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# Notes on Economic Depreciation of Natural Resource Stocks and National Accounting

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**Abstract**

We consider numerous variations on the theme that stock diminution weighted by marginal rent should be netted from basic NNP to account for economic depreciation of natural stocks. Mineral discoveries, durable exhaustible resources, and mining pollution effects are examined. Capital appreciation from transforming land in virgin forest to land in agriculture is considered. Attention is paid to second best prices.

**Notes on Economic Depreciation of Natural  
Resource Stocks and National Accounting**

In Hartwick [1990] I presented a methodology and formulas for incorporating changes in values of natural resource stocks in an economy's national accounts. The aim was to account for the depletion of stocks from extraction (e.g., oil), from "overuse" (e.g., fishing) and from degradation (e.g., pollution). Essential to the analysis was the assumption that (a) prices reflected true scarcity (perfect competition or optimal planning) (b) property rights were well defined and universal (this is related to (a) via the notion of market failure or departures from perfect competition) and (c) technological progress was, in possibly a statistical sense of averaging, correctly anticipated.

When the first two assumptions fail to hold, one is in the world of the third best, a world in which observed market prices generally fail to reflect basic scarcity. In practice one cannot make good policy prescriptions with observed prices in a world of the third best. When the third assumption fails, current prices again fail to reflect basic scarcities and policy actions are of dubious merit when based on those current prices. Much of the analysis below circles back to this theme of national accounting with "imperfect" prices. But early on we actually make use of some prices to calculate economic depreciation of oil stocks in the U.S.

First, I will set out my approach (derived from arguments of Cass and Shell [1976], Weitzman [1976], Solow [1986] and others). An essential

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\* I am indebted to John Livernois, Ngo van Long, and Gerard Gaudet for helpful comments.

ingredient of formulas for depreciating natural resource stocks in the national accounts is the marginal cost of using the resources over the current period, e.g., marginal extraction costs and/or extraction costs and discovery costs. Adelman [1986] presents such rarely reported costs. Another peculiarity of mineral stocks is that new commercially attractive deposits are being discovered each period usually by the purposeful expenditure of resources. We discuss how to incorporate these increments to stocks in the national accounting measures incorporating economic depreciation of natural resource stocks. We also observe how to treat durable exhaustible resources such as gold, silver, copper, etc., as opposed to nondurable resources such as oil, coal, uranium, etc. An important durable non-renewable resource is land and we discuss "mining" land held in virgin forests and transforming it to use in agriculture.

We then consider explicit externalities such as mining activity as it pollutes a fishery and how these externalities show up in our formulas for economic depreciation. Curious "netting effects" show up in our formulas. We discuss the issue of incorporating pollution as a bad in alternate possible places; in particular introducing the stock in the utility functions of agents. And we take up another externality, namely mining directly polluting the environment. We note that property rights failures, non-competitive behavior, and improperly priced pollution activity result in third best outcomes and we relate these equilibria to second best equilibria analogous to the second best equilibria in static analysis. However we do not pursue this tangled issue. We do however note emphatically that actual prices used in national accounting are distorted from true scarcity prices and we discuss difficulties in this terrain.

## The Model of Dynamic Competitive General Equilibrium

In order to develop principles of national accounting, one needs a benchmark economy and the usual standard is that of a well-behaved competitive general equilibrium. However the static general equilibrium model of the Arrow-Debreu sort will not do because investment plays an essential role in national accounting. We need an inherently dynamic competitive general equilibrium model. The simplest of these is the aggregate Solow-Koopmans-Cass optimal growth model. The solution path in such an economy reflects efficiency or competitive price-taking behavior by agents. Moreover the savings-consumption decision is made endogenous so that the levels of investment period by period (or instant by instant) are derived from behavioral assumptions and not a consequence of rules-of-thumb. These optimal savings rules imply general time paths of investment and different parameterizations of the underlying tastes and technology will imply distinct investment for a real economy. We can envisage an abstract optimal growth problem which can reproduce the observed economy "on paper". The main point is that optimal growth models can be viewed correctly as dynamic competitive general equilibrium models. We are not contending that any real world economy is optimal. Rather we are arguing that a competitive economy with an essential savings-investment decision can be interpreted as a realization of an abstract optimally growing economy. Aggregate optimal growth models lump the thousands of distinct commodities (e.g., apples, tires, computers, etc.) into a composite commodity and focus essentially on the tradeoff at each instant between allocating produced output to current consumption and current investment. Routine extensions of these early models permit one to focus in addition on the trade-off between dis-investing in exhaustible resource stocks (current consumption of say oil) and "investing" in such stocks by not

consuming from them currently. The same notion carries over to renewable and environmental stocks: stock reduction is associated with consumption in excess of the renewal "output" from nature's bounty and non-consumption or investment is associated with stocks increasing because use is less than the natural renewal "output". The unifying notion is that at each instant agents purposefully decide to split various flows into current consumption (a form of disinvestment) and current investment (generally associated with capital stock growth).

These prefatory remarks are intended to explain why we leap into basic optimal growth theory when we wish to discuss national accounting principles. We need a benchmark economy in order to see what notions of GNP (gross national product), NNP (net national product), NI (national income) etc mean in principle and that benchmark economy is a competitive general equilibrium model with endogenous savings and investment decisions. In competitive general equilibrium, we have a fairly good idea what relative prices mean and how they reflect basic scarcity. In other contexts the meaning of prices is less clear and thus accounting entities constructed with such prices are difficult to interpret. This point of course is not new. Kuznets [1948], Samuelson [1950], Weitzman [1976] and others worked out the meaning of national accounting concepts and that meaning derives essentially from meaningful prices which are really meant to be those observed in the context of an abstract competitive general equilibrium.

Let us take up the simplest case - commodities aggregated to one composite, say "wheat" which is durable and can be consumed or added to a capital stock of existing "wheat", and citizens aggregated to one "agent" with a specific single utility function and discount rate  $\delta$ . The optimal growth problem is to find a sequence of  $C(t)$ 's (and outputs  $Q(t)$  and

investments  $\dot{K}(t)$ ) which maximize the agent's present value of felicity into the indefinite future. We'll suppose that the population (labor force) moves in an exogenous way, so that  $L(t)$  is a given time path.

We have then output  $Q(t) = F(K(t), L(t))$  where  $F(\cdot)$  is the production function and  $K(t)$  the current stock of accumulated "wheat". Commodity balance has use equal to production,

$$C(t) + \dot{K}(t) = F(K(t), L(t))$$

or

$$\dot{K}(t) = F(K(t), L(t)) - C(t)$$

This is our so-called equation of motion for our dynamic system, relating investment to consumption. The present value  $\int_0^{\infty} U(c) \exp(-\delta t) dt$  is maximized by choice of path  $\{C(t)\}$ , where  $U(\cdot)$  is the utility function of the society. Associated with this problem is the current value Hamiltonian

$$H(t) = U(C(t)) + \lambda(t) [F(K(t), L(t)) - C(t)]$$

where  $\lambda(t)$  is a shadow price of  $K(t)$  labelled the co-state variable (essentially a time varying Lagrange multiplier). The problem is an optimum when the Hamiltonian satisfies certain necessary conditions (the canonical equations). For our purposes the key canonical equation is: the first order condition corresponding to maximizing  $H(t)$  at each date with respect to "control"  $C(t)$ . That is  $\partial H / \partial C = 0$  implies  $U_c = \lambda$  at each date. Thus

$$\frac{H(t)}{U_c(t)} = \frac{U(C(t))}{U_c(t)} + \dot{K}$$

If in addition if we approximate  $U(C)$  by  $CU_c$ , we obtain

$$\frac{H(t)}{U_c(t)} = C(t) + \dot{K}(t)$$

or the normalized current value Hamiltonian defines at each date the NNP function,  $C + I$ .  $H(t)$  is measured in utils and  $H/U_c$  is measured in dollars. This transformation is our "normalization" of the current value Hamiltonian. We have demonstrated that in optimal growth, the current value Hamiltonian represents NNP. This is the basic fact which we dilate upon when we consider natural capital goods in addition to made-man capital goods.

We have glossed over the labor market to this point. This is reasonable since it is assumed to be functioning optimally at each date given the exogenous supply,  $N^S(t) = \bar{N}^S(t)$ . We can be more precise and formal about this. So doing will help later on when we consider an economy with pollution. Labor supply equals labor used or hired in our economy. This can be treated formally as a static constraint on the current value Hamiltonian

$$\mathcal{H} = U(C) + \lambda(t) [F(K, N) - C] + \Omega(t)[\bar{N}^S(t) - N]$$

where  $\Omega(t)$  is a Lagrangian multiplier (distinct from co-state variable  $\lambda(t)$ ) and  $\bar{N}^S(t) - N = 0$  is the labor supply equals labor demand constraint. Now  $\frac{\partial \mathcal{H}}{\partial N} = 0$  implies  $F_N = \frac{\Omega(t)}{\lambda(t)}$ . Recall that  $\lambda(t) = U_c$ . Thus  $F_N = \frac{\Omega(t)}{U_c} = w$ , i.e., the marginal product of labor equals the util shadow price  $\Omega(t)$  normalized by the util value of a unit of consumption. If, as before, we write  $U(C) = CU_c$  and divide  $\mathcal{H}$  by  $U_c$ , we get the NNP function

$$NNP(t) = C + \dot{K}$$

Note the labor constraint takes value zero in equilibrium, since labor demanded equals labor supply, and so the labor constraint does not show up in the NNP function, as we expect. We will continue to ignore the labor variable until we discuss pollution and property rights.

## NNP and Exhaustible Resources

Suppose now output uses say oil  $R(t)$  from a known stock in the ground  $S(t)$ . That is  $Q = F(K, L, R)$  at each date. Stock size  $S(t)$  declines by an amount equal to current extraction, or

$$\dot{S}(t) = -R(t)$$

Let us assume that  $f(R)$  is the amount of "wheat" required to extract oil,  $R(t)$ . Then,

$$\dot{K} = F(K, L, R) - C - f(R)$$

We now have two state variables,  $S(t)$  and  $K(t)$ , and two control or decision variables  $C(t)$  and  $R(t)$ . Our current value Hamiltonian is now

$$H(t) = U(C) + \lambda(t) [F(K, L, R) - C - f(R)] + \psi(t) [-R(t)]$$

where  $\lambda(t)$  is a co-state variable or shadow price of  $K(t)$  and  $\psi(t)$  is a co-state variable or shadow price of  $S(t)$ . Optimality requires that  $\partial H / \partial C = 0$  and  $\partial H / \partial R = 0$ . These conditions imply that  $\lambda(t) = U_c$  and  $\lambda(t)[F_R - f_R] = \psi$  and  $\psi / U_c = [F_R - f_R]$ . Then,

$$\frac{H}{U_c} = C + \dot{K} - [F_R - f_R]R$$

given  $U(C) \approx CU_c$ . Our new NNP,  $H/U_c$  has a price,  $F_R$ , minus marginal extraction cost  $f_R$ , multiplied by current extraction  $R$  (equal to stock diminution  $-\dot{S}(t)$ ).  $F_R - f_R$  is called rent or user cost on the marginal ton extracted. Thus  $[F_R - f_R]R$  is an aggregate rent on the stock of the exhaustible resource which is currently used up or "wasted". Since  $F_R - f_R$  is sometimes called dynamic or Hotelling rent,  $[F_R - f_R]R$  is total Hotelling rent on the amount currently extracted. This rent is to be netted out from

$C + \dot{K}$  to allow for the using up of the exhaustible resource stock  $S(t)$  over the period.<sup>1</sup> We have

Principle on Depreciating Natural Resource Stocks in the National Accounts:

To depreciate natural resource stocks, subtract from "basic" NNP the amount of stock used up over the accounting period weighted by the marginal value of a unit, namely its price net of the marginal cost of "producing" a unit of the stock.

An abbreviation of this rule is: deduct the quantity of stock used up weighted by its dynamic rent per unit. This rule presumes that markets are competitive so that price minus marginal cost includes no monopoly or oligopoly component.

What is the intuition underlying economic depreciation? It is simply the loss in value of a durable asset from optimal use. The change in the market value of a mine at date  $t+1$  and at date  $t$  after the mine has been extracted from efficiently is precisely economic depreciation. In taxation, the principle followed is to allow mining companies a "depletion allowance"

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<sup>1</sup> An early reference to the relationship between GNP, NNP and natural resources is A. Marshall [1936, Book VI, Chapt. I, pp. 523-24].

"The labour and capital of the country, acting on its natural resources, produce annually a certain net aggregate of commodities, material and immaterial, including services of all kinds. The limiting word "net" is needed to provide for the using up of raw and half-finished commodities, and for the wearing out and depreciation of plant which is involved in production: all such waste must of course be deducted from the gross produce before the true or net income can be found. And net income due on account of foreign investments must be added in. This is true net annual income or revenue; or, the national dividend: we may, of course, estimate it for a year or for any other period".

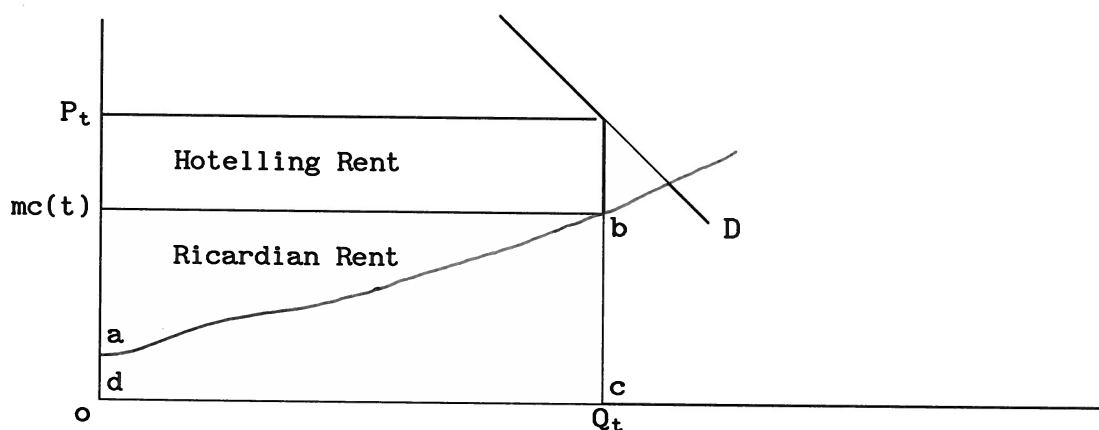
because the market value of their company is shrinking each year as extraction proceeds. Taxation is based on the current market value of the company, the value net of the "depletion allowance". That is, taxes are levied on profits corresponding to a firm of a specific size or value, namely one for which shrinking or a "depletion allowance" has been factored in the estimated value of the firm. These considerations seem in principle straightforward for mining companies because one can readily see the physical shrinking of the deposits owned and conceptualize a decline in value for the firm in a world of zero inflation. In an ideal world "depletion allowances" would be correctly calculated economic depreciation.

For non-mining firms, the notion of optimally using one's durable assets and calculating their loss in value from optimal use seems too tricky to arrive at. But this is what economic depreciation is in fact for such firms. One can see why tax authorities end up with somewhat arbitrary rules of thumb for actually allowing depreciation of durable capital to be written off. With mining firms it is somewhat easier to conceptualize the decline in the value of the firm as the known reserves are worked or run down. In Hartwick and Lindsey [1989] I demonstrated that Hotelling Rent does capture the decline in value of a firm, for the particular case of a known, homogeneous stock of reserves. In Hartwick [1991] I addressed this issue for the case of a firm which jointly explores and extracts. In this latter case Hotelling Rent will not quite equal economic depreciation.

The now standard procedure for introducing heterogeneity in stocks is to postulate that each ton of extracted and refined ore involves a distinct cost of extraction and processing. The well behaved cases involve unit costs rising as the deposit is worked. Formally one has extraction and processing costs depending on where one is in the deposit or extraction and processing

depend on the amount of stock remaining at a given date. In our model we will have extraction (and processing) costs  $f(R)$  now depend on stock remaining or  $f(R, S)$  with  $f_s(\cdot) < 0$ . This formalizes the idea that stocks are heterogeneous. Remarkably enough our rule for depreciating stocks is unchanged. We still must deduct  $[F_R - f_R]R$  from "gross" NNP as before. The formula remains unchanged; we do not claim that the time path of variables is the same with or without heterogeneity in the stock.<sup>2</sup>

The issue of getting unit costs correct in economic depreciation measures can be made clear with a familiar diagram. At any date in a program of on-going extraction, extraction costs might be as in Figure 1.



Industry Snapshot  
Figure 1

We assume each producer has slightly different costs of extraction per ton. One can think of each unit of homogeneous, processed output as coming from more ore as the total stock in the economy is worked. More ore means

<sup>2</sup> In work with partial equilibrium models of the classic Hotelling sort (e.g., Hartwick and Lindsey [1989]), we did not observe "heterogeneity-of-stock" effects washing out of the economic depreciation formulas. It is unclear why this is so.

increased cost per unit of salable output. The deposit as an aggregate is becoming thinner in terms of useful output as it is worked. The correct unit cost figure for our calculations is  $mc(Q(t))$  in Figure 1.  $[p_t - mc(Q_t)]Q_t$  is of course Hotelling Rent and is the basic entity in measures of stock depreciation. Any cost other than  $mc(Q(t))$  will not yield the current basic economic depreciation measure. The correct marginal cost can be interpreted as the unit cost of operating the marginal deposit, that with the currently highest unit extraction cost. Any averaging of extraction cost will yield "Hotelling Rent" in excess of what it should be.

Figure 1 provides a convenient guide as to how a national accounts would be in an economy specialized in extraction of mineral. The large quadrilateral  $pQ$  is the gross national product (GNP) concept. It corresponds on the income side to dynamic rents (Hotelling Rent in Figure 1), Ricardian Rent and payment to other inputs such as wages and rentals to machine owners (area  $abcd$ ) representing total direct extraction costs. NNP is GNP net of economic depreciation or  $pQ$  minus Hotelling Rent.

Discoveries of new stock represent increments to known supplies and should be incorporated in a figure for net changes in stock size and value. Suppose stock on any date is represented by known stock,  $S$ , discovered earlier plus current discoveries,  $D(t)$ . Then  $\dot{S} = -R + D$  and extraction costs-from-inventory are as before  $f(R, S)$ . But discoveries depend on how difficult or costly it is to locate remaining ore. A simple formulation of this idea is to say current discovery costs rise as an increasing function of remaining potential discoveries. Let cumulative discovery to date  $X(t)$  be a proxy for potential discoveries. Then discovery  $D(t)$  costs  $g(D, X)$  and of course  $\dot{X}=D$ . Our Hamiltonian is now:

$$H = U(c) + \lambda(t) [F(K(t), L(t), R(t)) - C(t) - f(R, S) - g(D, X)] + \psi[D-R] + \phi D$$

Then

$$NNP = C + \dot{K} - [F_R - f_R]R + Dg_D$$

Clearly discovery costs  $Dg_D$  "dissipate" the earlier depreciation charge,  $[F_R - f_R]R$  somewhat. This is intuitively correct of course. Discoveries make depreciation charges arising from resource "use", less than when no discoveries exist.<sup>3</sup>

Actually  $-[F_R - f_R]R + g_D D$  turns out to be the sum of the depreciation charge associated with a unit of  $S(t)$  namely  $-[F_R - f_R][R - D]$  and the charge associated with a unit of cumulative discoveries,  $X(t)$ , namely  $\{g_D - [F_R - f_R]\}D$ . In an efficient, competitive economy exploration will be pursued until its marginal cost  $g_D$  equals the gross marginal profit  $[F_R - f_R]$  plus a marginal profit term for future discoveries. Then the depreciation term associated with  $X(t)$  should be negative. Static marginal benefits of exploration equalling marginal costs imply  $g_D - [F_R - f_R] = 0$ . This static approach leaves economic depreciation at  $-[F_R - f_R][R - D]$ , an intuitively plausible entity. The correct value however is  $-[F_R - f_R]R + Dg_D$ .<sup>4</sup>

#### An Example

U.S. Production in 1978 was 2353.91 million barrels. Discoveries were 562 million barrels. Thus R-D was 1791.91 million barrels. Price in 1978

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<sup>3</sup> Imports of exhaustible resources represent an "outside" supplier depleting its capital or current stock. Thus one must deal only with domestic production, including exports, if one is going to obtain domestic economic depreciation of natural resource stocks.

<sup>4</sup> I am indebted to John Livernois for the above formulation of exploration. He may well disagree with my emphasis in the above rendering.

was \$12.15 per barrel and in 1979 rose to \$23.50. We will take the average as an estimate of true 1978 value: i.e. 1978 price is estimated at \$17.80. The extraction cost for the marginal barrel was \$8.06 (Adelman [1986; Table 3]). Hence net price, rent, or unit royalty was \$9.74.

We obtain a value for  $-(F_R - f_R)R + Dg_D$ .  $F_R - f_R$  is 9.74.  $g_D$  is \$3.83 per barrel in 1978 (Adelman [1986; Table 3])  $R$  was 2353.91 million barrels. Then the economic depreciation term is  $-2353.91 \times \$9.74 + 562 \times \$3.83 = -\$20774.6$ . The Capital Consumption Allowance for the U.S. in 1978 was \$225,500. Thus economic depreciation of oil is  $\{20774.6 / [225,500 + 20774.6]\} \times 100 = 8.4\%$  of the enlarged capital consumption allowance. NNP excluding natural resource stock adjustments was \$1,941,400 million in 1978. Our oil depletion stock adjustment is  $20,774.6 / [1,941,400 - 20,774.6] = 1.1\%$  of NNP.

### Market Failures

It is well known that observed market prices only reflect basic welfare-optimizing scarcities when there are no market failures anywhere in the economy. This proposition is often expressed by the assertion that perfect competition must prevail. For our purposes this is a very distressing proposition. The Adelman data we used were taken from an article whose central theme was that resource stock owners do not follow wealth maximizing extraction programs. We might add that world oil markets are generally regarded as oligopolistic or are imperfectly competitive. These factors suggest that the prices and probably the marginal costs we used above are not true measures of scarce resources in the economy. Thus on a priori grounds we believe that our estimates of economic depreciation of oil stocks in the U.S. in 1978 are flawed. How do we proceed?

There are two courses of action: easy and difficult or practical and impractical. We can assert that distortions in the economy move in opposite directions and on average cancel each other out leaving observed prices as good approximations to competitive or socially optimal prices. This is the easy or practical approach. A variant of this approach is to make adjustments to prices in the sector under study when obvious distortions are known. Such a procedure runs counter to the message of the theory of the second best which here might be summarized as "two wrongs repaired in isolation, do not make a right". The alternative approach is to analyze the complete economy as a web of distorted prices and to derive a series of adjustments or corrections to be made to observed prices, so that adjusted prices reflect genuine scarcity (as in Mirrlees [1969]). This is the hard and seemingly impractical approach, though researchers have been pursuing it with the aid of computable general equilibrium models in recent years [as in Whalley [1982]]. But let us reflect on further evidence of the basic economic efficiency or lack of it in the oil extraction sector.

Using data for a cross-section of U.S. oil and gas producers, Miller and Upton [1985] indicate that a reasonable conclusion is that firms are following competitive wealth maximizing extraction strategies. Roughly speaking, if one assumes Hotelling's  $r\%$  rule for efficient extraction is being met by the firms, then the data have a plausible interpretation. A purist can fault the empirical analysis in Miller and Upton in many places but the criticism might be termed nit-picking by an optimist. In another study, Nordhaus [1974] constructed an intertemporal global energy supply and demand model and concluded that the then prevailing retail price of oil in the U.S. was unreasonably high relative to prices generated in a socially optimal extraction and use program. Nordhaus transformed the intertemporal

multi-source, multi-use trajectory into the solution of a linear program, easy to solve on large computers. He conjectured that the observed price departed from the true scarcity price for one or more of these reasons: actual energy supply was not competitive, but rather oligopolistic; futures markets for oil, coal, natural gas etc. were not complete and this incompleteness could distort future prices which are projected backwards by agents as current prices; or speculative activity could cause observed price trajectories to depart from true scarcity future price paths. Of interest is that Nordhaus did not consider that the wrong choice of key parameters of his analysis were resulting in anomalous results. He performed sensitivity tests and was satisfied with his estimates of key parameters.

We could add other reasons for efficiency in energy markets failing to prevail. For example, common pool situations generally involve some form of extractive racing by competitors. No extractor wishes to defer exploitation and end up with a disproportionately smaller overall "take". Stock size uncertainty and uncertain technical progress can each tilt extraction paths away from those calculated under the assumption of certainty in stock size and technical change. Unanticipated (drastic uncertainty) shocks in the form of new discoveries or new techniques of extraction or a new technology for a substitute imply that current prices are not precise measures of scarcity over the longer term. If one believes that such unanticipated shocks are common, then all calculations done with current prices will be in error; to an extent that is difficult to pin down in general. We remind ourselves that the inadequacy of current prices to reflect true scarcity makes not only the calculation economic depreciation suspect but makes the meaning of traditional national accounting procedures extremely difficult to work out.

## Economic "Depreciation" of Durable Exhaustible Resources.<sup>5</sup>

Economic depreciation of durable exhaustible resources raises a special issue. In this case, though the stock in the ground shrinks and as such represents a decline in value of a capital good, the stock above ground is augmented by mining, and being durable, represents a rise in capital value. The market analysis of these two opposing tendencies yields a net rise in society's capital or an economic appreciation as a result of extraction. The increment in stock and value above ground more than compensates for the decline in value below ground. For perfectly durable exhaustible resources, the actual economic appreciation is the change in valuation of the amount extracted from being below ground to being above ground at the appropriate scarcity or shadow prices and this change in value turns out to equal the amount currently extracted, weighted by the marginal cost of extracting it. We demonstrate this.

For a durable exhaustible resource, we have  $S(t)$  tons below ground at date  $t$  and  $B(t)$  tons above ground. Then  $-\dot{S} = R(t)$  when  $R(t)$  is current extraction and  $\dot{B}(t) = R(t)$ .  $B(t)$  and  $S(t)$  are state variables.  $R(t)$  is a control variable. Extraction costs are  $f(R, S)$  as before. We will assume that cumulative gold (the durable exhaustible resource) is used as an input in production as in electronics. Then aggregate output is  $F(K, N, B)$ . Our current value Hamiltonian is now

$$\mathcal{H} = U(C) + \psi(t)[F(K, N, B) - C - f(R, S)] + \phi[-R] + \eta[R]$$

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<sup>5</sup> I am indebted to Gerard Gaudet for invaluable suggestions concerning the issue of durability of exhaustible resources for evaluating economic depreciation. He also urged that I consider this issue.

Now  $\frac{\partial \mathcal{H}}{\partial C} = 0$ , yields  $U_c = \psi$  and  $\frac{\partial \mathcal{H}}{\partial R} = 0$ , yields  $f_R = \frac{\eta - \phi}{\psi}$ . Expressing  $U(C)$  as  $CU_c$  allows us to write  $\mathcal{H}/U_c$  as

$$NNP = C + \dot{K} + Rf_R.$$

Now  $f_R > 0$  is extraction cost of the marginal ton. Hence "gross" NNP,  $C + \dot{K}$ , must be augmented by  $Rf_R$  to reflect a gain to society from moving a part of a capital good  $S(t)$  from below ground to above ground as a new capital good,  $B(t)$ .

If the gold deteriorated above ground at rate  $\rho(t)$  then  $\dot{B}$  would equal  $R(t)$  minus  $\rho(t)B(t)$  and economic depreciation would become  $Rf_R - [\eta(t)/\psi(t)]\rho B$ . That is economic appreciation would be less, other things being the same. The polar case is of course one with no deterioration or perfect durability and this case involves net economic appreciation in national income from extraction. Mining gold is analogous to investing in new plant and equipment since NNP is augmented by the activity.

#### Land in Virgin Forest Transformed to Land in Agriculture

A particular durable exhaustible resource is land in a virgin state. Typically in economic growth with an increasing population, land is brought out of virgin forests and transformed to use in growing non-forest crops. Often this transformation is known as "slash and burn" and frequently occurs with imperfect property rights on the land in virgin forest. This is assuming that the pace and style of land transformation would be different if private property rights were secure. Then the transformation would typically be slower and take place under say: "harvest and clear" rather than "slash and burn". Our investigation here assumes perfect private property rights. A consequence is that land is only transformed to

agriculture when there is a net increase in its value as a capital good. This can be diluted if agricultural use degrades the land's quality (i.e., opens the land to erosion). We combine land use change with potential quality decline in the following sketch.

There is a fixed amount of land  $\bar{L}$  and at any date  $L$  is the amount in agriculture ( $\bar{L} - L$  is in virgin forest).  $\dot{L} = R$  is the amount shifted to agriculture at cost  $f(R)$  in terms of composite produced good, wheat.

$A$  is a fertility index for land in agriculture. The quality adjusted agricultural land is then  $AL$ . Fertility increases naturally at exponential rate  $b$ . Fertility can be increased  $\rho(Z)$  by fertilizing land with  $Z$  units of fertilizer. Fertility is diminished by wheat production  $F(\ )$  in amount  $\gamma F(\ )$ . The net fertility change at a date  $t$  is then

$$\dot{A} = bA + \rho(Z) - \gamma F(K, N, AL).$$

Thus land  $L$  in agriculture and fertility level  $A$  are two state variables. Fertilizer level  $Z$  is a control variable.

We also have produced capital  $K$  and the increase in  $K$  at level  $\dot{K}$  results from foregoing consumption of wheat currently as in

$$\dot{K} = F(K, N, AL) - C - f(R) - g(Y)$$

where  $C$  is current consumption of wheat,  $R$  is the amount of land brought into agriculture ( $R = \dot{L}$ ) from virgin forest use at cost  $f(R)$ , and  $g(Y)$  is the cost of fertilizing at level  $Y$ .

There are consumption services to citizens from having virgin forests. These might be the sap and nuts gathered. The utility function for society is  $U(C, G(\bar{L} - L))$  where  $G(\bar{L} - L)$  are the services from the virgin forest.

Our current value Hamiltonian is now

$$\mathcal{H} = U(C, G(\bar{L} - L)) + \phi(t) [F(K, N, AL) - C - f(R) - g(Y)] + \psi(t)[R] \\ + \eta(t) [bA + \rho(Y) - \gamma F(K, N, AL)]$$

Now  $\frac{\partial \mathcal{H}}{\partial R} = 0$  implies  $\frac{\psi}{\phi} = f_R$ .  $\frac{\partial \mathcal{H}}{\partial C} = 0$  implies  $\phi = U_C$ .  $\frac{\partial \mathcal{H}}{\partial Y} = 0$  implies  $\eta = \phi g_Y / \rho_Y$ . We represent  $U(C, G)$  as  $CU_C + GU_G$ . Then our NNP function  $\mathcal{H}/U_C$  is

$$NNP = C + \frac{U_G}{U_C} G + \dot{K} + f_R R + \frac{dg}{d\rho} \dot{A}$$

The first two terms capture static aggregate consumption at prices 1 for wheat and  $U_G/U_C$  for virgin forest services. The next term is net investment in new buildings, infrastructure and machines and  $f_R R (= f_R \dot{L})$  is economic appreciation arising from bringing virgin forest into agricultural use (assuming  $\dot{L} > 0$ ).  $f_R$  is the marginal cost of transforming a hectare to agricultural use and this marginal cost can be viewed as a wedge between the shadow price of a unit of land in virgin forest and the shadow price of a unit of land in agriculture.  $f_R$  captures the net increase in the value of a unit of land.

$\dot{A}$  will generally be negative during economic growth as erosion and soil degradation takes place with intensive use in agriculture.  $dg/d\rho (= (dg/dY)/(d\rho/dY))$  is the marginal cost of fertilizer in terms of wheat with fertility effect  $\rho$ . Alternatively, it is the marginal cost of increasing the fertility index  $A$  one unit when the current improvement level from fertilizer is  $\rho$ . Then  $(dg/d\rho)\dot{A}$  is the cost of degradation  $\dot{A}$  (negative) in terms of wheat. It is the economic depreciation in the quality of land in agriculture.

The striking result is of course that a unit of land in agriculture is implicitly more valuable and hence here costly "slash and burn" yields an

economic appreciation. Instead of costly clearing, one can envisage profitable clearing as the virgin forest is harvested and the land cleared ("harvest and clear"). One can envisage the value of the forest harvested and marketed as exceeding harvesting and clearing costs. Then our costs  $f(R)$  above become net benefits. For  $f(R)$  a net benefit, one observes that the economic change in the capital value of land in agriculture is now negative and thus that "harvest and clear" introduces economic depreciation of agricultural land in amount  $Rf_R$ . Again  $f_R$  is capturing the difference between the shadow price of a unit of land in agriculture and in virgin forest. However now the unit value wedge of land at the margin declines as measured by the marginal benefit of harvesting the virgin timber.

Fertility deterioration is formally similar to environmental stock deterioration from pollution caused by economic activity. In our formulation above, there are implicit Pigovian taxes being charged on inputs to wheat production to reflect the marginal damage caused from increases in wheat production. We discuss these sorts of optimal taxes below when we discuss pollution or degradation of environmental capital.

#### Renewable Resources (No Market Failures)

In a growing economy it may be optimal to harvest fish at rates which erode the current stocks. In the very long run, this would involve extinction or an asymptotic variant of extinction and we will not pursue this issue here. We are simply interested in factoring in economic depreciation of the renewable resource stocks as the stocks are run down from rational harvesting.

The price of fish in the market will emerge from the interaction of supply and consumer demand. Thus the aggregate utility function will depend

on wheat consumed  $C$  and fish  $H$  as in  $U(C, H)$ . The stock at any date is  $Z(t)$ . Natural processes result in the stock increasing by  $\phi(Z)$  in a period including a  $\phi_z(Z_{\max}) = 0$  for a carrying capacity corresponding to  $Z_{\max}$ . At any date  $\dot{Z} = \phi(Z) - H$  is the net change in fish stock. With exhaustible resources  $\phi(Z)$  is not present because there is no natural growth in the stock as there is with fish reproducing. Thus we have two differences here from the analysis with exhaustible resources. First we have fish harvested appear as a consumer good with implicit price (which will be the market price)  $U_H/U_C$  and secondly our stock has a capacity to renew itself. Harvesting does not necessarily reduce the current known stock over the period in question. Natural growth can offset the effect of harvesting. Despite these changes, economic depreciation of the stock turns out to be the rent on the net stock decline  $\dot{Z}$ . To see this we write down our current value Hamiltonian corresponding to the socially optimal or competitive markets solution.

$$\mathcal{H} = U(C, H) + \lambda(t) [F(K, N) - C - h(H, Z)] + \eta(t) [\phi(Z) - H]$$

where  $h(H, Z)$  is the cost of harvesting  $H$  units of fish given stock size  $Z(t)$ . The two local in time maximizing conditions  $\frac{\partial \mathcal{H}}{\partial C} = 0$  and  $\frac{\partial \mathcal{H}}{\partial H} = 0$  yield  $U_C = \lambda$  and  $U_H = \lambda h_H + \eta$ . Thus  $\eta(t) = \frac{U_H}{U_C} - h_H$  and the

$$NNP = C + \frac{U_H}{U_C} H + \dot{K} + \left[ \frac{U_H}{U_C} - h_H \right] \dot{Z}$$

where we approximate  $U(C, H)$  by  $U_C C + U_H H$ . Since  $\dot{Z}$  will generally be negative in a growing economy, gross NNP is reduced by the price of fish,  $\frac{U_H}{U_C}$ , minus its marginal cost  $h_H$  or rent per unit of fish multiplied by stock diminution  $\dot{Z}$ . Obviously this is in principle identical to the concept we obtained for economic depreciation exhaustible resource stocks.

Gordon [1954] made clear how property rights failure in fish stock ownership (common property) resulted in over-fishing. Inadequate property

rights implies an inefficiently low shadow price on stock (rent dissipation). To a rough approximation, this property rights failure will show up as an observed harvest in excess of the socially optimal level and an excessively small unit rent  $[U_C/U_H - h_H]$ .  $Z(t)$  will be awry also, probably smaller than its socially optimal level. These guesses are bothersome enough to make, but we know that the property rights failure will spill over into all other magnitudes, e.g.,  $\lambda$ , the shadow price on  $K$  and  $\dot{K}$  the level of investment in new produced capital  $K$ . In addition the time path  $\{C(t)\}$  will be distorted by the property rights failure in fish stocks. The observed "equilibrium" would be a third best; i.e., one in which social welfare unconstrained or constrained is not being maximized. A second best involves maximizing social welfare subject to constraints. With a rent dissipation constraint each scarcity price or shadow price would differ from its market price by an optimal "wedge", essentially emanating from the price equals average cost constraint. Such an outcome is a second best economy. The real world is however a third best equilibrium in which there are no optimal "wedges" or true scarcities reflected in market prices. We return to these issues below. Immediately below we introduce an explicit distortion in the form of pollution and solve the implicitly competitive solution with optimal "wedges". This will illustrate a second best equilibrium.

#### Renewable Resources (Pollution from Mining)

A well-circulated idea is that extraction of minerals frequently pollutes the air and the water. Chemicals used in extraction often end up in nearby rivers and kill off aquatic life. Perhaps more pervasive is the pollution caused by the refining per se of ores. Let us formalize these external costs and see how they affect our formulas for economic depreciation of natural

resource stocks. We have our formulas above derived under the assumption of no externalities as benchmarks.

We assume that  $R$  tons of exhaustible resource extracted inhibits growth of the renewable resource stock by  $P(R)$ ,  $P(\cdot)$  for pollution.  $P_R > 0$  or more mining results in greater choking off of fish growth. To keep matters transparent, we will ignore exploration activity in the exhaustible resource sector. Our current value Hamiltonian is now

$$\mathcal{H} = U(C, H) + \lambda(t) [F(K, L, R) - C - f(R, S) - h(H, Z)] + \psi(t)[-R(t)] \\ + \eta(t)[\phi(Z) - H - P(R)]$$

Maximization with respect to  $C, H$  and  $R$  for each date yields

$$NNP = C + \frac{U_H}{U_C} H + \dot{K} - \left\{ [F_R - f_R] - \left[ \frac{U_H}{U_C} - h_H \right] P_R \right\} R + \left[ \frac{U_H}{U_C} - h_H \right] \dot{Z}$$

Two observations are in order concerning this new statement of NNP. First the shadow price of mineral is lower because each unit mined causes pollution, i.e., has negative economic consequences. This new shadow price is  $F_R - f_R - \left( \frac{U_H}{U_C} - h_H \right) P_R$  where  $\frac{U_H}{U_C} - h_H$  is the shadow price on the fish stock. Since  $P_R > 0$ ,  $\left[ \frac{U_H}{U_C} - h_H \right] P_R$  is positive. This translates into each unit of mineral stock being worth less than before because each unit causes pollution as it is extracted. Given  $R$  the same in a model with and without pollution, we have the result that economic depreciation is smaller in the polluted economy! A lower value should be netted out from "gross" NNP when pollution is caused by mineral extraction.<sup>6</sup> We observe this again below when

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<sup>6</sup> This result "goes through" at the level of the firm and as such implies that the tax base for polluting mining firms should be larger than that for non-polluting mining firms. That is allowable depreciation is smaller for a polluting mining firm than for a non-polluting mining firm.

we consider mineral extraction polluting "the environment". Our second observation is that the formula for economic depreciation of the fish stock is unchanged.

The formulas point to economic depreciation of stocks being less when there is pollution caused by mineral extraction. However the time paths of endogenous variables will be much different in the different cases; namely an economy with a pollution "cycle" and one without. Thus we cannot assert that actual economic depreciation of mineral stocks will be lower in the polluted economy. Both the formulas and the values of the variables differ between the economies in the two cases.

This is a second best economy because (a) social welfare is being optimized albeit in the face of an externality or pollution constraint and (b) the solution values ( $C, H, \dot{K}, R, K, Z$ ) reflect a constrained optimal program because optimal "wedges" are present. In this case each resource owner is treating his or her stock as if its marginal value was  $[F_R - f_R] - \left[ \frac{U_H}{U_C} - h_H \right] P_R$  and not simply  $F_R - f_R$ . The optimal wedge here is  $\left[ \frac{U_H}{U_C} - h_H \right] P_R$ . Immediately below we take up another second best equilibrium, a generic pollution scenario in which productive activity causes pollution as a negative by-product.

#### Depreciation of Environmental Capital

Pollution or stress on environmental capital is, from an economic point of view, an intrinsic by-product of otherwise productive economic activity. One's reflex is to say pollution should be eliminated; but zero emissions or residuals can usually only be associated with zero productive activity. The negative by-products will always be present. Pollution control is asking that the flow of emissions or residuals be set at a level such that natural

dispersal and natural degradation prevent build up of the emissions of residuals in the environment. Constraining the flows of emissions and residuals involves the familiar array of instruments: taxes or prices for flows or quantity controls (standards). A useful view is that instruments of control change the institutional setting from one of no property rights enforced on the environmental capital (common property) to one of enforced rights. Since environmental capital can correctly be viewed as a capital good useful in production in the sense that it cleans the production process and leaves the production process in good order for additional production, it should be rationed so that its services go to the highest bidder and secondarily its services are not choked off by over-use. One interpretation of rationing the services of a capital good is that an owner "lets out" its services as in a piece of land so as to maximize the present value of "surplus" from the capital good. In a well-functioning economy rationing via the granting of secure private property rights is generally enough to cause an optimal use of resources in the economy. With environmental problems there is the problem of assigning the property rights and having such rights enforced. Economists generally view property rights enforcement as approximately costless but with environmental capital, one needs an administrative apparatus to set "prices" and/or standards and collect charges and/or ensure that standards are being met.

A short-hand way for expressing degradation of environmental capital is to say that the stock of pollutants suspended in air and/or watersheds has increased. Since pollution is more easily measured than environmental capital, we pursue our analysis of economic depreciation of environmental capital in terms of pollution levels. The environment is used to dump residuals from productive economic activity and it becomes polluted as long

as it cannot dissipate the residuals as fast as they are "dumped". We might have, then,  $\dot{E} = -bE + \gamma Q$  where  $E$  is the amount of pollution suspended (a stock notion) and  $Q$  is productive output elsewhere in the economy.  $\gamma Q$  is pollution added to airsheds and watersheds by output  $Q$  and  $-bE$  is the "evaporation" of pollution by natural processes. In this formulation  $b$  is the rate of "radioactive decay" of the pollution stock by natural processes. More generally we might postulate  $-b(E)$  as the amount of "evaporation" of pollution per unit time given current pollution stock  $E$ . Pollution here is a negative by-product of other productive economic activity.

By pollution abatement we might mean taking action to make  $\gamma$  a smaller number, corresponding to each unit of productive output causing less pollution. We can introduce flow costs  $\alpha(\gamma)$  to keep  $\gamma$  at same current level. The larger is  $\alpha(\gamma)$ , the lower will be  $\gamma$ .

Pollution can reasonably impinge negatively in two places. It can constrain current production in the sense that the larger is the pollution stock  $E$ , the lower is the output of "wheat",  $Q$ . In addition we might argue that pollution has direct negative effects on welfare so that  $U(C, E)$  reflects the fact that a larger  $E$ , more pollution, corresponds to a lower level of utility, given  $C$  constant. The current value Hamiltonian for this problem is

$$\mathcal{H} = U(C, E) + \lambda(t)[F(K, N, E) - C - \alpha(\gamma)] + \psi(t)[-b(E) + \gamma F(K, N, E)]$$

Key first order conditions,  $\partial \mathcal{H} / \partial C = 0$  and  $\partial \mathcal{H} / \partial \gamma = 0$  yield  $\lambda(t) = U_C$  and  $\psi(t)/U_C = \frac{d\alpha}{d\gamma} / \left( \frac{d\gamma F}{d\gamma} \right)$ . Since  $\gamma F$  is units of pollution (the extra residuals resulting from current "wheat" production  $F$ ) we can express  $\psi(t)/U_C = \Delta\alpha/\Delta E$  which is the wheat value of an extra unit of pollution. We approximate  $U(C, E)$  by  $CU_C + EU_E$ . Then the current value Hamiltonian ( $H/U_C$ ) can be expressed as

$$NNP = C + EU_E/U_C + \dot{K} + (\Delta g/\Delta E)\dot{E}.$$

Since  $d\alpha/d\gamma < 0$ ,  $\Delta\alpha/\Delta E < 0$ . Thus  $(\Delta\alpha/\Delta E)\dot{E}$  is the economic depreciation of environmental capital as the pollution stock  $E$  increases during economic growth. I.e. gross NNP must be reduced by the increase in the pollution stock valued at its marginal "draw-down" of wheat or amount of wheat foregone at the margin.  $\dot{E}$  is the increase in pollution stock over the accounting period.

In addition there is a netting out of the consumer disutility from having to live with the current stock. That is,  $U_E < 0$  and  $(U_E/U_C)$  is the wheat price of a unit of pollution and  $E U_E/U_C$  is the consumer value (negative) of the current stock,  $E$ . There are then two nettings out: one for economic depreciation of environmental capital, namely  $(\Delta g/\Delta E)\dot{E}$  and one for damages to "consumers" from the pollution stock, namely  $EU_E/U_C$ .

What about the general result of stock reduction weighted by price minus marginal cost? First our environmental stock reduction is here pollution stock increase. This results in price being negative (i.e.,  $U_E/U_C < 0$ ), marginal cost being negative (i.e.,  $\Delta g/\Delta E$ ), and stock size change being positive (i.e.,  $\dot{E} > 0$ ). Thus symmetry with early economic depreciation results obtains, though with sign reversals throughout. One other variation. Here marginal cost and stock size change are flows. This is what we observed for earlier economic depreciation formulas. Now however price  $(U_E/U_C)$  relates to the stock and not to the flow. This causes our adjustment to the national accounts to segment into a separate stock value component  $(EU_E/U_C)$  and a separate stock increment component  $(\dot{E}\Delta\alpha/\Delta E)$ . This separation is new to us because we are dealing with market prices related to stocks for the first time and not market prices related to flows as we did earlier.

Usher [1981; pp. 130-134] has reservations about introducing certain

variables representing "atmosphere" (our term) into agents utility functions. He takes average hours of sunshine as an example. First he objects to having items in the utility function which are largely unaffected by human activity. Since ultimately it is economic activity vis-a-vis human welfare one is interested in measuring, one should omit variables largely unaffected by human activity, such as average hours of sunshine. If however human activity does alter average hours of sunshine, even in a once over change, then the resulting change in welfare for those who "consume" sunshine should be calculated. (This first contention of Usher is present but not emphasized in his book but has been explained to me in helpful discussions with him). The second reservation he has is that calculations of the growth of welfare with average hours of sunshine included can be less meaningful than the same calculation with hours of sunshine omitted. Thus suppose we do a calculation and average hours of sunshine remain approximately constant over the interval.

"When sunshine is treated as an environmental condition, the fact, if it be so, that hours of sunshine have not increased over the years...has no effect whatsoever on our measure of the rate of economic growth. If sunshine were inputted as an ordinary commodity, we would have to say that the failure of the number of hours of sunshine to increase means that the true rate of economic growth is lower than a computation that did not take sunshine into account would show it to be". (p. 133-34)

And further on Usher defends the introduction of pollution as an argument in the utility function if there are definite changes in the levels of pollution over time. Pollution as an "environmental variable" (Usher's

term) is principally a result of human activity and as such passes Usher's first hurdle for introduction in the utility function. Whether to put changes in levels in the utility function or levels per se is not an issue provided the correct implicit "price" is assigned to the variable. The "price" in question should be different for stocks and for flows. In earlier analyses of economic depreciation of environmental capital (Hartwick [1990], [1991a]), I introduced changes in the stock of pollution into the utility function. This "worked" but is not elegant and upsets those who feel intuitively that it is the stock which matters to consumers and not just increments in the stock. Netting out a valuation of current disamenities such as pollution and traffic congestion seems practicable and appropriate. These nettings out are however distinct from those involved with economic depreciation of environmental capital.

To carry out a correct accounting with degrading environmental capital, one needs consumer prices  $U_E/U_C$  as well as a solid measure of pollution stock  $E$ .  $E$  is presumably easier to quantify than is  $U_E/U_C$ . This price is the dollar value of the disutility of an extra unit of pollution. Careful observers have been pondering how to estimate such prices for at least a decade and good progress has been made (see for example Pearce and Turner [1990]). Small scale empirical studies have been done in many instances. In national accounting, one needs agreed upon prices for a diversity of pollutants, "averaged" over different types of consumer and those in different regions. Questionnaires have been used by some to obtain such prices. These surveyors get estimates of willingness-to-pay to have a pollutant reduced at the margin and willingness-to-receive marginally more pollution with appropriate dollar transfers as compensation. Appropriate prices emerge in these exercises.

The other price we require is the marginal reduction in NNP or in output of wheat required to reduce the pollution stock by a unit. This is a measure of marginal "defensive" expenditure in pollution control: for current emissions to decline by one unit, we require a commitment of  $k$  dollars to new technology of emissions control.  $k$  is the datum we require.  $k$  corresponds to the best technical approach (least cost) for achieving a unit reduction in emissions. In fact "pollution" is a vector of diverse emissions and/or residuals and so we would be in fact considering the least cost way of reducing a group of pollutants, each by a unit. It is this number  $k$ , weighting current pollution stock increases, which is our measure of economic depreciation of environmental capital.  $\Delta\alpha/\Delta E$  is a marginal cost entity. There is no price counterpart because the flow  $\dot{E}$  appears neither as an input in production ( $E$  might however) nor as an argument in the utility function (again where  $E$  has been placed).

### The Real World is Third Best

A general welfare result is the following: if sector  $i$  is not competitive and all other sectors of the economy are, observed prices do not reflect a quasi optimal allocation of resources. Either all sectors must be competitive in order for market prices to reflect genuine scarcities or if one is distorted, all remaining prices must, in general, be adjusted by a planner in order for the allocation occurring at those adjusted prices to reflect a constrained social optimum (a second best).

One might be inclined to say: we know sector  $i$  is a monopoly and so we will lower its observed price before we construct NNP. Lowering the price requires that an adjustment also be made in its quantity. This is a perilous procedure because in general equilibrium a distortion in sector  $i$  spills

over, implying distortions in most other sectors in the economy. They also need adjustments in their prices and quantities. The directions and magnitudes of the changes are very difficult to estimate or to guess at intelligently. A variant of this problem is known in the literature as "piecemeal welfare analysis". (See Hatta [1977].)

Some idea of "the distorted pricing problem" can be obtained from considering second best economies in more detail. Our economy with a pollution sector has in fact been modeled as a second best economy. In our situation, we introduced a distortion in the form of production yielding output plus a "bad" or negative externality, namely pollution. Implicit in our modeling was that correct Pigovian taxes for pollution effects were being charged to make the resource allocation (physical flows of goods and services) a second best optimum.

The easiest way to see that prices reflect marginal pollution effects is to consider the wage rate. Recall that the labor supply  $\bar{N}^S(t)$  equals labor demanded in the production of wheat. Formally the wage rate is the Lagrangian multiplier on this constraint, normalized by marginal util value, and now adjusted for the pollution damage caused by an increase in labor in the wheat sector. That is, with the labor constraint, our current value Lagrangian (constrained Hamiltonian) is

$$\mathcal{L} = U(C, E) + \lambda(t)[F(K, N, E) - C - \alpha(\gamma)] + \xi(t)[-b(E) + \gamma F(K, N, E)] + \Omega[N^S - N]$$

Now  $\frac{\partial \mathcal{L}}{\partial N} = 0$  implies  $F_N + \frac{\xi}{\lambda} \gamma F_N = \frac{\Omega}{\lambda}$ . The wage rate defined inclusive of corrective taxes for internalizing the pollution damage caused by a larger  $F$  induced by a marginal increase in labor is  $w = \frac{\Omega}{\lambda} - \frac{\xi}{\lambda} \gamma F_N$  where  $-\frac{\xi}{\lambda} \gamma F_N$  is a tax on a unit of labor. (Recall that  $\xi < 0$  because more pollution reduces

welfare). Implicit in this formulation is the notion that the marginal product of labor,  $F_N$ , overestimates labor's true worth because besides producing output, labor indirectly causes more pollution. As our model economy operates, labor is implicitly priced to reflect its contribution at the margin to pollution. It is as if perfect property rights on pollution rights or environmental capital are in effect. A lack of property rights on environmental capital would be reflected in wage rates not being defined inclusive of marginal pollution damage. That is labor is overused or under-priced when its indirect pollution effect is not taken account of in the price charged for labor.

In order to incorporate the more realistic case of property rights failure we need to constrain the shadow price on pollution to lie below its first best value. Pollution "problems" are generally associated with an underpricing (over-use) of the environmental capital. To formally capture these real world pricing situations, we need to add a constraint to our model indicating that the price charged for pollution is artificially (non-first best) low. This is an exercise in the theory of the second best (see for example Green [1961] or Dixit [1975]) and we leave such an investigation for another occasion.

The technology of abatement is treated as state-of-the-art and given. Presumably technical change in pollution abatement technology may be the principal source of gains in environmental cleanliness in the future. The whole subject of R & D and technical change requires explicit treatment in our framework and will be taken up at a future date. We turn to an explicit externality involving environmental capital and the associated economic depreciation formulas.

## Mining Causing Pollution

We illustrate the derivation of formulas for depreciating environmental capital when there is an explicit negative spillover from mineral activity, in addition to our generic pollution spillover from the production of "wheat". Somewhat paradoxically, our new formula suggests once more that the economic depreciation of mineral stocks should be valued less when mining not only produces minerals but also pollution. We do not see double damage being calculated: namely damage from stock depletion per se and additional damage from pollution produced in mining. Rather, we see that minerals are somewhat less valuable when their use results in pollution and the same diminution in mineral stocks is treated as causing less "damage" (net economic depreciation) when mining involves pollution than when mining causes no pollution. There remains as before, terms for the damage caused by pollution to consumers and a term for economic depreciation of the stock of environmental capital.

Our new problem includes a pollution production process  $\beta(R)$  increasing with the amount of mineral  $R$  currently mined. Our new current value Hamiltonian is

$$\mathcal{H} = U(C, E) + \lambda(t) [F(K, N, R, E) - C - f(R, S) - g(\gamma)] + \psi(t)[-R] \\ + \xi(t)[-b(E) + \gamma F(K, N, R, E) + \beta(R)]$$

Then  $\frac{\partial \mathcal{H}}{\partial C} = \frac{\partial \mathcal{H}}{\partial \gamma} = \frac{\partial \mathcal{H}}{\partial R} = 0$  yield  $\lambda = U_C$ ,  $\frac{\xi}{U_C} = \frac{g_\gamma}{F}$  and

$\frac{\psi}{U_C} = [F_R - f_R] + \frac{dg}{d\gamma}/F [\gamma F_R + \beta_R]$ . Representing  $U(C, E)$  by  $U_C C + U_E E$  and

substituting yields our NNP function

$$\frac{H}{U_C} = C + \frac{U_E}{U_C} E + \dot{K} + \left\{ [F_R - f_R] + \frac{\Delta g}{\Delta E} (\gamma F_R + \beta_R) \right\} \dot{S} + \frac{\Delta g}{\Delta E} \dot{E}$$

where  $\frac{\Delta g}{\Delta E} \equiv \frac{dg}{d\gamma}/F = \left( \frac{dg}{d\gamma} \right) / \left( \frac{d\gamma F}{d\gamma} \right)$ . Recall  $\dot{S} = -R$  or  $\dot{S} < 0$ . The new term is

$\frac{\Delta g}{\Delta E} (\gamma F_R + \beta_R) \dot{S}$  a positive entry, since  $\dot{S} < 0$  and  $\frac{\Delta g}{\Delta E} < 0$ . Mining causes pollution in two ways. First mining produces say "oil" which increases the output of "wheat" and "wheat" production causes pollution. Hence the term  $\frac{\Delta g}{\Delta E} \gamma F_R \dot{S}$ . Secondly mining causes pollution directly via  $\beta(R)$ . Hence the term  $\frac{\Delta g}{\Delta E} \beta_R \dot{S}$ . Economic depreciation of mineral stocks is

$$[F_R - f_R] \dot{S} + \frac{\Delta g}{\Delta E} (\gamma F_R + \beta_R) \dot{S}$$

The first term is our familiar Hotelling rent term and is negative since  $\dot{S} < 0$ . The second term relates to pollution caused by mining and is positive because  $\dot{S} < 0$  and  $\Delta g/\Delta E < 0$ . Economic depreciation for a given stock depletion  $\dot{S}$  is less when mining causes pollution! Roughly speaking each ton mined is worth less to the economy in utils because each ton mined causes pollution. In the absence of pollution effects from mining, for a given  $\dot{S}$ , each ton mined is worth more to the economy. The higher valuation per ton mined under no pollution effects means of course for any stock diminution  $\dot{S}$ , economic depreciation is more. Recall that  $\left( [F_R - f_R] + \frac{\Delta g}{\Delta E} (\gamma F_R + \beta_R) \right)$  is the dollar-value shadow price of an extra unit of mineral stock to the economy. It is this  $\psi/U_c$  which is the valuator for the  $\dot{S}$  used up over an accounting period.

Note that we still deduct  $\frac{U_E}{U_C} E$  from a measure of gross NNP to allow for the negative effects on utility directly from the stock of pollution. Also the environmental stock depreciation term  $(\Delta g/\Delta E) \dot{E}$  is unchanged. Also we must keep in mind that though the formulas may be the same, the competitive dynamic paths of the economy will be different for the case of pollution introduced into the model or not introduced. Thus the time paths of  $C$ ,  $\dot{K}$ ,  $\dot{S}$ ,  $\dot{E}$  etc will be different if pollution is present. We cannot say that because the economic depreciation term for pollution is unchanged from

one form of the economy (without mining pollution) to another (with mining pollution) that its magnitude will be the same in the two cases.

### Concluding Remarks

In this paper, we have outlined a "method" for calculating economic depreciation of natural resource stocks and we have taken up a number of complications. However we have not dealt explicitly with general property rights failures or oligopoly problems. These require an extension of our method or the incorporation of new static constraints into the model. We have seen how exploration activity and externalities caused by mineral extraction affect our basic economic depreciation measures. The basic rent measures become complicated with externality or price "wedges". As a practical matter how should one proceed? Should one search for good measures for the refinements or proceed with basic unrefined rent formulas? The answer turns on the magnitudes anticipated. Will unrefined measures be poor or good approximations to refined measures?

There are two additional practical problems. Our formulas presume that environmental capital is being efficiently rationed or the Pigovian user charges are in effect and are reflected in the observed prices for inputs and outputs as well as in the quantities used in the calculations. In actual economies these Pigovian "taxes" are generally not in effect and thus observed prices and quantities are distorted versions of true scarcity or efficiency prices and quantities. These distortions generally spill over to all sectors of the economy. That is, distortion in sector  $i$  imply that the satisfaction of efficiency conditions in sector  $j$  will generally not yield a constrained optimal allocation. One distortion generally requires further departures from efficiency in other sectors in order for a constrained

optimum to obtain. How should the practicing national income accountant proceed?

One approach is to make calculations with observed data which certainly are distorted prices and quantities. Assume the distortions are small and that economic depreciation magnitudes are good approximations to correct values. Another approach is to attempt to correct the distorted prices a priori and then to calculate economic depreciation magnitudes. This latter approach is sometimes referred to as the shadow price approach and has been used to try to estimate the true value of foreign exchange in an underdeveloped country with many departures from competition in its economy. In our case the distortions are rooted in unpriced use of environmental capital, possible property rights failures in the fishery and oligopoly behavior likely in the resource extractive sectors. The current way to deal with the shadow pricing approach in practice is to construct a computable general equilibrium model of the economy in question (as in Whalley [1982]) and to obtain estimates directly from perturbations to the empirically articulated economy. Clearly other ad hoc procedures could be used at lower cost to arrive at adjusted distorted prices. Reliability is the obvious desideratum.

The other issue involves technical change or unanticipated shocks to the economy in the future. It is only when one focuses directly on NNP as an artifact of a growing economy that one confronts the issue of the meaning of shadow prices on the capital stock (e.g., Weitzman [1976]). The focusing becomes more pronounced when one considers changes in natural resource stocks, obviously, over time. Shadow prices on resource stocks (co-state variables) become an intrinsic part of deriving expressions for economic depreciation. Our main point is that these efficiency prices reflect a

discounting back of the entire future history of the economy. Thus any change in the future history of the economy will be reflected in the calculation of correct scarcity prices on capital stocks. (These prices influence the values of "non-dynamic" prices as well.) Provided the future history is correctly anticipated, current prices will accurately reflect basic scarcities. Correctly anticipated involves anticipating future changes arising from technical progress. If technical change (or any other future shock) is incorrectly anticipated, current prices and quantities inadequately reflect basic scarcity. Prices and quantities will jump to new values when the unanticipated shock occurs. But the pre-shock prices will in no way reflect the post-shock prices and in an important sense are inadequate reflections of fundamental scarcity.

To repeat, these difficulties with unanticipated shocks become clear when one considers national accounts in an inherently dynamic context. They have always been present in arriving at a meaningful estimate of aggregate economic activity but become focal points when one considers an economy as an entity changing over long periods. The matter can be expressed most simply if one ponders the fact that today's price of oil reflects currently known world reserves. Unanticipated discoveries will cause the current price of oil to drop suddenly, other things being the same. Thus the current price before the discoveries fails to reflect the basic scarcity of oil in the economy.

How is one to deal with this problem? By definition the shifts cannot be anticipated. The best one can say is that our estimates of NNP net of economic depreciation of natural resources are approximations subject to future uncertainty. Of course anticipated shifts in the "environment" of the economy will be capitalized in current prices and quantities - the more

accurate the anticipation, the more precisely will current prices reflect basic scarcities.

National accountants make do with the prices they observe in the market place. The less distorted these prices are in the sense that they emerge in an economy characterized by perfect competition and complete property rights, the better will these prices reflect basic scarcities. Undistorted prices permit us to attach welfare significance to NNP and the economic depreciation terms as in Weitzman [1976] and Solow [1986].

We summarize. We have explored "variations" on the theme that rent on current stock use (diminution) should be deducted from gross NNP to obtain NNP net of economic depreciation of natural resource capital. The variations include considerations of exploration activity for new mineral stocks, new estimates of economic depreciation of oil stocks in the U.S., incorporating durable exhaustible resources in our formulas, incorporating externality effects such as mining polluting fisheries and mining polluting the environment directly. New formulas were presented and interpreted. We also considered transforming land in virgin forest to land in agriculture. Open for subsequent research is dealing further with explicitly distorted economics via systematic use of the theory of price distortions. Then we could relate the distortions from say imperfect property rights in the fishery to prices and quantities used in calculating economic depreciation from natural resource stock diminution. Also open is a more detailed examination of how technical change affects measures of economic depreciation. And no doubt patient readers can think of a host of other matters to explore and/or clarify.

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