TWO MOMENT DECISION MODELS AND EXPECTED UTILITY MAXIMIZATION:

SOME IMPLICATIONS FOR APPLIED RESEARCH

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I. Introduction

The sometimes confusing and usually controversial discussion of the relationship between an expected utility (EU) and a mean standard-deviation (MS) ranking of a set of random variables was recently revived. Another condition was presented which is sufficient to ensure that any EU ranking of a set of random variables could instead be represented by one which is based upon only their means and standard deviations. (Sinn [1983], Meyer [1987]). That this linear class or location and scale (LS) condition had remained largely unrecognized, even though it was pointed out and used much earlier, is evidence that the condition, and the relationship it implies for these two approaches, is not very well understood. The discussion here focuses on the LS condition and its implications for applied research involving risk, especially that being conducted in agricultural economics. The impact for theoretical analysis and empirical analysis is discussed. To accomplish this the outline below is followed.

In the next section, the literature concerning the relationship between the EU and MS ranking procedures is reviewed very briefly. Most of the review focuses on the linear class or location and scale condition. Next, in section III, the implications of this condition for the construction and analysis of theoretical models involving risk is discussed. Section IV goes on to examine the implications for empirical work. A description is given of how one can statistically examine models where the LS condition is not able to be verified or rejected on theoretical grounds, to see if the data suggests that it is likely to hold. Within this context, the impact on stochastic dominance based empirical work is discussed, and the subject of estimation error is addressed.

II. Literature Review

The work of Tobin [1958] and Markowitz [1959] some thirty years ago made it clear that an expected utility ranking of a set of random variables can instead be represented by one based on only their means and standard deviations if: i) the utility function is quadratic, or ii) all the random variables are normally distributed. Either of these

two conditions, by itself, is __sufficient__ for this consistency.[1] Since the work of Tobin and Markowitz, much research effort has been devoted to examining the extent to which either of these two conditions can reasonably be assumed to hold. The conclusion reached is familiar; for most problems neither quadratic utility nor normality has theoretical or empirical support. Thus, neither assumption provides an adequate basis for using MS ranking methods when consistency with the axioms of expected utility is desired.

In the search for other conditions which are sufficient for representing an EU decision function with an equivalent MS function, Baron [1977] has shown that when the random variables are not restricted in any way, then quadratic utility is not only a sufficient condition for this representation to exist, but is necessary as well. Thus, if one wishes to restrict only the preference side of the decision model, a necessary and sufficient condition is known to be the quadratic utility assumption.

Concerning the choice set side of the decision problem, the situation is less clearly or completely resolved. The assumption of normality has long been recognized as one which can be generalized, and thus, it is not a necessary condition even in the absence of restrictions on preferences. Uniformly or lognormally distributed random variables, for instance, can also be ranked using a MS function which is consistent with the axioms of expected utility. Chipman [1973] and others have referred to this type of generalization of the normality assumption, as the "two parameter family" condition. This is because for these examples, each member of the family of random variables can be uniquely identified by specifying the family name and exactly two parameters.

Rothschild and Stiglitz [1970] attempt to formalize this "two parameter family" condition and extend its meaning so that it applies to unnamed as well as to named families of distribution functions. In doing so, they find that the two parameter family condition, when sufficiently generally defined, does imply a relationship between MS and EU, but allows that relationship to exhibit properties which are unacceptable. As an example of this, Rothschild and Stiglitz define a two parameter family of distribution functions such that risk averse agents prefer those with increased variance, even when the mean value is held fixed. A similar but less distressing example is given by Feldstein [1969], who shows that if the random variables in the choice set are lognormally distributed, then the resulting $V(\sigma,\mu)$ need not be quasiconcave even when the agent is risk averse. This violation of the convexity of preferences assumption is often presented as a case where

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[1] It appears that many researchers act as if one of these two assumptions is necessary. For instance, Hawawani [1978] states "This assumption implies that profits are normally distributed. We exclude quadratic utility curves since they display increasing absolute risk aversion."

"plunging" behavior can arise; that is, corner solutions can be optimal in the portfolio problem.

In the process of concluding that the "two parameter family" condition is too loosely or imprecisely specified to yield a relationship between EU and MS which maintains certain desired properties, Rothschild and Stiglitz mention in passing that if the random variables in the choice set are all equal in distribution[2] to one another except for location and scale, then those random variables satisfy the two parameter family condition. Rothschild and Stiglitz do not go on to develop results concerning this particular restriction.

It appears that the location and scale condition lay dormant for many years following its discussion by Rothschild and Stiglitz. Recently, Sinn [1983] and Meyer [1987] each present research in which the location and scale, or as Sinn calls it, the linear family condition, plays a large role. Before discussing this research and interpreting this condition, the following formal statement of it is in order.

The Location and Scale Condition: Random variables Y_i are equal in distribution to one another except for location and scale if there exists a random variable X such that each Y_i is equal in distribution to $\mu_i + \sigma_i X$.

In the above definition it is assumed that the random variables Y_i have finite means and variances. The random variable X has been normalized to have mean of zero and a variance of one. Thus, the location and scale parameters, μ_i and σ_i, are the mean and standard deviation of Y_i, respectively. The same definition can be stated in terms of cumulative distribution functions (Feller [1966]). $F(\cdot)$ and $G(\cdot)$ are equal to one another except for location and scale if $F(x) = G(a+bx)$ for some a, some b > 0 and at all points x.

This LS condition is satisfied by many, but not all, two parameter families of distribution functions which have been named and described in statistics textbooks. The normal and uniform families, for instance, do satisfy the condition, while the lognormal family does not. This further emphasizes the fact that the LS condition is a generalization of the normality assumption, but is not as general as the two parameter family condition.

While extending the normality restriction to include other named families of two parameter distribution functions may be of some value, it is the generalization to an infinite number of unnamed families that is most significant. One can generate a two parameter family of random variables which satisfies the LS condition by beginning with any

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[2] Random variables ae equal in distribution to one another if they are represented by the same cumulative distribution function. This is a weaker condition than being equal to one another.

Show graphically with a CDF

distribution function, and transforming its argument in a positive linear fashion. Each possible choice for the parameters a and b leads to a new element of the family. The LS condition requires that the random variables comprising the choice set be represented by distribution functions which are obtainable from one another by a shifting and/or rescaling process. It does not require that the starting distribution function take on any particular form. This is a considerable generalization over the normality assumption. Symmetry, infinite tails, a single mode, and many other characteristics of the normal, which the data may not satisfy, are no longer required. It is this aspect of the LS condition that allows it to be met in instances where normality is rejected.

That the LS condition implies a relationship between an EU and MS ranking of the random variables is obvious. A formal statement of this relationship is given below.

> The Location and Scale Implication: If the random variables in the choice set satisfy the LS condition, then any expected utility ranking of them can instead be represented by one based only on he means and standard deviations of those random variables. That is, $Eu(Y_i) = Eu(\mu_i + \sigma_i \cdot X) = V(\sigma_i, \mu_i)$ for some function V.

Sinn and Meyer each show that the LS condition puts some interesting structure on the $V(\sigma, \mu)$ preference function, and allows one to associate properties of the vonNeumann-Morgenstern utility function, such as decreasing absolute risk aversion, with properties of V. These properties are not of concern here and are not reviewed.

Sinn and Meyer also note that many important economic models, because of the structure of the model, represent the economic agent as facing a choice set which automatically satisfies the LS condition. Sandmo's [1971] model of the competitive firm facing a random output price, and Tobin's [1958] riskless and single risky asset portfolio model are two very important examples. Feder, Just and Schmitz' [1980] and Holthausen's [1979] extensions of Sandmo's model, which add forward markets, also possess the appropriate structure. What it is about these models that implies that the LS condition is satisfied is discussed in the next section.

III. Implications For Building Theoretical Models

Given the LS condition and its implication concerning the relationship between MS and EU, this section addresses three questions. First, what is the gain, in terms of ability to do and present theoretical analysis, from having models where the choice set satisfies the LS condition? Then, how can one construct models so that the agent faces such a choice set? Finally, what improvements or generalizations of the LS assumption are likely?

When analyzing a decision model in which the choice set satisfies the LS condition, one has the option of using either an EU or a MS ranking function. That is, the agent can be assumed to maximize expected utility from alternatives which are described by cumulative distribution functions, or to maximize $V(\sigma,\mu)$ subject to a choice set which lies in (σ,μ) space. An important aspect of this statement to recognize is that the "or" which is used is an inclusive "or". Thus, one need not choose between the EU and MS approaches, but can and often should use them in combination.

The advantages of EU over MS and vice versa are well known enough to be mentioned only briefly. MS has advantages which stem primarily from its two dimensional nature. This allows one to graph opportunity sets and preferences, and to present arguments and proofs using more elementary mathematics. EU on the other hand, has been much more intensively studied over the last twenty five years, resulting in a rich set of definitions and propositions concerning various measures of risk and risk aversion. Many of these terms, and such concepts as general stochastic dominance, have not been translated into MS terminology, and thus are not easily used in the MS context. Thus, if one must choose between the two approaches, the selection, as often is the case, depends on the need for simplicity versus richness. When the LS condition is satisfied one can take advantage of each approach's particular strengths. This is one gain from working with a model in which the choice sets satisfy the LS condition.

In a very practical sense, another reason for wanting to be able to use both approaches arises because of our limited abilities. It sometimes is "easier" to demonstrate a particular proposition in one framework than the other. As an example of this consider the model of the competitive firm facing a random output price and able to forward contract the sale of output. This is a model which yields choice sets satisfying the LS condition. Feder, Just and Schmitz, and Holthausen, each present and analyze this model assuming the firm maximizes expected utility. They derive a variety of interesting propositions concerning this model. Recently, Meyer and Robison [1988] reexamine the same model using MS techniques. Certain propositions of Feder, Just and Schmitz or Holthausen are presented graphically, and presumably in a manner which yields additional insight into the workings of that model. As Meyer and Robison note however, other propositions can be rederived in the MS context only with considerable effort. For some questions the answers seem to be more easily derived using one approach, while for others the reverse is true.

A similar point can be made by examining the analysis of the riskless and single risky asset portfolio model that Arrow [1971], Cass and Stiglitz [1972] and Fishburn and Porter [1976] have carried out in an EU model, and which Adler [1969] presents in MS terms. The Sandmo model of the competitive firm facing a random output price, and Hawawani's [1978] analysis of it in MS terms can also be used to make this point.

In summary, the gain from having a model where the choice set satisfies the LS condition is that two modeling techniques rather than one are made available. Since neither dominates the other in all dimensions, being able to use both of them improves our ability to present and carry out an analysis of the model.

In order to make use of the gain described above, one must be able to recognize and/or construct models in which the choice set satisfies the LS condition. For the most part, the published models with choice sets which satisfy this condition, are models in which there is only one random parameter, and the outcome variable is a linear function of this parameter. That is, the model specifies that the objective of the agent is to maximize expected utility from some outcome variable z, where z may depend on many parameters and choice variables, but exactly one of those parameters, x, is modeled as a random variable. Furthermore, z is a linear function of this parameter.

For instance, in Sandmo's model of the competitive firm, the firm maximizes expected utility of profits π. Profits depend on the output level selected by the firm, and the parameters describing costs and demand. Among these parameters only that representing output price, p, is assumed to be random, and π is a linear function of this parameter. Checking for linearity is as simple as noting that $\partial^2\pi/\partial p^2 = 0$. This same characteristic is present in the extension of this model presented by Feder, Just and Schmitz or Holthausen, in Tobin's portfolio model, in Feder's [1977] "general economic decision model", and in many other models in the literature.

To see why this model structure implies that the choice set satisfies the LS condition, note that when the outcome variable z is a linear function of x, and x is the only source of outcome randomness, then the choice set contains random variables of the form $a + b \cdot x$. Thus, each is distributed in the same fashion as x, except for location and scale. In most such models, the values for the location and scale parameters, a and b, depend on choices made by the agent and on other nonrandom parameters in the model. This implies that each of the random variables z, which the agent can choose among, are equal to random variable x, and one another, except for location and scale. Since equal implies equal in distribution, the LS condition is satisfied.

This single source of randomness feature appears to be present in virtually all models which have been presented and analyzed in an EU context. Some of these models however, do not represent the outcome variable as being a linear function of that parameter. How to deal with this, and how one can impose the LS condition on the agent's choice set when there is more than one random variable, are discussed next.

If the randomness of the outcome variable arises from one source, a single random parameter, but the relationship between the outcome variable and the random parameter is nonlinear, then the LS condition need not hold. It may be however, that one can either transform the

outcome variable so that the resulting variable is linear in the random parameter, or can write the outcome variable as a two parameter nonlinear function of the random parameter. For either of these cases, the approach used in Meyer can likely be modified to establish a relationship between the EU and MS ranking criteria.

For instance, if each outcome variable in the choice set can be written as $z = e^{a+bx}$, then $\ln z = a+bx$, and $\ln z$ satisfies the LS condition. Thus, when calculating expected utility, one need only recognize that $u(z) = v(\ln z)$ is the appropriate transformation to apply to the preference function. This implies, for instance, that even if $u(z)$ is concave, $v(\cdot)$ need not be concave in z. It is precisely this lack of concavity, even under risk aversion, that allows Feldstein to present the violation of convexity of preferences when the choice set is composed of lognormally distributed random variables. (Recall that if x is normally distributed, then e^{a+bx} is lognormally distributed).

When the choice sets contains outcome variables of the form $z = a + b \cdot \ln x$, where x is random, then z is linear in $\ln x$. Since $\ln x$ is itself random, the LS hypothesis holds, and one can establish a relationship between an expected utility ranking of such a choice set and the MS ranking.

It appears that for most models with a single source of randomness, one of the two types of transformations just described allow a link between EU and MS to be established. This relationship is not necessarily one with the nice properties demonstrated by Sinn and Meyer, however.

For the case of multiple sources of randomness the story is far less encouraging. Even if the outcome variable is linear in only two random parameters, it does not appear that a relationship between EU and MS can be established, except under the strictest of conditions. The portfolio problem with two risky assets is an example with such outcome variables. Chamberlain [1983], Cass and Stiglitz [1970], and others have rigorously addressed this issue in the context of finding a separation theorem, and have shown that such separation theorems exist only under very strong assumptions concerning the random variables involved. Since separation theorems exist in "reasonable" MS models, it does not appear that one will be able to establish the LS condition in models with multiple sources of randomness.

One exception to the above negative statement is the case where the multiple sources of randomness combine in a fixed fashion. That is, the outcome variable z can be written as $a + b \cdot h(x_1, x_2)$, where x_1 and x_2 are each random, but enter only through the function $h(\cdot)$. An example of this is the Capital Asset Pricing Model (CAPM) used in finance. In that model the rate of return on any asset or portfolio has a mean value which depends only on market parameters, and the asset's correlation with the market portfolio.

The single index model version of this specifies that the random rate of return for asset or portfolio i is given by $r_i = \phi + (r_m - \phi)\beta_i + \epsilon_i$ where r_m is the random rate of return on the market portfolio, ϕ is the risk free rate of return, and β_i is the ratio of the assets covariance with the market portfolio to the market portfolio's variance. The ϵ_i term represents variation in the asset or portfolio's rate of return not explained by variation in the market's rate of return. It is a random variable which is assumed to have a zero mean and to be independent of the market's rate of return. The ϵ_i's are also assumed to be independent across assets.

Within this model, if one is willing to assume that the ϵ_i terms are identically distributed <u>when scaled by $1/\beta_i$</u>, then r_i can be written as $r_i = \phi + (r_m - \phi + \delta_i)\beta_i$, where the δ_i terms are independent and identically distributed. The two sources of randomness, r_m and δ_i, are now combined into a single factor. For all assets or portfolios, the only random term is $(r_m - \phi + \delta_i)$, and these random variables, although not equal to one another, are equal to one another in distribution. Thus, for this model, the rate of return on assets or portfolios are equal in distribution to one another except for location and scale. Although models with multiple sources of randomness, where all of the random parameters are combined into a single term, can satisfy the LS condition; this model is the only example of this which I have at this time. Thus, the news concerning satisfaction of the LS condition in theoretical models with multiple sources of randomness and is not very encouraging. Only under very restrictive conditions is the LS condition satisfied. For empirical analysis, many sources of randomness seem to present less of a difficulty. This discussion is the subject of the next section.

IV. Implications For Empirical Studies Involving Risk

When ranking a set of random variables on the basis of observations concerning them, the nature of the gains from working with alternatives which satisfy the LS condition are not much different than those just described for theoretical analysis. If the LS condition is satisfied by the random alternatives to be ranked, then both the MS and EU methods are available, and each has the advantages mentioned earlier. In addition however, the problem of estimation is better understood and more adequately resolved for means and standard deviations than for expected utility. This seems especially true in examining the ability to test hypotheses and/or establish confidence intervals about estimated values.

While the above indicates that the gain from meeting the LS condition in empirical work is similar to that for theoretical analysis, this does not imply that its <u>impact</u> must be of the same magnitude. In fact, I conjecture that the LS condition will have a much larger impact on empirical work. This is because the LS condition is more likely to be "satisfied" when doing empirical analysis. That is, while the gain for empirical work may be similar in nature and magnitude to that for

theoretical analysis, the impact will be far larger due to the frequency with which the LS condition is satisfied. The reasoning behind this conjecture is given below.

In many models it is not possible to verify or reject the LS condition on purely theoretical grounds. Instead, one must obtain data concerning the random alternatives and test the hypothesis that the LS condition holds. There is a difference between verifying that the LS condition is true in a theoretical model, and testing the LS hypothesis in empirical work. This difference arises because the random alternatives are estimated with some degree of imprecision. This allows the LS hypothesis to not be rejected for reasons other than it being true; this includes not rejecting because of estimation error. Since all empirical work involves some estimation error, it is more likely that the LS hypothesis will not be rejected, than it is for the LS condition to be verified in the corresponding theoretical model.

Suppose one examines samples from a set of random alternatives and concludes that with the given data, the LS hypothesis cannot be rejected. What does this mean? This implies that it is not unlikely that those samples could have been obtained from random alternatives which satisfy the LS condition. What does this imply concerning the true random alternatives? Certainly, they could satisfy the LS condition. It is also possible however, that the true alternatives do not satisfy the LS condition, but are not sufficiently different from alternatives which do to be distinguished from one another on the basis of the samples and the statistical test employed.

While the latter conclusion does not verify that the true alternatives satisfy the LS condition, it does indicate that acting as if they do does not lead to statistically significantly different conclusions than otherwise. Thus, if one tests the LS hypothesis and does not reject it, then differences in the ranking of the random alternatives which arise from using MS rather than EU ranking criteria are not significant differences in a statistical sense.

This suggests that as a first step in examining data concerning random alternatives one should check to see if the EU and MS ranking criteria can possibly give statistically significantly different results. I suspect that for many data sets the answer will be no, and this will provide a justification for using MS ranking methods. Certainly, this is one implication of this work which those wishing to use MS methods in their empirical analysis will emphasize.

Before briefly describing the statistical tests which can be employed to examine the LS hypothesis, two further comments are in order. The first concerns estimation error, and the second involves the continued use of stochastic dominance procedures with small samples.

The work of Pope and Ziemer [1984], Stein, Pfaffenberger and Kumar [1983] and others, tells us that with too few observations there appear

to be rather severe amounts of estimation error in the various stochastic dominance procedures which have been employed. It is conjectured that this same estimation error is likely to cause one to not reject the LS hypothesis in those instances. If this is true, then when estimation error is severe one can use MS methods, confident that the results obtained are not significantly different in a statistical sense from those obtainable using EU. Furthermore, it then appears to be easier to specify the size of this estimation error is terms of standard errors on the estimated means and standard deviations than for expected utility. Quite simply, large estimation error is likely to allow one to use MS ranking methods, which in turn allows estimation error to be more easily quantified.

Concerning stochastic dominance, recall that the LS condition implies that the EU and MS ranking techniques are each available to augment the other. Thus, stochastic dominance is not ruled out or made invalid if the LS hypothesis is not rejected. For certain problems, especially those involving ranking alternatives for individuals whose risk aversion measures are known or assumed to fall in specific ranges and/or have certain slopes, there is no MS technique to carry out the ranking. Thus, while MS ranking methods may be appropriate, it may be that EU methods are better suited to the task at hand. It should be clear though that if the LS hypothesis is not rejected, the comparison of EU and MS efficient sets cannot lead to statistically significant differences. I suspect that many such differences pointed out in past empirical work fall into this category.

To carry out tests of the LS hypothesis, the k-sample Kolmogorov-Smirnov (KS) statistic, denoted D, can be used. D is the maximum difference between any pair of the k empirical distribution functions (EDF) formed from the k samples. Formally, D is given by $D = \sup |F_i(x) - F_j(x)|$, where the supremum is taken across x, i, and j. $F_i(x)$ and $F_j(x)$ are the EDFs from samples i and j. The statistic D always takes on rational values between zero and one.

Under the assumption that the random variables from which the samples are drawn are independent and identically distributed, the probability distribution function for D has been derived or can be estimated using Monte Carlo procedures. D is distribution free under this set of conditions. Published work has dealt primarily with the two and three sample cases. Meyer and Rasche [1988] have extended this to an arbitrary number of samples and an arbitrary number of observations.

Given samples from k independent random variables, one can use this multisample KS statistic to test the hypothesis that the random variables are equal in distribution to one another. A high value for D indicates a large difference in the EDFs formed from the data, and is unlikely under the null hypothesis. Thus, observing a high value leads one to reject the hypothesis that the samples are from random variables which are equal to one another in distribution. In order to use this test to examine the LS hypothesis with specified location and scale

parameters, one must first normalize the data using the given location and scale parameters, and then apply the test.

The KS test compares quite favorably with other nonparametric tests of the same hypothesis, and parametric versions as well. Conover [1971] indicates this for the two sample case, and the Cramer-von-Mises, Lillifors and Chi-square tests. When compared with these tests, KS is similar in power, and sensitive to all forms of deviation from the null hypothesis. The test appears to be quite powerful if the rejection level is set in the 10-20% range. Since the KS statistic is distribution free under the null hypothesis, information obtained concerning its probability function using Monte Carlo procedures is of a general nature. This, plus the ease in computing D make finding its probability function well suited to Monte Carlo methods. Gardner, Pinder and Wood [1980] use Monte Carlo methods to tabulate the probability distribution for D for a few selected cases. As mentioned earlier, Meyer and Rasche have dealt with the general case of an arbitrary number of samples and observations.

In attempting to carry out the test of the LS hypothesis, one must deal with two difficult issues. First, the location and scale parameters are often not known and must be estimated. Second, the samples may not be independent of one another. Each of these problems arises for the portfolios of common stock model to be described next.

The single index version of the CAPM model described earlier specifies that the rate of return on any asset or portfolio of assets can be written as $r_i = \phi + (r_m - \phi + \delta_i)\beta_i$. These rate of return variables are not independent of one another because of the market factor which is random and common to all r_i. Also, each of the r_i differ from one another by unknown location and scale parameters ϕ and β_i. As indicated earlier, the δ_i terms are assumed to be independent of one another, and identically distributed. This is sufficient for the r_i to satisfy the LS hypothesis. To make a long story very short, Meyer and Rasche use <u>independent data</u> to estimate the unknown location and scale parameters, ϕ and β_i. The cause of dependence, the rate of return on the market, is represented by the rate of return on the CRSP market index. With these parameters one can then transform samples from r_i into samples from δ_i. The LS hypothesis can then be tested for these samples. A very loose statement of the results of this test is that the LS hypothesis cannot be rejected with 50 or fewer observations (monthly), but is rejected with 100 or more observations. This is with a rejection level of 20%.

The above result indicates that the use of MS methods to form portfolios of common stock is likely to lead to no statistically significant differences if the number of observations is less than 50. On the other hand if more than 100 observations are used, it <u>might</u> make a difference.

Meyer and Rasche go to formulate another version of the single index CAPM model. In this version, the LS condition is no longer satisfied. Another parameter, rather than $1/\beta_i$, is introduced as a coefficient on ϵ_i. With this added parameter the total number is two, since the original single index model has the same location parameter, the risk free rate of return, for each r_i. Hence, that model is actually a one parameter model. Tests of this new model never reject the null hypothesis at the 20% level. The LS hypothesis is not being tested, but one sufficiently similar to establish the fact that for portfolios of common stock, even 150 monthly observations are not sufficient to allow EU and MS to generate statistically significantly different efficient sets. Details of this result are available from Meyer and Rasche.

In summary, for empirical work the role of estimation error in testing the LS hypothesis is clearly significant. It appears that for many of the problems which have been examined by agricultural economists and others, the estimation error is large enough to suspect that EU and MS ranking procedures cannot give statistically significantly different results. I suspect that for those wanting to use MS ranking procedures, but fearing inconsistency with EU, this a desired finding.

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