Queen's Economics Department Working Paper No. 815

Contractual Design with Correlated Information Under Limited Liability

Dominique M. Demougin Devon a. Garvie

Department of Economics
Queen's University
94 University Avenue
Kingston, Ontario, Canada
K7L 3N6

5-1991

CONTRACTUAL DESIGN WITH CORRELATED INFORMATION UNDER LIMITED LIABILITY

Dominique M. Demougin
University of Toronto
and
Devon A. Garvie*
Queen's University

Discussion Paper No. 815

May, 1991

* Financial support was provided in part by a Social Science and Humanities Research Council of Canada Doctoral Fellowship.

An earlier version of this paper was presented at the Sixth World Congress of The Econometric Society, Barcelona, Spain, August, 1990. The authors thank Nancy Gallini, Ben Hermalin, Jeffrey Perloff, Michael Riordan, John Riley, Roger Ware, Ralph Winter and Brian Wright and an anonymous reviewer for helpful discussions and comments.

Abstract

We examine contractual design in a principal-agent model under two forms of limited liability: non-negative constraints on the transfer payments to and the profits of the agent. We show that when limited liability is a binding constraint the principal cannot implement the first-best solution and the agent earns rents from private information. Limited liability is a binding constraint in the non-negative transfers model only when a signal is insufficiently responsive to type (inelastic). Further, as the production rule and profits are continuous in the type elasticity of the signal density function, the level of inefficiency is less than that which obtains with no signal. If a signal is sufficiently responsive to type (elastic) in this environment, then the principal can implement the first-best allocation and the value of the agent's private information is zero.

§1. Introduction

When we think about making principal-agent contracts contingent on information available after an agreement has been reached, a natural conclusion is that the value of the information and the level of efficiency achievable by the contract should depend on the quality of the information. It is rather surprising then that recent work by McAfee and Reny (1989), Riordan and Sappington (1988) and Demougin (1988) shows that even "coarse information" may be sufficient to eliminate the distortionary consequences of information asymmetries in principal-agent relationships.[1] Four conditions are required to achieve efficient production and reduce the value of private information to zero: (i) the principal can condition the payments to the agent on an *ex-post* observable signal correlated with the agent's private information; (ii) the signal is costless; (iii) payments are unbounded; and (iv) all parties to the contract are risk neutral. This paper considers the consequences of condition (iii) and studies some alternatives.

We set up an expository model to highlight some of the critical features of the standard results for a principal-agent model with and without an *ex-post* observable correlated signal. In the standard problem, the principal's task is to design an optimal contract or incentive scheme for an agent with private information about the production environment.[2] In the absence of an *ex-post* observable signal, the solution is characterized by a sub-optimal production allocation and rents accruing to the holders of the private information. If, however, the payments to the

---

[1] Related results have been obtained in auction models (McAfee and Reny (1989), Cremer and McLean (1985,1988) and McAfee and McMillan and Reny (1988)), regulatory auditing models (Baron and Besanko(1984), Garvie (1989)), and multiple agent models (Demski and Sappington (1984) and Demski, Sappington and Spiller (1988)).

[2] A large number of examples of economic relationships exhibiting information asymmetries have been analyzed in the recent literature. These include the regulation of a natural monopoly with unknown cost (see Baron and Myerson (1982)); nonlinear pricing in the case of a monopoly with incomplete information (see Maskin and Riley (1984)); optimal labour contracts with asymmetric information (see Hart (1983)); and, the choice of an optimal selling mechanism for a single good (see McAfee and McMillan (1987)).

agent can be conditioned on an *ex-post* observable signal and the conditions (i)-(iv) are satisfied, then implementing the first-best allocation becomes optimal. Also the value of private information is reduced to zero. We briefly characterize these results from the literature in an environment with a binary signal. In particular, we illustrate how the latter powerful but economically unappealing mechanism works.

The latter result is unappealing mainly because the effectiveness of the signal is independent of the degree of correlation.[3] That is, the principal can sustain the first-best solution from any non-zero measure of informativeness.[4] While this is not surprising from a statistical perspective, given that all players are assumed to be risk neutral, it is less appealing from an economic point of view. As we show here, it implies that both parties have unlimited wealth: as the signal becomes less informative the principal's payment to the agent can fluctuate between minus and plus infinity.

For a truly positive theory of contracts, a more reasonable assumption is that buyers and sellers operate in an environment of limited liability. We examine two forms of limited liability: non-negativity constraints on the transfer payments to and the profits of an agent. The former model can describe a regulatory relationship wherein a regulator does not have the power to tax a firm under any conditions or the design of disaster compensation schemes. The latter model can describe any economic relationship wherein a principal must cover at least the costs incurred by an agent (e.g. defense procurement).

---

[3] A second difficulty with the above result is that the solution is discontinuous at the point at which the signal becomes noninformative. Further, making a contract depend on a noninformative signal amounts to pure randomization, which does not improve the efficiency of the mechanism; see for example Baron and Myerson (1982).

[4] The first-best solution is implementable provided that certain technical requirements are met. These are not, however, conditions on the informativeness of the signal but rather on the distribution of the signal.

We find that a principal cannot implement first-best production or fully extract informational rents from an agent by conditioning transfer payments on an *ex-post* observable signal when limited liability is a binding constraint. Further, we show that the optimal contractual arrangements which emerge in both models are sensitive to the information content of the signal as measured by the rate of change in the conditional probability density function of the signal with respect to the agent's type. We prove that production and welfare are in either case increasing in the informativeness of the signal. Finally, we show that welfare and production are higher in the first model with non-negative transfer.

Our paper is related to two strands in the existing literature. First, it is related to some of the contributions discussed above on correlated information with limited liability. In this context we present a simple extension of Riordan and Sappington (1988) to a continuum of types. The study of a continuum of types is not just for mathematical completeness. Indeed, if results derived for the case of discrete distributions could not be extended to the continuous case, their usefulness would become questionable. Also, since the majority of the literature on asymmetric information assumes a continuum of types, our study allows for better comparison.

Second, our paper is related to the literature on optimal auditing of a privately informed regulated firm when there are limited liability constraints. The model most closely related to ours is by Demski, Sappington and Spiller (1987, 1988). They assume a binary-valued private information to derive a demand for auditing when the signal is costly and transfers are constrained to yield non-negative *ex-post* profits.[5] We simplify the problem by assuming that

[5] In the case of unbounded penalties and costly auditing, the principal can implement the first best solution by setting the probability of auditing close to zero and the penalty arbitrarily large.

an audit takes place with certainty. We improve upon their analysis in two ways. First, we examine the case of a continuum of types, to which the same observations as above apply. Second, we extend their analysis by examining an alternative limited liability constraint which also allows for further comparison. Within the literature on optimal auditing, our paper is also related to Baron and Besanko (1984). As in our paper, they study the case of a continuum of types. Using the first order approach, they characterize locally incentive compatible mechanisms. They are, however, unable to find a mechanism which is globally incentive compatible.

We present the model in Section 2. The standard results from the literature for a principal-agent model with and without a correlated signal are discussed in the next section. We examine the consequences of two different forms of limited liability and derive our main results in section 4. Our concluding remarks are found in section 5.

§2. The Model

Consider the following principal-agent model with risk neutrality.[6] The principal has a separable welfare function $W(q,T) = v(q) - E(T)$, where q is the quantity produced by and E(T) is the expected transfer made to the agent. The principal's surplus v(q) is assumed to be increasing and concave in output. We also assume that $\dot{v}(0) = +\infty$ to ensure that the principal will always require strictly positive production.[7] The agent has a payoff or profit function $\pi(q,T) = E(T) - C(c,q)$. The agent's type, parameterized by c, and the incurred costs C(c,q) are assumed to be observable only by the agent. The principal, however, knows the support of

[6] The model, absent the introduction of a signal, is essentially the model analyzed by Baron and Myerson (1982).

[7] Following standard notation in optimal control a "·" will denote a derivative.

c, $c \in [\underline{c},\bar{c}]$, and forms a prior distribution $f(c)$ on the unknown parameter. $F(c)$ is the cumulative distribution function of the agent's type corresponding to $f(c)$. The agent's reservation profit is zero.

We assume that at the end of the production period the principal and agent can both costlessly observe a verifiable signal s which is correlated with the agent's true type c. Thus, the transfer to the agent can be conditioned on the realization of the signal. We adopt a parsimonious model for analytical tractability, with the following simplifying assumptions. First, the cost function is linear in the agent's type c, that is, $C(c,q) = cq$. Second, the signal, s, is binary, where $s \in \{s_1, s_2\}$. We denote the joint distribution over type and signal as $f(c,s_i)$, which we assume to be differentiable in type and strictly positive over the entire support. Following the definition from Milgrom (1981), we assume that s_2 is *more favourable than* s_1, meaning that s_1 signals relatively high cost whereas s_2 signals relatively low cost. In terms of the joint distribution, it means that for all $c_2 > c_1$

$$\frac{f(c_1,s_2)}{f(c_2,s_2)} > \frac{f(c_1,s_1)}{f(c_2,s_1)} \tag{1}$$

We denote the probability of observing the signal s_i conditional on c as $p_i(c)$. The above assumption on the joint distribution guarantees that $p_i(c)$ is differentiable everywhere and that $p_1(c)$ is increasing in type.[8] We denote the type elasticity of $p_i(c)$ by $\theta_i(c)$, where $\theta_i(c) = c\dot{p}_i(c)/p_i(c)$.

The principal constructs a mechanism $M = \{q,T\}$ which works as follows: the

[8] By definition $p_1(c) = f(c,s_1)/\{f(c,s_1)+f(c,s_2)\}$. Thus, $p_1(c)$ is increasing if and only if the ratio $f(c,s_2)/f(c,s_1)$ is decreasing in type for all c, but this follows immediately from equation (1).

principal offers the agent a contract specifying a production rule q(r) and a transfer T(r,s), where r is the agent's report about his type c.[9] The agent can accept or reject the contract. By the Revelation Principle, we restrict attention to direct revelation mechanisms. Therefore, the optimal mechanism must induce the agent to report truthfully and to accept the contract.

§3. Existing Results

Two extreme cases have been extensively studied in the literature. First, in the absence of a correlated signal, the model is a special case of Baron and Myerson (1982). We summarize their findings, as they apply for our model, in Theorem 1 (for a proof see Baron and Myerson):

Theorem 1: *Assume that* $\frac{d}{dc}\left[c + \frac{F(c)}{f(c)}\right]$ *is strictly positive.*[10] *If the mechanism can only be conditioned on the report of the agent, then the optimal contract can be characterized by the following equations:*

$$\dot{v}(q^*(c)) - c = \frac{F(c)}{f(c)} \tag{2}$$

$$\pi^*(c) = \int_c^{\bar{c}} q^*(z)dz \tag{3}$$

[9] We assume without loss of generality that T(·) and q(·) are differentiable almost everywhere. See Baron and Myerson (1982) for a complete discussion of assuming differentiability.

[10] The requirement that the expression d/dc[·] > 0 is standard. It ensures that the production rule is decreasing in marginal costs, given that the principal's surplus function is concave in output. This in turn guarantees that the second-order condition of the agent's reporting problem is satisfied. This condition will be satisfied if the density function does not increase too rapidly, since F(c) is an increasing function of c. The property that F(c)/f(c) is nondecreasing is satisfied for many distributions, including the uniform, the beta (with means greater than ½), the normal, the exponential, the Pareto and the logistic.

The optimal contract is characterized by "no production distortion at the top" and "no rent distortion at the bottom": that is, only the most efficient type of agent produces the first-best output and receives the maximum attainable profit or rents from private information. This solution derives from the principal's trade-off between efficient production and information rent reduction.[11]

A second result in the literature states that if the principal can condition transfer payments on the realized value of the *ex-post* signal and there are no constraints on the magnitude of the transfers, then the first-best outcome is attainable (see McAfee and Reny (1989), Riordan and Sappington (1988) and Demougin (1988)). We summarize this second result in the context of our model in Theorem 2:[12]

Theorem 2: *Assume that* $p_1(c)$ *is concave c.*[13] *If the mechanism can be conditioned on the agent's report and a correlated signal in an environment of unlimited liability, then there exists a pair of transfer functions* $(t_1^{**}(c), t_2^{**}(c))$ *such that the optimal mechanism implements the first-best production* $q^{**}(c)$ *and the value of the agent's private information is zero. The optimal contract can be characterized by the following set of equations:*

$$\dot{v}(q^{**}(c)) - c = 0 \tag{4}$$

[11] Truth-telling and the envelope theorem yield the following relation: $\dot{\pi}(c) = -q(c)$. The principal can and will reduce the agent's profit by changing the production rule as his welfare is nonincreasing in payments to the agent.

[12] See footnote 1 for other related work. Theorem 2 is an extension of Riordan and Sappington (1988) to a continuum of type. The result of the theorem trivially extends to a class of models where the support of the signal is continuous, simply by appropriately dividing the support of s into intervals. McAfee and Reny (1990) and Demougin (1988) study more general alternatives.

[13] The concavity restriction on $p_1(c)$ ensures that the optimal mechanism is globally incentive compatible.

$$\pi^{**}(c) = 0 \tag{5}$$

$$t_i^{**}(c) = cq^{**}(c)[1-\theta_j^{-1}(c)], \text{ where } i \neq j \tag{6}$$

Proof: See Appendix.

To understand Theorem 2, consider the regulator's problem in the absence of information asymmetry. The first-best mechanism is simply $M^{**} = \{q^{**}(c), T^{**}(c) = cq^{**}(c)\}$. This mechanism is not feasible in the standard asymmetric information case, because it is not incentive compatible.[14] For the case of a binary signal, consider the transfer functions

$$\hat{t}_1(c) = T^{**}(c) + \frac{\tau(c)}{p_1(c)} \tag{7}$$

$$\hat{t}_2(c) = T^{**}(c) - \frac{\tau(c)}{p_2(c)} \tag{8}$$

If the principal uses the transfer functions $\hat{t}_1(c)$ and $\hat{t}_2(c)$ and the agent reveals the truth, then the expected transfer is equal to total realized costs and is independent of the function $\tau(\cdot)$. It is precisely this independence which gives the principal one degree of freedom to choose $\tau(\cdot)$ to induce the agent to reveal the his type honestly. Consider the first-order condition of the agent's reporting problem[15]:

---

[14] The first-best or symmetric information solution M^{**} is not incentive compatible, because the agent can increase his payoff by overstating c:

$$\frac{d}{dr}\big(T^{**}(r) - cq^{**}(r)\big)\big|_{r=c} = q^{**}(c) > 0$$

[15] Note that equation (9) is a local incentive compatibility constraint, that is, given a marginal cost level c, the mechanism is constrained to be a truth-telling mechanism for small deviations of the report around the true cost level. For the mechanism to be globally incentive compatible, equation (9) must be satisfied for the entire support of the distribution of c. The concavity of p_1 guarantees global incentive compatibility (see the appendix).

$$\dot{t}_1(c)p_1(c) + \dot{t}_2(c)p_2(c) - c\dot{q}(c) = 0 \tag{9}$$

Substituting $\hat{t}_1(c)$ and $\hat{t}_2(c)$ into the agent's first-order condition yields a first-order differential equation. Solving for $\tau^{**}(\cdot)$ and substituting yields (6). The principal effectively uses $\tau^{**}(\cdot)$ as a wedge between the two possible payments to the agent, a wedge that is inversely related to θ_i. Further, since $\tau^{**}(\,)$ enters $\hat{t}_2(c)$ negatively, whether $\hat{t}_2(c)$ is positive or negative also depends upon the information content of the signal.

Referring to equation (6), we find that if $|\theta_i| < 1$, then the signal is insufficiently responsive to type, or inelastic, to sustain positive transfers to the agent for all realizations of the signal. In such an event, the principal offers the agent a contract wherein a transfer made to the agent conditional on observing a bad signal (s_1) is exactly offset by a transfer from the agent when a good signal (s_2) is observed. This contract keeps the individual rationality constraint binding for all types of agent.

The usefulness of Theorem 2, from an economic perspective, is dubious. An obvious criticism is that the functions $t_1^{**}(\cdot)$ and $t_2^{**}(\cdot)$ become unbounded as the signal becomes less and less informative. It is easy to see that as $\theta_1(c)$ approaches zero, $t_1^{**}(\cdot)$ goes to plus infinity and $t_2^{**}(\cdot)$ goes to minus infinity.[16] Hence, the agent makes an infinite profit and the principal an infinite loss in state s_1 and vice versa in state s_2. A second criticism is that the result is discontinuous when the signal is noninformative, that is when $\theta_i(c)=0$. We respond to both criticisms of this result in the next section.

---

[16] As $\theta_1(c) \to 0$, the agent's private information becomes less correlated with the signal s, that is, the signal is increasingly perceived as noise.

§4. Limited liability

The result in the previous section is unsatisfactory because the assumption of unlimited liability, upon which it depends, is not feasible in most institutional environments. Realistically, parties to a contract must face some form of limited liability. For example, the liability of a firm is often restricted to the assets of the firm rather than the wealth of the agent. Similarly, neither managers nor workers can receive (excessively) negative salaries or wages.[17] In this section, we examine the consequences of conditioning contractual arrangements on an *ex-post* observable signal correlated with the agent's private information in an environment of limited liability. We examine two forms of limited liability: non-negativity constraints on the transfers to and the profits of the agent.[18] These legal rules require the principal to offer the agent an *ex-ante* or *ex-post* individually rational contract, respectively.

§4.1 Non-negative Transfer Payments

The problem of the principal is to find a triplet of functions $\{q(\cdot),t_1(\cdot),t_2(\cdot)\}$ which maximizes his expected welfare subject to the *ex-ante* incentive compatibility and individual rationality constraints and the *ex-post* non-negative transfer payment. The problem can be formulated as an optimal control problem[19]

---

[17] A theoretical justification might be the prohibitive costs of observing the wealth of an agent.

[18] Alternatively, one can think of the agent as being risk-neutral over all non-negative transfers or profits but as infinitely risk-averse beyond this point.

[19] We omit the argument of the function unless confusion is possible.

$$\underset{q, t_1, t_2}{\text{Max}} \int_{\underline{c}}^{\bar{c}} (v(q) - t_1 p_1 - t_2 p_2) f(c) dc$$

subject to

$$t_1 \geq 0 \tag{10}$$

$$t_2 \geq 0 \tag{11}$$

$$t_1 p_1 + t_2 p_2 - cq \geq 0 \tag{12}$$

$$c \in \underset{r}{\text{ArgMax}}\ t_1(r)p_1(c) + t_2(r)p_2(c) - cq(r) \tag{13}$$

where equations (10) and (11) are the limited liability constraints, and equations (12) and (13) are the individual rationality and the incentive compatibility constraint, respectively.

From equation (6) in the foregoing section, we see that if $\theta_1(c) \geq 1$ for all types, the principal will be able to implement the mechanism derived in Theorem 2, because the transfers will be non-negative in all cases. Thus for the purpose of this section we impose the following restriction:

$$\theta_1(c) < 1 \qquad \forall\ c \in [\underline{c}, \bar{c}] \tag{14}$$

We state the solution to the above optimal control problem in Theorem 3:

Theorem 3: *Assume that* $\frac{d}{dc}\left[c + [1-\theta_1(c)]\frac{F(c|s_1)}{f(c|s_1)}\right]$ *is strictly positive.*[20] *If the mechanism can be conditioned on the report of the agent and a signal correlated with the agent's private information and payments are constrained to be non-negative, then the optimal mechanism is defined by the following set of equations:*

$$\dot{v}(q^{***}) - c = [1-\theta_1(c)]\,\frac{F(c|s_1)}{f(c|s_1)} \tag{15}$$

$$\pi^{***}(c) = \int_{\underline{c}}^{\bar{c}} [1-\theta_1(z)]\frac{p_1(c)}{p_1(z)}q^{***}(z)\,dz \tag{16}$$

$$t_1^{***}(c) = \int_{c}^{\bar{c}} \frac{[1-\theta_1(z)]}{p_1(z)}q^{***}(z)\,dz + \frac{cq^{***}(c)}{p_1(c)} \tag{17}$$

$$t_2^{***}(c) = 0 \tag{18}$$

where F(c | s_1) *and* f(c | s_1) *are the posterior distribution and associated density function of the agent's type c conditioned on the signal* s_1. *Social efficiency and production is increasing and the informational rent is decreasing in c.*

---

[20] The requirement that d/dc[·]>0 ensures that the production rule is decreasing in marginal cost. The requirement that $p_1(c)$ is everywhere increasing avoids a signal switching problem. To see this, write the expected transfer to the agent as

$$p_1(c)t_1^{***}(c) = \int_{c}^{\bar{c}} [1-\theta(z)]\frac{p_1(c)}{p_1(z)}q^{***}(z)\,dz + cq^{***}(c)$$

The monotonicity of $p_1(c)$ and equation (14) guarantees that the terms $[1-\theta(z)]$ and $p_1(c)/p_1(z)$ are less than one everywhere. If there was an interval of marginal cost for which $p_1(c)$ was not increasing, we can easily predict the outcome: the principal would switch signals and set $t_1^{***}(c) = 0$ and $t_2^{***}(c) > 0$ in this interval since the probability ratio in the expected transfer would be larger than one.

Proof: See Appendix.

We can think of the optimal contract as a lottery over two possible payments. The contract fully specifies the conditions under which each reward will be paid. The introduction of limited liability in the form of non-negativity constraints on the transfer payments, however, removes the principal's ability to tax the agent when a good signal is observed. With this form of limited liability and an inelastic signal, the best the principal can do when a good signal is observed is transfer nothing, that is, $t_2^{***}(c) = 0$ for all types.[21] Correspondingly, the worst the agent can do is lose incurred costs. Hence, although the lottery is attractive to the agent *ex-ante*, the *ex-post* outcome of the lottery can be unfavourable. Further, note that limiting the liability of the agent by bounding t_2^{***} from below results in an upper bound on t_1^{***}, that is, the principal's liability is also limited.

Two alternative applications of this model include regulatory strategies and design of compensation schemes. First, the model can describe a regulatory relationship wherein the regulator does not have the power to tax a firm under any conditions. When a signal is inelastic with respect to the firm's type, the optimal regulatory contract entails subsidizing the firm in the event that a signal of relatively high costs is observed while a signal of efficiency prompts a zero payment. Moreover, the subsidy will be bounded from above, that is, even with an extremely noisy signal the payment has a finite upper bound. This is appealing given the liquidity constraints faced by most regulatory agencies. Further, we cannot think of any regulatory agency with a legal charter which permits the raising of infinite funds.

---

[21] Recall that we have restricted the model with equation (14) in such a way that, in the case with unlimited liability, $t_2^{**}(c)$ would always be negative. That $t_2^{***}(c)$ is always zero reflects the fact that the limited liability constraint is always binding.

Alternatively, one could apply the results to the design of compensation packages. In particular, consider the case of agricultural subsidies. Farmers possess private information concerning their productivity while the government has access to the farmer's tax and bank records, agricultural scientists capable of making assessments of farm productivity and various microeconomic and macroeconomic productivity indicators. The compensation scheme prescribed by the model would result in positive payments if and only if a bad signal is observed. Once again, this is appealing provided the government wants all types of farmers to produce but does not want to make excessive payments to inefficient types. Further, the government would not want to design a scheme which allows the possibility of high levels of taxation given the wealth and credit constraints of farmers.

The solution, characterized by equations (15) - (18), suggests that in this model the type elasticity, θ_1, is a natural measure of the economic informativeness of the signal. First, the solutions discussed in Theorem 1 and 2 are extreme solutions nested in our model. As θ_1 converges to zero for all types, the signal becomes noninformative and our solution converges to that presented in Theorem 1, the case with no signal. Conversely, as the type elasticity becomes greater than one for all types, the signal becomes sufficiently responsive to type (elastic) and our solution converges to that presented in Theorem 2, the case with a correlated signal and unlimited liability.[22]

This result can be extended the following way. We denote by ι the information content of the signal. Comparing two signals s and s', we will say that s *is more informative than* s' if $\theta_1(c,\iota) > \theta_1(c,\iota')$ for all c. This definition allows for a partial ordering of the distributions

---

[22] Referring to equation (6), if $\theta_1(c) \rightarrow 1$, then $t_2^{**}(c) \rightarrow 0$.

of the signal.[23]

In the appendix we prove the following proposition:

Proposition 1: *Welfare and production are increasing in the information content of the signal.*

The result can be interpreted as follows. As the quality of the information contained in the signal improves, the information asymmetries between the principal and agent are reduced. Consequently, the production rule advances toward the first-best solution. This effect has a positive impact on the principal's welfare. Thus, our solution, being continuous and monotonic in θ, behaves in an economically appealing way.

In order to further interpret this result, we rewrite the welfare induced by the mechanism:

$$W^{***} = \int_{\underline{c}}^{\bar{c}} \{v(q^{***}(c)) - [c + H'(c,s_1,\iota)]q^{***}(c)\} f(c)dc \tag{19}$$

$$\text{where} \qquad H'(c,s_1,\iota) = [1-\theta_1(c,\iota)]\frac{F(c|s_1,\iota)}{f(c|s_1,\iota)}$$

$H'(c,s_1,\iota)$ denotes the adjusted hazard rate, which can be interpreted as the necessary "bribe" per unit of output required to keep the mechanism incentive compatible. We show in the appendix, that this adjusted hazard rate is decreasing in the informational content of the signal. Thus, an increase in the informativeness of the signal works exactly like a decrease in the per unit cost, raising production and welfare. We note that a change in ι affects the expected rent - and thus the rent - in an indeterminate way. This results since the expected rent is $E[H'(c,s_1,\iota)q^{***}(c)]$ and a change in the informativeness of the signal affects the adjusted hazard

[23] The ordering is only partial, because a change in the distribution of s may affect $\theta_1(c)$ differently for different types.

rate and production in opposite directions. Which of these effects dominates depends upon H' and the curvature of v. However, as the signal becomes almost perfect, that is as θ approaches 1 for all c, it is easy to show that the expected rent becomes decreasing in the information content of the signal.

Proposition 1 allows for a comparison between Theorems 1, 2 and 3: Theorem 1 can be thought as the case of a completely non-informative signal; Theorem 2 as the case of a perfectly informative signal, and; Theorem 3 as the intermediary case. Thus, we have the following ordering:

Corollary: *Given functions that satisfy the requirements of Theorems 1, 2 and 3, it follows almost everywhere that (i)* $q^{*}(c) \leq q^{***}(c) \leq q^{**}(c)$ *and (ii)* $W^{*} \leq W^{***} \leq W^{**}$

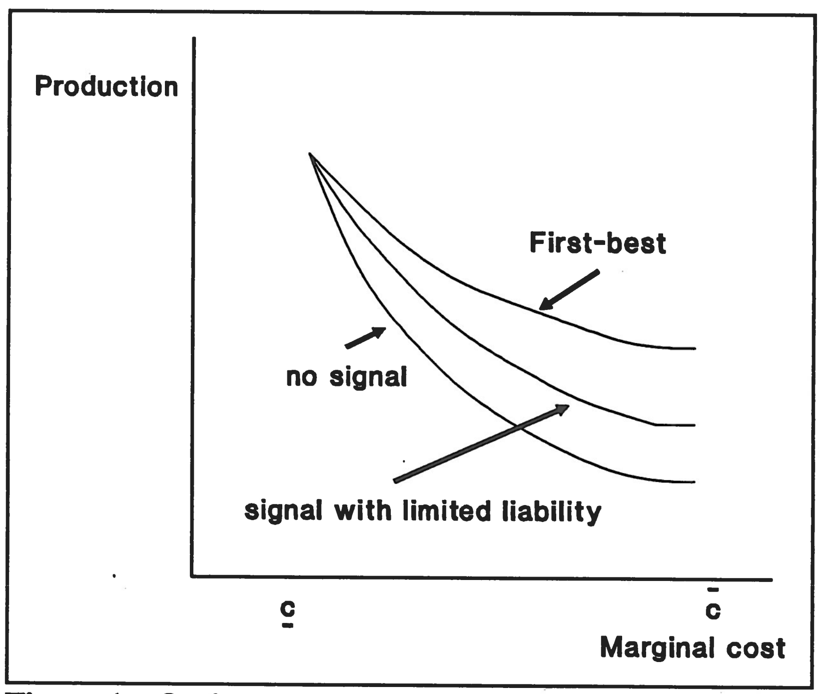

Figure 1. Optimal Production Rules

The ordering of the production schedules for the three cases is shown graphically in Figure 1. The optimal production schedule when a signal is available but the liability of the agent is limited lies between the first-best schedule and the schedule when no signal is available. Note that the contracts with or without a signal are characterized by "no production distortion at the top".

§4.2 Non-negative Profits

How would stricter limited liability rules affect contractual design? Suppose that a principal is required by law to cover at least the costs incurred by an agent. This limited liability rule constrains the principal to design an *ex-post* individually rational contract, that is, profits must be non-negative for either realization of the signal. Equations (10) - (12) must be replaced by the following constraints:

$$\pi_1 = t_1 - cq \geq 0 \tag{20}$$

$$\pi_2 = t_2 - cq \geq 0 \tag{21}$$

Theorem 4 summarizes the solution to the principal's mechanism design problem:[24]

Theorem 4: *Assume that* $\frac{d}{dc}\left[c + \frac{F(c|s_1)}{f(c|s_1)}\right]$ *is strictly positive.*[25] *If the mechanism is constrained to be ex-post individually rational, then the optimal mechanism is defined by the following set of equations:*

---

[24] Theorem 4 is an extension of Demski, Sappington and Spiller (1987, 1988) to a continuum of types.

[25] The requirement that $d/dc[\cdot]>0$ ensures that the production rule is decreasing in marginal cost. For a discussion see the footnotes 10 and 20.

$$\dot{v}(q^{\diamond}) - c = \frac{F(c|s_1)}{f(c|s_1)} \tag{22}$$

$$\pi^{\diamond}(c) = \int_{c}^{\bar{c}} \frac{p_1(c)}{p_1(z)} q^{\diamond}(z)\,dz \tag{23}$$

$$t_1^{\diamond}(c) = \int_{c}^{\bar{c}} \frac{q^{\diamond}(z)}{p_1(z)}\,dz + cq^{\diamond}(c) \tag{24}$$

$$t_2^{\diamond}(c) = cq^{\diamond}(c) \tag{25}$$

The social efficiency of the mechanism and the production rule are increasing and the information rent is decreasing in c.

Proof: See Appendix.

Note first that the optimal mechanism is no longer a function of the type elasticity of the agent, θ_1. In particular, whatever the information content of the signal the principal can never attain the first-best solution. The result is not surprising since *ex-post* individual rationality guarantees that the expected rent of the agent will be positive, allowing for the standard trade off between informational rent and efficiency. Alternatively, given $\theta_1 > 0$ and equation (6), we see that in the state $s = s_2$ the first-best mechanism requires the firm to give a transfer which is less than the cost incurred by the firm. Thus, in its present form the limited liability constraint will always be binding for any finite θ_1 and for all c.

The informativeness of the signal remains important. It is captured through the hazard rate. In order to see this, we rewrite the welfare induced by the mechanism

$$W^{\diamond} = \int_{\underline{c}}^{\bar{c}} \{v(q^{\diamond}(c)) - [c + H(c,s_1,\iota)]q^{\diamond}(c)\} f(c)dc \qquad \text{where} \qquad H(c,s_1,\iota) = \frac{F(c|s_1,\iota)}{f(c|s_1,\iota)} \tag{26}$$

Just as in the foregoing section, we interpret $H(c,s_1,\iota)$ as the necessary "bribe" per unit of output required to keep the mechanism incentive compatible. Once again, the hazard rate is decreasing in the informational content of the signal. The remainder of the argument is the same as in §4.1. Finally, since the adjusted hazard rate is always less or equal to the hazard rate, tightening the limited liability constraint induces an increase in the required "bribe" to keep the mechanism incentive compatible and, thus, works exactly as an increase in the per unit cost. This allows for a comparison between the optimal mechanism of §4.1 and §4.2:

Proposition 2: *Given functions that satisfy Theorem 3 and 4, it follows that tightening limited liability rules reduces production and social efficiency.*

§5. Conclusion

In this paper, we have re-examined the design of an optimal mechanism in a principal-agent model when a costless signal correlated with the agent's private information is observable *ex-post*. We defined the institutional environment by introducing two forms of limited liability: non-negativity constraints on the transfer payments to and the profits of the agent. Limited liability eliminated two economically implausible results found in the literature: first, a mechanism that implements the first-best solution with transfer payments which can fluctuate between plus and minus infinity, and second, a mechanism which is discontinuous at the point at which the signal becomes noninformative.

We show that when limited liability is a binding constraint, the principal cannot implement the first-best solution and the agent receives positive rents. If the profits of an agent are constrained to be non-negative, then limited liability is always a binding constraint on contractual design. However, if transfer payments are constrained to be non-negative, then limited liability is a binding constraint only when a signal is insufficiently responsive to type (inelastic). However, as the production rule and profits are continuous in the type elasticity of the signal, the level of inefficiency is less than that which obtains with no signal. When a signal is sufficiently responsive to type (elastic), then the principal can implement the first-best allocation and the value of the agent's private information becomes zero.

There are many possible extensions of this paper. We have found that the information content of the signal has a positive value to the principal. A principal's demand for information content or efficiency in a signal could be examined by introducing costly auditing in the present model. If one maintains the assumption of risk neutrality, then one could introduce an economic definition of the precision of auditing. An auditing procedure would be said to have a higher economic precision if it yields a higher level of informativeness. This is a very different approach from that found in Baron and Besanko (1984). The choice variable in Baron and Besanko is the probability of auditing as opposed to the precision of auditing.

A further extension is introducing risk averse players. This would yield a statistical measure of informativeness. This seems to be a more natural environment in which to study the auditing problem. Alternatively, one could introduce an auditor as a third player. Garvie (1989) examines such a case and assumes that the auditor is risk averse in a Bayesian statistical decision sense. She finds that the first-best solution is implementable only over a well-defined region of the principal's prior beliefs. The optimal auditing region depends upon the accuracy of the auditing technology, the prior beliefs and the loss function of the auditor.

Appendix

Proof of Theorem 2: *If the type of the agent is c and his report is r, then denote the profit by $\pi(r,c)$. We must show that $\pi(c,c) - \pi(r,c) \geq 0$ to prove that the mechanism is globally incentive compatible. Since the mechanism is individually rational by construction, that is, $\pi(c,c) = 0$ $\forall$ c, we need only show that*

$$\pi(r,c) = t_1^{**}(r)p_1(c) + t_2^{**}(r)p_2(c) - cq^{**}(r) \leq 0 \qquad \textbf{(A1)}$$

Substituting t_1^{**} and t_2^{**}, as defined by equation (5) in the text, into equation (A1) we obtain

$$\pi(r,c) = q^{**}(r)\left[\frac{p_1(c) - p_1(r)}{\dot{p}_1(r)} - (c - r)\right] \qquad \textbf{(A2)}$$

$$\textit{Suppose } r < c, \textit{ then } \pi(r,c) = q^{**}(r)\int_r^c \left[\frac{\dot{p}_1(z)}{\dot{p}_1(r)} - 1\right] dz \leq 0 \qquad \textbf{(A3)}$$

The conditional probability ratio is less than 1 since $p_1(c)$ is increasing and concave in c. Analogous reasoning shows that the inequality also holds when $r > c$.

Q.E.D.

Proof of Theorem 3: *(i) We adopt the first order approach for solving the principal's mechanism design problem. We replace the truth telling requirement by its first order condition, thus, initially ignoring the second-order requirement. After deriving the solution we will check that the mechanism is indeed globally incentive compatible. We use constrained calculus of* *variation (for a reference see Elsgolds (1977)).*[26] *The optimal control problem can be written as follows:*

$$\max_{q,t_1,t_2} \int_{\underline{c}}^{\bar{c}} (v(q)-t_1p_1-t_2p_2)f(c)+\phi(t_1p_1+t_2p_2-cq)+\gamma_1t_1+\gamma_2t_2+\mu(\dot{t}_1p_1+\dot{t}_2p_2-c\dot{q})\,dc$$

where γ_1, γ_2, ϕ *and* μ *are multipliers. The Euler conditions of the problem are*

$$\dot{v}(q)f(c) - \phi c = \frac{d}{dc}[-\mu(c)c] \tag{A4}$$

$$-p_1(c)f(c) + \phi p_1(c) + \gamma_1(c) = \frac{d}{dc}[\mu(c)p_1(c)] \tag{A5}$$

$$-p_2(c)f(c) + \phi p_2(c) + \gamma_2(c) = \frac{d}{dc}[\mu(c)p_2(c)] \tag{A6}$$

(i) $\phi = 0$ *almost everywhere. Assume not, suppose* $\exists$ *an interval I with* $\pi(c) = 0 \;\forall\; c \in I$. *Then* $\dot{\pi}(c) = 0 \;\forall\; c \in I$. *Solving for* t_1 *and* t_2 *results in a contradiction of the nonnegativity requirement. In particular, we have that* $t_2^{***}(c) = cq^{***}(c)[1 - \theta_1^{-1}] < 0$.

(iii) Equations (10) and (11) cannot be slack simultaneously. Assume not, that $\gamma_1 = \gamma_2 = 0$. *Adding equations (A5) and (A6) gives*

---

[26] *Alternatively, we could use a standard optimal control approach applying the Hamiltonian principle. In order to do this, one introduces the expected rent for an agent of type c,* $\pi(c)$, *and substitute* $q(c)$. *This yields the following optimal control problem:*

$$\max_{\{t_1,t_2,\pi\}} \int_{\underline{c}}^{\bar{c}} \left[v\left(\frac{1}{c}(t_1p_1 + t_2p_2 - \pi)\right) - t_1p_1 - t_2p_2 \right] f(c)\,dc$$

subject to

$$\dot{\pi} = t_1\dot{p}_1 + t_2\dot{p}_2 - \frac{1}{c}(t_1p_1 + t_2p_2 - \pi)$$

$$\pi \geq 0$$

$$t_i \geq 0, i = 1,2$$

$$-f(c) = \dot{\mu}(c)$$
$$\Rightarrow \mu(c) = \tilde{C} - F(c) \tag{A7}$$

Substituting μ back into (A5) leads to an immediate contradiction since $p_1(c)$ is strictly increasing by assumption.

(iv) The limited liability constraints (10) and (11) in the text cannot bind simultaneously. Indeed, setting $\gamma_1, \gamma_2 > 0 \Rightarrow t_1 = t_2 = 0$. This in turn implies, by equation (12) in the text, that $q = 0$. This cannot be a solution since we assume $\dot{v}(0) = +\infty$. That is, either $\gamma_1 = 0$ and $\gamma_2 > 0$ or $\gamma_2 = 0$ and $\gamma_1 > 0$. It is not possible that $\gamma_2 = 0$ because setting $\gamma_2 = 0$ yields $\gamma_1 < 0$.

(v) Substituting $\phi = \gamma_1 = 0$ into equations (A4) - (A6), we can solve the differential equation (A5) to yield the multiplier μ[27]

$$\mu(c) = \int_{\underline{c}}^{c} -\frac{p_1(z)}{p_1(c)} f(z)dz \tag{A8}$$

(vi) We can now implicitly solve for output. Substituting the derivative of μ we obtain

$$\dot{v}(q(c))f(c) = -\left[-f(c) - \frac{\dot{p}_1(c)}{p_1(c)}\mu(c) \right] c - \mu(c) \tag{A9}$$

where the square bracket is simply $\dot{\mu}$. Rearranging the terms yields

$$\dot{v}(q^{***}) - c = [1 - \theta_1(c)] \frac{\int_{\underline{c}}^{c} \frac{p_1(z)}{p_1(c)} f(z)\, dz}{f(c)} \tag{A10}$$

Applying Bayes rule yields equation (13) in the text.

(vii) From equation (A5) we have that $\gamma_2(c) > 0$, which implies $t_2(c) = 0$. This result simplifies equation (12) in the text, allowing us to solve for the transfer function

---

[27] *The constant of integration is zero. To see this consider a variation in t_1. t_1 is unconstrained at $\underline{c}$. A standard result in calculus of variations states that $\mu(\underline{c}) = 0$. At $\bar{c}$ the transfer will be constrained to satisfy $t_1(\bar{c})p_1(\bar{c}) = \bar{c}q(\bar{c})$ which explains why there is no restriction on $\mu(\bar{c})$.*

$$\dot{t}_1 p_1 - c\dot{q} = 0 \tag{A11}$$

$$\Rightarrow t_1^{***}(c) = -\int_c^{\bar{c}} \frac{z\dot{q}^{***}(z)}{p_1(z)} dz + \hat{C} \tag{A12}$$

where $\hat{C}$ is the constant of integration. In order to solve for the constant, denote the expected profit of a type c agent as $\pi(c)$. That is,

$$\pi(c) = p_1(c)t_1(c) - cq(c) \tag{A13}$$

$\pi(c)$ is increasing in $\hat{C}$, for all c. We know that the principal will set the constant as small as possible since the profit of the producer enters negatively into the objective function of the principal. The requirement that the expected profit be non-negative at $\bar{c}$ implies that $\hat{C} \geq \bar{c}q(\bar{c})/p_1(\bar{c})$. Assume we set

$$\hat{C} = \frac{\bar{c}q(\bar{c})}{p_1(\bar{c})} \tag{A14}$$

We now show that $\pi(c) \geq 0$, proving that $\hat{C}$ must indeed be defined by equation (A14)

$$\pi^{***}(c) = p_1(c)\left[\int_c^{\bar{c}} -\frac{z\dot{q}^{***}(z)}{p_1(z)} dz + \frac{\bar{c}q^{***}(\bar{c})}{p_1(\bar{c})}\right] - cq^{***}(c)$$

$$= p_1(c)\int_c^{\bar{c}} \frac{d}{dz}\left[\frac{z}{p_1(z)}\right] q^{***}(z)dz \tag{A15}$$

$$= \int_c^{\bar{c}} [1-\theta_1(z)]\frac{p_1(c)}{p_1(z)} q^{***}(z)dz \geq 0$$

Finally, to conclude the proof we need to show that the contract offered by the principal is globally incentive compatible. The contract is globally incentive compatible if and only if $\pi(c,c) - \pi(r,c) \geq 0$.

$$\pi(c,c) - \pi(r,c) = [t_1^{***}(c) - t_1^{***}(r)]p_1(c) - c[q^{***}(c) - q^{***}(r)]$$

$$= p_1(c)\int_r^c \frac{z\dot{q}^{***}(z)}{p_1(z)}dz - c\int_r^c \dot{q}^{***}(z)dz \tag{A16}$$

$$= -\int_r^c \dot{q}^{***}(z)\left[c - z\frac{p_1(c)}{p_1(z)}\right]dz$$

Suppose that $c > r$. A sufficient condition for the difference to be negative is that the bracket term in the integral be positive, given that production is decreasing. A sufficient condition for this is that $c/p_1(c)$ be non-decreasing in c. It is easy to check that the monotonicity of $p_1(c)$ guarantees this. Analogous reasoning holds to show that the inequality holds when $c < r$.

Q.E.D.

Proof of Proposition 1: *We define a higher information content of the signal by $\theta_{1\iota}(c,\iota) > 0$. Following the notation in the text, we now show that, according to the partial ordering induced by the relation "more informative than", the adjusted hazard rate $H'(c,s_1,\iota)$ is decreasing in the information content. First, note:*

$$\theta_{1\iota}(c) = \frac{c}{p_1^2(c)}[p_{1c\iota}(c)p_1(c) - p_{1c}(c)p_{1\iota}(c)] \geq 0 \tag{A17}$$

Thus a signal can only have a higher information content if the square bracket is positive. This is equivalent to requiring that the ratio $p_{1\iota}(c)/p_1(c)$ be everywhere increasing in c. Therefore:

$$\frac{dH'}{d\iota}(c,s_1,\iota) = [1-\theta]\int_{\underline{c}}^{\bar{c}} \frac{p_{1\iota}(z)p_1(c) - p_1(z)p_{1\iota}(c)}{[p_1(c)]^2}\frac{f(z)}{f(c)}dz$$

$$- \theta_{1\iota}\int_{\underline{c}}^{\bar{c}} \frac{p_1(z)f(z)}{p_1(c)f(c)}dz \geq 0 \tag{A18}$$

The remainder of the proof follows immediately. First, applying the implicit function theorem on equation (15) with respect to ι yields the effect of a change in the information content on the production rule. Second, totally differentiating equation (19) with respect to ι yields the derivative of welfare with respect to the information content of the signal.

Q.E.D.

Proof of Theorem 4: *(i) Define* $\pi_i(c) = t_i(c) - cq(c)$ *so that* $\dot{t}_i(c) = \dot{\pi}_i(c) + q(c) + c\dot{q}(c)$. *Thus, the first-order condition of the agent's reporting problem becomes*

$$\dot{\pi}_1 p_1 + \dot{\pi}_2 p_2 + q = 0 \qquad \textbf{(A19)}$$

The regulator's problem can now be written as

$$\underset{q,\,\pi_1,\,\pi_2}{\underline{Max}} \int_{\underline{c}}^{\bar{c}} \underline{(v(q) - \pi_1 p_1 - \pi_2 p_2 - cq) f(c) + \gamma_1 \pi_1 + \gamma_2 \pi_2 + \mu(\dot{\pi}_1 p_1 + \dot{\pi}_2 p_2 + q) dc}$$

where γ_1, γ_2 *and* μ *are multipliers. The Euler conditions of the problem are*

$$\left(\dot{v}(q) - c\right) f(c) + \mu = 0 \qquad \textbf{(A20)}$$

$$-p_1(c) f(c) + \gamma_1(c) = \frac{d}{dc}[\mu(c) p_1(c)] \qquad \textbf{(A21)}$$

$$-p_2(c) f(c) + \gamma_2(c) = \frac{d}{dc}[\mu(c) p_2(c)] \qquad \textbf{(A22)}$$

(ii) $\mu(c) \neq 0$ *for any interval I where production is non-zero. Assume* $\mu(c) = 0 \; \forall \; c \in I$. *This implies that* $d/dc[\mu p_1] = 0 \Rightarrow \gamma_1, \gamma_2 \neq 0 \Rightarrow \pi_1 = \pi_2 = 0 \Rightarrow \dot{\pi}_i = 0 \Rightarrow q = 0$. *For the remainder of the proof we assume positive production which yields* $\pi_i > 0$.

(iii) The rest of the proof follows the proof outlined in Theorem 3.

Q.E.D.

References

Baron, D.P. and D. Besanko. (1984) "Regulation, asymmetric information, and auditing". Rand Journal of Economics. Vol.15, No.4, p.447-470.

Baron, D.P. and Myerson, R.B. (1982) "Regulating a Monopolist with Unknown Costs". Econometrica Vol.50, No.4, p.911-930.

Cremer, J. and R. McLean. (1988) "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions". Econometrica. Vol.56, No.6, p.1247 -1257.

________________. (1985) 'Optimal Selling Strategies under Uncertainty for aDiscriminating

Monopolist When Demands are Interdependent". Econometrica. Vol.53, No.2, p.345-361.

Demougin, D.M. (1988) "An *Ex-post* Efficient Mechanism for a Principal-Agent Model with Asymmetric Information". Mimeo. University of Toronto.

Demski, J. and D. Sappington. (1984) "Optimal Incentive Contracts with Multiple Agents". Journal of Economic Theory. Vol.33, p.152-171.

Demski, J., D. Sappington and P. Spiller. (1988) "Incentive Schemes with Multiple Agents and Bankruptcy Constraints". Journal of Economic Theory. Vol.44, p.156-167.

Garvie, D. (1989) "A Bayesian Auditing Strategy for Regulation". Mimeo. University of California - Berkeley.

Hart, O.D. (1983) "Optimal Labour Contracts Under Asymmetric Information: An Introduction". Review of Economic Studies. Vol.50, No.160, p.3-35.

Maskin, E. and J. Riley. (1984) "Monopoly with Incomplete Information". The Rand Journal of Economics. Vol.15, No.2, p.171-196.

McAfee, P.R. and J. McMillan. (1987) "Auctions and Bidding". Journal of Economic Literature. Vol.25, No.2, p.699-738.

McAfee, P.R., J. McMillan and P.J. Reny. (1988) "Extracting Surplus in Common Value Auctions". Mimeo. University of Western Ontario.

McAfee, P.R. and P.J. Reny. (1989) "Correlated Information and Mechanism Design". Mimeo. University of Western Ontario.

Sappington, D. (1983) "Limited Liability Contracts Between Principal and Agent". Journal of Economic Theory. Vol.29, p.1-21.

Riordan, M.H. and D. Sappington. (1988) "Optimal Contracts with Public *Ex-post* Information". Journal of Economic Theory. Vol.45, No.1, p.189-199.