is given by $P\Pi = P$. Therefore,

$$\begin{aligned} P_L(1-\pi_L) + \sum_{R=1}^{\infty} P_R(1-\pi_R) &= P_L, \\ P_L\pi_L &= P_1, \\ P_1\pi_1 &= P_2 \\ P_{R-1}\pi_{R-1} &= P_R \qquad (R = 2, 3, \ldots). \end{aligned} \tag{20}$$

This gives the relative values of P_L and P_R elements. Note in particular that

$$\frac{P_1}{P_L} = \pi_L \quad \text{and} \quad \frac{P_{R+1}}{P_R} = \pi_R \qquad (R = 1, 2, \ldots). \tag{21}$$

The absolute values may be obtained by noting that $P_L + \sum P_R = 1$. Therefore,

$$\begin{aligned} P_L &= [1 + \pi_L \sum_{R=1}^{\infty} \pi_1\pi_2 \cdots \pi_{R-1}]^{-1}, \\ P_R &= P_L\pi_L\pi_1\pi_2 \cdots \pi_{R-1}. \end{aligned} \tag{22}$$

This characterizes the population distribution by family history. A couple of special cases are of some interest.

i. **π_R independent of genetic history.** The probability of being high ability is independent of parental ability, $\pi_L = \pi_R = \pi$ for all R. In this case,

$$\frac{P_1}{P_L} = \frac{P_{R+1}}{P_R} = \pi. \tag{23}$$

ii. **π_R depends only on own parent's type.** The probability of being high ability depends only on the ability of one's own parent, in other words, $\pi_R = \pi_H$ for all R. In this case, π_H can be greater or smaller than π_L. In our discussion, we arbitrarily assume the former to be the case ($\pi_H > \pi_L$).

We refer to this second case as *one-generation dependence.* In principle, we could consider two-generation dependence, and so on. In turn, genetic history determines lifetime utility. In our simple model, a person of type L obtains V^U, whereas persons such that $1 \leq R < R^*$ obtain V^C, which increases with R. Thus, constrained persons are distributed along the curve of Figure 1. Finally, persons in state $R \geq R^*$ realise V^E. Thus, in the stationary regime utilities are distributed according to the probability distribution P, from the uneducated level, V^U, through the constrained educated, V^C, up to the unconstrained level, V^E. At least part of this distribution (that among persons of type $R < R^*$) is accounted for by the liquidity constrained. It is natural to ask how policy may be used to address this inequality. We turn to this next.

V. Redistributive Tax Policy in the Simple Two-Ability Model

The distributional problem arises partly because of differences in parental income and partly because of differences in ability. We design tax policies intended to address the former problem. The *informational assumptions* are important here. We assume that the Planner can assess both the income and past educational expenditures of adults, but cannot observe the ability type of children. Lump sum taxes and transfers are made during adulthood and are contingent on state $(L; R = 1, 2, \cdots)$.[14] An alternative informational assumption would be to assume that the Planner can observe income in adulthood, but not educational attainment. This would lead to distortionary taxes if the educational investment decision were affected. For simplicity, we avoid this distortion in our model. Let T_L and T_R denote the lump-sum tax levied on persons in states L and R, $(R = 1, 2, \ldots)$. These may be positive or negative.

To analyze the Planner's problem we must first incorporate the taxes into the behavioural and utility functions of the households. The tax will simply reduce adult income. Thus period one savings become:

i. Unconstrained educated persons $(R \geq R^*)$:[15]

$$s_1^R = \frac{\omega(e^*) - [e^* + c_0^E][1+r] - T_R}{2}. \tag{24}$$

ii. Constrained educated persons $(R < R^*)$:

$$s_1^R = \frac{\omega(s_1^{R-1} - c_0^R) - s_1^{R-1}[1+r] - T_R}{2}. \tag{25}$$

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[14]There would be no point in making transfers in retirement since that would be too late to lend to children. Making them directly to children, but contingent on the same parental information would be equivalent to our procedure. Note also that parental type is enough to determine all relevant family history.

[15]Note that s_R is constant for all $R > R^*$.

iii. Uneducated persons (L):

$$s_1^L = \frac{w[2+r]-T_L}{3}. \tag{26}$$

Notice that regardless of the value of T_L (positive or negative), the uneducated person will never become educated, since in this simple two-ability case, it is never profitable for low ability individuals to invest in education.

The Planner's goal is to maximise the sum of the utilities of each of the various types of agents in the community, weighted by the proportion of total population that each type represents. Recall that the utility of uneducated individuals can be written as:

$$V^U = \log(s_L^3)$$

and similarly, the utility of unconstrained educated persons may be expressed as:

$$V^R = \log(s_R^3) + H. \qquad (R \geq R^*)$$

Noting that the first-order condition with respect to education for the constrained educated agent implies that $\omega'(s_{R-1} - c_0^C) = \frac{c_1}{c_0}$, and recalling that $s_R = c_1^C = c_2^C[1+r]$, we can express the utility of the constrained individual as:

$$\begin{aligned} V^R &= \log(\frac{s_R}{\omega'(s_{R-1} - c_0^C)}) + \log(s_R) + \log(s_R[1+r]) + H \\ &= \log(s_R^3 \frac{[1+r]}{\omega'(s_{R-1} - c_0^C)}) + H. \end{aligned}$$

Weighting utilities by P_L or P_R, and recalling that at the unconstrained solution $w'(e^*) = [1+r]$, the Planner's problem is:[16]

$$\max_{\substack{s_L, T_L \\ s_R, T_R}} W = P_L \log(s_L^3) + \sum_{R=1}^{\infty} P_R \left[\log(s_R^3 \frac{[1+r]}{\omega'(s_{R-1} - c_0^C)}) + H\right]$$

subject to:

$$\begin{aligned} s_R &= \frac{\phi_H \omega(e^*) - [e^* + c_0^*][1+r] - T_R}{2}, && \text{if } s_{R-1} \geq e^* + C_0^* \\ s_R &= \frac{\phi_H \omega(s_{R-1} - c_0^C) - s_{R-1}[1+r] - T_R}{2}, && \text{if } s_{R-1} < e^* + c_0^C \\ s_L &= \frac{w[2+r] - T_L}{2}, \end{aligned}$$

and

$$P_L T_L + \sum_{R=1}^{\infty} P_R T_R = 0.$$

[16]Note that since P_R tends to zero as R tends to infinity, this infinite sum is well-defined.

The last constraint represents the government's budget constraint; it could be replaced by the resource constraint for the economy.

After some simplication, the first-order conditions of the Planner's problem can be expressed as:

$$\frac{3P_R}{s_R} + \lambda_R = 0, \tag{26.1}$$

$$\frac{3P_R}{s_R} - P_{R+1}\frac{\omega''(s_R - c_0^C)}{\omega'(s_R - c_0^C)} + \lambda_R - \lambda_{R+1}\,[\frac{\phi_H\omega'(s_R - c_0^C) - [1+r]}{2}] = 0, \tag{26.2}$$

$$\frac{3P_L}{s_L} - P_1\frac{\omega''(s_R - c_0^c)}{\omega'(s_R - c_0^C)} + \lambda_L - \lambda_1[\frac{\phi_H w'(s_L - c_0^C) - [1+r]}{2}] = 0, \tag{26.3}$$

$$\frac{\lambda_R}{2} + \mu P_R = 0, \tag{26.4}$$

$$\frac{\lambda_L}{2} + \mu P_L = 0. \tag{26.5}$$

After substituting from (26.5) into (26.3) and from (26.4) into (26.2) and (26.1), and using the fact that $P_{R+1}/P_R = \pi_R$, we obtain:

For $R \geq R^*$:

$$\frac{3}{2s_R} = \mu. \tag{27.1}$$

For $R < R^*$:

$$\frac{3}{2s_R} + \frac{\pi_R}{2}(\frac{-\omega''(s_R - c_0^C)}{\omega'(s_R - c_0^C}) = \mu[1 - \pi_R[\frac{\phi_H\omega'(s_R - c_0^C) - [1+r]}{2}]]. \tag{27.2}$$

For L :

$$\frac{3}{2s_L} + \frac{\pi_L}{2}(\frac{-\omega''(s_L - c_0^C)}{\omega'(s_L - c_0^C)}) = \mu[1 - \pi_L[\frac{\phi_H w'(s_L - c_0^C) - [1+r]}{2}]]. \tag{27.3}$$

The interpretation of these first-order conditions is quite straightforward. When agents are unconstrained, lump-sum taxes must be chosen so that the marginal benefit of an additional unit of resources to the present generation (the left-hand sides of relations (27)) is equated with the marginal cost of public funds. When agents are constrained, the first-order conditions are somewhat more complicated because the inter-generational effect must also be taken into account. Thus, as compared with (27.1), equations (27.2) and (27.3) have additional terms on both the left- and right-hand sides. The second term on the left-hand side reflects the expected marginal benefit to the high ability children of the current generation, who can better smooth their life-cycle consumption when parental savings increase.[17] The additional term on the right-hand side analogously captures the expected increase in the economy's resources when a higher level of savings for the current generation enables their children to purchase more education, and thus become more productive.

In analysing the implications for redistribution, it should first be noted that if $s_R < s_{R^*}$, then the left-hand side of (27.2) is greater than that of (27.1), and both equations cannot be satisfied simultaneously. Thus, if R^* exists, all high ability agents must be unconstrained. If R^* does not exist then, comparing (27.2) and (27.3), it is also clear that if $\pi_L = \pi_R$ for all R, then the optimal tax policy is that of full redistribution. This is because, if all parents are equally likely to have high ability children, then by equalising income the marginal benefit to future generations of an additional unit of parental saving is equalized also. If π_R is increasing in R, and $\frac{\omega''(s_R-c_0^C)}{\omega'(s_R-c_0^C)}$ is non-decreasing, then redistribution should be less than full since the expected marginal benefit for the next generation of an additional unit of parental savings is no longer independent of parental ability.

It is perhaps useful to remark that the optimal tax policy is identical to the complete insurance solution, under the hypothesis that insurance is paid out after the individual is born, and thus knows his/her parents, but before learning his/her type. Thus, if all newborns are equally likely to be of high ability, the complete insurance solution dictates that the expected marginal benefit of an additional unit of resources be equalized, and so also parental savings should be equalized. However, if π_R is increasing in R, then the expected marginal benefit of additional parental savings is also increasing in R, and insurance payouts will also rise with R.

V. General Analysis of Optimal Tax Policy

In view of the results obtained in the above analysis of optimal tax policy when only high ability individuals can profit from education, it is clearly of interest to establish the generality of these results when the assumption that low ability individuals do not benefit from education is relaxed. What makes the analysis of the general case difficult, however, is that for a given fiscal policy, it is no longer possible to characterise a family's wealth and welfare at any point in time by purely genetic characteristics; instead, the utility of a particular family i at time t must be characterised by the level of parental savings, s_{t-1}^i along with the ability level of the current generation, $\theta_t \in \{H, L\}$. In the simple case analysed above, although the actual income distribution depended upon the choice of instruments, the state space did not, and in particular each family regularly returned to the uneducated state every time a member of the family was of low ability, regardless of the choice of instruments. In analysing the general case, however, since the income

[17] We can measure the 'missing' period 0 consumption by calculating the difference between actual first period consumption, and the discounted value of second period consumption, i.e., $\Delta = c_1^C \left[\frac{1}{1+r} - \frac{1}{\omega'(s_{R-1}-c_0^C)}\right]$. Then $\left.\frac{\partial \Delta}{\partial s_{R-1}}\right|_{c_1 \text{ constant}} = c_0^C \frac{\omega''}{\omega'}$.

distribution will depend upon tax policy, the set of states visited by each dynasty in the steady state will depend upon the choice of instruments.[18] Consequently, the simple technique used above cannot be adapted to solve the general case, and instead dynamic programming techniques are exploited to solve the Planner's problem.

It is simplest to study initially the Planner's problem under the hypothesis that *all dynasties are always constrained*. Thus, the problem that the Planner wishes to solve is to choose lump-sum taxes for each family i at each time t so as to maximise the unweighted and undiscounted sum of utilities of each family from the present period until infinity, i.e.,

$$\max_{\substack{T_t^i \\ i=1,\dots N,\ t=1,\dots,\infty}} \quad W = E\sum_{t=1}^{\infty}\sum_{i=1}^{N} U(s_{t-1}^i, \theta_t^i)$$

subject to

$$\sum_i T_t^i = 0 \quad \forall t$$

where θ_t^i the ability of generation t of family i, and T_t^i denotes the tax (perhaps negative) paid by family i at time t.

Due to the additivity of the Planner's objective, once the sequence of Lagrange multipliers of the budget constraint, $\mu_t(t = 1, 2, \dots)$, is known, the problem boils down to as many separate problems as there are dynasties ($i = 1, 2, \dots N$). If the economy has reached a steady state and the number of dynasties is large enough for the law of large numbers to apply, the Lagrange multipliers are constant over time ($\mu_t = \mu, t = 1, 2, \dots$). Thus, dropping the superscript i and subscript t, and letting $\mathcal{V}(s;\theta)$ denote the value function, the Planner must choose a policy function which satisfies the following functional equation:[19]

$$\mathcal{V}(s;\theta) = \max_T \Bigg(\log\left(s(\theta)^3 \frac{[1+r]}{\omega'(s-c_o^C)}\right) + H + \pi_\theta \mathcal{V}(s(\theta);H) + [1-\pi_\theta]\mathcal{V}(s(\theta);L) + \mu T\Bigg), \qquad \theta \in \{H, L\} \tag{28}$$

where $s(\theta) = \dfrac{\phi_\theta \omega(s-c_o^C) - s[1+r] - T}{2}$. Thus, when T is chosen optimally, it must be the case that

$$\frac{3}{2s(\theta)} + \frac{\pi_\theta}{2}\mathcal{V}'(s(\theta);H) + \frac{[1-\pi_\theta]}{2}\mathcal{V}'(s(\theta);L) - \mu = 0 \tag{29}$$

where $\mathcal{V}'$ denotes the derivative of $\mathcal{V}$ with respect to s. To simplify this equation, it is useful to obtain an expression for $\mathcal{V}'$. Using the envelope theorem, it is straightforward to show that

$$\mathcal{V}' = 2\mu\left[\frac{\phi_\theta \omega'(s-c_o^C) - [1+r]}{2}\right] - \frac{\omega''(s-c_o^C)}{\omega'(s-c_o^C)} \tag{30}$$

[18] Although, for any given values of the tax parameters, there will exist a steady-state distribution of wealth which is independent of the savings level of the 'original ancestor' of each dynasty, there will be a continuum of states that can be visited by each dynasty in this steady state.

[19] It should also be noted that we have not introduced any discounting here; however, as the first-order conditions would not change if we adopted the overtaking criterion, we have chosen to merely present the functional equation.

and substituting (30) into (29) we can see that

$$\frac{3}{2s(\theta)}+\frac{1}{2}\left[\frac{\omega''(s-c_o^C)}{\omega'(s-c_o^C)}\right]=\mu\left[1-\pi_\theta\left[\frac{\phi_H\omega'(s-c_o^C)-[1+r]}{2}\right]\right.$$
$$\left.-[1-\pi_\theta]\left[\frac{\phi_L\omega'(s-c_o^C)-[1+r]}{2}\right]\right]. \tag{31}$$

Comparing this expression with equation (27.2), it should be noted that the weight attached to the second expression on the left-hand side is now 1 rather than π_θ, and that there is an additional term on the right-hand side. These differences are straightforward to interpret; as before, the ω''/ω' term measures the expected benefit to the next generation of more equally distributed consumption over the life-cycle, but now this benefit is of importance to generation t's children, regardless of their actual ability. Analogously, the additional term on the right-hand side measures the expected relaxation of the economy-wide resource constraint in the next period due to the fact that low ability children are less constrained when their parents obtain an additional unit of resources.

As in the analysis of the simple case above, it is easy to see that if $\pi_L=\pi_H$, then the right-hand side of (31) is independent of θ, and thus the Planner must choose T so that $s_L=s_H$, i.e., there is full redistribution. Similarly, as in the simple case analysed in the previous section, if $\pi_H>\pi_L$, then redistribution should be less than full. One would expect that if π_θ depended in a more general way on the genetic history of dynasty i, then this would reinforce the tendency away from full redistribution.[20]

In contrast, if for a particular level of s the current generation will be constrained only if it is of high ability, then equation (30) applies only if $\theta=H$, whereas if $\theta=L$ then $V'(s,L)=0$. Consequently, after simplification, equation (29) becomes:

$$\frac{3}{2s(\theta)}+\frac{\pi_\theta}{2}\left[\frac{\omega''(s_{-1}-c_o^C)}{\omega'(s_{-1}-c_o^C)}\right]=\mu\left[1-\pi_\theta\left[\frac{\phi_H\omega'(s_{-1}-c_o^C)-[1+r]}{2}\right]\right] \tag{32}$$

which is exactly the same as relations (27.2) and (27.3) obtained in the analysis of the simple two-ability case above. This once again implies that when the probability of having a child of high ability is independent of parental capacity, the optimal tax policy is to redistribute income fully, whereas if $\pi_H>\pi_L$, taxes should be chosen so as to leave high ability parents with greater savings than low ability ones.[21]

Of course, if neither high nor low ability individuals are constrained, then neither equation (31) nor (32) applies; instead, optimal tax policy must be chosen so that:

$$\frac{-3}{2s(\theta)}+\mu=0. \tag{33}$$

[20] For example, we could consider $\theta\in\{HH,HL,LH,LL\}$, that is, two-generation dependence.

[21] Remark that, in the absence of tax policy, if education is socially profitable for both types of agents, and the unconstrained equilibrium exists for both low and high ability individuals then, even if low ability agents can be caught in a poverty trap, each dynasty will eventually have a sufficiently long string of high ability members to pull the family out of poverty and onto the stable path for low ability individuals. Thus, in the steady state without redistributive tax policy, no low ability individual would be constrained.

This again implies that income should be equalised, here regardless of whether or not the probability of having high ability children is positively correlated with parental ability.

VI. Conclusion

This paper has studied a simple overlapping generations economy in which individual wealth and welfare are related to educational attainment, and in which liquidity constraints may induce children to invest in a lower quality of education than that which they would choose if they could borrow in capital markets against future earnings. One of the principal goals of this analysis has been to examine the circumstances under which children whose parents are unequally wealthy will rationally choose to invest in education, and the different equilibria that may occur in the stationary steady state. More specifically, if individuals can either work or attend school during their youth, but must borrow from their parents to finance both their tuition and their consumption during childhood, and repay this loan during their working life, then the children of sufficiently poor parents may not find it worthwhile to acquire an education; they, and their children, will then be caught in a poverty trap. Furthermore, even if the children of uneducated parents find it worthwhile to invest in education, they may themselves save less than their parents, and the family may then be trapped in a perpetual cycle of relative indigence. When parental saving is sufficiently high, however, or the education technology is very productive, subsequent generations will attain greater levels of wealth and utility. If an unconstrained optimum exists, it will be attained in a finite number of periods; otherwise, the family will converge to a stable, constrained equilibrium with a relatively high level of educational attainment.

The second major concern of this analysis has been to study optimal redistributive tax policy when individuals differ with respect to their capacity to profit from a given quality of education. It is assumed that the Planner can observe parental ability and earnings, but not the ability of children. If the probability of having high ability children is the same for all parents, the Planner will generally equalize income across families. However, if the ability of successive generations is positively correlated, so that able parents are more likely to have able children, then optimal redistribution is less than full. The optimal redistributive tax policy gives rise to exactly the same income distribution as the complete insurance solution, when insurance must be paid out before the uncertainty with respect to the child's type is resolved.

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