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# Public Firms as Regulatory and Auditing Instruments

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PUBLIC FIRMS AS REGULATORY AND  
AUDITING INSTRUMENTS

by

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**Abstract:** We re-examine the regulatory role of a public firm in an environment of private but correlated information about industry costs. We study three regimes of mixed market interaction involving both public and private firms: a symmetric Bayesian-Nash equilibrium, an asymmetric Bayesian equilibrium in which the public firm is able to commit to production before the private firms and a mechanism in which the regulator designs an incentive compatible schedule for the industry. We find that a public firm plays an important strategic informational role which strengthens its role as a disciplinary regulatory instrument. Further, we find that this strategic informational role is considerably enhanced as we move from indirect regulatory schemes to direct regulation.

## §1. INTRODUCTION

This paper addresses the problem of regulating an oligopolistic industry with private but correlated information about the production environment. In such an environment there are two sources of market failure: *imperfect competition* and *incomplete information*. Our purpose in this paper is to explore the implications of incomplete information for the design of regulatory mechanisms for a market structure involving public and private firms, that is, *mixed market equilibria*. In particular, we ask if a public firm can play two corrective roles: a *disciplinary role* as a competitive strategic supplier and an *auditing role* by generating information about industry technology enabling more effective regulatory control. Further, we investigate the relative importance of these two roles in different institutional environments. We examine three models involving increased commitment power and sophistication of the public firm.

A re-examination of mixed enterprise markets is both appropriate and timely given the recent theoretical advances which have generated a renewed interest in alternative industrial organizational forms. There has long been interest in the role of state or public enterprise in indirectly regulating oligopolistic industries. Much of the writing on this subject has been informal, and has stemmed from the work of European Socialist thinkers.<sup>1</sup> A small body of analytical literature also exists, most notably the contribution of Harris and Wiens (1980), and more recently, Cremer, Marchand and Thisse (1989), and De Fraja and Delbono (1989).<sup>2</sup>

The regulatory role of a public firm producing in an oligopolistic industry has, hitherto, been largely explored in the context of complete information. Harris-Wiens showed that if the

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<sup>1</sup>See, for example, Holland, Stuart, *Beyond Capitalist Planning*, Oxford, Blackwell, 1978.

<sup>2</sup>See also Beato and Mas-Colell (1984), Bös (1981), Ware and Winter (1986) and Ware (1986).

public firm could make a credible announcement of a reaction function to the output choices of the private oligopolists, the industry equilibrium could be restored to a first-best allocation. They also briefly consider a dynamic adjustment model in which private firms costs are not initially known to the regulator. More recent papers, still in the complete information context, have focussed on the credibility of a public firm's strategy, and the possible inefficiency effects associated with entry of a public firm. Pint (1991), in a study which complements this paper, analyses different regulatory schemes for monopoly in an environment of incomplete information about costs.

Spurred on by changing political fashions, much of the recent interest in public firms (and diverse services hitherto thought to be in the realm of public production) has focussed on *privatization* (Shapiro (1989), Vickers and Yarrow (1988), Bös and Peters (1988)). It is probably fair to say that the interest in privatization stems largely from a belief that managers of public firms pursue objectives other than cost minimization or profit maximization. Absent is the discipline imposed by a competitive market structure which results in the static and dynamic cost advantages of private production. Although most of this work focusses on Natural Monopoly industries, Demski, Sappington and Spiller (1987) have studied the value of auditing and second source production when private entry into an industry hitherto supplied by a regulated monopoly is possible.

Our focus is on the disciplinary and auditing effects of mixed enterprise in industries where a private firm may indeed have a cost advantage. We extend the Harris-Wiens framework to allow for incomplete information about industry costs. However, although the cost levels of individual firms are unknown to the regulatory authorities, the distribution of those costs may

be known. Further, costs are likely to be *correlated* across firms in an industry, given common technologies and the use of common inputs. As a result, an observation on the costs of a public firm provides a signal of industry costs. In a sense, this is equivalent to an auditing function of the public firm. In this paper, we show how this signal may be incorporated into the design of a regulatory strategy for the public firm. By incorporating the informational role as an intrinsic part of the strategy of the public firm, the power of this regulatory strategy is much enhanced.

The remainder of the paper is organized as follows. The next section describes the market environment and stochastic assumptions, which form the common basis for all the analytical models. We examine models from the linear-quadratic class of models in Section 3 in order to investigate the effects of different firm objectives and strategic structures. In particular, we derive the Bayesian Nash equilibria for a private duopoly as well as mixed markets of public and private firms with symmetric and asymmetric strategic structures. We construct a revelation game in section 4 in which the public firm is given access to an even more sophisticated strategy: the ability to commit to a reaction function to the private firm's choice of quantity. This last model is closely analogous to the Harris and Wiens approach, except that incomplete information about costs is explicitly built into the model and the solution concept. A simulation is performed in Section 5 to facilitate a comparison of models presented in Sections 3 and 4. Section 6 draws some conclusions.

## **§2. MARKET STRUCTURE AND STOCHASTIC ENVIRONMENT**

An exogenously given number of firms supply a market for a homogeneous good  $q$ . On

grounds of simplicity, we restrict consideration to two firms. The following assumptions are made about demand, costs and the stochastic environment.

**Assumption 1:** Demand is linear and is given by

$$P = \alpha - \beta Q, \text{ where } Q = \sum_{i=0}^1 q_i \quad (1)$$

**Assumption 2:** Each firm's costs are linear and are given by

$$C_i = c_i q_i, \quad \forall i = 0, 1 \quad (2)$$

Firm marginal costs are private information to the firm. Both firms, however, know the joint distribution function of marginal costs.

**Assumption 3:** Marginal costs are distributed asymmetrically but are correlated. The joint distribution function is  $F(c_0, c_1; \mu_0, \mu_1, \sigma, \rho)$  with an associated probability density function  $f(c_0, c_1; \mu_0, \mu_1, \sigma, \rho)$ . The mean of the cost distribution for firm 0 is above that of firm 1 by an amount  $\mu_\Delta$ . The magnitude of  $\mu_\Delta$  is of the order of at least one standard deviation of costs.

Following standard notation,  $\mu_i$  and  $\sigma$  are the unconditional mean and standard deviation of marginal costs for firm  $i$  while  $\rho$  measures the degree of cost correlation in the bivariate distribution. We further restrict  $\sigma=1$  and  $\rho$  to be contained in the interval  $[0,1]$ . We assume costs are asymmetrically distributed for two reasons. First, we often observe firms in private oligopolies being taken into public ownership precisely because they have an *intrinsic cost*



*inefficiency* and are losing money. As a positive exercise, our study can be seen as an investigation of the consequences of such a decision.<sup>3</sup> Second, we require a mean cost differential to ensure the existence of equilibrium.

When choosing a production strategy, each firm forms a prior distribution over the other firm's costs  $f(c_j | c_i, \mu_0, \mu_1, \sigma, \rho)$ , hereafter denoted  $f(c_j | c_i)$ , which is conditioned on the observation of its own costs.

**Assumption 4:** First moments of the conditional distribution possess the following linear structure<sup>4</sup>

$$E(c_j | c_i) = \mu_j + \rho(c_i - \mu_i), \quad \forall i \neq j, i, j = 0, 1 \quad (3)$$

**Assumption 5:** The support of the bivariate distribution  $f(c_0, c_1)$  is bounded within the rectangle  $[\underline{c}, \bar{c}] \times [\underline{c}, \bar{c}]$ .<sup>5</sup>

### §3. BAYESIAN-NASH MODELS

We begin our investigation in this section by deriving the private equilibrium for an asymmetric duopoly with private but correlated information about costs. We impose symmetry

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<sup>3</sup>Moreover, in existing mixed markets, public sector unions are often successful in forcing up wages, relative to comparable private firms, creating a cost differential. Cremer et al justify a similar assumption of asymmetric costs on the grounds that higher wages in the public sector take the form of *pure rents*, which have no effect on the marginal opportunity cost of resources, only on the budget constraint. But the distinction is a dubious one, since there is a real opportunity cost of raising tax revenue, and this can be considered part of the firms costs.

<sup>4</sup>The bivariate normal, with  $\sigma_1 = \sigma_2$ , satisfies equation (3).

<sup>5</sup>Strictly, the assumption of a finite bounded support for  $c_i$ ,  $i=0,1$  makes it impossible to satisfy expression (3), linear conditional expectations, exactly. But provided that the bounds only take a small probability mass from the joint density, the conditional expectation will still satisfy equation (3) to a very close approximation.

on the strategic structure of the private duopoly game. The natural equilibrium concept for this kind of environment is that of Bayesian Nash equilibrium. Each firm chooses an action contingent on its type (costs). In equilibrium these type contingent strategies must be best responses to rival strategies. We next examine how the equilibrium strategies are affected when the relatively inefficient firm is taken into public ownership, holding the institutional structure of the game constant. We then relax the assumption of strategic symmetry in the last model of this section by giving the public firm the power to commit to a strategy.

### §3.1. THE PRIVATE EQUILIBRIUM

As a benchmark for our study of mixed market equilibria, we begin by deriving the Bayesian Nash equilibrium strategies for the private duopoly case.<sup>6</sup> Each firm chooses its own quantity of production, contingent on its own costs, so as to maximize its expected profits. Thus, firm  $i$  solves the following optimization problem

$$\max_{q_i} E\pi_i = \int_{\underline{c}}^{\bar{c}} [P(q_i + q_j(c_j))q_i - c_i q_i] f(c_j | c_i) dc_j \quad (4)$$

Substituting in demand from equation (1) and differentiating equation (4) for  $i = 0, 1$  yields the following first order condition

$$\alpha - 2\beta q_i - \beta E(q_j(c_j) | c_i) - c_i = 0 \quad (5)$$

for firm 0 and firm 1, respectively.

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<sup>6</sup>These results are reported, albeit in a slightly different form, in Shapiro (1986).

It is known that there is a unique Bayesian equilibrium to this problem, and that it involves linear strategies, that is, strategies of the form  $q_i = a_i - b_i c_i$ , where  $a_i$  and  $b_i$  are positive constants.<sup>7</sup> To solve for  $a_i$  and  $b_i$ , take expectations through each firm's decision rule to yield

$$E(q_i | c_j) = a_i - b_i E(c_i | c_j) \quad (6)$$

Combining (6) and (3) with (5) for  $i = 0, 1$ , we obtain two linear equations as functions of  $a_0$ ,  $b_0$ ,  $a_1$  and  $b_1$ , and the equilibrium coefficients can be evaluated as

$$a_i = \frac{1}{3} \left[ \frac{\alpha}{\beta} + \frac{1}{\beta} \mu_j - \frac{2(1 - \rho^2) + 3\rho}{\beta(4 - \rho^2)} \mu_i \right] \quad i = 0, 1, i \neq j \quad (7)$$

$$b = \frac{1}{\beta(2 + \rho)} \quad i = 0, 1 \quad (8)$$

We should note that the equilibrium strategies are valid only if they yield positive quantity choices over the support of the joint distribution. These constraints, which we call shutdown constraints, are an important restriction and are discussed further in Appendix 1. In order to meet the no shutdown condition, the cost distributions for both firms must be bounded. The necessary bounds are given in the appendix.

There are two sources of market failure in the above described environment: *imperfect competition* and *incomplete information*. While the former results in a contraction of industry output, the latter affects output variability and the efficiency of production allocation across

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<sup>7</sup>See Basar and Ho (1974), and Basar (1978).

firms. To see how these separate phenomena affect producers and consumers, we decompose expected welfare into terms which are a function of average production quantities and terms which are a function only of the degree of cost correlation.

Industry profits can be written as  $\pi = (\alpha - \beta Q)Q - \sum_i c_i q_i$ . Taking the expectations operator through this expression and rearranging terms we have that

$$E(\pi) = (\alpha - \beta \bar{Q} - \bar{\mu})\bar{Q} + \sum_{i=0}^1 (\bar{\mu} - \mu_i)\bar{q}_i - \beta E[(Q - \bar{Q})^2] - E \left[ \sum_{i=0}^1 (q_i - \bar{q}_i)(c_i - \mu_i) \right] \quad (9)$$

where  $\bar{q}_i$  is average production for firm  $i$ , average industry production is  $\bar{Q} = \sum_i \bar{q}_i / 2$  and average industry mean marginal costs is  $\bar{\mu} = \sum_i \mu_i / 2$ . The first two terms in equation (9) represent the level of average industry profits without cost uncertainty, that is, when both firms produce at their mean cost level. The first term represents profits at average mean industry costs while the second term is the effect of the reallocation of production toward the least cost producer. The last two terms in equation (9) measure the contribution to expected producer surplus from cost uncertainty. The third term measures the variance of industry output while the final term, the sum of (negative) covariances between each firm's production and costs, again measures the degree of efficiency of production allocation across firms. Producers benefit from a more efficient allocation of production but not from output variability.

With linear demand, consumer surplus is  $CS(Q) = \beta Q^2 / 2$ . Again taking expectations yields

$$E(CS) = \frac{\beta}{2} \bar{Q}^2 + \frac{\beta}{2} \text{var}(Q) \quad (10)$$

Consumers, unlike producers, benefit from output variability.

Adding up expected producer and consumer surplus yields expected welfare

$$E(W) = \left[ \alpha - \frac{\beta}{2} \bar{Q} - \bar{\mu} \right] \bar{Q} + \sum_{i=0}^1 (\bar{\mu} - \mu_i) \bar{q}_i - \frac{\beta}{2} \text{var}(Q) - \sum_{i=0}^1 \text{cov}(q_i, c_i) \quad (11)$$

The above expression indicates that welfare gains are obtainable with output expansion, reduced output variability and a more efficient distribution of production between firms. These components are now evaluated in turn.

Average industry production for the private duopoly can be computed as

$$\bar{Q}^p = \frac{2\alpha - \mu_0 - \mu_1}{3\beta} \quad (12)$$

The corresponding average industry price is  $\bar{P}^p = (\alpha + \mu_0 + \mu_1)/3$ , above the industry mean marginal cost level.

To solve for the cost uncertainty terms, take expectations of the linear production rule  $q_i^p = a_i - b_i c_i$  to yield  $\bar{q}_i^p = a_i - b_i \mu_i$ .<sup>8</sup> Rewrite the production rule as  $q_i^p = \bar{q}_i - b_i(c_i - \mu_i)$ . Adding up across firms gives  $Q^p = \bar{Q} - \sum_i b_i(c_i - \mu_i)$ . We can now derive the variance of output as

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<sup>8</sup>Expected market shares can easily be derived, showing that, on average, the low cost firm has the larger market share. See Shapiro (1986).

$$\text{var}(Q^p) = 2b^2(1 + \rho) \quad (13)$$

and the sum of covariances between firm costs and production as

$$\sum_{i=0}^1 \text{cov}(q_i^p, c_i) = - E \left[ b \sum_{i=0}^1 (c_i - \mu_i)^2 \right] = - 2b \quad (14)$$

Note that the net contribution to expected welfare from cost correlation is positive given that  $\rho \in [0,1]$ . However, as  $\rho$  increases, the ability to exploit cost differences diminishes.

Two major motivations can now be distinguished for investigating the merits of an industrial policy which employs a public firm as a regulatory instrument in a mixed market. First, a public firm might play a *disciplinary role*, that is, improve the efficiency of a market with imperfect competition. Given that oligopolistic firms tend to restrict output, one would expect that the presence of a public firm motivated by social rather than private efficiency would result in an expansion of average industry output. The first two terms of expression (12) measure expected welfare associated with this disciplinary role. The final two terms of expression (12) measure the *auditing effect*; that is, the effect of cost correlation, which can provide informational input into more effective regulatory control. In many industries, the complexity of the production technology means that regulators are at a severe informational disadvantage relative to producers, which may impair the efficiency of regulatory control. Hence, a public firm could provide a "*window on the industry*" for regulators.<sup>9</sup> We are primarily interested in whether a public firm can manipulate information garnered through

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<sup>9</sup>One example of this rationale in practice is the Canadian case of Petro Canada: when the government purchased a number of private firms to form Petro Canada, one of its major justifications was to learn more about the oil and gas industry for industrial policy purposes.

production in a welfare-enhancing way.

### §3.2. THE PUBLIC FIRM AS REGULATORY AND AUDITING INSTRUMENT I:

#### Public/Private Bayesian Nash Equilibrium

In this section we investigate the consequences on the industry equilibrium of taking one of the private duopolists, firm 0, into public ownership. We hold the strategic structure of the game constant in order to examine the effects of different objectives of public and private firms. The public firm's objective is to choose its own quantity, contingent on its own costs, so as to maximize expected industry welfare, defined as the sum of industry consumer plus producer surplus. Thus, the public firm solves the following optimization problem

$$\max_{q_0} E(W) = \int_{\underline{c}}^{\bar{c}} \left[ \int_0^{q_0} P(q_1(c_1) + t) dt - c_0 q_0 - c_1 q_1 \right] f(c_1 | c_0) dc_1 \quad (15)$$

The private firm, firm 1, chooses its quantity  $q_1$  so as to maximize expected profits.

$$\max_{q_1} E(\pi_1) = \int_{\underline{c}}^{\bar{c}} [P(q_0(c_0) + q_1) q_1 - c_1 q_1] f(c_0 | c_1) dc_0 \quad (16)$$

Substituting in demand from (1) and differentiating equations (15) and (16), we obtain the first order conditions,

$$\alpha - \beta q_0 - \beta E(q_1(c_1 | c_0)) - c_0 = 0 \quad (17)$$

$$\alpha - 2\beta q_1 - \beta E(q_0(c_0 | c_1)) - c_1 = 0 \quad (18)$$

for the public and private firm, respectively. As discussed above, by combining equations (6) and (3) with equations (17) and (18), respectively, we obtain two linear equations as functions of  $a_0$ ,  $b_0$ ,  $a_1$ , and  $b_1$ , and the equilibrium coefficients can be evaluated as

$$a_0 = \frac{\alpha}{\beta} + \frac{1}{\beta} \mu_1 - \frac{\rho + 2(1 - \rho^2)}{\beta(2 - \rho^2)} \mu_0 \quad (19)$$

$$b_0 = \frac{2 - \rho}{\beta(2 - \rho^2)} \quad (20)$$

$$a_1 = \frac{1}{\beta} \mu_0 - \frac{\rho + (1 - \rho^2)}{\beta(2 - \rho^2)} \mu_1 \quad (21)$$

$$b_1 = \frac{1 - \rho}{\beta(2 - \rho^2)} \quad (22)$$

Note that the equilibrium strategies must meet the shutdown conditions presented in Appendix 1.

Using the same procedures as used in section 3.1, we can derive the following characteristics of the equilibrium:

$$\bar{Q}^s = \frac{\alpha - \mu_0}{\beta} \quad (23)$$



$$\bar{P}^s = \mu_0 \quad (24)$$

$$\bar{s}_0^s = \frac{\alpha + \mu_1 - 2\mu_0}{\alpha - \mu_0} \quad (25)$$

$$\bar{s}_1^s = \frac{\mu_0 - \mu_1}{\alpha - \mu_0} \quad (26)$$

$$\text{var}(Q^s) = b_0^2 + b_1^2 + 2b_0b_1\rho \quad (27)$$

$$\sum_{i=0}^1 \text{cov}(q_i^s, c_i) = - (b_0 + b_1) \quad (28)$$

Two observations follow immediately from the properties of the symmetric public/private duopoly equilibrium reported above. First, since the public firm is concerned to maximize industry surplus, it adopts an aggressive pricing policy, driving the industry price down, on average, to the level of its own costs. The result is directly analogous to the case of full information (where the private firm has lower costs), where it can be shown that price always equals the public firm's marginal costs. Second, provided that demand is sufficiently strong or the mean cost differential between firms is not too large, the public firm will produce a larger share of industry output on average.<sup>10</sup>

We are now in a position to assess the contribution of a public firm through a comparison of the private duopoly and the symmetric public/private duopoly. The salient features of this comparison are summarized as Proposition 1.

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<sup>10</sup>Strictly, the public firm will produce a larger share of industry output if and only if the following inequality holds:

$$\alpha - \mu_0 > 2(\mu_0 - \mu_1)$$

**Proposition 1:** *Provided that Assumptions 1-5 hold and the strategic structure of the game is symmetric, we have that*

- (i) *A positive disciplinary role for the public firm will occur only if demand is sufficiently large or the mean cost differential between public and private firms is sufficiently small.*
- (ii) *A public firm can play a positive auditing role.*
- (iii) *Taking a relatively inefficient private firm into public ownership in a duopolistic market will raise ex ante expected welfare provided that the disciplinary effect is not too negative. A negative disciplinary effect can occur if the conditions outlined in (i) above are not met.*
- (iv) *The distribution of ex ante expected welfare gains from public production is not symmetric. Consumers always gain from the creation of the public firm, whereas producers do not.*

**Proof:** See Appendix 2.

Recall that the disciplinary role of a public firm captures effects not due to asymmetric information. Part (i) of the Proposition reflects the fact that the public firm captures a larger share of industry output, compared to a private firm with the same costs. Given that the public firm has higher costs on average, too much production may be allocated to the relatively inefficient public firm.<sup>11</sup> Part (ii) shows that the ability of the public firm to use its own cost information as a signal of private firm costs is valuable. Although the cost signal enables the public firm to make its production strategy more responsive to costs, another effect is to increase

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<sup>11</sup> De Fraja and Delbono (1989) obtain a similar result with a model of symmetric, increasing marginal costs.

output variability. The latter effect works in the opposite direction from the former, resulting in a small overall auditing effect. Part (iii) reflects the same phenomenon as part (i) and incorporates the informational effects of the two equilibria as well. A larger output share for the public firm causes average industry output to be higher and average price lower, which explains the first part of (iv). Producers, however, may suffer because the misallocation of production towards the high cost public firm lowers industry profits.

### §3.3. THE PUBLIC FIRM AS REGULATORY AND AUDITING INSTRUMENT II:

#### The public firm as Stackelberg leader

Instead of announcing quantities in a symmetric Nash game, in this section we give the public firm a *strategic advantage* by allowing it to commit to a quantity after observing its own costs. The private firm then chooses its own quantity. Because of the sequential nature of the game, the private firm is able to infer the public firm's costs precisely from observing its quantity decision. Only the public firm faces a problem of statistical inference: that of using its own cost information to condition the expectation of the private firm's costs.

The private firm now chooses  $q_1$  to maximize actual, not expected profits.

$$\max_{q_1} \pi_1 = P(q_0 + q_1) q_1 - c_1 q_1 \quad (29)$$

Substituting for demand from equation (1) and differentiating, we obtain a standard Cournot reaction function for firm 1.

$$q_1 = \frac{\alpha - c_1 - \beta q_0}{2\beta} \quad (30)$$

The public firm's problem, as in section 3.2, is to maximize expected welfare, which now appears in a slightly different form.

$$\max_{q_0} E(W) = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left[ \int_0^{q_0 + q_1(q_0, c_1)} P(q_0 + q_1(q_0, c_1)) dq_0 - c_0 q_0 - c_1 q_1(q_0, c_1) \right] f(c_1 | c_0) dc_1 \quad (31)$$

Differentiating, we obtain

$$\frac{\partial E(W)}{\partial q_0} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left[ P(q_0 + q_1(q_0, c_1)) \left( 1 + \frac{\partial q_1}{\partial q_0} \right) - c_0 - c_1 \frac{\partial q_1}{\partial q_0} \right] f(c_1 | c_0) dc_1 \quad (32)$$

Substituting into equation (32) from equations (1), (30) and its derivative  $(-1/2)$ , and setting the resulting expression equal to zero, we obtain

$$\frac{\alpha}{4} - \frac{1}{4} \beta q_0 - c_0 + \frac{3}{4} E(c_1 | c_0) = 0 \quad (33)$$

Finally, substitute for  $E(c_1 | c_0)$  from equation (3) to obtain the equilibrium quantity for firm 0.

The equilibrium quantity for firm 1 is then obtained by substituting this value in equation (30).

The equilibrium coefficients can be derived as:

$$a_0 = \frac{\alpha + 3(\mu_1 - \rho \mu_0)}{\beta} \quad (34)$$

$$b_0 = \frac{(4 - 3\rho)}{\beta} \quad (35)$$

$$a_1 = \frac{3(\rho\mu_0 - \mu_1)}{2\beta} \quad (36)$$

$$b_1 = \frac{1}{2\beta} \quad (37)$$

$$d_1 = \frac{4 - 3\rho}{2\beta} \quad (38)$$

Thus, while the equilibrium strategy for firm 0 takes the same form,  $q_0^a = a_0 - b_0c_0$ , the equilibrium strategy for firm 1 now takes the form  $q_1^a = a_1 - b_1c_1 + d_1c_0$ . Firm 1's equilibrium strategy can depend on  $c_0$  as well as  $c_1$ , because  $c_0$  is completely inferred from firm 0's choice of quantity. Note that existence of equilibrium in the Stackelberg model also requires satisfaction of a set of shutdown restrictions, which are set out in appendix 1.

Employing by now familiar methods, we can again derive the following characteristics of the equilibrium:

$$\bar{Q}^a = \frac{\alpha - \mu_0 - (\mu_0 - \mu_1)}{\beta} \quad (39)$$

$$\bar{P}^a = \mu_0 + (\mu_0 - \mu_1) \quad (40)$$

$$\bar{s}_0^a = \frac{\alpha - 4\mu_0 + 3\mu_1}{\alpha - 2\mu_0 + \mu_1} \quad (41)$$

$$\bar{s}_1^a = \frac{2(\mu_0 - \mu_1)}{\alpha - 2\mu_0 + \mu_1} \quad (42)$$

$$\text{var}(Q^a) = (b_0 - d_1)^2 + b_1^2 + 2(b_0 - d_1)b_1\rho \quad (43)$$

$$\sum_{i=0}^1 \text{cov}(q_i^a, c_i) = - (b_0 + b_1 - d_1\rho) \quad (44)$$

Note that the average price in the Stackelberg regime is higher than the public firm's mean marginal cost by an amount  $(\mu_0 - \mu_1)$ . The higher price is a reflection of the Stackelberg public firm's commitment to a lower level of production in order to induce a higher output from the relatively efficient private firm. However, provided that demand is large or the mean cost differential is small, the public firm will produce a larger share of industry output on average.<sup>12</sup>

Next we turn to examine how the regulatory powers of a public firm affect its ability to play a disciplinary and auditing role in a mixed market. To this end, we compare the equilibrium properties of the symmetric and asymmetric regimes with a public firm. Our results are summarized in Proposition 2.

**Proposition 2:** *Provided that Assumptions 1-5 hold, we have that*

- (i) *Average industry output and the average market share of the public firm are smaller in the Stackelberg regime than in the symmetric regime with a public firm.*
- (ii) *The power of commitment enables a public firm to play a more effective disciplinary role. The efficiency losses from a lower average industry output are smaller than the gains*

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<sup>12</sup>The public firm will produce a larger share of industry output if and only if the following inequality holds:

$$\alpha - \mu_0 > 5(\mu_0 - \mu_1)$$

*from a more efficient allocation of production between firms.*

- (iii) The auditing role of a public firm is strengthened by the power of commitment.*
- (iv) Giving a public firm an increased regulatory power of commitment in a duopolistic industry will raise ex ante expected welfare.*
- (v) The distribution of ex ante expected welfare gains depends upon the commitment power of a public firm. Producers are better off in a mixed market with a strong public firm while consumers are worse off.*

**Proof:** See Appendix 2.

The ability to commit gives the public firm greater control over the allocation of resources in the industry, both with respect to known aspects of production and to its use of information. Parts (i) and (ii) of Proposition 2 reflect that a Stackelberg public firm's disciplinary power derives from the leverage to promote higher output from the private firm by committing itself to a lower level of production. Part (iii) illustrates a Stackelberg public firm's ability to manipulate information more efficiently. Strategies are more responsive to cost realizations, resulting in a higher covariance between costs and output. A higher covariance has two effects: a more efficient allocation of output and increased output variability. In the Stackelberg regime, both of these effects as well as the positive net auditing effect are larger than in the symmetric regime. Part (iv) is easy to derive as an analytical result. The equilibrium allocation in the symmetric model is a feasible choice for the public firm in the asymmetric model; hence, ex ante expected welfare in the asymmetric regime must always at least (weakly) dominate that in the symmetric regime. Finally, Part (v) notes that, despite overall welfare gains from a strong public firm, the distribution of these gains between producers

and consumers is uneven.

#### §4. THE PUBLIC FIRM AS REGULATORY AND AUDITING INSTRUMENT III:

In this section, we further increase the scope of regulatory intervention. We allow the regulator to announce a mechanism for the industry, consisting of a rule for allocating production between a public and private firm. The production allocation rule is made contingent on the public firm's known costs and on a cost report by the private firm.

The problem bears some similarities to the classic exposition of Baron and Myerson (1982) on regulating a monopolist with unknown costs. There are three major differences in our approach. First, we consider the regulatory problem for an industry with more than one firm. Second, the possibility of taking one of the firms into public ownership and obtaining a perfect observation on its costs, and a signal of industry costs, is not considered by Baron and Myerson. Third, we restrict the set of instruments available to the regulator by assuming that lump sum transfers or subsidies to the private firm are not feasible. We only consider price-quantity schedules which are sustained through production by the public firm. We believe that this specification is a more realistic description of the policy instruments available to regulators.<sup>13</sup>

After observing the public firm's cost, the regulator forms a conditional distribution on the private firm's costs  $f(c_1 | c_0)$ . A production allocation rule  $\{q_0(c_0), q_1(c_1 | c_0)\}$  is chosen by the regulator to maximize expected net industry surplus. The optimal rule is best interpreted as a

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<sup>13</sup>Our model specification is also the closest incomplete information analogue to the model studied by Harris and Wiens (1980). In Harris and Wiens, the regulator announces a reaction function  $q_0(q_1)$  from which the private firm, acting (in a restricted sense) as a Stackelberg leader, makes a production choice. There is no private information, and the mechanism is able to produce a first best price and output allocation between firms. The restriction to a linear pricing scheme is also adopted by Pint (1991).



*direct revelation mechanism.* The mechanism works as follows: the private firm announces a cost level and receives a corresponding quantity and (implicitly) a price allocation.<sup>14</sup> The equilibrium price is the price which clears the market for the sum of public and private production.

The analysis in this section is greatly simplified by the following assumption:

**Assumption 6:**  $M(\underline{c}) > \bar{c}$ , where  $M(c)$  denotes the monopoly price corresponding to some cost level  $c$ .

The regulator's mechanism design problem can be written as:

$$\max_{q_0(c_1), q_1(c_1)} \int_{\underline{c}}^{\bar{c}} \left[ \int_{P(q_0+q_1)}^{\infty} P^{-1}(z) dz + \pi_0(c_0, c_1) + \pi_1(c_0, c_1) \right] f(c_1 | c_0) dc_1 \quad (45)$$

subject to the following constraints

$$c_1 = \underset{r}{argmax} \pi_1(r; c_0, c_1) \quad (46)$$

$$\pi_1(c_0, c_1) \geq 0 \quad (47)$$

where  $r$  is the private firm's "report" of its cost level. Equation (46) constrains the private firm to truthfully report its cost level to the regulator. Equation (47) constrains the regulator to design a mechanism which ensures the private firm's participation, that is, a non-negative level of ex-post profits.

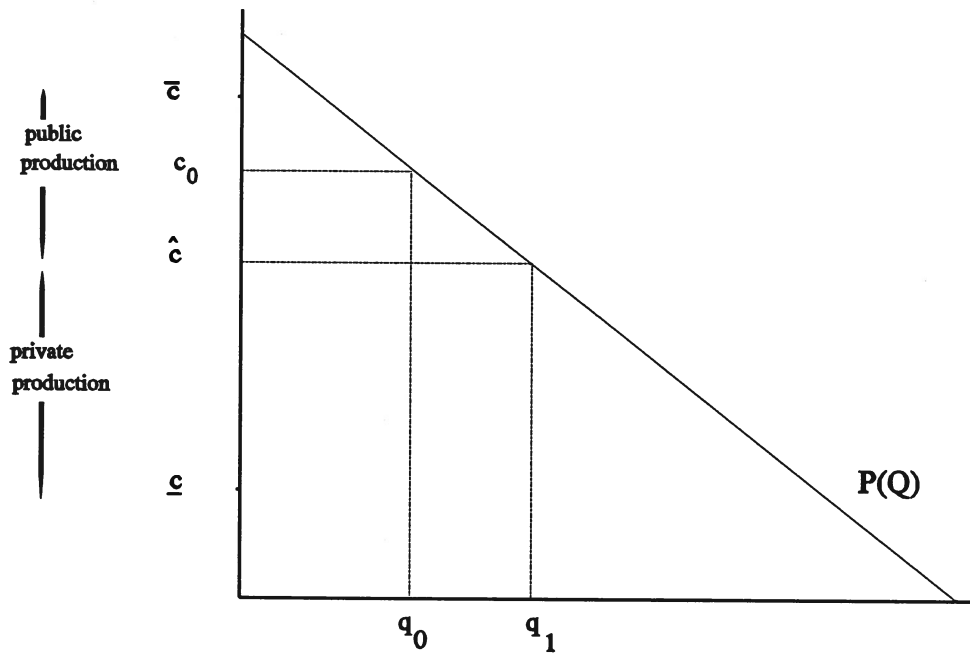
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<sup>14</sup>The mechanism is also equivalent to offering the private firm a schedule  $\tilde{P}(q_1)$  from which the private firm can make a profit maximizing choice. Baron (1989, p1359) refers to this as the "delegation" approach.

**Proposition 3:** *Provided that Assumptions 2,3, 5 and 6 hold, the optimal regulatory mechanism entails either public or private production for any given cost realization - it is never optimal for both firms to produce simultaneously. In particular, since the regulator knows the level of public costs  $c_0$ , public production is optimal in a cost interval  $c_1 \in [\hat{c}, \bar{c}]$ , otherwise the private firm supplies the market.*

**Proof:** See Appendix 2.

The solution to the mechanism design problem defined by equations (45)-(47) is illustrated in Figure 1. In the region of public production, the optimal production level is  $P^{-1}(c_0)$ ,



**Figure 1:** Optimal Production Schedule given Public Costs  $c_0$

the level of production which maximizes net surplus. In the region of private production, the optimal production level is  $P^{-1}(\hat{c})$ , the largest quantity that a firm with a cost realization in this

interval is willing to supply. It remains to determine  $\hat{c}$ . As  $\hat{c}$  is lowered below  $c_0$ , the regulator is trading off increased production by the private firm against having the 'wrong' firm produce in a cost interval just below  $c_0$ .

Expected welfare for an arbitrary value of  $\hat{c}$  is given by the following expression:

$$E(W) = \int_{\underline{c}}^{\hat{c}} [GS(P^{-1}(\hat{c})) - c_1 P^{-1}(\hat{c})] f(c_1 | c_0) dc_1 + \int_{\hat{c}}^{\bar{c}} [GS(P^{-1}(c_0)) - c_0 P^{-1}(c_0)] f(c_1 | c_0) dc_1 \quad (48)$$

where  $GS(\bullet)$  is gross surplus. Maximization of equation (48) with respect to  $\hat{c}$  yields optimal values for  $\hat{c}$  for a given value of  $c_0$ . Inspection of the first order condition, presented in Appendix 3, shows that  $\hat{c}$  is strictly less than  $c_0$ .<sup>15</sup>

The essence of this result is that the public firm can do no better than impose a "limit price" on the private firm. If the private firm has costs below this limit price, then it will choose quantity exactly as would any monopolist confronted by the same limit price. That is, the private firm will choose a quantity which clears the market at the limit price,  $\hat{c}$ .<sup>16</sup> The public firm can do no better in *disciplining* the private firm, because its own costs are too high for it to be efficient to produce at low cost levels. However, the limit price is not set equal to the public firm's costs. It is efficient for the public firm to supply the market when it is not the low cost producer, that is, when  $c_1 \in [\hat{c}, c_0]$ . This pricing strategy induces the private firm to supply a higher quantity

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<sup>15</sup>The reader may wonder whether the regulator has an incentive to use the announcement of  $\hat{c}$  as a signal in order to misrepresent the public firm's costs  $c_0$ . This is not the case, however, since the announcement of  $\hat{c}$  is sufficient for the private firm to compute its own profits with certainty; the public firm's true costs are, at this point, irrelevant to the private firm.

<sup>16</sup>Private cost levels so low that the private firm would choose an unconstrained monopoly price are ruled out by Assumption 6.

when its costs are below  $\hat{c}$ .<sup>17</sup>

The effect of correlated costs works in this mechanism through the conditional density  $f(c_1|c_0)$ . Although closed form solutions for the optimal production rules are not obtainable for this model, we can make some qualitative observations. The distance between  $\hat{c}$  and  $c_0$  is decreasing in the degree of cost correlation. As  $\rho$  increases, the expected costs of having the 'wrong' firm produce in the interval  $[\hat{c}, c_0)$  are high relative to the expected benefits of increasing private firm output when private costs are below  $\hat{c}$ . This result will be reversed with a low degree of cost correlation.

In summary, the regulator is able to utilise two aspects of public production in order to improve the efficiency of regulatory control. First, private producers tend to restrict output while a public firm always chooses production efficiently. Second, the observation of the public firm's costs allows the regulator to manipulate private production, through the choice of  $\hat{c}$ , resulting in increased average welfare. The first of these two effects is analogous to the *disciplinary* effect and the second to the *auditing* effect in our discussion of Bayesian Nash models in section 3.

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<sup>17</sup>It is interesting to compare our mechanism involving a public firm with an analogous scheme which retains private ownership. Formally, this problem is equivalent to asking both firms to report their costs  $r = \{r_0, r_1\}$ , and offering a production scheme conditioned on the vector of reports  $\{q_0(r_0, r_1), q_1(r_0, r_1)\}$ . The solution to this problem is a Vickrey auction. The contract to serve the industry is offered to the firm announcing the lowest costs, with an industry price equal to the costs announced by the high cost firm. This is incentive compatible; neither firm has an incentive to overbid its costs, because doing so would not affect profits if the firm wins the contract, but would lower the probability of winning. Similarly, neither firm has an incentive to underbid costs, because this would increase the probability of winning (the payoff to winning again does not vary with the firm's report), but only in those outcomes in which the rival firm had *lower* costs, in which case the winning firm would be forced to accept the contract at a loss.

The Vickrey auction is the optimal mechanism whether or not the firms costs are correlated. This may seem surprising, in view of results of Cremer and Maclean (1985), Riordan and Sappington (1988) and McAfee and Reny (1989) that, if a Principal has access to a correlated signal on which payments can be conditioned, then first best efficiency can be achieved, that is, the value of private information goes to zero. The result depends on the ability to make unbounded payments to and from the agent, based on the signal. In our model, however, the only instrument available to effect transfers is the market price.

## §5. SIMULATION RESULTS

In the foregoing discussion, equilibrium strategy choices have been derived conditional on a specific institutional structure. Equilibrium strategies are based upon *posterior* distributions of costs since each firm is able to observe its own costs prior to choosing its action. When comparing alternative institutional structures, the appropriate criteria for evaluating these mechanisms is *ex ante* expected welfare, that is, the expected welfare before either of the firms has entered and observed their own cost levels. Thus, mechanism choice is based upon *prior* distributions of costs. We can write this as:

$$E(W) = \int_{\underline{c}}^{\bar{c}} \int_{\underline{c}}^{\bar{c}} \left[ \int_0^{q_0^* + q_1^*} P(t) dt - c_0 q_0^* - c_1 q_1^* \right] f(c_0, c_1) dc_0 dc_1 \quad (49)$$

Some preliminary conclusions on the *ex ante* welfare properties of the Bayesian Nash models are contained in Propositions 1 and 2. Because the mechanism of section 4 is not amenable to closed form solutions for quantities and hence for expected welfare, a direct analytical comparison of the Bayesian Nash regimes with the mechanism of section 4 is difficult to make. Nevertheless a simple ranking of *ex ante* expected welfare in the three regimes involving a public firm can be made, and is summarised as Proposition 4.

**Proposition 4:** *Ex ante expected welfare in the revelation mechanism of section 4 (weakly) exceeds that of the asymmetric Bayesian Nash equilibrium with a public firm. Expected welfare in the asymmetric public/private regime in turn (weakly) dominates that in the symmetric public/private*

*regime.*

**Proof:** See Appendix 2.

We perform a numerical simulation in order to enable further comparison between the different regimes. For the Bayesian Nash models, expected welfare can be computed by substitution of parameter values into the closed form expressions presented in section 3. In the case of the mechanism of section 4, numerical values for expected welfare were obtained from a computer simulation exercise, assuming a bivariate normal distribution of costs. We chose parameter values such that Assumptions 3,4 and 6 are satisfied to a close approximation. Computed values of expected welfare are reported in Table 1.

<b>Table 1. <i>Ex Ante</i> Expected Welfare: <math>\alpha=20</math>, <math>\beta=1</math>, <math>\mu_0=6</math>, <math>\mu_1=4</math></b>				
<b><math>\rho</math></b>	<b>Private Duopoly</b>	<b>Symmetric Public/Private</b>	<b>Asymmetric Public/Private</b>	<b>Revelation Mechanism</b>
0	102.75	102.88	106.38	126.77
.1	102.70	102.81	106.09	126.72
.2	102.66	102.75	105.82	126.68
.3	102.62	102.70	105.58	126.65
.4	102.59	102.65	105.36	126.63
.5	102.56	102.61	105.16	126.64
.6	102.53	102.58	104.98	126.66
.7	102.51	102.55	104.83	126.71
.8	102.48	102.53	104.70	126.79
.9	102.46	102.51	104.59	126.88
1	102.44	102.50	104.50	128.42

First, the reported simulation values illustrate the welfare ranking reported in Proposition 4. For the chosen parameter values, the private duopoly is always ranked with the smallest welfare;

but, as we have stated above, this is not a general result and with higher public firm costs or smaller demand the welfare ranking of equilibrium in the first two columns could be reversed.

Next, the revelation mechanism performs significantly better than the Bayesian Nash regimes. The reason is that the regulator is able to restrict production almost completely to the low cost firm, utilizing production from the high cost public firm only when it performs a valuable incentive role.

Finally, consider the effect on expected welfare of changing the degree of cost correlation. Although the public firm's ability to manipulate its strategy with respect to  $\rho$  across regimes is substantive, in all cases the net welfare effects of changes in  $\rho$  within a regime are small. Broadly speaking, the reason for this is that two effects are at work with opposing impacts on expected welfare. First, a higher degree of cost correlation increases the information available to the regulator concerning private costs and is a factor leading (through the corresponding public firm's strategy) to higher expected welfare. Greater cost correlation, however, implies a smaller dispersion in industry costs, and reduced potential for obtaining gains from reallocating production towards the low cost firm. The latter effect contributes to lower expected welfare with increased cost correlation.

## §6. CONCLUSION

The purpose of this paper has been to re-examine the regulatory role of a public firm in a mixed market environment with private but correlated information about industry costs. We find that a public firm can play important disciplinary and auditing roles in regulating a private oligopoly. Further, we find that these roles are considerably enhanced as we move from symmetric

to asymmetric strategic regimes. If the cost differential between public and private firms is sufficiently large or demand for a product is weak, then a mixed market structure can actually be welfare inferior to a private oligopoly. This result obtains only if the strategic structure of the oligopoly is symmetric. Welfare gains can be achieved in such an environment only if the public firm is given a strategic advantage.

We examined two regimes involving increased commitment power for the public firm: the ability to commit to a production level and the ability to sustain a price-quantity schedule for the industry. We found that expected welfare gains are significantly higher in the latter regime. In the former regime, the public firm induces higher production from the relatively efficient private firm by committing itself to a lower production level, leading to an expected price above its own mean marginal cost level. In the latter regime, the public firm is able to sustain a limit price below its own cost realization.

An important aspect of mixed markets as an organizational form is the preservation of market rivalry. The emphasis in the recent regulation literature on sophisticated direct intervention strategies has narrowly focussed attention on the agency relationship between regulator and firms and neglected the important role of firm interaction at the strategy level. We show here that strategic interaction needs to be more clearly understood in order to better assess the performance of a public firm as an instrument of industrial policy.



## Appendix 1: No Shutdown Conditions

### (i) Private-Private Bayesian Duopoly Equilibrium

We require that  $\bar{c} - \underline{c} \leq (\alpha - \underline{c})/2$  (see Shapiro (1986), p436). Provided that demand is large enough, this will always be satisfied.

### (ii) Public-Private Bayesian Nash Equilibrium

Evaluate the inequality  $q_1^s \geq 0$ , initially for  $\rho = 0$ . The expression can be arranged as  $c_1 - \mu_1 \leq 2\mu_\Delta$ . Given our assumption that  $\mu_\Delta$  is at least one standard deviation of the cost distribution, the distribution of costs can be bounded such that  $\bar{c} \leq \mu_1 + 2\mu_\Delta$ , without violating (approximately) our assumption on linear conditional expectations.

We also require  $q_0^s \geq 0$ . A modified version of the expression in (i) above can be derived, namely  $\bar{c}_1 - \underline{c}_1 \leq ((\alpha - \mu_\Delta) - \bar{c}_1)/2$ . Again, provided that demand is large enough, this will be satisfied.

### (iii) Public-Private Bayesian Stackelberg Equilibrium

We require that  $q_1^s \geq 0$ . Shutdown is most likely to occur if  $\rho = 0$ . Rearrange the inequality as  $c_1 - \mu_1 \leq 4(c_0 - \mu_0) + 4\mu_\Delta$ . We require  $\mu_\Delta$  to be of the order of  $2^{1/2}$  standard deviations to ensure that shutdown does not occur in this case. The reason that this condition is more stringent than in case (i), is that the private firm's strategy depends on the public firms costs as well as its own. If the private firm knows both that the public firm has very low costs, and that its own costs are high, it is likely to shut down.

## Appendix 2: Proofs of Propositions

### Proof of Proposition 1:

(i) Substitute the equilibrium quantities into the first two terms of equation (11). The public firm will play a positive disciplinary role if and only if the following inequality holds:

$$\alpha - \mu_0 > 7(\mu_0 - \mu_1) \quad (\text{A1})$$

(ii) Substitution of the cost uncertainty terms for the different regimes into the last two terms of equation (11) yields analytically clumsy expressions. Since these expressions do not provide any further insight into the auditing role, we simply present simulated values in the table below to illustrate the Propositions in the text.

Table A1: Numerical Simulation of Auditing Terms: $\beta=1$						
$\rho$	Variance of Industry Output			Covariance of Firm Output and Costs		
	p	s	a	p	s	a
0	.50	1.25	4.25	1.00	1.50	4.50
.1	.50	1.20	3.86	.95	1.41	4.02
.2	.50	1.16	3.38	.91	1.33	3.56
.3	.49	1.12	3.12	.87	1.26	3.14
.4	.49	1.09	2.77	.83	1.20	2.74
.5	.48	1.06	2.44	.80	1.14	2.38
.6	.47	1.04	2.12	.77	1.10	2.04
.7	.47	1.02	1.82	.74	1.06	1.74
.8	.46	1.01	1.53	.71	1.03	1.46
.9	.45	1.00	1.26	.69	1.01	1.22
1	.44	1.00	1.00	.67	1.00	1.00

To obtain the auditing effect  $A$  for each regime, substitute the auditing terms into the last two terms of equation (11) in the text. Taking differences, we have that  $A^s - A^p > 0$ .

(iii) Follows from (i) and (ii).

(iv) An implication of the shutdown condition for the Public Private Bayesian Nash Equilibrium is that  $\alpha - \mu_1 \leq 2(\alpha - \mu_0)$ . By substituting equilibrium values into (11) for the two equilibria, it is easy to show that within the above range,  $E(CS^s) - E(CS^p) > 0$ .

**Q.E.D.**

**Proof of Proposition 2:**

(i) Using equations (39) and (23), we have that  $\bar{Q}^a - \bar{Q}^s = (\mu_0 - \mu_1)/\beta$ . The difference is positive given Assumption 3. Using equations (41) and (25), we have that the difference between  $\bar{s}_0^s$  and  $\bar{s}_0^a$  is always positive since  $\alpha > \mu_1$ .

(ii) Define average production rules for both firms in the Stackelberg regime in terms of the average production rules in the symmetric regime so that  $\bar{q}_1^a = 2\bar{q}_1^s$  and  $\bar{q}_0^a = \bar{q}_0^s - 2\bar{q}_1^s$ . Substituting these expressions into the first two terms of equation (11) and taking differences, we find that the Stackelberg public firm always exerts a stronger disciplinary role by an amount  $(\mu_0 - \mu_1)^2/2\beta$ .

(iii) Following steps outlined in Proof of Proposition 1, part (ii) yields the required result.

(iv) Follows from (ii) and (iii).

(v) Consider consumers first. By substituting into equation (10) expressions for  $\bar{Q}$  and  $\text{var}(Q)$  from the text, corresponding to the Symmetric and Asymmetric Bayes equilibrium, we obtain

$$E(CS^a) - E(CS^s) = \frac{\beta}{2} \left\{ \left[ \frac{\alpha - \mu_0 - (\mu_0 - \mu_1)}{\beta} \right]^2 - \left[ \frac{\alpha - \mu_0}{\beta} \right]^2 - \frac{3(4 - 3\rho)^2 + 2\rho(4 - 3\rho)}{4\beta^2} \right\} \quad (A2)$$

which is always negative. Since Expected Welfare is greater for the Asymmetric equilibrium, it follows immediately that  $E(\pi)$  is greater.

**Q.E.D.**

**Proof of Proposition 3:** The assumption of linear costs implies that the solution consists of a cost interval  $c_1 \in [\hat{c}, \bar{c}]$  in which only public production occurs, and an interval  $c_1 \in [\underline{c}, \hat{c}]$  in which only private production occurs. The critical value  $\hat{c}$ , and the production schedule for each firm within each interval, remain to be determined.

In the region of public production, the optimal value is  $P^1(c_0)$ , since  $c_0$  is known, and this is the level of production which maximizes net surplus. Secondly, the critical value  $\hat{c}$  can at most be equal to  $c_0$ , since higher private cost realizations are strictly dominated by public production.

In the region of private production, the derivation is more involved. The truth-telling constraint given by equation (46) in the text imposes the restriction that, in any mechanism,  $\pi(c_1; c_0, c_1)$

$$\frac{\partial \pi}{\partial c_1} = -q_1 \quad (\text{A3})$$

but by definition  $\pi(c_1) = R(q_1) - c_1 q_1$  so that

$$\frac{\partial \pi}{\partial c_1} = \left[ \frac{\partial R}{\partial q_1} - c_1 \right] \frac{\partial q_1}{\partial c_1} - q_1 \quad (\text{A4})$$

It is immediate that  $\partial \pi / \partial c_1 = -q_1$  if and only if either  $\partial q_1 / \partial c_1 = 0$  or  $\partial R / \partial q_1$ , that is, the condition for a profit maximizing output choice by firm 1. Consider the latter possibility. It represents

monopoly output choices by the private firm, for any realised value of  $c_1$ . But given our assumption 6 on the support of the cost distribution, such a solution would be strictly welfare dominated by the *constant quantity* solution  $q_0 = P^{-1}(c_0)$ . Thus only solutions involving constant private production can be optimal.

Finally, whatever level is chosen for  $\hat{c}$ , it is immediate that the optimal level of private production is  $P^{-1}(\hat{c})$ . Any smaller level could not be welfare maximizing, and any larger value would violate the participation constraint given by equation (47) in the text, and leave an interval of cost realizations for which the market would not be served.

It remains to determine  $\hat{c}$ , the critical level of private costs, below which only private production occurs, and above which only public production occurs. Realized welfare for an arbitrary value of  $\hat{c}$  (recall that  $c_0$  is known) is:

$$E(W) = \int_{\underline{c}}^{\hat{c}} [GS(P^{-1}(\hat{c})) - c_1 P^{-1}(\hat{c})] f(c_1 | c_0) dc_1 + \int_{\hat{c}}^{\bar{c}} [GS(P^{-1}(c_0)) - c_0 P^{-1}(c_0)] f(c_1 | c_0) dc_1 \quad (\text{A5})$$

where  $GS(P^{-1}(c))$  is the Gross Surplus generated by production  $P^{-1}(c)$

Differentiating with respect to  $\hat{c}$ , yields the first order condition for  $\hat{c}$ , for a given  $c_0$

$$\begin{aligned} \frac{dE(W)}{d\hat{c}} = & \left( [GS(P^{-1}(\hat{c})) - GS(P^{-1}(c_0))] + [c_0 P^{-1}(c_0) - \hat{c} P^{-1}(\hat{c})] \right) f(\hat{c} | c_0) \\ & + P^{-1'}(\hat{c}) \left[ \hat{c} F(\hat{c} | c_0) - \int_{\underline{c}}^{\hat{c}} c_1 f(c_1 | c_0) dc_1 \right] = 0 \end{aligned} \quad (\text{A6})$$

**Q.E.D.**

**Proof of Proposition 4:** For the first inequality, simply note that the Stackelberg strategy of a fixed quantity, independent of the private firm's costs, is available to the regulator as a solution to problem (46) and satisfies the constraints (47) and (48). For the second inequality, the allocation in the symmetric equilibrium is feasible for the public firm acting as a leader (since the private firm's strategy is the same in the two regimes).

**Q.E.D.**

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