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The Significance of the Probabilistic Voting Theorem

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Abstract

Public decision-making by majority rule is open to the danger of exploitation of minorities by majorities. Since any majority can employ the vote to expropriate the corresponding minority, it would seem that there can be no electoral equilibrium allocation of income or transfers in a democratic society and that democracy itself might be unstable. Much of democratic theory is devoted to the study of how this danger can be averted. The probabilistic voting theorem establishes that a degree of voter insensitivity to offers of rival political parties imparts an electoral equilibrium where it would not otherwise exist. The theorem is valid on its assumptions, but those assumptions are considerably stronger, and the theorem is less comforting about the prospects for the stability of democratic government, than one might at first suppose.

> "a pure democracy, by which I mean a society consisting of a small number of citizens, who assemble and administer the government in person, can admit of no cure from the mischiefs of faction. A common passion or interest will, in almost every case, be felt by a majority of the whole; a communication and concert, results from the form of government itself; and there is nothing to check the inducements to sacrifice the weaker party, or an obnoxious individual. Hence it is, that such democracies have ever been spectacles of turbulence and contention; have ever been found incompatible with personal security, or the rights of property; and have, in general, been as short in their lives, as they have been violent in their deaths"[1]

James Madison, 1787

This quotation is probably the most famous statement of the principle argument against democratic government, that, in a democracy with majority rule voting, there is nothing to stop a majority - any majority - from expropriating the minority altogether. If the national income, Y, is to be divided among three voters, A, B, and C and if all public decisions are made by majority rule voting, a majority coalition of, for instance, voters A and B can adopt a platform that assigns the entire national income to themselves, leaving poor C with nothing. Since any pair of voters can form a majority coalition, a democratic form of government is precarious for everybody.

The force of the anti-democratic argument has been recognized by the friends of democracy as well as by its enemies. Much speculation has been devoted to the task of weakening the argument by identifying other aspects of democratic society that make exploitation of minorities by majorities less feasible or less attractive to the majority than the crude paradigm of exploitation by voting would suggest. Speculation has also been devoted to the question of how democratic societies might be organized to keep the mischief of "faction", to use the old-fashioned term, at bay.

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* With thanks for comments from Dan Bernhardt, Robin Boadway, Raj Jha and members of the public finance workshop at Queen's University

There is an old tradition in democratic political theory-going back at least as far as Aristotle - that democracy is unstable unless it is circumscribed. The range of issues about which people vote must be constrained by a consensus in society (Berg, 1956), by the establishment of a compound republic[2], by legislative institutions that keep excessively divisive issues off the agenda (Shepsle and Weingast, 1981), by limiting franchise to property-holders[3], by a strong property rights (Usher, 1981, among many others) or more generally, by rules of entitlement that voters have come to respect (Brennan and Buchannan, 1986). The range of issues about which men vote must be constrained, on this view of democracy, because the alternative is discord, faction, instability and, ultimately, the dissolution of democracy itself.

To this list has recently been added a new item. It is claimed that a degree of voter insensitivity to the platforms of rival political parties is sufficient to generate an "electoral equilibrium" allocation of the national income in which nobody is expropriated altogether and which may even be socially-optimal in the sense that a full-blown social welfare function is maximized subject to a production constraint. The basis of the claim is the "probabilistic-voting theorem". For an excellent introductory exposition and bibliography, see Mueller, (1989, Chapter 11).

I show in this paper that, though the probabilistic-voting theorem is correct on its assumptions, it is nothing like as strong a refutation of the anti-democratic argument as one might at first suppose. To develop this point I begin by reviewing the anti-democratic argument, I then state and prove the probabilistic-voting theorem in a simple case, and I conclude by showing that the assumptions of the theorem are very much stronger than they seem and are not descriptive of actual democratic politics.

The Exploitation Paradigm and the Anti-Democratic Argument

Imagine a society with these characteristics:

(i) A total national income, Y, is to be allocated among three groups of voters, A, B and C, where everyone within a group is treated identically but income per head may differ among groups. The total population is n and there are equal numbers of people in each group. Thus, as an accounting identity

$$(n/3)\ y_A + (n/3)\ y_B + (n/3)\ y_C = Y \tag{1}$$

where y_A, y_B and y_C are incomes per head in the three groups. The assumption that everybody's income is the same within each group is for analytical convenience and may be relaxed without lessening the force of the example.

(ii) There are two political parties, R and D. The parties seek office but not incomes. On attaining office, a party is empowered to choose the incomes of citizens, y_A, y_B and y_C. Parties can commit themselves at election time to a choice of incomes. They do so by announcing platforms, $\{y_A^R,\ y_B^R,\ y_C^R\}$ and $\{y_A^D,\ y_B^D,\ y_C^D\}$ where, for instance, y_A^R is the income that party R will supply to group A in the event that party R wins the election. All platforms must be consistent with the national accounting identity in equation (1).

(iii) Define v_A^R as the number of people in group A who vote for party R, and define v_A^D, v_B^R, v_B^D, v_C^R and v_C^D accordingly. By definition, all of these numbers are positive or zero and

$$v_A^R + v_A^D, = v_B^R + v_B^D = v_C^R + v_C^D = n/3 \tag{2}$$

since the size of all three groups is the same. Numbers of votes are determined in response to the offers of income in the platforms of the political parties.

(iv) (To be modified presently) Each person votes for the party that offers him the largest income.

Strictly-speaking, there is no electoral equilibrium in this model. If party R chooses a platform $\{y_A^R, y_B^R, y_C^R\}$, then party D can win the election by a two-to-one majority with a platform

$$\{y_A^D, y_B^D, y_C^D\} = y_A^R + \epsilon, y_B^R + \epsilon, y_C^R - 2\epsilon\} \tag{3}$$

for any feasible positive value of ε. And, of course, party R can play the same game, for there is no platform that beats all other platforms in a pair-wise vote.

Voters who can be expected to understand the essential instability of this process might behave differently than is assumed in (iv). They might be prepared to stick with a political party that offers to share the entire national income equally among its supporters, with nothing left over for anybody else. Such behaviour leads to a "nasty" equilibrium where, for instance, party R wins with a platform {Y/2n, Y/2n, 0} which provides all of the available income to groups A and B, and nothing to group C. To get such an equilibrium, it must be assumed that party R is the first to offer a sufficiently attractive platform to the two groups in what has become the majority coalition and that members of groups A and B are "loyal" to party R because they know that any switch of allegiance to party D will set off a train of events that could lead eventually to their exclusion from a new majority coalition. Obviously, this nasty equilibrium is bad news for the prospects of the liberal society. Nobody can be expected to adhere to the conventions of majority rule voting if total impoverishment of minorities is to be expected.[4]

The exploitation problem is really two connected difficulties. The first is that voting about incomes is <u>capricious</u> in the sense that there is no telling who the members of the majority coalition will turn out to be.

Any sufficiently large collection of people or groups will do. The second is that voting about incomes precarious in that the losers in the process - those who find themselves among the excluded minority - appear in danger of being wiped out completely, of being left with nothing, of losing all that they might otherwise possess. Thus consideration of the anti-democratic argument suggest two questions about voting: Is there, or under what circumstances might there be, an electoral equilibrium platform that defeats all other platforms in a pairwise vote? and, What, if anything constrains the majority from expropriating the minority completely?

The Probabilistic-Voting Theorem

A more or less acceptable equilibrium can emerge if it is assumed that voters are somewhat insensitive to the platforms of the political parties. Assumption (iv) above carries the implication that, for instance, v_A^R can take only three extreme values: $v_A^R = n/3$ when $y_A^R > y_A^D$, $v_A^R = n/6$ when $y_A^R = y_A^D$, and $v_A^R = 0$ when $y_A^R < y_A^D$. Everybody in group A votes for party R when party R offers the higher income, the vote is split evenly when both parties offer the same income, and everybody votes for party D when party D offers the higher income. As an alternative, it might be reasonable to introduce a degree of continuity in voting behaviour. Perhaps party platforms are misperceived. Perhaps voters have a degree of party loyalty; the most loyal supporters require a big disparity in offers of income to induce them to change their vote, while others are induced to switch parties on the basis of a small difference in offers of income. Perhaps voters within groups differ in their assessment of the abilities of the rival party leaders.

With identical groups of voters and identical vote-maximizing political parties, it can be shown that a degree of insensitivity of voters to offers of incomes in the platforms of the rival political parties may lead to the emergence of an electoral equilibrium in which everybody's income is the same, an equilibrium in which each party offers a platform {Y/n, Y/n, Y/n} and the winner of the election is determined by tossing a coin. The requirement for such an equilibrium is that, starting from the electoral equilibrium, voters be more sensitive to reductions than to increases in income, a form of behaviour that would in certain circumstances be a reflection of diminishing marginal utility of income. To see this, imagine that party D holds to the electoral equilibrium platform, but that party R offers $1 more to everybody in groups A and B, and $2 less to everybody in group C. This was a winning strategy when all voters were assumed to be exquisitely sensitive to small differences in income. It need not be a winning strategy when each group's vote for party R varies continuously with the income offered to that group in the party platform. It is not a winning strategy when, for example, party D holds to the equilibrium strategy and the responses of any group to changes in the offer of income by party R are as follows: provision of <u>one</u> extra dollar induces <u>one</u> extra member of the group to vote for party R; withdrawal of <u>one</u> dollar leads to the withdrawal of <u>two</u> votes; withdrawal of <u>two</u> dollars leads to the withdrawal of <u>five</u> votes; withdrawal of <u>three</u> dollars leads to the withdrawal of <u>nine</u> votes; and so on. On this assumption, party R no longer gains the vote of the entire groups A and B by offering $1 more than its rival. It gains only one vote from each of the two beneficiaries of its manoeuvre and it loses five votes from the injured party. Party R loses two votes on balance, which, under our extreme assumptions, is enough to swing the election to party D.

The platform {Y/n, Y/n, Y/n} is an equilibrium because any small deviation by either party causes that party to lose the votes when the other party holds firm. Suitably generalized and formalized, this result would seem to carry very strong and very attractive implications about the prospects for democratic government. It would seem to reverse the dour implication of our analysis of the exploitation paradigm.

To formalize the degree of sensitivity of voters to offers by rival political parties, replace assumption (iv) above with assumption (iv^1), which is a special case of a much wider set of possibilities.[5]

$$(iv^1)\quad v_A^R = n/6 + S_A\left[(U_A(y_A^R) - U_A(y_A^D)\right] \tag{4}$$

$$v_B^R = n/6 + S_B\left[(U_B(y_B^R) - U_B(y_B^D)\right] \tag{5}$$

$$v_C^R = n/6 + S_C\left[(U_C(y_C^R) - U_C(y_C^D)\right] \tag{6}$$

provided v_A^R, v_B^R and v_C^R lie within the admissible bounds of 0 and n/3. [The number of voters in group A who vote for party R cannot be less than 0 or greater than the number of people in group A]. The terms S_A, S_B and S_C are sensitivity parameters; they are assumed to be positive. The U_A, U_B and U_C are ordinary utility of income functions with an assumed diminishing marginal utility of income i.e. $U'_A > 0$ and $U''_A < 0$, etc. Diminishing marginal utility of income will prove critical in establishing the electoral equilibrium.

The meaning of equation (4), for example, is that half the members of group A, n/6, vote for party R if the offers, y_A^R and y_A^D, are the same, and that otherwise, the vote for party R increases with y_A^R and decreases with y_A^D, as indicated in Figure 1. The vertical axis shows the number of votes for party R among members of group A. The horizontal axis shows the income offered by party R to members of group A. The curve shows how these are related for a given value of the income offered to members of group A by party D.

The Response of Votes to Offers in the Platform of a Political Party

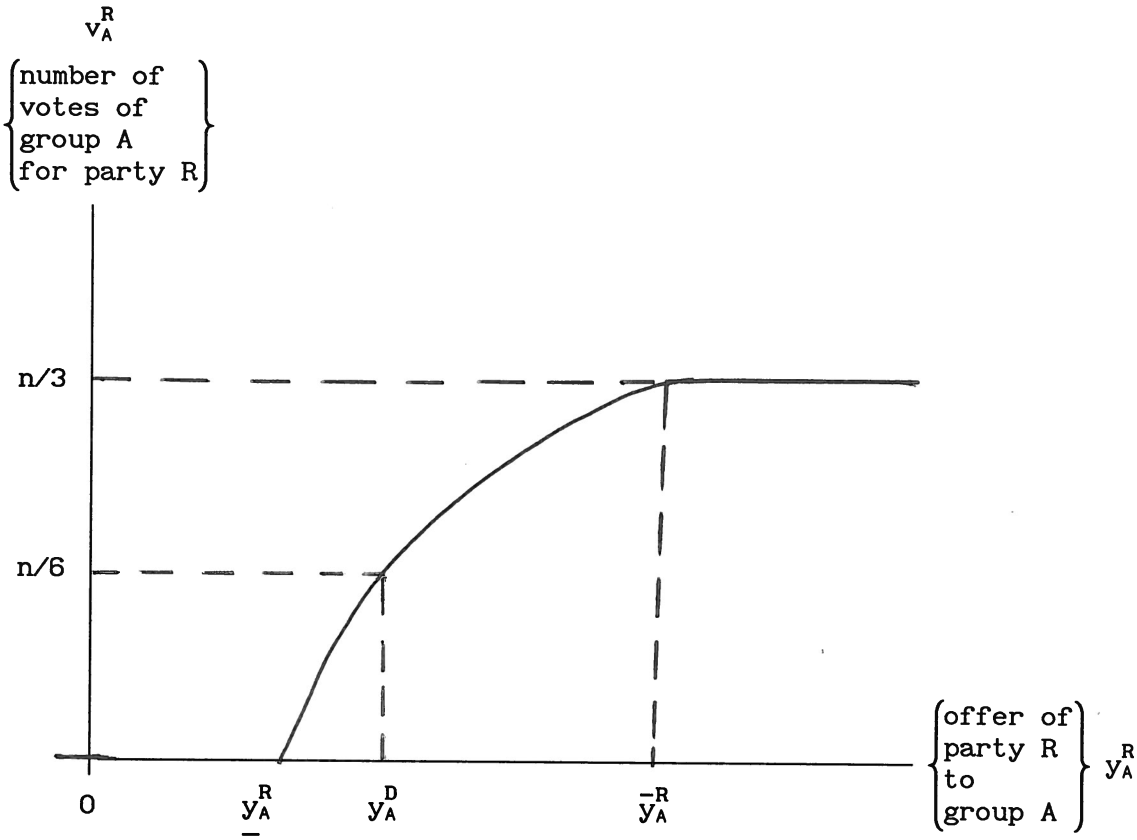

Figure 1

A final assumption is necessary. Each party is assumed to choose its platform to maximize its vote. Though questionable in many contexts, this assumption is probably innocuous here.[6]

(v) Party R chooses $\{y_A^R, y_B^R, y_C^R\}$ to maximize its total vote, V^R, where

$$V^R \equiv v_A^R + v_B^R + v_C^R \tag{7}$$

and party D behaves analogously.

The proof of the "probabilistic-voting theorem" is now straightforward. If there is an electoral equilibrium, it is represented by a pair of platforms, $\{y_A^R, y_B^R, y_C^R\}$ and $\{y_A^D, y_B^D, y_C^D\}$ each obtained by maximizing total vote, V^R and V^R, subject to the income constraint, and with due regard to the platform chosen by the other political party.[7] Since there is assumed to be no intrinsic difference between the parties, their electoral equilibrium platforms have to be the same. They are identified by the first order conditions

$$S_A U_A'(y_A^*) = S_B U_B'(y_B^*) = S_C U_C'(y_C^*) \tag{8}$$

where y_A^*, y_B^* and y_C^* are the incomes in the common equilibrium platform.

It follows immediately that the income of each group in the electoral equilibrium is an increasing function of its sensitivity to offers; the greater S_A, for instance, the smaller is U_A' in equilibrium, and the larger y_A^* must be. Suppose Y = 3000, n = 300 and all utility functions are logarithmic. By symmetry, if S_A, S_B and S_C are all the same, then $y_A^* = y_B^* = y_C^* = Y/300 = 10$. But, if $S_A = S_B = 100$ while $S_C = 200$, then equation (8) requires that $1/y_A^* = 1/y_B^* = 2/y_B^*$, so that $y_A^* = y_B^* = 7.5$ while $y_C^* = 15$.

Local and Global Equilibrium

Having proved the probabilistic-voting theorem, albeit for a very simple case, it is time to examine the assumptions of the theorem to see if and to what extent the theorem really does overthrow the exploitation paradigm. The critical assumption, without which the theorem becomes invalid, is the concavity of the relation between, for example, v_A^R and y_A^R, that is between the number of votes by group A for party R and the income offered group A in the platform of party R. Concavity in this context means that the value of v_A^R increases with the value of y_A^R, but at a decreasing rate. As long as the sensitivity parameter, S_A, is constant, the concavity in the relation between v_A^R and y_A^R is inherited from the assumed concavity (that is, from the diminishing marginal utility of income) of the utility of income function, $U_A(y_A)$.

The probabilistic-voting theorem would hold unambiguously and without qualification if v_A^R, the share of the votes of group A that are cast for party R, were a concave function of y_A^R for all feasible values of y_A^R. This could possibly be so, but need not be as is illustrated in Figure 1. The break in concavity occurs because the value v_A^R lies between 0 and n/3 and because there is assumed to be a minimum offer, $\underline{y}_A^R$, of party R to group A below which nobody in group A votes for party R. The function relating v_A^R and y_A^R simply cannot be concave once that minimum is reached. The electoral equilibrium breaks down at the point where concavity fails.

To see how the break down occurs, consider once again a political party that is attempting to win a majority of votes by offering high incomes to groups A and B at the expense of group C. Assume that party D holds to the original electoral equilibrium platform, and that there is for each group a certain minimal offer of income by party R below which nobody in the group votes for party R. As the offer of income by party R to any group decreases, the vote of that group for party R decreases by a progressively larger amount. The first reduction of one dollar costs one vote, the second costs two extra votes, the third costs three extra votes, and so on. As long as that is so, there is no advantage to party R in transferring dollars from group C to groups A and B. But as the vote of group C for party R decreases by an ever larger amount, there must come a time when poor group C is casting all of its votes party D, and can reduce its vote for party R no farther. From then on, party R has nothing to lose by squeezing group C, but may have much to gain by increasing the incomes of groups A and B if their votes for party R are not already at 100%.

An extension of our numerical example might help at this point. Let all three utility functions be logarithmic, that is, $U_A = \ln(y_A)$ etc. Let $S_A = S_B = S_C = 100$, let $Y = 3000$, and let $n = 300$, so that average income per person is 10 and the electoral equilibrium platform is {10, 10, 10}. Assume that party D adheres to this platform. If party R does so too, each party gets 150 votes and the winner is determined by tossing a coin. If party R deviates by a small amount from this platform, it loses votes and loses the election. But if party R deviates a great deal from this platform, it gains votes and wins the election!

Party R is considering a strategy of offering groups A and B the same income per head, which may be greater or less than 10, and offering group C whatever is left. For instance, party R might choose a platform {11, 11, 8} or a platform {13, 13, 4}. The total vote, V^R, for party R is plotted on Figure 2 as a function of the common value of y_A^R and y_B^R, where y_C^R is

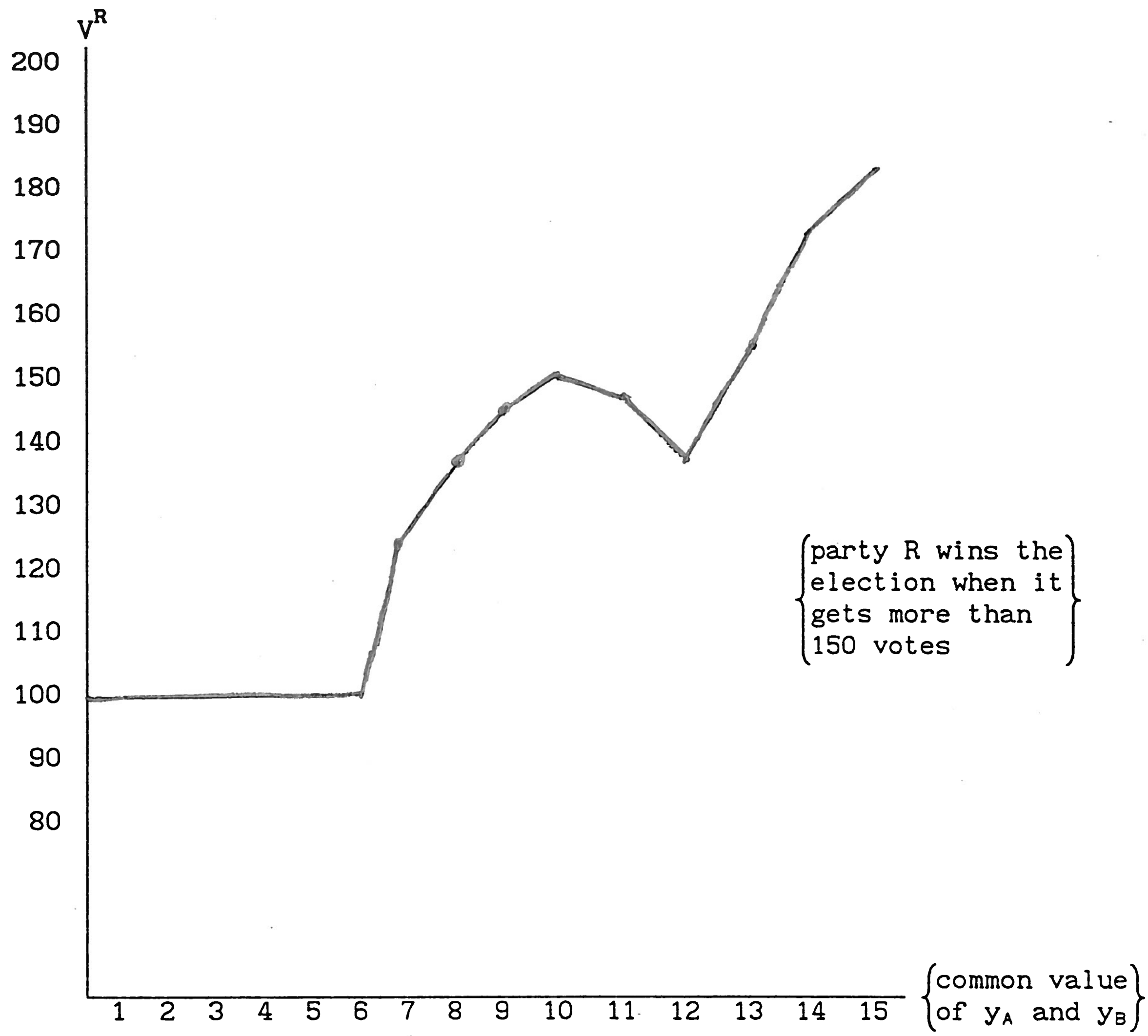
V^R
200
190
180
170
160
150
140
130
120
110
100
90
80
party R wins the election when it gets more than 150 votes
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
common value of y_A and y_B

Figure 2

necessarily equal to $(30 - y_A^R - y_B^R)$. The plot is for integral values of the common income, y_A^R and y_B^R. The corresponding values of V^R are easily computed from equations (4), (5) and (6).

As may be read off the graph in Figure 2, a platform for party R of {9, 9, 12}, when the platform for party D is {10, 10, 10}, yields party R a vote of only 146 so that party D wins the election with 154 votes. Similarly, a platform of {11, 11, 8} yields party R a vote of 148 which again implies a win for party D. Party R does even worse with a platform of {12, 12, 6}. But from there on, party R gains votes by raising the common value of y_A and y_B in its platform because groups A and B increase their vote for party R, while group C cannot reduce its vote any further. If party D holds firm to the "equilibrium" platform of {10, 10, 10}, then party R maximizes its vote with a platform {15, 15, 0} winning the election by a count of 182 to 116. Since there is no predicting _a priori_ which two of the three groups will constitute the winning coalition, the situation is essentially that in the original exploitation paradigm. The electoral equilibrium is essentially unsatisfactory because it is _local_ rather than _global_; small deviations reduce a party's vote but large deviations can increase its vote to the point where it is a clear winner in the election.

From a purely mathematical point of view, the reason why the equilibrium in Figure 2 is local rather than global is the presence of the nonconcavity at $\underline{y}_A^R$. This occurs because the curved portion of Figure 1 intersects with the horizontal axis rather than with the vertical axis. Whether v_A^R is globally concave depends entirely on the size of the sensitivity parameter, S_A. If S_A is small the function relating v_A^R to y_A^R is quite flat, the curved portion of Figure 1 intersects with the vertical axis rather than with the horizontal axis and the running-out-of-votes as described above would not occur. If S_A is quite large, we return for all practical purposes to the original exploitation paradigm. The effect of a reduction in S_A is to flatten the curve while swinging it clockwise around the point $(n/6,\ y_A^D)$. Eventually, the concave portion of the curve would touch the horizontal axis and, when the same is true for all three groups, the local electoral equilibrium becomes global as well. Whether the curve cuts the horizontal or the vertical axis is ultimately an empirical matter.

There is reason to believe that the shape of the function connecting votes and offers in Figure 1 is the more realistic representation of electoral politics. It is not plausible that substantial numbers of people would vote for a party threatening to dispossess them if a better alternative were available. When one party is prepared to allocate the national income more or less evenly, there is a limit, well above zero, to which the other party can reduce the income of any group without losing the votes of all of the members of that group.

The concavity assumption in the probabilistic-voting theorem is also open to the objection that, in the context of the model developed here, there is no strong economic or political justification for the assumption that the sensitivity parameters, S_A, S_B and S_C are really parameters rather than functions that vary with income. Concavity with respect to income would reinforce the original implications of the theorem, but convexity could destroy the theorem altogether.

Electoral Equilibrium and the Status Quo

In a sense, the theorem proves too much. The core of the proof of the probabilistic-voting theorem is the link from the concavity of the utility-of-income function to the concavity of the function connecting offered income to voting behaviour. The presence of such a link is inconsistent with the presence of large disparities of income among voters unless it can be assumed that the rich have high sensitivity parameters, S, or that they have steep utility of income functions. The probabilistic-voting theorem requires that people be rich or poor because of their tastes - as represented by S or by U′ in equation (8) - rather than because of inherited wealth, personal ability, luck or other factors commonly believed to affect the distribution of income.

The probabilistic-voting theorem determines the entire distribution of income, without reference to civil rights, property rights (which are always unequally distributed), rights of beneficiaries of public programs such as the old age pension and unemployment insurance, skill differentials or plain luck. The absence of these determents of the distribution of income is, in one sense, a strength rather than a weakness of the probabilistic-voting theorem. An electoral equilibrium distribution of income has been found in the most unpropitious circumstances. But there remains the question of whether the assumptions of the probabilistic-voting theorem can be extended to account for the ordinary determinants of income. There is no obvious way of incorporating the two theories of the income distribution into a unified whole. It is as yet an open question whether the theorem can be watered down to the point where politics exerts an influence on the market without displacing the market altogether as the principal mechanism for production of goods and allocation of income among people.

There is a dilemma here. If the sensitivity-of-votes-to-offers functions are as postulated in equations (4), (5) and (6), where voting shares, such as v_A^R, depend in the first instance on utilities, such as U_A, and only indirectly on incomes, then the concavity required for the probabilistic-voting theorem is inherited from the concavity of utility itself, but the distribution of income is determined entirely in the electoral market with no place for property rights, civil rights or a status quo of entitlements. One can get around this problem by postulating that voting shares depend on transfers rather than incomes, so that equation (4), for example, would be replaced by an equation of the general form

$$v_A^R = f(T_A^R, T_A^D) \tag{9}$$

where v_A^R remains as the proportion of group A that votes for party R and the arguments, T_A^R and T_A^D, are offers of transfers of income in the platforms of parties R and D. The difficulty now is in defining a pre-transfer status quo, and in justifying the required concavity of the function f in equation (9), now that utility can no longer do the job. (Suppose members of group A are old. Is the present old age pension part of the status quo, so that T_A^R = 0 unless party R offers to increase it. Or is the old age pension itself a transfer within the terms of the theorem?)

Nevertheless, it may be in the context of transfers that the true political significance of the probabilistic-voting theorem is to be found. If there is a status quo of civil rights, property rights and acquired entitlement to public programs, if voters are more responsive to platforms of political parties that entail small losses in welfare than to platforms that entail small gains, and if large reallocations of welfare are ruled out by constitutional restrictions that most people are inclined to respect or simply because people know that you cannot reallocate income radically without reducing the size of the pie or placing democratic government in jeopardy, then the status quo may represent a political equilibrium. It would represent an equilibrium in the sense that no political party could command the votes of a majority of the electorate with a platform representing any significant deviation from the <u>status quo</u>. This cannot be the whole story. Allowance must be made for changes in economic and social conditions and for swings of public opinion that affect the political equilibrium from time to time. But the significance of this line of reasoning is that the probabilistic-voting theorem is not on its own sufficient to break the exploitation paradigm. Only in the context of a wide-spread respect for established rights can it serve to explain how a political equilibrium can arise and why a political party may be reluctant to rely too heavily in its platform on measures that benefit a majority of voters at the expense of the rest.

Footnotes

1. The quotation is from The Federalist Papers #10 (Hacker, page 20).

2. op. cit. #51, page 121.

3. This was a common view among the classical economists. Their reasons for distrusting universal franchise, and their general preference for property qualifications, are examined in Grampp, 1948. The main articles in the famous debate on universal franchise between James Mill, who favoured it, and T.B. Macaulay, who was opposed, are reprinted in Lively and Rees, 1978.

4. There are six possible nasty equilibria: groups A and B voting for party R, groups A and B voting for party D, groups A and C voting for party R, and so. These are known in game theory as bargaining sets. See, Shubik, 1987, p. 342.

5. Strictly speaking, the function is

$$v_A^R = \max \left[0, \min \left[n/3, n/6 + S_A\left(U_A(y_A^R) - U_A(y_A^D)\right)\right]\right]$$

and similarly for v_B^R and v_C^R.

6. Political parties do not seek to maximize votes. They seek to win elections, which may or may not amount to the same thing. To say that political parties maximize votes is in some respects like saying that hockey teams maximize their scores. In a sense they do, but a team is, as a rule, not less satisfied with a win of 4 goals to 3 than with a loss of 8 goals to 9. Similarly, a party that seeks to maximize votes would presumably maximize expected votes as well, in which case, in a two party race, it would prefer a 30% chance of winning 80% of the votes together with a 70% chance of winning 49% of the vote to a certainty of winning 51% of the votes. This implication of the hypothesis is clearly false. Furthermore, as William Riker pointed out many years ago, vote-maximization is at variance with the "size principle". A rationally administered political party may prefer to win by a margin that is safely above the 50% mark but not overwhelming. The reason is that a party with the support of an overwhelming majority of the electorate is prone to fission, as a bare majority of its supporters comes to realize that they could do better for themselves in a new party with a smaller base of support and no need to placate superfluous adherents. A party with too large a base in the electorate is intrinsically undisciplined and unstable. On the size principle, see Riker (1962). The vote-maximization hypothesis is usually attributed to Downs (1957).

7. The Lagrangian of the problem is

$$\begin{aligned}\pounds &= V^R - \lambda\left[y_A^R + y_B^R + y_C^R - Y/3n\right] \\ &= n/2 + S_A\left[U_A\left(y_A^R\right) - U_A\left(y_A^D\right)\right] + S_B\left[U_B\left(y_B^R\right) - U_B\left(y_B^D\right)\right] \\ &\quad + S_c\left[U_C\left(y_C^R\right) - U_C\left(y_C^D\right)\right] - \lambda\left[y_A^R + y_B^R - y_C^R - Y/3n\right]\end{aligned}$$

Differentiating with respect to y_A^R, y_B^R and y_C^R, the first order conditions

$$\pounds y_A^R = S_A U_A'\left(y_A^R\right) - \lambda = 0$$

$$\pounds y_B^R = S_B U_B'\left(y_B^R\right) - \lambda = 0$$

$$\pounds y_C^R = S_C U_C'\left(y_C^R\right) - \lambda = 0$$

from which follows equation (21) in the text. If all three utility functions, U_A, U_B and U_C, are the same and all sensitivity parameters, S_A, S_B and S_C, are the same as well, then, by symmetry, the equilibrium platform has to be {Y/n, Y/n, Y/n}.

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