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Cointegration Error Correction and the Dynamics of Canadian M2 and M2+ Demand

Aly F. Jiwan

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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DISCUSSION PAPER #825

COINTEGRATION, ERROR CORRECTION AND THE DYNAMICS OF CANADIAN M2 AND M2+ DEMAND*

by

Aly F. Jiwan
Department of Economics
Queen's University, Kingston, Ontario
Canada K7L 3N6

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Abstract

Using the properties of integrated and cointegrated economic time series, this paper derives an error correction model (ECM) of money demand from a dynamic optimization problem. A general form of the ECM is estimated for Canadian M2 and M2+ over the period 1968:I to 1989:IV. The ECM appears to be a stable representation of broad money demand in Canada in terms of parameter constancy and forecasting ability. It is found that the demand for broad money in Canada is sensitive to the expected returns to holding foreign money. This result implies that the monetary authority should account for currency substitution in setting policy rules. The very low interest rate elasticities estimated indicate that a policy of interest rate targetting may be difficult to conduct since the spread between competing and own rates would have to be controlled.

1 Introduction

The specification of a parsimonious and well-behaved demand for money function is an integral component of an effective monetary policy. For this reason the demand for money has been much studied both theoretically and empirically. Therefore, further work in this area requires justification. Recent studies at the Bank of Canada by Caramazza (1989) and Caramazza, Hostland and Poloz (1990) have investigated the short-run dynamics of empirical money demand models in an effort to find a tractable representation of Canadian money demand for monetary policy purposes.

The primary motivation for much of this work has been the Bank's unsuccessful experience with monetary targeting over the period from 1975 to 1981 when M1 demand became increasingly unpredictable. At the time, the breakdown of the relationship between M1 demand and its determinants was attributed to financial innovations such as the introduction of daily interest savings accounts (DISAs) and daily interest chequing accounts (DICAs) in 1979 and 1981 respectively. The current work at the Bank has been focused on finding a richer dynamic specification for money demand in Canada. The underlying philosophy in these studies is that the conventional money demand models previously used for targeting purposes were dynamically misspecified.

This paper applies the recent advances in time series analysis and cointegration theory by Engle and Granger (1987), Engle and Yoo (1987) and Phillips (1987) to modelling the dynamics of broad money demand in Canada. Using an optimizing framework in which agents are assumed to minimize the expected future discounted costs of deviating from target money balances a forward-looking error correction model (ECM) of the short-run demand for money is derived. This model is then estimated for Canadian M2 and M2+ over the period 1968:I to 1989:IV. The estimation procedure is general to specific following the methodology of Davidson, Hendry, Srba and Yeo (1978), Hendry (1979) and Hendry and Ericsson (1990). The ECM is proposed to be a more stable representation of the demand for money function in terms of parameter constancy and forecasting ability than more restrictive dynamic specifications such as the partial adjustment model.

The paper is organized as follows. In the next section, I illustrate the derivation of the ECM from an optimization problem and its relation to integrated and cointegrated stochastic processes. The estimation methods, data and tests of structural stability and parameter constancy are presented in section three. In section four, I present the empirical estimations of the ECMs of M2 and M2+ along with results of the stability tests and out-of-sample forecasts. The conclusions and suggestions for further work are given in section five.

2 Deriving the ECM

2.1 A Dynamic Adjustment Model

Following Domowitz and Elbadawi (1990), Otto and Wirjanto (1990), Cuthbertson and Taylor (1990) and Gregory (1991) I derive the forward-looking ECM assuming that agents choose a sequence $\{m_{t+j}\}_{j=0}^{\infty}$ of the logarithm of real money balances to minimize a multi-period cost function. Agents choose this sequence in order to attain a long-run target level of real balances m_t^* which has a law of motion given by a static equilibrium theory. Thus agents choose the actual m_t to minimize the expected future discounted sum of the losses associated with deviating from the target and the costs of adjusting towards the target.

Therefore, the agent is assumed to solve the following stochastic dynamic programme:

$$\min_{\{m_{t+j}\}_{j=0}^{\infty}} \mathcal{L}_t = E_t \sum_{j=0}^{\infty} \beta^j \{\delta(m_{t+j} - m_{t+j}^*)^2 + (m_{t+j} - m_{t+j-1})^2\} , \quad (1)$$

where $\beta \in (0,1)$ is a discount factor, $\delta > 0$ is a weighting parameter and E is the rational expectations operator where it is assumed that expectations are formed conditional on some information set Ω_t available to the agent. The static equilibrium relationship describing the law of motion for the target variable can be represented generally as:

$$m_t^* = z_t \gamma + \epsilon_t , \quad (2)$$

where ϵ is an independently, identically distributed error with mean zero and constant variance σ_ϵ^2, γ is a $k \times 1$ parameter vector and z_t is a $1 \times k$ vector of forcing variables.

As Gregory (1991) shows, the forward solution to (1) is:

$$m_t = \lambda m_{t-1} + (1-\lambda)(1-\beta\lambda) E_t \sum_{j=0}^{\infty} (\beta\lambda)^j m_{t+j}^* , \quad (3)$$

where $\lambda < 1$ is the stable root of the Euler equation obtained from the first order conditions for the soluton of (1) (see Cuthbertson and Taylor [1990]). In order to replace the expectations in (3) with observables we must specify a law of motion for the forcing variables. Here we are interested in the case where z_t is a $1 \times k$ vector of processes that are integrated of order one ($I(1)$). To simplify notation, assume that $k = 1$ so that z_t is a scalar. Therefore, we can write:

$$\Delta z_t = \xi_t , \quad (4)$$

where ξ is a stationary, white noise error term and Δ is the first difference operator. Given this stochastic process for the forcing variable, (3) can be solved to obtain the ECM:

$$\Delta m_t = (\lambda - 1)(m_{t-1} - z_{t-1}\gamma) + (1-\lambda)\Delta z_t \gamma + (1-\beta\lambda)(1-\lambda)\epsilon_t . \quad (5)$$

The error correction term $(m_{t-1} - z_{t-1}\gamma)$ represents last period's disequilibrium in planned money holdings. The absolute value of the error correction parameter $(\lambda - 1)$ represents that portion of the disequilibrium corrected this period. Thus the ECM can be interpreted as a short-run demand for money where adjustments in current money holdings are made up of reactions to shocks, Δz_t, and corrections of past deviations from a long-run equilibrium.

Equation (5) is the simplest form of the ECM where only the contemporaneous change in z_t and no past changes in m_t are included on the right hand side. More general ECMs include both contemporaneous and lagged values of Δz_t and lagged values of Δm_t. The exact number of lags is chosen according to certain model selection criteria. Hendry (1979) and Hendry and Ericsson(1990) recommend starting with a general form of the ECM where the initial number of lags is chosen depending on the frequency of the data and then "testing down" using selection criteria to reach a specific ECM.[1]

2.2 Cointegration and the ECM

Deriving the ECM in (5) required a law of motion for the target variable and an assumed stochastic process for the forcing variables. We assumed that $\boldsymbol{z}_t$ is a $1 \times k$ vector of $I(1)$ variables and that there exists some vector $\boldsymbol{\gamma}$ such that (2) holds for all time periods. Intuitively these assumptions amount to specifying a cointegrating relationship between m_t and $\boldsymbol{z}_t$. Engle and Granger (1987) show that if this relationship holds, then by the *Granger Representation Theorem*, there exists a valid ECM for m_t which is given in (5). In a steady-state, actual money balances converge to target balances so that $m_t^* = m_t$. The implication is that the long-run equilibrium relationship between m_t and $\boldsymbol{z}_t$ is characterized by (2). More formally, if m_t and $\boldsymbol{z}_t$ are both $I(1)$ and there is some vector $\boldsymbol{\gamma}$ such that $m_t - \boldsymbol{z}_t\boldsymbol{\gamma} = \epsilon_t \sim I(0)$, then m_t and $\boldsymbol{z}_t$ are cointegrated and $\boldsymbol{\gamma}$ is the cointegrating vector.

An important result from the theory of cointegration is that if m_t and $\boldsymbol{z}_t$ are cointegrated then the ordinary least squares (OLS) estimates of the elements of the vector $\boldsymbol{\gamma}$ will be *super-consistent*. That is, they will approach their true values at a rate faster than $n^{-1/2}$. In fact, as Engle and Granger (1987) show, $\hat{\boldsymbol{\gamma}}$ approaches its true value at a rate proportional to n^{-1}.

Since the derivation and existence of the ECM relies on the time series properties of m_t and the forcing variables $\boldsymbol{z}_t$, it is important to verify that the data do exhibit these properties in order to use the ECM in an empirical

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[1] When using quarterly data, for example, one would start with four to six lags of Δz_t and Δm_t. To test-down one could use Akaike's Information Criterion (AIC) and other similar test statistics along with F tests of the significance of the lagged values.

excercise. Dickey and Fuller (1979) and (1981) have developed tests for the null hypothesis that a time series follows a random walk and is therefore $I(1)$. Engle and Granger (1987) have extended these procedures to tests of whether two or more variables are cointegrated where the null hypothesis is that of non-cointegration. The Engle and Granger (1987) procedure essentially tests that the residuals from the cointegrating regression (2) are $I(1)$ against the alternative that they are $I(0)$. These tests will be discussed further in section 3.1.

3 Data, Estimation Method and Stability Tests

3.1 The Data and Choice of Forcing Variables

In choosing the forcing variables I take a conventional transactions approach to money demand where m_t is a function of some scale variable and the opportunity costs of holding money:

$$m_t = f(y_t,\ R10_t,\ SDR_t,\ FP_t,\ I_t)\ . \tag{6}$$

where f_1' and $f_3' > 0$ and f_2', f_4' and $f_5' < 0$.

The definitions of all the variables and their sources are given explicitly in appendix A. m is the logarithm of real M2 or M2+. The scale variable is chosen as the logarithm of real GDP, y. The yield on ten year and over Government bonds is used as the competing rate. Since I am considering monetary aggregates that have components that bear interest, it is necessary to allow for an own rate. I have proxied the own rate on M2 and M2+ by the rate on savings deposits at chartered banks, SDR. The inflation rate, I, proxies the return to real assets. I have also considered the possibility that foreign money may be a close substitute to Canadian M2 and M2+. The expected return to holding foreign money is proxied by the ninety-day forward premium on the US dollar in Canada, FP. Therefore, the effect of currency substitution is accounted for in modelling Canadian money demand (see Daniel and Fried [1983]).

The GDP price index is used to deflate the monetary aggregates. All variables are unadjusted for seasonality except for the GDP price index.

The price index is taken seasonally adjusted since, according to CANSIM, the unadjusted series is more prone to changing weights and is, therefore, a less reliable indicator of price changes. I have included three seasonal dummy variables denoted by S_1, S_2 and S_3 in the estimated ECMs to account for seasonal factors. Any varaiable that was reported monthly was transformed to quarterly frequency using a three-month average. The opportunity cost terms are in levels rather than logarithms so that the net opportunity cost elasticity is allowed to vary with the level of the net opportunity cost. Therefore, the coefficients on $R10_t$, SDR_t, FP_t and I_t are interpreted as semi-elasticities.

There are two issues with respect to the data that I am not considering. These are the effects of postal strikes (see Gregory and MacKinnon [1981]) and sales of Canada Savings Bonds (CSBs). Recent studies have found that postal strike dummy variables have not performed well in Canadian money demand models (see Ebrill [1989]). Since CSBs are usually sold in the fourth quarter of each year, using variables such as the stock of CSBs outstanding as a measure of the effect that these sales have on money demand in Canada has not proved succesfull.

3.1.1 Unit Root Tests

To ensure that all of the variables used in the estimations follow a unit root process, I have used the tests developed by Dickey and Fuller (1979) which are commonly referred to as Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) tests. The DF test regression for the null hypothesis that a time series x_t is $I(1)$ is given by:

$$\Delta x_t = a + \alpha x_{t-1} + cT + e_t \,, \tag{7}$$

where a is a constant, T is a time trend and e is the error term. The regression (7) can be run with or without the time trend. The test statistic $\tau_{\hat{\alpha}}$ is the t ratio for the null hypothesis that $\alpha = 0$. This t statistic follows a nonconventional distribution derived by Fuller (1976). Original critical values were tabulated by Fuller (1976) and later extended by Dickey and Fuller (1979). MacKinnon (1991) has recently estimated more accurate critical values and has provided response surface regressions allowing one to estimate finite-sample critical values.

The major assumption of the DF test is that the error e be serially uncorrelated. In practice this assumption is often violated. To overcome this problem Dickey and Fuller (1979) recommended adding enough lagged values of Δx_t to the right hand side of (7) to ensure that e is white noise. Thus we have the ADF test regression:

$$\Delta x_t = a + \alpha x_{t-1} + \sum_{i=1}^{p} \eta_i \Delta x_{t-i} + cT + e_t \,, \tag{8}$$

where the test statistic and asymptotic critical values are the same as for the DF test.

For the variables used in this paper, an ADF test with $p = 4$ was required to remove any serial correlation in the test regression. The results are given in table 3.1 for the two cases of trend and no trend in the test regression. It is evident from these results that the null hypothesis of a unit root cannot be rejected for any of the variables.

3.1.2 Cointegration Tests

To test whether m_t is cointegrated with the variables specified in (5), Engle and Granger (1987) have recommended estimating the cointegrating regressions:

$$\log(M2/P)_t = \gamma_0 + \gamma_1 y_t + \gamma_2 R10_t + \gamma_3 SDR_t + \gamma_4 FP_t + \gamma_5 I_t + u_t \tag{9}$$

and

$$\log(M2+/P)_t = \gamma_0' + \gamma_1' y_t + \gamma_2' R10_t + \gamma_3' SDR_t + \gamma_4' FP_t + \gamma_5' I_t + v_t \tag{10}$$

and performing ADF tests on $\hat{u}_t$ and $\hat{v}_t$ where the null hypothesis is that the residuals are $I(1)$. That is, the null hypothesis is one of non-cointegration.

As in the ADF tests for unit roots in the individual variables, the cointegration ADF tests required four lags of the dependent variable on the right hand side to ensure serial independence of the errors in the test regressions. The results are presented in table 3.2. For M2 it is clear that the null hypothesis of non-cointegration is rejected at all significance levels for both the trend and no trend cases. Thus (9) is a cointegrating relationship for M2. The results for M2+ are not as strong. The $\tau_{\hat{\alpha}}$s for both the trend and no trend cases are just below the ten percent finite-sample critical values. However, this result may be attributed to the low power of the ADF test rather than lack of cointegration, especially when the root is just inside the unit circle. The calculated $\tau_{\hat{\alpha}}$s do exceed the asymptotic ten percent critical values.

3.2 Estimation Procedure

The ECM as written in equation (5) is nonlinear in the parameters. To avoid computationally burdensome nonlinear estimation, I will estimate the linear but statistically equivalent form of the ECM given by:[2]

$$\Delta m_t = \psi + \rho m_{t-1} - \rho \boldsymbol{z}_{t-1}\boldsymbol{\gamma} + \Delta \boldsymbol{z}_t \boldsymbol{\theta} + \xi_t \ , \tag{11}$$

where $\rho = (\lambda - 1)$ is the error correction parameter, $\boldsymbol{\theta}$ is a vector of parameters to be estimated and ξ is an independently and identically distributed error. I have now included a constant ψ in the ECM. From (11), the elements of the estimated parameter vector $\hat{\boldsymbol{\gamma}}$ can easily be derived and their standard errors can be calculated using the δ-method.

As noted in section two, equation (11) can be generalized to include lagged values of both Δm_t and $\Delta \boldsymbol{z}_t$ on the right hand side. In practice this is usually necessary to ensure that ξ is serially uncorrelated. The exact number of lags is chosen so that this condition is satisfied. Therefore I will begin by estimating the following general form of equation (11):

$$\Delta m_t = \psi + \rho m_{t-1} - \rho \boldsymbol{z}_{t-1}\boldsymbol{\gamma} + \sum_{i=0}^{6} \Delta \boldsymbol{z}_{t-i}\boldsymbol{\theta}_i + \sum_{j=1}^{6} \phi_j \Delta m_{t-j} + \xi_t \ . \tag{12}$$

Equation (12) will be estimated and tested down by ordinary least squares (OLS). There is a possibility that income and the opportunity cost terms may be determined endogenously by both the supply and demand for money. The resulting simultaneity would, of course, bias the OLS estimates. Poloz (1980) has considered this endogeneity problem in the context of varying monetary policy regimes. However, the results generally indicate the simultaneous equation bias to be quite small (see Ebrill [1989]).

3.3 Stability Tests

To test the estimated ECMs for structural stability and parameter constancy, I will employ the recursive stability tests developed by Brown, Durbin and

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[2] The reader will appreciate this point after the stability tests are described in the next section. These tests would potentially require close to two hundred nonlinear estimations for each ECM estimated.

Evans (1975). These tests involve estimating the regression equation recursively, adding observations and performing OLS until the last step uses all of the observations. The recursive estimations can be performed starting at the beginning of the sample and working forward or starting at the end of the sample and proceeding in reverse. The backward recursion is often necessary if the regression equation under test contains many regressors. Since the forward resursion will necessarily begin the stability test at observation $k+1$, the stability of the regression function over the first k observations of the sample will be indeterminant unless a backward recursion is also performed.

Brown, Durbin and Evans (1975) suggested two tests based on the recursive residuals generated in the sequential estimation procedure described above. These are known as the CUSUM and CUSUMSQ tests. Since the power of the CUSUM test in finite samples is sometimes quite low (see Ploberger, Krämer and Alt [1989]) I will use the CUSUMSQ test which is based on a test statistic calculated from the cumulative sum of squared recursive residuals defined as:

$$WW_t = \frac{\sum_{t=k+1}^{t} \vartheta_t^2}{\sum_{t=k+1}^{T} \vartheta_t^2}\,, \tag{13}$$

where ϑ_t is the recursive residual for the t^{th} observation. WW_t is essentially the ratio of the cumulative sum of squared recursive residuals for the first t observations to the cumulative sum of squared recursive residuals for the entire sample beginning at $t = k+1$ in a forward recursion. When $t = k$, the numerator in (13) is zero so that $WW_k = 0$ and when $t = T$, $WW_T = 1$. The plot of the CUSUMSQ statistic under the null hypothesis of parameter constancy lies on the diagonal from zero to one. The stability test is most conveniently conducted graphically where the calculated CUSUMSQs are plotted within a confidence band. The null hypothesis is rejected if the calculated CUSUMSQ plot deviates significantly from the zero-one diagonal.

4 Empirical Results

4.1 M2 Demand

Following the procedure outlined in section three, the tested-down version of the linear form of the ECM for M2 was found to be:

$$\begin{aligned}\Delta m_t \;=\;& \psi + \rho m_{t-1} + \rho\gamma_1 y_{t-1} + \rho\gamma_1 R10_{t-1} + \rho\gamma_3 SDR_{t-1} + \rho\gamma_4 I_{t-1} + \\ & \rho\gamma_5 FP_{t-1} + \phi_1 \Delta m_{t-1} + \phi_4 \Delta m_{t-4} + \theta_{4_{R10}} \Delta R10_{t-4} + \\ & \theta_{4_{SDR}} \Delta SDR_{t-4} + \theta_{0_{FP}} \Delta FP_t + \theta_{2_{FP}} \Delta FP_{t-2} + \theta_{4_{FP}} \Delta FP_{t-4} + \\ & \theta_{0_I} \Delta I_t + \theta_{1_I} \Delta I_{t-1} + \theta_{2_I} \Delta I_{t-2} + \theta_{4_I} \Delta I_{t-4} + c_1 S_1 + c_2 S_2 + \\ & c_3 S_3 + \xi_t \,, \qquad (14)\end{aligned}$$

where the parameter estimates and summary statistics are given in table 4.1. AIC is Akaike's Information Criterion, ARCH(p) is a χ^2 statistic for autoregressive conditional heteroskedasticity of order p and $\chi^2(23)$ is the Lagrange multiplier statistic for the null hypothesis that the first twenty-three autocorrelations are zero.

For an ECM, equation (14) displays relatively good fit. The error correction term is significant and negative as expected adding support for the finding of a cointegrating relationship between M2 and the forcing variables. The testing-down procedure has produced an ECM free of residual autocorrelation and ARCH errors.

It is evident that past shocks in the forcing variables do matter in the way agents adjust their level of M2 holdings. The estimate of the error correction parameter, $\hat{\rho}$, indicates that agents correct for approximately 7.0 percent of the disequilibrium in M2 holdings every quarter.

The cointegrating, long-run equilibrium equation (9) for M2 is given in table 4.2. The estimates of the cointegrating parameters are significant and have the expected signs. the long-run income elasticity of 1.67 is relatively high. However, this result is not inconsistent with theories of scale economies in cash management. Thus we would expect the broader monetary aggregates to be more sensitive to changes in the scale variable than narrower aggregates such as M1 (see Ebrill [1989]). The net interest rate semi-elasticity $(\hat{\gamma}_2 - \hat{\gamma}_3)$ of -0.04 is very low indicating that the broader aggregate is less sensitive to changes in the opportunity cost of holding money. This result is intuitive since M2 contains interest-bearing components that are not included in the definitions of the narrower aggregates.

It is evident from the significant and negative coefficient on the forward premuim variable that the expected return to holding foreign money represents an opportunity cost to domestic M2 balances. This result has important implications for policy. Currency substitution in a freely floating exchange rate regime can result in a loss of domestic monetary policy independence. If the monetary authority conducts policy using money demand models that fail to account for domestic agents holding foreign money and foreign agents holding domestic money, then currency substitution could result in those models being misspecified. Further, the sensitivity of M2 demand to the expected returns to foreign money is strikingly close to the net interest rate semi-elasticity. Therefore, this currency substitution effect is by no means negligible.

Figure 1 shows the forward recursion of the CUSUMSQ test for M2. The test begins in the third quarter of 1974. The calculated CUSUMSQs lie fairly close to the zero-one diagonal for the entire test period. Therefore, the null hypothesis of parameter constancy is not rejected. To get a better picture of the stability of the M2 ECM over the early part of the sample, I have plotted the backward recursions of the CUSUMSQs in figure 2. Now we see that the stability of the ECM from the first quarter of 1975 to the first quarter of 1976 is questionable. This period coincides with the introduction of cash management packages by the major banks and the development of over night money market instruments.

4.2 M2+ Demand

The ECM for M2+ is given by:

$$\begin{aligned}\Delta m_t \;=\;& \psi + \rho m_{t-1} + \rho\gamma_1' y_{t-1} + \rho\gamma_1' R10_{t-1} + \rho\gamma_3' SDR_{t-1} + \rho\gamma_4' I_{t-1} + \\ & \rho\gamma_5' FP_{t-1} + \phi_1 \Delta m_{t-1} + \phi_2 \Delta m_{t-2} \phi_3 \Delta m_{t-3} + \theta_{0_y} \Delta y_t + \\ & \theta_{3_{R10}} \Delta R10_{t-3} + \theta_{2_{SDR}} \Delta SDR_{t-2} + \theta_{4_{SDR}} \Delta SDR_{t-4} + \\ & \theta_{2_{FP}} \Delta FP_{t-2} + \theta_{0_I} \Delta I_t + \theta_{2_I} \Delta I_{t-2} + \theta_{4_I} \Delta I_{t-4} + c_1 S_1 + \\ & c_3 S_3 + \xi_t \;, \end{aligned} \tag{15}$$

where the parameter estimates and diagnostics are given in table 4.3. As for the M2 case, the error correction parameter estimate is significant and negative. Equation (15) also displays fairly good fit and is free of residual autocorrelation and ARCH error processes up to order four.

The significant estimate for the error correction parameter provides evidence that the inabiltiy to find a cointegrating relationship for M2+ in section 3.1.2 was probably due to the low power of the conventional tests. The long-run cointegrating equation (10) for M2+ is given in table 4.4. Here we see that M2+ has a higher income elasticity of 1.73 compared to that for M2 lending support for the hypothesis that there exists scale economies in cash management. Also, the net interest semi-elasticity of -0.03 is lower than that for M2 which is expected since the "plus" component of M2+ is mainly composed of interest-bearing deposits at near banks. The forward premium semi-elasticity of -0.012 is also lower for M2+ than for M2 indicating that the broader aggregate is less sensitive to changes in the expected returns to foreign money holdings.

The forward recursion of the CUSUMSQ test for M2+ is presented in figure 3. Just as in the M2 case, this test shows a stable function for the period 1974:III to 1989:IV. However, the backward recursion in figure 4 indicates a potential shift in the M2+ ECM from the first quarter of 1976 to the first quarter of 1977. This result is consistent with that for the M2 ECM.

4.3 Out-of-Sample Forecasts

This section presents the results of out-of-sample on-step-ahead forecasts for the M2 and M2+ ECMs. The sample period for estimation is set at 1968:I to 1987:IV. Forecasts are then made for the period 1988:I to 1989:IV. The forecasts for M2 and M2+ are plotted in figures 5 and 6 respectively. The summary statistics are given in table 4.5. Theil's (1978) inequality coefficient, U, compares the one-step-ahead forecasts to mere "no-change" extrapolations. The inequality coefficient must be less than one if the forecasts are to be considered better than the no-change extrapolations.

Both the ECMs seem to forecast with fairly good fit. In general, the forecasts lie within one standard error of the actual values for all eight quarters.

5 Conclusions

This paper has applied recent advances in studying integrated and cointegrated economic time series to deriving a dynamic model of the short-run demand for broad money in Canada. The error correction model (ECM) seems to be a relatively well-behaved representation of M2 and M2+ demand in terms of parameter constancy and forecasting ability for the period 1968:I to 1989:IV.

The substantial financial innovations that occured during this period were originally cited as the primary reason for the poor performance of standard money demand models in prediciting M1 demand in the late 1970s and the subsequent departure from M1 targetting in late 1981. Following the recent work at the Bank of Canada, this paper has investigated an alternative approach to modelling the dynamics of money demand and has provided evidence that the ECM could be used for the purposes of targetting M2 and M2+. The ECM has the attractive feature of combining short-run dynamics with a long-run static equilibrium relationship in a single equation while still permitting parsimonious inference on certain relationships that have importance economic implications.

It is found that that both M2 and M2+ demand have very low net interest elasticities. This result indicates that a policy of interest rate targetting may prove difficult since the monetary authority would have to control the spread between competing and own rates of return. Previous studies (see Daniel and Fried [1983]) have found that the effect of currency substitution on Canadian money demand has been significant but close to negligible in terms of its magnitude in relation to that of the interest rate effect. The parameters estimated on the forward premium varable in this paper indicate that the currency substitution effect is by no means small. For M2, this effect is approximately the same as the net interest rate effect. However, M2+ demand displays less sensitivity to the expected returns to holding foreign money. Therefore, if the monetary authority is interested in using M2 as a policy tool, it would have to take account of the potentially destabilizing effects of currency substitution in constructing targets.

A Data Appendix

This appendix describes the data used in the empirical estimations and their sources. The CANSIM series identifiers refer to the University Base version of the CANSIM databank.

M2: currency outside banks and chartered bank demand, chequeable, notice and personal term deposits, unadjusted for seasonal variation, in millions of dollars. CANSIM series B2031, monthly.

M2+: M2 plus deposits at near banks, unadjusted for seasonal variation, in millions of dollars. Bank of Canada, monthly.

Y: Gross Domestic Product (GDP) at 1981 prices, unadjusted for seasonal variation, in millions of dollars. CANSIM series D10031, quarterly.

P: implicit GDP price index (1981=100), seasonally adjusted at annual rates. CANSIM series D20337, quarterly.

SDR: chartered bank non-chequeable savings deposit rate, per cent per annum. CANSIM series B14019, monthly.

FP: 90-day forward premium (+) or discount (−) on the US dollar in Canada, per cent per annum. CANSIM series B14043, monthly.

R10: average yield on 10 year and over Government of Canada bonds, per cent per annum. *IMF International Financial Statistics,* databank identifier (subject code) No.61, quarterly.

I: rate of inflation ($= \log P_t - \log P_{t-1}$).

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Table 3.1
Unit Root Tests

Variable	$\tau_{\hat{\alpha}}$ (No Trend)	$\tau_{\hat{\alpha}}$ (Trend)
$\log(M2/P)$	0.005	−2.394
$\log(M2+/P)$	−0.309	−2.355
y	−0.894	−2.193
$R10$	−1.974	−1.995
SDR	−2.233	−2.230
FP	−2.325	−3.034
I	−2.239	−2.423

Critical Values for $T = 88$

No Trend	Trend	Size
−3.506	−4.065	1 %
−2.894	−3.461	5 %
−2.584	−3.156	10 %

Asymptotic Critical Values

No Trend	Trend	Size
−3.900	−3.964	1 %
−3.338	−3.413	5 %
−3.046	−3.128	10 %

Table 3.2
Cointegration Tests

Residual	$\tau_{\hat{\alpha}}$ (No Trend)	$\tau_{\hat{\alpha}}$ (Trend)
$\hat{u}$	−6.225	−7.344
$\hat{v}$	−4.490	−4.857

Critical Values for $T = 88$

No Trend	Trend	Size
−5.544	−5.869	1 %
−4.901	−5.215	5 %
−4.576	−4.887	10 %

Asymptotic Critical Values

No Trend	Trend	Size
−5.240	−5.513	1 %
−4.705	−4.977	5 %
−4.424	−4.700	10 %

Table 4.1
M2 ECM–OLS Estimation

Sample Period: 1968:I–1989:IV

Variable	Parameter	Estimate	Standard Error
m_{t-1}	ρ	−0.0697	0.0223
y_{t-1}	$\rho\gamma_1$	0.1163	0.0277
$R10_{t-1}$	$\rho\gamma_2$	−0.0660	0.0008
SDR_{t-1}	$\rho\gamma_3$	0.0038	0.0006
FP_{t-1}	$\rho\gamma_4$	−0.0028	0.0006
I_{t-1}	$\rho\gamma_5$	−0.6869	0.1548
Δm_{t-1}	ϕ_1	0.2641	0.0836
Δm_{t-4}	ϕ_4	−0.1491	0.0458
$\Delta R10_{t-4}$	$\theta_{4_{R10}}$	0.0070	0.0016
ΔSDR_{t-4}	$\theta_{4_{SDR}}$	−0.0037	0.0009
ΔFP_t	$\theta_{0_{FP}}$	−0.0024	0.0007
ΔFP_{t-2}	$\theta_{2_{FP}}$	0.0029	0.0007
ΔFP_{t-4}	$\theta_{4_{FP}}$	0.0025	0.0009
ΔI_t	θ_{0_I}	−0.9783	0.0711
ΔI_{t-2}	θ_{1_I}	0.1567	0.0785
ΔI_{t-4}	θ_{4_I}	−0.0026	0.0009
S_1	c_1	0.0384	0.0024
S_2	c_2	0.0290	0.0027
S_3	c_3	0.0100	0.0033
constant	ψ	−0.7889	0.1634

Summary Statistics and Diagnostics

$R^2 = 0.8718$	$\bar{R}^2 = 0.8305$	$\hat{\sigma} = 0.0074$	AIC= 0.00007
ARCH(1)= 1.70	ARCH(4)= 7.5504	$\chi^2(23) = 15.487$	
$\chi^2_{23,0.01} = 41.638$	$\chi^2_{23,0.05} = 35.173$	$\chi^2_{1,0.01} = 6.635$	$\chi^2_{1,0.05} = 3.842$
$\chi^2_{4,0.01} = 13.277$	$\chi^2_{4,0.05} = 9.488$		

Table 4.2
M2 Demand
Long-Run Equilibrium Equation

Variable	Parameter	Estimate	Standard Error
y_t	γ_1	1.6680	0.3973
$R10_t$	γ_2	−0.0953	0.0118
SDR_t	γ_3	0.0549	0.0093
FP_t	γ_4	−0.0397	0.0088
I_t	γ_5	−9.8520	2.2200
constant	γ_0	−11.3150	2.3440

Table 4.3

M2+ ECM–OLS Estimation

Sample Period: 1968:I–1989:IV

Variable	Parameter	Estimate	Standard Error
m_{t-1}	ρ	−0.0767	0.0222
y_{t-1}	$\rho\gamma_1'$	0.1328	0.0331
$R10_{t-1}$	$\rho\gamma_2'$	−0.0051	0.0007
SDR_{t-1}	$\rho\gamma_3'$	0.0027	0.0006
FP_{t-1}	$\rho\gamma_4'$	−0.0009	0.0005
I_{t-1}	$\rho\gamma_5'$	−0.4380	0.0998
Δm_{t-1}	ϕ_1	0.4097	0.0763
Δm_{t-2}	ϕ_2	0.2088	0.0850
Δm_{t-3}	ϕ_3	−0.2476	0.0791
Δy_t	θ_{0_y}	0.1188	0.0185
$\Delta R10_{t-3}$	$\theta_{3_{R10}}$	0.0059	0.0013
ΔFP_{t-2}	$\theta_{2_{FP}}$	0.0019	0.0007
ΔSDR_{t-2}	$\theta_{2_{SDR}}$	−0.0032	0.0008
ΔSDR_{t-4}	$\theta_{4_{SDR}}$	−0.0018	0.0005
ΔI_t	θ_{0_I}	−0.8622	0.0491
ΔI_{t-2}	θ_{2_I}	0.1952	0.0925
ΔI_{t-4}	θ_{4_I}	−0.0030	0.0007
S_1	c_1	0.0168	0.0020
S_3	c_3	−0.0115	0.0026
constant	ψ	−0.8890	0.2105

Summary Statistics and Diagnostics

$R^2 = 0.8985$	$\bar{R}^2 = 0.8679$	$\hat{\sigma} = 0.0055$	AIC= 0.00004
ARCH(1)= 0.42	ARCH(4)= 7.1016	$\chi^2(23) = 22.900$	
$\chi^2_{23,0.01} = 41.638$	$\chi^2_{23,0.05} = 35.173$	$\chi^2_{1,0.01} = 6.635$	$\chi^2_{1,0.05} = 3.842$
$\chi^2_{4,0.01} = 13.277$	$\chi^2_{4,0.05} = 9.488$		

Table 4.4
M2+ Demand
Long-Run Equilibrium Equation

Variable	Parameter	Estimate	Standard Error
y_t	γ_1'	1.7326	0.4318
$R10_t$	γ_2'	−0.0667	0.0096
SDR_t	γ_3'	0.0355	0.0080
FP_t	γ_4'	−0.0120	0.0060
I_t	γ_5'	−5.7140	1.3020
constant	γ_0'	−11.5980	2.7460

Table 4.5
Out-of-Sample Forecasts
Diagnostics

Forecast Period: 1988:I–1989:IV

Statistic	M2 ECM	M2+ ECM
R^2 between forecast and actual	0.9242	0.8029
Sum of squared errors	0.0023	0.0014
Root mean square error	0.00538	0.00412
Theil's (1978) inequality coefficient, U	0.325	0.307

Fig.1: M2 ECM, CUSUMSQ Test
Forward Recursion, 95% Confidence Band

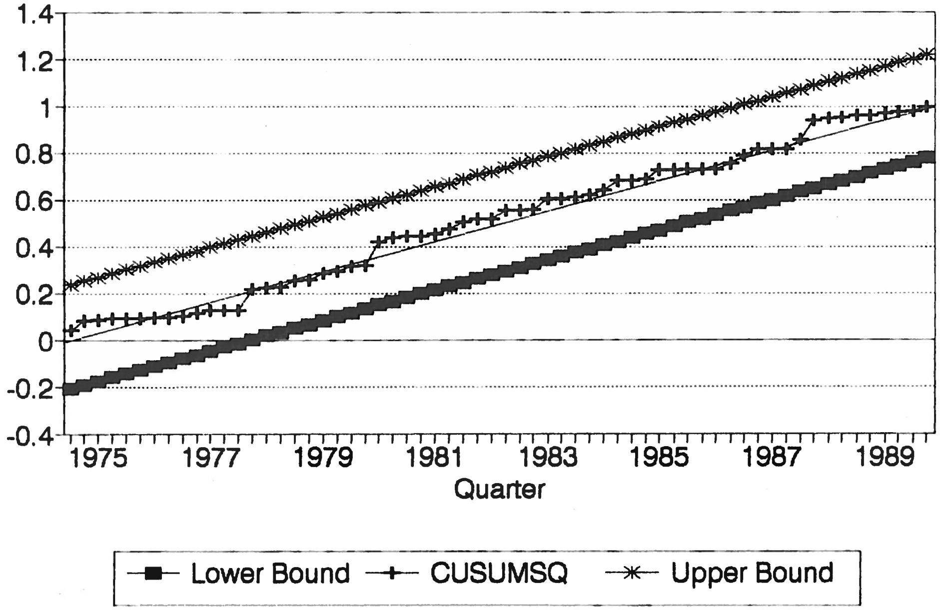

Fig.2: M2 ECM, CUSUMSQ Test
Backward Recursion, 95% Confidence Band

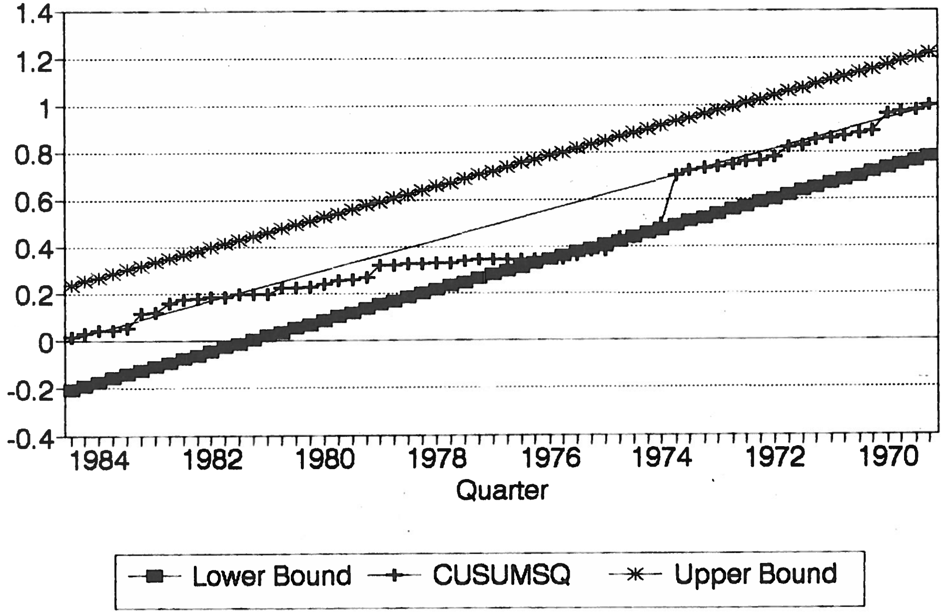

Fig.3: M2+ ECM, CUSUMSQ Test
Forward Recursion, 95% Confidence Band

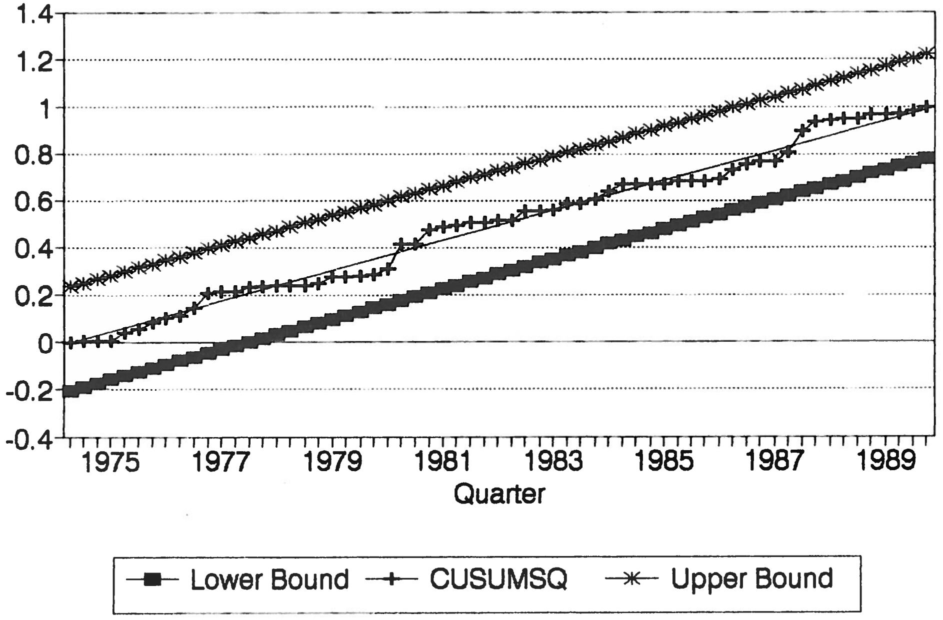

Fig.4: M2+ ECM, CUSUMSQ Test
Backward Recursion, 95% Confidence Band

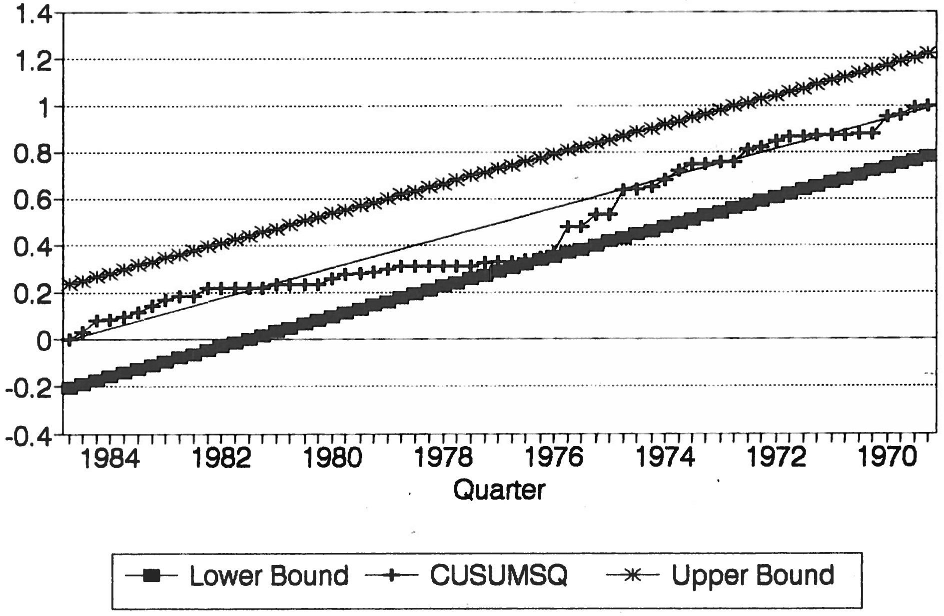

Fig.5: M2 ECM

Out-of-Sample Forecasts

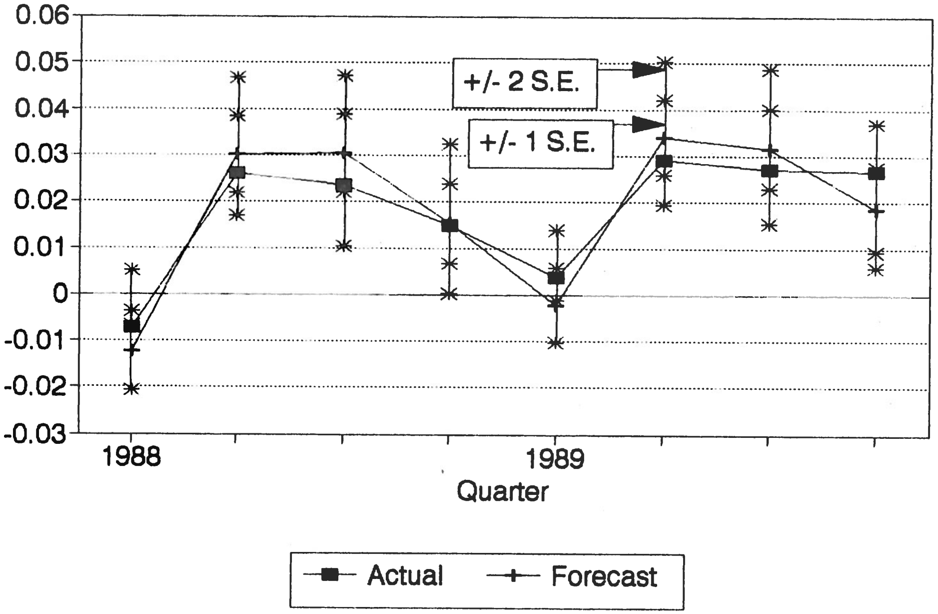

Fig.6: M2+ ECM

Out-of-Sample Forecasts