# SIMULTANEOUS PRODUCTION AND MARKETING DECISIONS OVER TIME[1]

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## A. Nature of the Agricultural Decision

Time is an important element in agricultural decisions. The results of decisions may not be observed until months after the decisions are made. Production, marketing, and investment decisions require explicit consideration of activities separated temporally. In addition, decisions are linked over time. Planting decisions necessarily follow land preparation activities. Harvesting decisions are based on production levels resulting from a series of decisions made over the production year.

This paper describes an application of a modeling approach that incorporates decisions over time, allows decisions to be made that are conditional on past actions, and determines optimal decisions depending upon observed outcomes of uncertain parameters important to the decision under consideration.

The problem addressed in this paper is the situation faced by a cow/calf producer in the fall of the year. Weaned calves may either be marketed at that time or fed over the winter and either marketed the following spring or placed on grass for summer grazing. The second section of this paper outlines the rancher's decision in greater detail.

The rancher's problem is modeled using the discrete stochastic programming model discussed by Cocks and by Rae (1971a, 1971b). The third section of the paper describes the characteristics of this model. Reference is made to previous development and applications of the formulation, and a cursory examination of the characteristics of the approach is given.

The fourth section of the paper provides greater detail on the components of the production/marketing decision problem. Results of the sequential formulation of the model are next presented and are contrasted with the single period formulation of the problem. Final observations on the advantages and use of the discrete stochastic programming model conclude the paper.

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## B. Problem statement

Cow/calf producers face several decisions in the fall of the year when weaner calves are traditionally marketed. The rancher may decide to market the 350-450 pound calves in the fall, or he may hold them over the winter for either sale the following spring or for placement on rangeland for summer grazing. If the decision is to hold the animals, optimal rates of gain over the winter must be determined. This production decision relies upon marketing and production options available in the spring.

Important considerations in the rancher's decision are current and expected future input and output prices, the relationship between price and weight, and animal performance on winter feed and subsequent summer range.

## C. The Discrete Stochastic Programming Model

Models that determine optimal activity levels in light of the sequential resolution of parameter uncertainty are a class of the stochastic linear programming problem developed by Tintner and called the "wait and see approach" by Madansky.

First model development is generally attributed to Cocks in 1968. Cocks described a multistage model in which the values of some or all coefficients within each stage acquire modified probabilities as a result of past events, or actions, within the model. The multistage model discussed by Cocks allocates resources to activities within a time period, and then, whatever event is observed over the period, optimizes allocation over the next period. Allocation over succeeding stages continues based on past decisions and observed outcomes of the initially uncertain events.

Rae (1971a,1971b) expanded the discussion of Cocks' model and described in detail the construction, solution and interpretation of results of a discrete stochastic programming model applied to a vegetable farm.

A limited bibliography of stochastic programming with recourse, a subset of which includes discrete stochastic programming, is provided in Hansotia.

The decision tree in figure 1 illustrates the discrete stochastic programming model. The decision maker is initially faced with several possible future events in time period A. He makes a decision, $X_A$, in light of his expectations of these future events. Event $E_{B1}$ or $E_{B2}$ then occurs. Assume $E_{B1}$ occurs. The decision maker must now reach an optimal decision, $X_{B1}$, conditioned on $X_A$, the occurrence of $E_{B1}$, and future uncertain events, $E_{C11}$ and $E_{C12}$.

The decision maker observes one of four possible events in period C ($E_{C11}$, $E_{C12}$, $E_{C21}$, or $E_{C22}$). The optimal decision given $E_{C11}$ is observed, for example, would be a function of all past decisions ($X_A$ and $X_{B1}$) and events ($E_{B1}$ and $E_{C11}$). The discrete stochastic programming model is formulated such that optimal decisions are reported at each time period based on prior decisions and events and on the expected distribution of future events.

Assume that the objective in figure 1 is the maximization of the expected utility of income over the three time periods. This can be represented:

$$\max \quad EU(Y) = \sum Pr(\theta_i)\; U(y_i)$$

subject to:

$$\begin{aligned}
Q^1(X_A,\ X_{B1},\ X_{C11}) - y_1 &= 0\\
Q^2(X_A,\ X_{B1},\ X_{C12}) - y_2 &= 0\\
Q^3(X_A,\ X_{B2},\ X_{C21}) - y_3 &= 0\\
Q^4(X_A,\ X_{B2},\ X_{C22}) - y_4 &= 0
\end{aligned}$$

and

$$\begin{aligned}
g^1(X_A,\ X_{B1},\ X_{C11}) &\leq 0\\
g^2(X_A,\ X_{B1},\ X_{C12}) &\leq 0\\
g^3(X_A,\ X_{B2},\ X_{C21}) &\leq 0\\
g^4(X_A,\ X_{B2},\ X_{C22}) &\leq 0\\
\text{All } X &\geq 0
\end{aligned}$$

The Kuhn-Tucker conditions derived in the appendix illustrate both the sequential nature of the model and the divergence of events over time. Changes made in any decision in period A, $X_A$, impact incomes in all states of nature since $X_A$ is common to all income rows, $Q^1$ - $Q^4$, and constraint rows, $g^1$ - $g^4$, of the model. Decisions made in period B, $X_{B1}$ or $X_{B2}$, depend on the occurrence of either $E_{B1}$ or $E_{B2}$. Changes in $X_{B1}$ have no influence on the outcome of decisions given $E_{B2}$ occurred. Finally, decisions made in period C, such as $X_{C11}$, only influence income along that branch of the event tree.

These characteristics of the discrete stochastic programming model confirm that the model meets Antle's three criteria that separate the static one period model from the dynamic multiperiod sequential problem. Antle's criteria pertain to the extent to which information is available and used by the dynamic decision maker:

a. Sequential dependence of decisions - optimal decision in period t is based on how the decision affects the optimal decision in period t+1

$$X_{t+1,s} = X_{t+1,s}^{o}\,(X_{t,s})$$

where s refers to the state of nature and the right hand side of the function states that the optimal decision in period t+1 is a function of the previous period's decision.

b. Information feedback - If information feedback exists, the decision in period t+1 is made after the results of the decision in period t are observed

c. Anticipated revision - I altered Antle's description to reflect the current model, but decisions made earlier, $X_{t,m}$, may be revised later, $X_{t+1,m}$, as new parameter information becomes available.

## D. The Ranch Decision Model

A model of the rancher's problem is described in this section. Production decisions are based on animal performance, costs and availabilities of different feeds, and expectations of future prices. Marketing decisions are based on current and expected future prices, past marketing decisions, and animal performance. The rancher is faced with marketing and production decisions throughout the one year period considered in the model, so long as some cattle remain. Past decisions continue to influence current decisions over the period.

The different components of the model are discussed individually below. The section concludes with a description of the complete model.

### 1. Animal performance:

Production activities over time for retained animals are a sequence of linked decisions. Input decisions are made on a periodic (daily, weekly) basis, and may rely on prior input decisions and subsequent animal weight gain, as well as on expectations or realizations of input and output prices.

The periodic production process employed in the feeding model is based on Fox and Black's (1977, 1984) net energy model for medium frame steer calf performance:

[1] $$VFI_t = (.1493[Ne_m]_t - .0460[Ne_m]_t^2 - .0196)\ W_t^{0.75}$$

$$Ne_{mt} = .077\ W_t^{0.75}$$

$$Ne_{gt} = (VFI_t - Ne_{mt} / [Ne_m]_t)\ [Ne_g]_t$$

$$ADG_t = 13.91\ Ne_{gt}^{0.9116}\ W_t^{-0.6837}$$

$$W_{t+1} = W_t + ADG_t$$

VFI is voluntary feed intake, determined by both the beginning live weight of the animal and the net energy concentration (Mcal/Kg) of the ration available for maintenance, $[Ne_m]$. The maintenance energy requirement ($Ne_m$) of the animal is a function of animal weight. The total amount of energy available for animal gain, $Ne_g$, is composed of excess energy after maintenance requirements have been met and the gain energy concentration of the ration, $[Ne_g]$. Actual daily gain, ADG, is a function of animal weight and the total amount of energy available in the ration for gain. Decisions made during period t with respect to ration and, consequently, animal gain, are then embodied in $W_{t+1}$.

Weight gains on summer range has been found to be influenced by production decisions made over the previous winter. Rogers and Malone gathered data from 72 steers over two Nevada growing seasons in the early 1960's and estimated the following rate of gain during the summer:

$$[2] \quad ADG_{summer} = \underset{}{2.3300} - \underset{(.0006)}{0.0014W_{winter}} - \underset{(.0755)}{0.6182ADG_{winter}}$$

$R^2 = 0.50$
Standard errors in parentheses (error on constant not reported)

Average daily gain over the entire summer period, $ADG_{summer}$, was found to be negatively related to $W_{winter}$, the weight of the steers at the beginning of the winter feeding season, and $ADG_{winter}$, the average daily gain of the animals over the winter period.

Although the $R^2$ value is relatively low, suggesting production uncertainty might be appropriate in the model for at least summer performance, production is considered deterministic in all periods in the current model.

2. Simulation of Steer Calf Prices for the Model

Two characteristics of steer prices had to be accounted for in simulating future price expectations for the model. First, prices vary over time (figure 2). Time series and ad hoc procedures were used to simulate these price movements. Second, steer prices are negatively related to animal weight. A simple regression was used to simulate this price-weight relationship.

Time series procedures were used on monthly Kansas City discounted steer prices (4-500 lbs.) from 1972 through the end of 1986. The following ARIMA model was deemed preferable to others estimated based on residual mean square and standard errors (in parentheses) of estimated coefficients:

$$(1-\underset{(.990)}{0.574}B)\ P_t = (1+\underset{(.069)}{0.683}B-\underset{(.117)}{0.308}B^2)\ (1+\underset{(.075)}{0.176}B^{12})\ a_t$$

Residual Mean Square = 23.290

Standard ARIMA notation (Pindyck and Rubenfeld) is used. $P_t$ is steer price in period t, $a_t$ is the error term of the estimation in period t, and B is the backspace operator applied to P and to a.

A five one-month ahead forecasts were made from the December, 1986 price. The final forecast represented one price outcome for May. The error characteristics of the ARIMA model were exploited to generate alternative price outcomes around the initial forecast. Estimation errors were assumed normally distributed with mean zero and standard deviation equal to $s_e$. The empirical distribution of observed errors was trisected, such that one-third of the errors were in the first segment, one-third in the middle, and one-third in the higher third of the distribution. Mean values from the top and bottom segments were then added to the initial five-month ahead forecasted value to represent the most and least favorable states of nature at the spring decision node.

Output prices at the end of the summer grazing season were then derived from the ARIMA model, using the top, middle, and low spring prices as the last "observed" price for forecasting an additional five months ahead. Higher and lower prices from each of these three forecasted prices were then calculated the same as was done for the spring prices. In all, nine different price states of nature were included in the model.

Price discounts resulting from higher calf weights were calculated using average prices over the 14 years of monthly price data for 4-500 pound animals, 5-600, 6-700, and 7-800. The following price-weight relationship was found:

$$[3] \qquad P_w = P_{450} + \underset{(.0150)}{(0.3160} - \underset{(.0001)}{0.0016\ W)}$$

$R^2 = 0.983$

where $P_w$ is steer price at weight w and $P_{450}$ is price for the 4-500 pound animal. The regression equation was added as a constraint to the programing model to adjust simulated prices by animal weights resulting from the production decisions.

3. The Marketing Decision

Alternative selling points are available to the producer: A (winter, year 1), B (spring, year 2), and C (fall, year 2). Profits at any one of these points under output price state of nature i may be represented as:

$$[4] \qquad \Pi_{ij} = N_{ij}((P_{ij} * W_{ij}) - \sum_{t=1}^{j} \sum_{k=1}^{m} (r_{tk} * x_{itk}))$$

Animal weight, $W_{ij}$, has been determined by a succession of past production decisions. The number of animals sold in the period under this state of nature, $N_{ij}$, depends upon the number of animals sold in earlier periods, the total number of animals initially available, and future marketing expectations, which in turn depend on animal performance and expectations of future prices.

4. Objective function

The objective of the model in its current form is the maximization of expected net returns over all price outcomes. Formally,

$$[5] \quad \text{Max } ER = \sum_{i=1}^{s} pr(\theta_i)\ (N_{Ai}P_{Ai}Q_{Ai} + N_{Bi}(P_{Bi}\ Q_{Bi} - \sum_{k=1}^{m} r_{Bik}X_{Bik}) + N_{Ci}(P_{Ci}Q_{Ci} - \sum_{k=1}^{m} r_{Bik}X_{Bik} - \sum_{k=1}^{m} r_{Cik}X_{Cik}))$$

The subscript i refers to the marketing and production decisions made under the price conditions existing in state of nature i. Expected returns ER are obtained by summing returns under each state, weighted by the probability $pr(\theta_i)$ of that state's occurrence. Each state is mutually exclusive and the set of states is collectively exhaustive.

5. Complete model

Production decisions were made on a monthly basis over the five month winter period for any calves not sold in period A (fall, year 1). Rations and subsequent rates of gain were solved simultaneously for each monthly period. Animals not sold in period B (spring, year 2) were placed on rangeland. Any animals not previously sold were sold in period C, the end of the summer grazing period.

Activities were chosen over all states of nature to maximize expected net returns, equation [5]. Constraints in the model included the winter feed relationships of equation [1] for the five month feeding period, animal performance on summer range (equation [2]), and the marketing activities of equation [4]. Output prices in the objective function were adjusted within the constraints by the price-weight relationships of equation [3].

## E. Results

### The Discrete Stochastic Programming Model

Characteristics of the expected return maximizing solution are shown in Tables I and II. Given the conditions assumed in the model, all animals are retained and fed over the winter. A ration of oat hay, rice bran and, for the last three months, corn is fed to achieve an average daily gain of roughly one kilogram. The production decision is altered each month due to increases in animal weights resulting from the previous month's ration.

The period B (spring, year 2) decision is dependent upon the realized price at that time. If the price observed in spring is $1.49 (adjusted downwards by the 350 Kg animal weight at the time), the optimal strategy is to sell all 100 animals at period B. If, however, the observed period B price is either $1.65 or $1.58 (again adjusted by weight), the optimal strategy is to retain all animals and put them on range for the summer grazing season. Given the initial weight of the animals at the beginning of the winter feeding season (period A) and the rates of gain achieved over the winter, the grazing animals will gain 0.66 Kg/day on range, and be sold in period C (fall, year 2) at 449 kilograms.

The maximum expected return that can be achieved given the prices and decisions points in the model is $43,328. Net returns under each state of nature are reported in Table II.

Single Period Formulation of the Model

It has been shown (Magnasarian) that problems in which decisions are allowed to be made over time as parameter uncertainty is sequentially resolved yield objective function values that are no worse, and are often better, than single period formulations of the problem. The rancher's problem was thus framed as a single period model to compare results with those obtained using the discrete stochastic programming approach.

Conversion to a single period model was easy, especially since a risk neutral objective function was used at this stage of the model development. Calculation of income under all price states of nature was replaced with a single objective in which the optimal strategy using expected values of prices at the three marketing nodes A, B, and C. The sequential objective function was replaced by:

$$\max ER = N_A P_A W_A + N_B\Big(P_B W_B - \sum_{k=1}^{m} r_{Bk} X_{Bk}\Big) +$$

$$N_C\Big(P_C W_C - \sum_{k=1}^{m} r_{Ck} X_{Ck} - \sum_{k=1}^{m} r_{Bk} X_{Bk}\Big)$$

where $P_t$ refers to the average of the prices at each time period in the sequential model. As before, output prices were adjusted by weight within the model.

Results of the single period model are presented in Table III. Ration, rates of gain, and animal weights are similar to those obtained in the sequential model. However, all sales occur in period C (fall, year 2) in the single period model. Net revenue for the single period model was $43,012, a very small drop from the expected returns of $43,328 in the sequential model.

Comparison of the Sequential and Single Period Formulations

Consistent with theory, the objective function value was higher under the sequential formulation of the model than it was under the single period model. But not by much. The question naturally arises of the need to undertake the extra trouble of the sequential model, which results in a much larger and more expensive to solve programming model?

The optimal strategy resulting from the single period model was only $16 less than the expected returns of the sequential. Certainly the additional value of the information generated would not cover the additional costs of the more complicated model.

The fallacy of this argument can be found in the realization of how uncertain information actually does become known over time. Average price expectations are seldom realized: a price is observed that represents a draw from the distribution of expected prices. And, based on the different marketing strategies observed between the single period and the sequential models, the observed future prices at the sequential model's decision nodes may trigger altogether different strategies.

To illustrate the importance of allowing decisions to be made sequentially, net returns were calculated for the optimal production and marketing strategy resulting from the single period model under each of the nine price states of nature of the sequential model. This comparison is reported in Table IV. Little difference in net returns is seen for the first six states. This is to be expected since production levels were similar under the two models and all cattle were sold in period C under the first six states of the sequential model.

Large discrepancies occur in the last three states, however. These states represent the lowest period B and C prices. Under the sequential model, when the low period B price is observed, cattle are sold. This option is not available in the single period formulation when only average prices are used. Net returns are maximized in the single period model by holding cattle to the third and last marketing period. Even were the low period B prices observed, the decision maker relying on the results of the single period model would be constrained to hold cattle until the third period. Consequently, he would incur net losses in two of the three last states of nature. The overall loss over all nine states of nature from conforming to the results of the single period model would be $3,239.

## F. Two Unsettling Characteristics of the Discrete Stochastic Programming Model

### 1. Solutions relatively insensitive to decision maker's risk attitudes

The sequential model was reformulated using a CARA expected utility function. No values tested of the negative-exponential utility function coefficient resulted in changes in the marketing and production activities of the model.

It would have been expected that the risk averse decision maker would have chosen a more diverse marketing plan. In the present case, the cow/calf model's insensitivity results from the output and input prices used and the potential for animal performance in the gain equations. Specifically, incomes in all states of nature at the optimum solution were higher than the income from selling the calves in period A (winter, year 1).

However, the discrete stochastic programming model's insensitivity to risk is well known (Anderson, Dillon and Hardaker; Lambert). Some of the problem arises from the all or nothing sales patterns discussed below. Relatively large changes in the value of the objective function may be required before significant basis changes occur.

Another contributing factor to the model's insensitivity to risk may be the small risk premia found in the marketing type discrete stochastic programming models. Lambert found the risk premium in his wheat marketing problem to be about 0.2 percent of the expected monetary value of income. Citing Tsiang's observation that if risk were small relative to total wealth, the influence of higher order moments of the uncertain outcome distributions are relatively unimportant to the problem's solution. These results are consistent with Anderson et al.'s observation (page 229) that it may not be important to account for nonlinear risk preferences in discrete stochastic programming models.

2. All or nothing sales

There was no diversification of sales over time in the sequential model. Either no animals were sold at a marketing node or all animals were sold. This may be expected in the risk neutral case. Consider figure 1. Assume decision $X_A$ is to sell all animals in the first period and $X_A$ is to sell no animals in the period. Assume further that A is the monetary outcome of $X_A$ and EA is the outcome of $X_A$. The marketing decision in period A is then to choose the action yielding the highest reward (or expected reward). Assuming no lumpiness or economies in sales of different lot sizes, there will be no value c such that $cA + E((1-c)A)$ is greater than the maximum of A or $E\bar{A}$. The optimal action will thus be either $X_A$ or $X_{\bar{A}}$.

As discussed in the preceding section, discrete stochastic programming model solutions are relatively insensitive to risk considerations. Due to the price and production assumptions in the current model, no change occurred in the model solutions as the hypothesized decision maker became more risk averse. The all or nothing nature of the risk neutral model did not change.

However, some diversification has been observed (Lambert) in other discrete stochastic programming models. Specifically, all or nothing sales patterns were observed in a wheat marketing model for neutral to moderate degrees of risk aversion. At a certain point, sales are shifted to the first period for a small range of increasing risk aversion, until all sales are made in the first period.

Arguments similar to those regarding the risk neutral decision rule seem to fit the nonlinear utility model as well. Assume the outcomes of $X_A$ and $X_{\bar{A}}$ are $U(A)$ and $EU(\bar{A})$. Comparisons are made at each decision node comparing $U(A)$ and $EU(\bar{A})$. Since von Neumann-Morgenstern utility functions are cardinal in the sense that they are defined up to an increasing linear transformation, one of the actions $X_A$ or $X_{\bar{A}}$ will dominate. Yet to be proven are the circumstances under which $U(cA) + EU((1-c)\bar{A}) \geq \max(U(A), EU(\bar{A}))$.

## G. Concluding statements

The value of the discrete stochastic programming model model as on approximation to the real world decision environment is hard to refute. Discrete events represent the decision maker's expectations of the future. Alternative optimal strategies are derived in the model contingent upon an event's occurrence. The structure of the model allows a large number of alternative future states limited only by computational capabilities and the analyst's ability to interpret the results for the decision maker. Even the computational difficulties can be overcome by decomposition techniques (Birge).

A decision problem common to many cow/calf producers was modeled in this paper: given prevailing input costs and expected output prices, should some or all weaned calves be retained in the fall? If so, to what rate of gain should they be fed over the winter and should they then be sold or placed on rangeland for additional gains over the coming summer?

In the optimal solution, all calves were retained and fed to about one kilogram over the winter. Dependent upon the observed price in the spring, calves were either placed on range or sold.

The sequential model was compared to a single period formulation of the same problem using expected prices rather than several possible prices. Although input decisions and incomes under six of the nine states of nature were similar between the two models, the use of parameter information as it became available substantially improved income under two of the three worse price scenarios.

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## Appendix I

Derivation of the Kuhn-Tucker Conditions for the following problem:

$$[1] \quad \max \quad EU(Y) = \sum_{i=1}^{4} Pr(\Theta_i)\, U(Y_i)$$

subject to:

$$[2] \quad \begin{aligned} &Q^1 (X_A, X_{B1}, X_{C11}) - Y_1 = 0 \\ &Q^2 (X_A, X_{B1}, X_{C12}) - Y_2 = 0 \\ &Q^3 (X_A, X_{B2}, X_{C21}) - Y_3 = 0 \\ &Q^4 (X_A, X_{B2}, X_{C22}) - Y_4 = 0 \end{aligned}$$

$$[3] \quad \begin{aligned} &g^1 (X_A, X_{B1}, X_{C11}) \le 0 \\ &g^2 (X_A, X_{B1}, X_{C12}) \le 0 \\ &g^3 (X_A, X_{B2}, X_{C21}) \le 0 \\ &g^4 (X_A, X_{B2}, X_{C22}) \le 0 \end{aligned}$$

forming the Lagrangian,

$$L = EU(Y) + \mu(Y - Q) + \tau(-G)$$

where, for simplicity, Y-Q represents [2], G represents [3] and EU(Y) represents [1].

Differentiating with respect to:

$Y_i$:

$$= \frac{\partial L}{\partial Y_i} = Pr(\Theta_i) \frac{\partial U(Y_i)}{\partial Y_i} + \mu_i = 0$$

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$X_A$:

$$= \frac{\partial L}{\partial X_A} = \sum_{i=1}^{4} Pr(\Theta_i) \frac{\partial U}{\partial Y_i} \frac{\partial Y_i}{\partial X_A}$$

$$+ \sum_{i=1}^{4} \mu_i \left(\frac{\partial Y_i}{\partial X_A} - \frac{\partial Q^i}{\partial X_A}\right)$$

$$- \sum_{i-1}^{4} \tau_i \frac{\partial g^i}{\partial X_A} \leq 0$$

$$= \sum_{i=1}^{4} Pr(\Theta_i) \frac{\partial U}{\partial Y_i} \cdot \frac{\partial Y_i}{\partial X_A} - \sum_{i=1}^{4} \tau_i \frac{\partial g^i}{\partial X_A} \leq 0$$

and

$$\left(\frac{\partial L}{\partial X_A}\right) X_A = 0$$

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$X_{B_j}$:

$$\frac{\partial L}{\partial X_{B_j}} = \sum_{i=k}^{l} \frac{\partial U}{\partial Y_i} \frac{\partial Y_i}{\partial X_{B_j}}$$

$$+ \sum_{i=k}^{l} \mu_i \left(\frac{\partial Y_i}{\partial X_{B_j}} - \frac{\partial Q^i}{\partial X_{B_j}}\right) - \sum_{i=k}^{l} \tau_i \frac{\partial g^i}{\partial X_{B_j}} \leq 0$$

$$= \sum_{i=k}^{l} \frac{\partial U}{\partial Y_i} \frac{\partial Y_i}{\partial X_{B_j}} - \sum_{i=k}^{l} \tau_i \frac{\partial g^i}{\partial X_{B_j}} \leq 0$$

and

$$\left(\frac{\partial L}{\partial X_{B_j}}\right) X_{B_j} = 0$$

If j = 1, [k,l] = [1,2]

else if j = 2, [k,l] = [3,4]

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$X_{C_{jm}}$:

$$\frac{\partial L}{\partial X_{C_{jm}}} = \frac{\partial U}{\partial Y_n} \frac{\partial Y_n}{\partial X_{C_{jm}}}$$

$$+ \mu_n \left(\frac{\partial Y_n}{\partial X_{C_{jm}}} - \frac{\partial Q^n}{\partial X_{C_{jm}}}\right)$$

$$- \tau_n \frac{\partial g^n}{\partial X_{C_{jm}}} \leq 0$$

$$= \frac{\partial U}{\partial Y_n} \frac{\partial Y_n}{\partial X_{C_{jm}}} - \tau_n \frac{\partial g^n}{\partial X_{C_{jm}}} \leq 0$$

and

$$\left(\frac{\partial L}{\partial X_{C_{jm}}}\right) X_{C_{jm}} = 0$$

If [jm] = [1 1], then n = 1

[1 2], then n = 2

[2 1], then n = 3

[2 2], then n = 4

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$\mu_n$:

$$\frac{\partial L}{\partial \mu_n} = Q^n - Y_n = 0 \qquad n = 1, \ldots, 4$$

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$\tau_n$:

$$\frac{\partial L}{\partial \tau_n} = g^n \leq 0$$

and

$$\left(\frac{\partial L}{\partial \tau_n}\right) \cdot \tau_n = 0 \qquad n = 1, \ldots, 4$$

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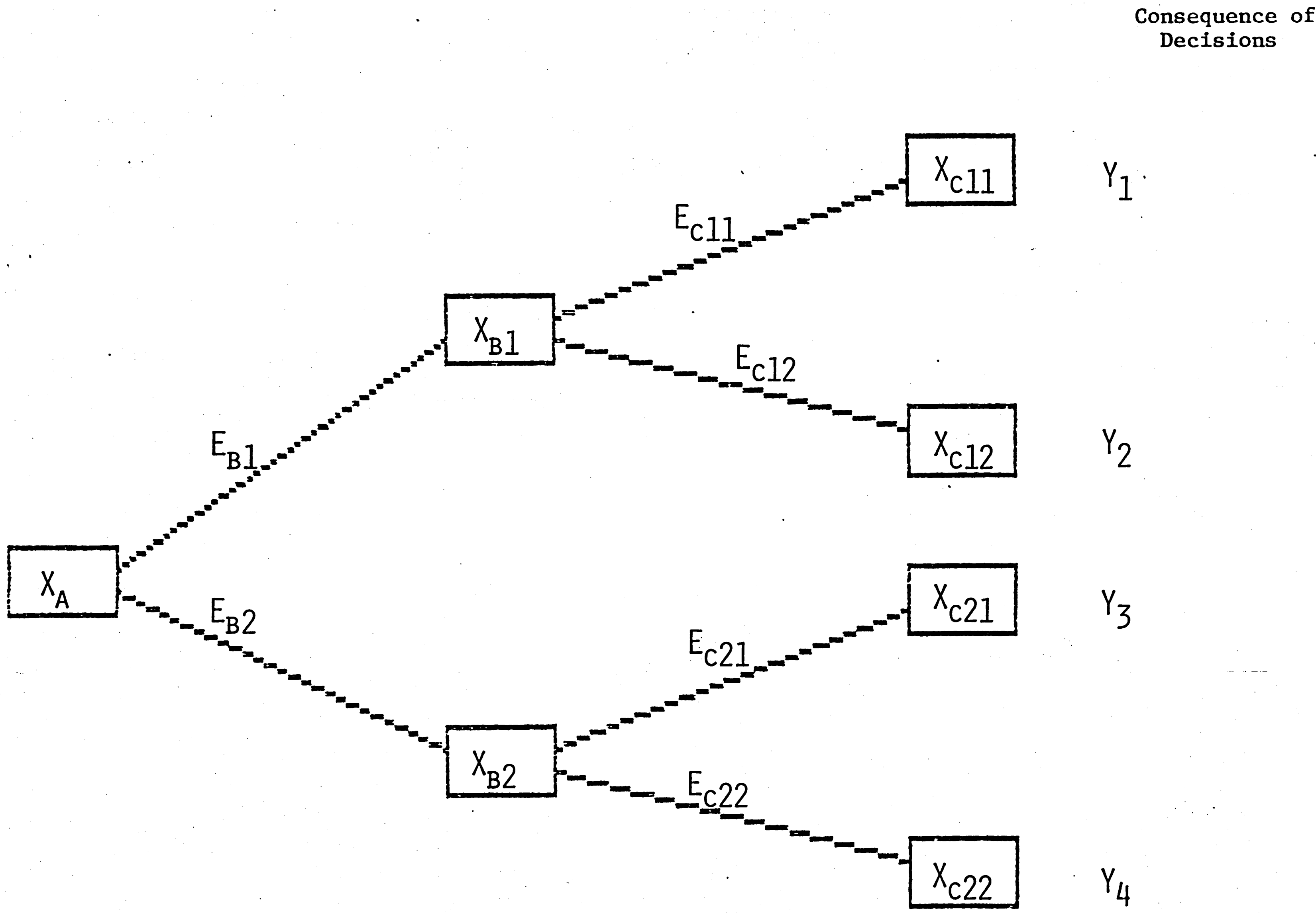


Figure 1. Discrete formulation of the sequential decision problem. $X_{ij}$ refers to decisions made in time period i given state of nature j.

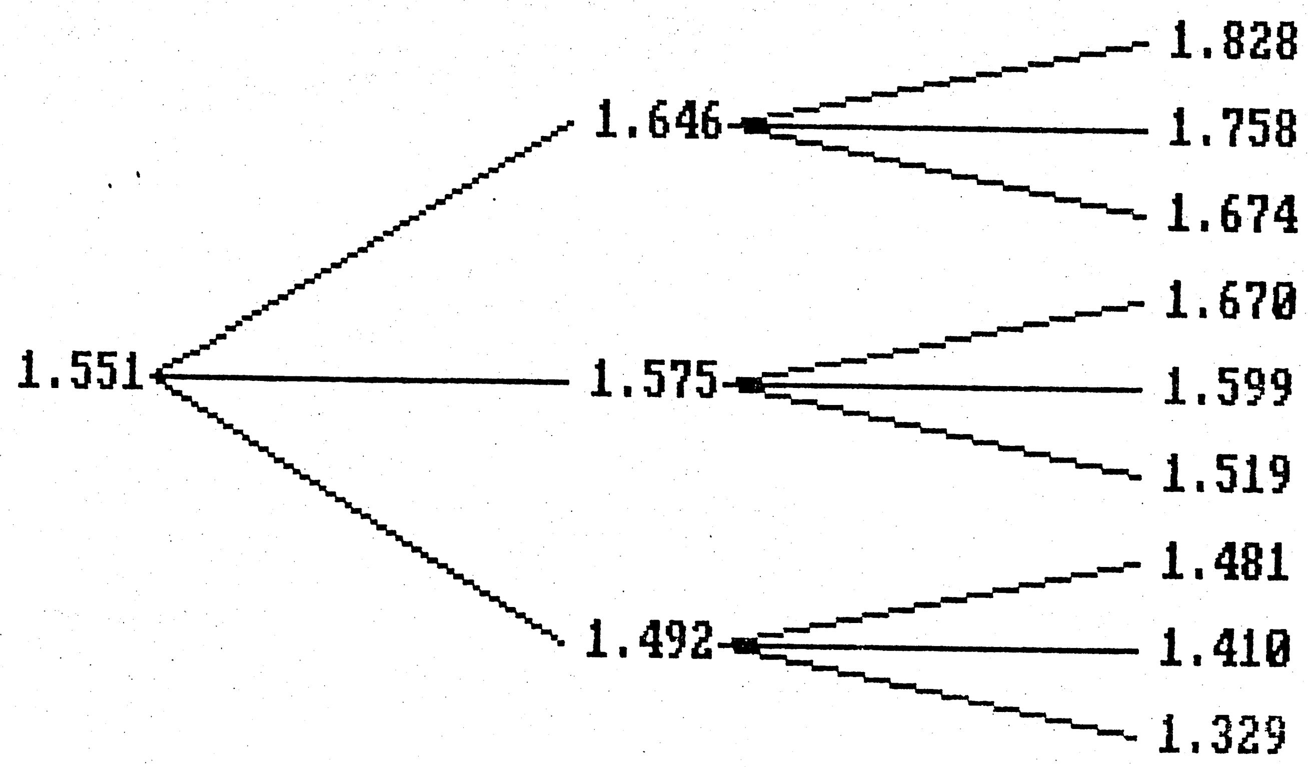


Figure 2. Price states for 200 Kg steer used in the sequential model (dollars per Kg).

Table I. Results of the discrete stochastic programming problem.

| Dec. | Jan. | Feb. | Mar. | Apr. | Summer |
|---|---|---|---|---|---|
| **Average Daily Gain** | | | | | |
| 0.871 | 0.909 | 0.973 | 1.056 | 1.114 | 0.658 |
| **Daily Ration (As fed basis)** | | | | | |
| **Oat Hay** | | | | | |
| 0.593 | 0.651 | 0.710 | 0.772 | 0.835 | --- |
| **Corn** | | | | | |
| --- | --- | 0.319 | 0.961 | 1.549 | --- |
| **Rice-Bran** | | | | | |
| 5.337 | 5.860 | 6.086 | 6.014 | 6.016 | --- |
| **Total** | | | | | |
| 5.930 | 6.511 | 7.115 | 7.747 | 8.400 | --- |
| **Feed Conversion Ratio (Dry matter basis)** | | | | | |
| 6.195 | 6.518 | 6.644 | 6.649 | 6.821 | --- |
| **Ending Weights** | | | | | |
| 226.6 | 254.3 | 284.0 | 316.2 | 350.1 | 448.9 |

Table 2. Net income by period under all states of nature.

| State of Nature | Period A | Period B | Period C |
|---|---|---|---|
| 1 | 0 | 0 | $53,703 |
| 2 | 0 | 0 | $50,561 |
| 3 | 0 | 0 | $46,790 |
| 4 | 0 | 0 | $46,610 |
| 5 | 0 | 0 | $43,423 |
| 6 | 0 | 0 | $39,698 |
| 7 | 0 | $36,390 | 0 |
| 8 | 0 | $36,390 | 0 |
| 9 | 0 | $36,390 | 0 |

Table III. Results of the single period programming problem.

| | Dec. | Jan. | Feb. | Mar. | Apr. | Summer |
|---|---|---|---|---|---|---|
| **Average Daily Gain** | | | | | | |
| | 0.839 | 0.875 | 0.903 | 0.924 | 0.937 | 0.683 |
| **Daily Ration (As fed basis)** | | | | | | |
| Fescue Hay | 0.586 | 0.641 | 0.698 | 0.753 | 0.810 | --- |
| Rice-Bran | 5.331 | 5.834 | 6.345 | 6.858 | 7.370 | --- |
| Total | 5.390 | 5.899 | 6.415 | 6.934 | 7.452 | --- |
| **Feed Conversion Ratio (Dry matter basis)** | | | | | | |
| | 6.424 | 6.742 | 7.104 | 7.504 | 7.953 | --- |
| **Ending Weights** | | | | | | |
| | 225.6 | 252.3 | 279.8 | 308.0 | 336.6 | 439.1 |

Table IV. Income under the sequential states of nature for the DSPM and the single period model.

| State of Nature | Sequential Model | Single Model | Difference |
|---|---|---|---|
| 1 | $53,703 | $53,637 | $66 |
| 2 | $50,561 | $50,565 | ($4) |
| 3 | $46,790 | $46,877 | ($87) |
| 4 | $46,610 | $46,701 | ($91) |
| 5 | $43,423 | $43,583 | ($160) |
| 6 | $39,698 | $39,939 | ($241) |
| 7 | $36,390 | $38,402 | ($2012) |
| 8 | $36,390 | $35,284 | $1,106 |
| 9 | $36,390 | $31,728 | $4,662 |