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The Comparison Between Ad Valorem and Specific Taxation under Imperfect Competition

Sofia Delipalla Michael Keen

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

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THE COMPARISON BETWEEN AD VALOREM AND SPECIFIC TAXATION UNDER IMPERFECT COMPETITION

By

Sofia Delipalla* and Michael Keen**

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* University of Essex, Wivenhoe Park, Colchester CO4 3SQ, UK.
** University of Essex and Queen's University, Kingston, Ontario K7L 3N6, Canada.

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Abstract: This paper compares ad valorem and specific taxation in two models of oligopoly, with and without free entry. Predominantly ad valorem taxation implies a relatively low consumer price, high tax revenue and (when entry is precluded) low profits. Ad valorem taxation dominates specific from the welfare perspective: the set of circumstances under which (with free entry) specific taxation raises welfare is a strict subset of that in which ad valorem taxation is welfare-improving, and in both models the maximisation of consumer welfare subject to a binding revenue constraint requires maximum reliance on ad valorem taxation.

1. INTRODUCTION

The comparison between ad valorem and specific (or 'unit') taxes is one of the oldest issues in the formal study of public finance. Their equivalence under perfect competition has long been recognised. Realisation that they require separate consideration under conditions of monopoly dates back to Cournot (1960; originally published 1838). Analysis of the monopoly case continued with Wicksell (1959; originally published 1896) - who showed, inter alia, that more tax revenue could be raised by ad valorem taxation than by specific - and culminated in the thorough treatment by Suits and Musgrave (1955). The issue is also one of continuing policy interest. In particular, the balance between ad valorem and specific taxes has been a (and until recently the)[1] central concern in the European Community's programme for the harmonisation of cigarette taxes. Successive stages of this programme, which began in the early 1970's, have involved a progressive narrowing of the permissible ratio of specific to total taxation, which now stands at 5 to 55 per cent. The contentiousness of the topic is indicated by four of the Member States currently being within five points of the upper bound and five within the same distance of the lower (whilst one is strictly below).

Little attention has been paid, however, to the comparative effects of ad valorem and specific taxes outside the polar cases of monopoly and perfect competition. The omission is clearly a significant one, not least in the particular context of cigarette taxation. In this paper we therefore begin to extend the comparison between the two to

intermediate market forms. Some of the issues that then arise have been studied by Kay and Keen (1983,1990) and Dierickx, Matutes and Neven (1988): the former examine the effects of the tax mix on product quality, the latter analyse the differential impact of the two forms of taxation on the way in which differences in costs across firms translate into differences in market share and profitability under Cournot competition.[2] Neither, however, attempts a systematic comparison between ad valorem and specific taxation in varying circumstances of homogeneous product oligopoly. That is the objective here.

The next section describes the framework of the analysis, which borrows heavily from the recent literature on tax incidence under imperfect competition, and derives the comparative statics results needed for the comparison of the two tax instruments. This begins in Section 3 with a focus on positive aspects, dealing with relative effects on prices, profits and entry. Welfare and revenue considerations are taken up in Section 4, which establishes, for a range of policy problems, the optimal balance between ad valorem and specific taxation. Section 5 concludes.

2. THE FRAMEWORK AND PRELIMINARY RESULTS

The framework is essentially that of Stern (1987), amended straightforwardly for the purpose in hand. The description here will therefore be brief.

We consider an industry consisting of n identical firms producing a single homogeneous product. The output of firm i is denoted by x_i and industry output by $X=\Sigma_i x_i$. All firms have the same cost structure, C(x). Fixed costs C(0) are taken to be strictly positive, and there are assumed to be increasing returns to scale in the sense that $C_x(x)x/C(x)<1$ (for which it is sufficient that marginal cost be non-increasing). The consumer price is given by an inverse demand function P(X), with derivative $P_X(X)<0$; the elasticity of (direct) demand is denoted by $e=-P/XP_X>0$. Introducing ad valorem taxation at the (tax-inclusive) rate t_v (assumed throughout to be strictly less than unity) and a specific tax of[3] t_s, the profits of firm i are thus

$$\pi_i = \{(1-t_v)P(X) - t_s\}x_i - C(x). \qquad (2.1)$$

In selecting its output each firm i conjectures that other firms' responses will be such that $dX/dx_i=\lambda$, this conjectural variation λ being taken as a fixed constant throughout. As is well-known, this formulation encompasses a wide range of possibilities. The case $\lambda=1$ corresponds to the Cournot conjecture. When $\lambda=0$, conjectures are 'competitive' in the sense that each firm takes industry output to be unaffected by its own output choice; 'Bertrand' might be a better label, since such conjectures lead to marginal cost pricing irrespective of the number of the firms. When $\lambda=n$, each firm

believes that all other active firms will behave exactly as it does; tacit collusion among incumbent firms then being perfect (in the sense that aggregate profits are maximised conditional on the number of firms), we refer to this case as that of joint profit maximisation. It will be assumed throughout that $\lambda \in (0,n]$, the limiting 'competitive' case - and associated familiar equivalence of the two forms of taxation - being omitted for brevity in stating results.

The first order condition of the representative firm is then (omitting arguments)

$$(1-t_v)P - t_s + (1-t_v)\lambda P_X x_i = C_x. \qquad (2.2)$$

Confining attention - as we shall - to symmetric equilibria, this becomes

$$(1-t_v)[P + \gamma P_X X] - t_s = C_x , \qquad (2.3)$$

where

$$\gamma = \lambda/n \in (0,1]. \qquad (2.4)$$

The second order condition requires that

$$2 + A - \gamma E > 0 \qquad (2.5)$$

where

$$A = -C_{xx}/\lambda(1-t_v)P_X \qquad (2.6)$$

and $E = -P_{XX}X/P_X$ denotes the elasticity of the slope of inverse demand.[4] For brevity, it will also be assumed throughout that $2+A>0$.

We refer to the case in which n is fixed as the Generalised Cournot model and to that in which entry and exit are free as that of Free Entry Oligopoly. In analysing the first of these we impose a restriction somewhat stronger than the second order condition by supposing the stability

condition of Seade (1980) to be satisfied, so that

$$1 + \gamma(1+A-E) > 0. \tag{2.7}$$

The free entry model is characterised by the addition of the zero profit condition

$$\{(1-t_v)P - t_s\}X - nC(X/n) = 0. \tag{2.8}$$

We follow the usual practice of treating n as a continuous variable.

Interest here centres on the comparison between ad valorem and specific taxation, and on the effects of changes in the balance between the two. To this end it is useful first to establish the comparative statics of each tax instrument considered in isolation. Taking the two models in turn:

PROPOSITION 1. *In the Generalised Cournot model, taxes affect prices as*

$$\frac{dP}{dt_s} = \frac{1}{(1-t_v)[1+\gamma(1+A-E)]} > 0 \tag{2.9}$$

$$\frac{dP}{dt_v} = \Phi\frac{dP}{dt_s}, \tag{2.10}$$

where

$$\Phi = \frac{C_X+t_s}{1-t_v}. \tag{2.11}$$

They affect aggregate profits $\Pi=n\pi$ *as*

$$\frac{d\Pi}{dt_s} = \frac{-\gamma X(2+A-E)}{1+\gamma(1+A-E)} \tag{2.12}$$

$$\frac{d\Pi}{dt_v} = P\frac{d\Pi}{dt_s} - \frac{\gamma(1-\gamma)PX}{e[1+\gamma(1+A-E)]}. \tag{2.13}$$

Proof: The first price part (2.9) is immediate on applying the implicit function theorem to (2.3); (2.10) follows similarly on noting from (2.3) that

$$P + \gamma P_X X = \Phi. \tag{2.14}$$

For the profit part, differentiate aggregate profits, defined from (2.1) for a symmetric equilibrium, to find

$$\frac{d\Pi}{dt_i} = (1-t_v)(1-\gamma)X\left[\frac{dP}{dt_i}\right] - \delta_i X, \quad i = s,v \tag{2.15}$$

where $\delta_v=P$, $\delta_s=1$ and use has been made of (2.3). Substituting (2.9) and (2.10) into (2.15) gives (2.12) and (2.13). QED

PROPOSITION 2. *In the model of Free Entry Oligopoly, taxes affect prices as*

$$\frac{dP}{dt_s} = \frac{2+A}{(1-t_v)(2+A-\gamma E)} > 0 \tag{2.16}$$

$$\frac{dP}{dt_v} = \alpha\frac{dP}{dt_s}, \tag{2.17}$$

where

$$\alpha = \frac{P(1+A)+\Phi}{2+A}. \tag{2.18}$$

They affect the number of firms as

$$\frac{dn}{dt_s} = \frac{-\gamma X(2+A-E)}{\theta(2+A-\gamma E)} \tag{2.19}$$

$$\frac{dn}{dt_v} = P\frac{dn}{dt_s} - \frac{\gamma(1-\gamma)PX}{e\theta(2+A-\gamma E)} \tag{2.20}$$

where $\theta(X) = C(X) - C_X(X)X > 0$.

Proof: Perturbing (2.3) and (2.8) gives

$$\begin{bmatrix} 1+\gamma(1+A-E) & -\gamma P_X X(1+A) \\ (P-\Phi)X_P+X & -\theta/(1-t_v) \end{bmatrix}\begin{bmatrix} dP \\ dn \end{bmatrix} = \frac{1}{1-t_v}\begin{bmatrix} 1 & \Phi \\ X & PX \end{bmatrix}\begin{bmatrix} dt_s \\ dt_v \end{bmatrix}, \tag{2.21}$$

where X_p is the derivative of direct demand X(P) and use has been made of (2.14). Solving (2.21), it is then a matter of using the first order and zero profit conditions to simplify. Denoting the matrix on the left of (2.21) by Σ, for instance, its determinant is

$$\text{Det}(\Sigma) = -\theta[1+\gamma(1+A-E)]/(1-t_v) + \gamma P_X x(1+A)[(P-\Phi)X_p+X] \quad (2.22)$$

$$= -\theta[1+\gamma(1+A-E)]/(1-tv) + x(1+A)(P-\Phi)(\gamma-1) \quad (2.23)$$

$$= -\theta(2+A-\gamma E)/(1-t_v) \quad (2.24)$$

where (2.23) follows from $P_X X_p=1$ and (2.14) whilst (2.24) then follows on noting that the zero profit condition (2.8) implies

$$(P-\Phi)x = \theta/(1-t_v) \quad (2.25)$$

and collecting terms. QED.

Previous analyses have tended to consider only specific taxation, which proves analytically rather more convenient than ad valorem. In this context the results above merely generalise those of others to allow for pre-existing ad valorem taxation. Consider for brevity the case in which costs are linear, so that A=0. Setting $t_v=0$ in (2.9) then shows that specific taxation is over-shifted ($dp/dt_s>1$) in the Generalised Cournot model iff E>1. This is as in Seade (1985) and Stern (1987). Indeed it is of interest to note that this condition can be traced all the way back to Cournot (1960) himself, who seems to have been the first to recognise the possibility of overshifting.[5] Other results on the incidence of specific taxation encompassed in Propositions 1 and 2 include (from (2.12)) Seade's (1985) observation that a tax increase raises profits iff E>2 and (setting $t_v=0$ in (2.16)) Besley's (1989) finding that over-shifting occurs with free

entry iff E>0.

The results on ad valorem taxation are more novel, and of some independent interest. In this case over-shifting might naturally be said to occur iff $dP/dt_v > P$, so that the induced price increase exceeds the increase in tax payable at the initial price. It then emerges from (2.10) and (2.17) that, in both the Generalised Cournot and Free Entry Oligopoly models, ad valorem taxes are less likely to be over-shifted than specific: since $P>\Phi$ from (2.14) and

$$P-\alpha = \frac{P-\Phi}{2+A} > 0 \qquad (2.26)$$

from (2.18), over-shifting of specific taxes is in each case necessary, but not sufficient, for over-shifting of ad valorem. Note too, comparing (2.12) and (2.13), that increasing ad valorem taxation is less likely to raise profits than is increasing specific. These observations begin to suggest, in broad terms, that predominantly ad valorem taxation will imply both a relatively low price and relatively low profits, themes that will emerge more sharply in the next section.

3. POSITIVE ASPECTS OF THE COMPARISON: PRICES, PROFITS AND ENTRY

Turning now to the explicit comparison of ad valorem and specific taxation, we consider in this section the way in which the balance between the two affects the key endogenous variables of the models described above: price, profits (in the Generalised Cournot model) and the number of active firms (in the model of Free Entry Oligopoly). For this it is necessary to choose a particular basis of comparison. An obvious possibility would be to compare mixtures of ad valorem and specific taxation that generate the same tax revenue. There seem, however, to be relatively few results of this kind that are both simple and general (though some will emerge in the next section). Here we focus instead on the effects of tax reforms which, while not generally fully revenue-neutral, have no 'first round' effect on tax revenue. Specifically, we consider a tax change of the form

$$Pdt_v = - dt_s > 0, \qquad (3.1)$$

which has the feature of tilting the balance towards ad valorem taxation whilst leaving total tax payments at the initial equilibrium price unchanged.[6] The reform (3.1) will be referred to as a 'P-shift' from specific to ad valorem taxation.

We again consider the two models in turn:

PROPOSITION 3. *In the Generalised Cournot model, a P-shift from specific to ad valorem taxation leads to*

(a) A strict reduction in the consumer price, and

(b) A strict reduction in profits, except in the polar case of joint profit

maximisation (in which they are unaffected).

Proof: (a) The effect on price of an arbitrary tax reform is given by

$$dP = \left[\frac{dP}{dt_V}\right]dt_V + \left[\frac{dP}{dt_S}\right]dt_S. \qquad (3.2)$$

Substituting into (3.2) for the particular reform in (3.1) and using (2.10) of Proposition 1 gives

$$dP = -(P-\Phi)\left[\frac{dP}{dt_S}\right]dt_V, \qquad (3.3)$$

and the result then follows from $P>\Phi$, $dP/dt_S>0$ (from (2.9)) and $dt_V>0$ (from (3.1)).

(b) Using (2.12) and (2.13) of Proposition 1 a similar argument to that in part (a) gives

$$d\Pi = -\left[\frac{\gamma(1-\gamma)PX}{e[1+\gamma(1+A-E)]}\right]dt_V. \qquad (3.4)$$

which is strictly positive for $\gamma\in(0,1)$ and zero in the joint profit maximising case $\gamma=1$. QED.

The intuition behind part (a) of this result is straightforward. Denote by MR_i^* the perceived after-tax marginal revenue of firm i, as on the left of (2.2). Perturbing for the P-shift in (3.1), holding both X and x_i constant, gives

$$d(MR_i^*) = -\lambda P_X x_i dt_V > 0 \qquad (3.5)$$

so that at the initial output configuration each firm now perceives marginal revenue to exceed marginal cost and so, by the second order condition, finds it optimal to expand output.

And the reason output expansion becomes profitable is simply that (as is evident from (2.2)) under ad valorem taxation, but not under specific, part of the associated reduction in sales revenue on intra-marginal units is borne by the Exchequer rather than by the firm.

Part (b) of Proposition 3 would lead one to expect firms to lobby for specific rather than ad valorem taxation. Interestingly, this is precisely what has been observed over the past decade or so in the context of cigarette tax harmonisation in the European Community, with the private manufacturers consistently pressing for convergence on a high ratio of specific to total taxation.[7] The result itself is easily explained. Consider the representative firm $\underline{i}$. Recalling (2.2), a P-shift affects $\underline{i}$'s profits only in so far as it leads to a change in $x_{\underline{i}}$ and/or in the collective output of other firms; and from part (a) of Proposition 3 all firms will be expanding their output. As an envelope condition, profit maximisation implies that the effect on $\underline{i}$'s profit of the induced expansion in its own output is negligible. But the increased output of its rivals, depressing the market price, leads to a first-order reduction in $\underline{i}$'s profits. Only in the limiting case of joint profit maximisation does this latter effect - akin to an externality - vanish; in this case (and so, of course, in the monopoly case on which the existing literature has focussed) profits are entirely unaffected.

For the case in which the number of firms is endogenous we have:

PROPOSITION 4. *In the model of Free Entry Oligopoly, a P-shift from specific to ad valorem taxation leads to:*

(a) A strict reduction in the consumer price, and

(b) A strict reduction in the number of active firms, except in the polar case of joint profit maximisation (in which it is unaffected).

Proof: (a) Proceeding as Proposition 3(a) but now using (2.17) of Proposition 2 gives

$$dP = - (P-\alpha)\left[\frac{dP}{dt_S}\right]dt_V < 0, \qquad (3.6)$$

the inequality being from (2.26).

(b) The argument is as in Proposition 3(b), this time using (2.20) of Proposition 2. QED.

The price part of this result exactly parallels that for the Generalised Cournot model, and can be similarly explained. The conclusion is thus a strong one: in both models of imperfect competition, price is reduced by a movement towards ad valorem taxation that leaves the tax burden at the original price unchanged. In purely formal terms the result in part (b) is also parallel to that in (b) of Proposition 3: as can be seen from Propositions 1 and 2 (and as noted by Stern (1987) for specific taxation), an increase in either tax instrument increases the number of firms in the model of Free Entry Oligopoly iff it increases profits in the Generalised Cournot model. The reason for this is straightforward - incipient profits attract entry - so that the present result can be explained by simple extension of the argument given above for Proposition 3(b). There is though a more direct

(albeit cruder) intuition that will prove useful. Writing total costs C(x) as the sum of fixed cost C(0) and variable costs c(x), note that maximising profits π_i in (2.1) is equivalent to maximising

$$\frac{\pi_i}{(1-t_v)} = Px_i - c^*(x_i) - C^*(0), \qquad (3.7)$$

where $c^*(x) = (c(x)+t_s x)/(1-t_v)$ and $C^*(0) = C(0)/(1-t_v)$. Both specific and ad valorem taxation thus have an effect equivalent to an increase in marginal cost. But ad valorem taxation also has another effect, akin to raising fixed costs.[8] A P-shift towards ad valorem taxation is in this sense analogous to an increase in fixed costs, and so might naturally be expected to induce exit.

4. OPTIMAL TAX STRUCTURES

In this section we address three sets of issues in the comparison between ad valorem and specific taxation that are of particular welfare and policy interest. The first is the possibility that they may usefully serve a purely corrective function. More precisely, suppose that the government is unrestricted in its ability to use lump sum taxation to meet any revenue constraint: might it nevertheless wish to deploy these two tax instruments simply as a response to the inefficiencies of imperfectly competitive outcomes? The second issue is closer to the standard optimal commodity tax problem. Suppose the government is unable to raise some required amount of revenue $\bar{T} \geq 0$ by lump sum taxation (or from the proceeds of corrective taxes): what balance between ad valorem and specific taxation will maximise consumer welfare subject to the satisfaction of this revenue constraint? We refer to this as the Ramsey problem. The third is the problem faced by an archetypal Leviathan: what balance between the two forms of taxation is required to maximise tax revenue?

For these purposes we now suppose the demand function $P(X)$ to be generated by a single representative consumer with indirect utility function $V(P,Y)$, where Y denotes lump sum income and the marginal utility of income V_Y is assumed strictly positive.[9] This consumer is the sole owner of all firms in the imperfectly competitive industry under study; in the background there is a second and untaxed good, taken as numeraire, the market for which is assumed to be perfectly competitive; this might for instance be labour that the consumer sells to these firms.

It may (in most cases will) be the case that these optimisation problems have no solution interior to the implicit requirement of the Generalised Cournot model that profits be non-negative or to the requirement of the model of Free Entry Oligopoly that there be at least one active firm, a feature familiar from Stern (1987). A natural strategy would be to impose these constraints explicitly while leaving the tax instruments themselves unrestricted. Here we adopt a rather different approach. For the Ramsey and revenue maximisation problems, we ignore these constraints (as indeed is commonplace) but instead require the specific tax to be non-negative. The advantage of this is the clarity of the dominance results that emerge. Having established results of this kind, we briefly consider optimal policy when the side constraints are recognised and the arbitrary restriction on t_S removed.

Once more, we deal with the two models in turn.

4.1 Generalised Cournot

When there are no restrictions on the use of lump sum taxation, the corrective role for commodity taxation is in this case straightforward and familiar:

PROPOSITION 5. *In the Generalised Cournot model, the optimal corrective fiscal policy is one of subsidisation - either ad valorem or specific - to ensure marginal cost pricing.*

Proof: In this case the consumer's lump sum income consists of profits Π plus a lump sum subsidy equal to the revenue $(t_S+t_VP)X$ raised by commodity taxation. Maximising $V[P,PX(P)-nC(X(P)/n)]$ with respect to P, using Roy's identity,

confirms that the first best requires marginal cost pricing. The assumption that $P_X<0$ and $C_{XX}\geq 0$ ensure that this optimum is unique. Denoting the associated price by P^*, the first order condition (2.2) implies that the first best will be attained by any combination of ad valorem and specific taxation satisfying

$$t_S/P^* = (1-t_V)[1-(\gamma/e(P^*))] - 1. \qquad (4.1)$$

Though (4.1) does not imply that both tax instruments be negative, (2.2) implies that revenue - and hence at least one of them - must be.[10] QED.

This result needs little comment, except to emphasise the irrelevance of the precise balance between the two tax instruments: when the number of firms is fixed, each is potentially as effective a corrective device as the other.

The equivalence collapses, however, when attention is turned to the Ramsey and revenue maximisation problems. Moreover there is then a clear ranking of the two instruments:

PROPOSITION 6. *Suppose the specific tax is restricted to be non-negative and the constraint that profits be non-negative ignored. Then for both the Ramsey problem and that of revenue maximisation, the optimal rate of specific taxation in the Generalised Cournot model is zero.*

Proof:[11] Consider any tax pair $\{t_S^a, t_V^a\}$ with $t_S^a>0$ and $t_V^a<1$; call this pair a. We first show that there exists a pair $\{t_S^b, t_V^b\}$ with $t_S^b=0$ that raises strictly more revenue than pair a; this establishes the revenue maximisation part. For the Ramsey part, we show that there exists another pair $\{t_S^c, t_V^c\}$,

again with $t_S^c=0$, that raises the same revenue as pair $\underline{a}$ but leaves consumer welfare strictly higher.

Set

$$t_V^b = \frac{t_S^a + C_X^a t_V^a}{t_S^a + C_X^a} \tag{4.2}$$

where C_X^a denotes $C_X(x^a)$, x^a being output per firm under pair $\underline{a}$, and note that

$$t_V^b - t_V^a = t_S^a/\Phi^a. \tag{4.3}$$

Using (4.2) in (2.2), wholly ad valorem taxation at the rate t_V^b leads to the same consumer price as the pair $\underline{a}$: $P^b=P^a$. Using (4.3), the difference in tax revenue $T=(t_S+t_VP)X$ is then

$$T^b - T^a = t_S^a(P^a-\Phi^a)X^a/\Phi^a, \tag{4.4}$$

which is strictly positive from (2.14) and the assumption that $t_S^a>0$.

The results of the preceding paragraph imply, assuming continuity, the existence of some $t_V^c<t_V^b$ such that wholly ad valorem taxation at t_V^c generates revenue of exactly T^a; moreover, since $dP/dt_V>0$, $P^c<P^b=P^a$. It therefore suffices to show that with revenue held constant (and retained by the government) welfare is strictly decreasing in P. Noting from (2.1) that $\Pi=PX(P)-C[X(P)]-T$, this follows on differentiating $V[P,\Pi]$ to find

$$\left[\frac{1}{V_Y}\right] \left.\frac{dV}{dP}\right|_T = (P-C_X)X_P \tag{4.5}$$

and noting from (2.2) that $\bar{T}\geq 0$ implies $P>C_X$. QED.

Removing the non-negativity restriction on t_S, Proposition 6 leads one to expect the optimal specific tax

rates for these problems to be strictly negative.[12] Matters become a little more complex, however, if one also recognises the requirement of non-negative profits. Combining the two, intuition suggests that optimality will require the specific tax to be set as low as is consistent with the profit constraint and (for the Ramsey problem) the revenue requirement. A referee has argued for an even stronger conclusion: that optimality requires a negative specific tax and an ad valorem tax arbitrarily close to unity.

4.2 Free Entry Oligopoly

Freedom of entry and exit fundamentally alters the potential for corrective fiscal policy. First, taxation rather than subsidisation may be optimal. This is established by Besley (1989) and de Meza (1982) for specific taxation, and by Kay and Keen (1983) - in a locational model somewhat different from the present - for ad valorem. Second, and more central to the present comparative concerns, there is again a clear ranking of the two instruments as corrective devices:

PROPOSITION 7. *In the model of Free Entry Oligopoly, when revenue is returned to the consumer as a lump sum:*

(a) Welfare is increased by the introduction of a small specific tax (conditional on $t_V=0$*) iff* $E < 0$;

(b) Welfare is increased by the introduction of a small ad valorem tax (conditional on $t_S=0$*) iff* $E < 1/e$.

Proof: For (a), note that since $\Pi=t_V=0$, welfare is simply $V[P,t_SX(P)]$. Evaluating the derivative of this with respect to t_S at $t_S=0$ gives, from Roy's identity and (2.16) of Proposition 2,

$$\left[\frac{1}{V_Y}\right]\frac{dV}{dt_S} = \frac{-\gamma XE}{2+A-\gamma E}\ , \qquad (4.6)$$

from which the result follows.

For (b), differentiation of $V[P,t_VPX(P)]$ at $t_V=0$, using (2.16) and (2.17) of Proposition 2, gives

$$\left[\frac{1}{V_Y}\right]\frac{dV}{dt_V} = X\left[\frac{(P-\alpha)(2+A) - \gamma EP}{2+A-\gamma E}\right] \qquad (4.7)$$

with

$$(P-\alpha)(2+A) - \gamma EP = \gamma P[(1/e)-E], \qquad (4.8)$$

from (2.26) and (2.14). QED.

Part (a) of Proposition 7 is as in Besley (1989) and de Meza (1982).[13] Comparing it with part (b), however, shows that the set of circumstances in which welfare is improved by specific taxation is a strict subset of that in which it is improved by ad valorem: the desirability of specific taxation implies that of ad valorem, but not conversely. There will be cases, for instance, in which the optimal ad valorem policy is taxation but the optimal specific policy is subsidisation.[14]

The reason that corrective considerations in this case point towards a relatively high rate of ad valorem taxation (though perhaps nevertheless a negative one) is clear from the results of Section 2. This is a model in which, in the absence of taxation, free entry leads to a socially excessive number of firms.[15] There are therefore welfare gains to be had not only from lowering price towards marginal cost but also from inducing exit, so reducing the aggregate of fixed costs. And for the intuitive reason given after Proposition 2 above, ad valorem taxation has a particularly direct discouraging impact on entry decisions and so is well-targeted to the pursuit of the latter effect.

Given the dominance of ad valorem taxation for the Ramsey and revenue maximisation problems in the Generalised Cournot model, and its particular corrective appeal when entry is free, it is no surprise to find:

PROPOSITION 8. *Suppose the specific tax is restricted to be non-negative and the constraint that there be at least one active firm ignored. Then for both the Ramsey problem and that of revenue maximisation, the optimal rate of specific taxation in the Free Entry Oligopoly model is zero.*

Proof: Consider a shift from specific to ad valorem taxation of the form

$$\alpha dt_v = - dt_s > 0, \qquad (4.9)$$

where α is as in (2.18). Substituting (4.9) into (3.2) and recalling (2.17) of Proposition 2, this reform has the feature that dP=0. Hence welfare V(P,0) is unchanged, whilst routine calculations show the change in tax revenue to be

$$dT = (P-\alpha)Xdt_v > 0, \qquad (4.10)$$

which establishes the revenue maximisation part. Given the existence of a reform with dV=0 and dT>0 (and since $dV/dt_i<0$, i=s,v, now that revenue is retained by the government), it is straightforward to show that there exists another reform with dV>0 and dT=0; which establishes the Ramsey part. QED.

Note that the proof of Proposition 8, unlike that of Proposition 6, does not require that $t_s>0$ in the initial position. The extension to the case in which the tax instruments are unrestricted but the constraint imposed that there be at least one active firm is then immediate: optimality requires that the specific tax be set as low as is consistent with that constraint and (for the Ramsey part) with the revenue requirement.

5. CONCLUSIONS

The classic treatments of the monopoly case referred to in the Introduction led to the view of predominantly ad valorem taxation as implying a relatively low consumer price and relatively high tax revenue. The analysis here has extended the same presumptions into the context of imperfect competition. It has also been seen that outside the monopoly case (but still with no entry) predominantly ad valorem taxation is also associated with relatively low profits, an observation echoed in the political economy of the European tobacco tax harmonisation debate. In welfare terms, ad valorem taxation has emerged as strictly dominant over specific. It has been shown, for instance, that whenever the introduction of a small specific tax would be welfare-improving (a possibility in the context of Free Entry Oligopoly noted by Besley (1989) and de Meza (1982)) so too would be the introduction of a small ad valorem tax; and that the converse is not true. The standard optimal tax problem of maximising welfare subject to a binding revenue constraint, it has also been argued, calls for maximum reliance on ad valorem taxation.

These results are strikingly unambiguous, and could hardly have been more favourable to ad valorem taxation. So it is appropriate to end by emphasising their limitations. First, they may not be robust to changes in the characterisation of imperfect competition. Endogenising product quality, for instance, the results of Kay and Keen (1983,1990) suggest that the optimal balance between the two tax instruments is liable to depend sensitively on the precise form of consumer

preferences. Second, there may exist other tax instruments with even more desirable properties. The results of Besley (1989) and Konishi, Okuno-Fujiwara and Suzumura (1990) for instance, and the intuition behind some of those here, suggest that discriminatory taxes on fixed and marginal costs may be particularly well-targeted for effective tax design. More fundamentally still, Tam (1990) notes that in the monopoly case it is possible to raise both consumer welfare and tax revenue by imposing a tax on price. In a wider choice of options, ad valorem taxation might itself be dominated.

FOOTNOTES

1. As part of its more recent and broader programme for commodity tax coordination, the Commission is now proposing the establishment of separate minima for both the specific and ad valorem component of the tax on cigarettes. This is intended as an intermediate step towards full convergence, the proposed target being a specific tax of ECU21.5 per 1000 cigarettes and a (VAT-inclusive) ad valorem rate of 54 per cent (COM(89) 525/fin). At current retail prices, the latter would imply a ratio of specific to total tax in the order of 35 per cent.

2. See also Venables (1986), who briefly considers the optimal mix between ad valorem and specific tariffs in the trading context of a non-producing country importing from Cournot-Nash oligopolists.

3. As Stern (1987) notes, his general formulation of a range of policy problems as ones of dual pricing encompasses both ad valorem and specific taxes (which correspond to a retention proportion of t_v combined with a retention price of $-t_s/t_v$). He does not though address the comparison between the two.

4. Stern (1987) works instead in terms of the price elasticity of the elasticity of (direct) demand S, while de Meza (1982) works in terms of the quantity elasticity of the elasticity, S^*. For comparison, these quantities are related as

$$\begin{aligned} S &= 1 + e(1-E) \\ &= -eS^*. \end{aligned}$$

5. In Chapter V, Cournot (1960) considers the effect on the price charged by a monopolist of a parallel upward shift in the marginal cost curve. He shows that price will increase by more than this addition to marginal cost iff (in the present notation) $1+A-E>0$, exactly as in Proposition 1 (with $t_v=0$ and $y=1$). Turning to explicit tax analysis in Chapter VI, Cournot notes the implication that "...according to circumstances, the increase of cost to the consumer may be greater or less than t_s" (p.70; notation changed).

6. This can be thought of as a local version of the concept of 'matched pairs' of ad valorem and specific taxes introduced by Suits and Musgrave (1955).

7. See for instance the evidence of the Tobacco Advisory Council in House of Lords (1986). Doubtless too there are considerations not captured in the present model underlying the private companies' position. In particular, they argue that high ad valorem taxation exacerbates the protective effects of the subsidies given to state monopolies in some Member States. It may also be that since the private companies' products tend to be of relatively high quality they hope to benefit from upgrading effects associated with predominantly specific taxation.

8. This is noted in Kay and Keen (1983).

9. Note that we are assuming away income effects in the demand for the good produced in the imperfectly competitive industry, though for simplicity this is not imposed as an explicit restriction on the indirect utility function.

10. There is no need to worry here about the constraint that profits be negative, since this can be ensured, if necessary, by lump sum subsidisation.

11. This method of proof - which extends that of Suits and Musgrave (1955) - was suggested to us by a refere.

12. One can indeed generalise Proposition 6 to show that any lower bound on t_S will prove a binding constraint. (Details available from the authors).

13. Though not stated by De Meza in the precise form of Proposition 7(a), the result follows on using the relations of footnote 4 above in his equation (40) and setting t=0.

14. A similar result appears in the literature on trade policy in the presence of imperfect competition: Helpman and Krugman (1989) show that the optimal policy for a non-producing country (or one with a perfectly competitive domestic industry) importing from a foreign monopolist may be a tariff if an ad valorem tax/subsidy can be deployed used but an import subsidy if it is restricted to use only a specific instrument. Though the two models are formally distinct - there, for instance, the number of firms is effectively fixed whereas here there is free entry - there is a common feature underlying this resonance: in each case welfare effects through profits can be ignored, there because they accrue to foreigners and here because they are eliminated by entry.

15. It is easily checked, for instance, that the sufficient conditions for excess entry of Proposition 1 in Mankiw and Whinston (1986) are satisfied.

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