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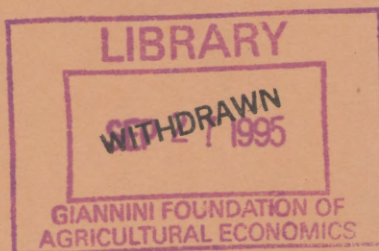
Exchange Rate Fluctuations and Commodity  
Price Instability: Simple Analytics of the  
Lognormal Model

S.M. Ravi Kanbur

Discussion Paper 78

March 1988

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## 1. Introduction and Summary

The first quarter of a century after the second world war was a period of fixed exchange rates. With the spectre of inter-war currency fluctuations and instability still haunting them, the architects of the Bretton Woods agreement on the new world financial order placed exchange rate stability high on the agenda. As is well known, the agreement held until the oil price shocks of the 1970's and the ensuing instability led to the demise of the Bretton Woods system and ushered in an era of floating exchange rates. And the extent of fluctuation has been extraordinary - with the latest rise and fall of the dollar as the leading example. What are the implications of this new world exchange rate regime for commodity price instability? Surprisingly little thought seems to have gone into this question. The object of this paper will be to broach the question and to suggest a possible framework of analysis.

The first organizing principle around which our analysis of exchange rate fluctuations and international commodity price stabilization is constructed is that while supply decisions are based on value of production in terms of the currency of the producing country, consumption decisions are based on the cost of the product in terms of the currency of the consuming country. The link between the two is the exchange rate. The second basic principle is that the exchange rate is determined by a host of factors lying well outside the particular market we are considering. Global financial flows determine the exchange rate and changes in this rate will affect the real income of farmers in the country. Given these principles, our task is to enquire how the (exogenous) fluctuations in the exchange rate will influence commodity price instability.

The plan of the paper is as follows. Section 2 sets out the basic model. We have chosen the simplest possible representation in this first cut. After describing the equilibrium in Section 2, in Section 3 we conduct some basic comparative static exercises on the effects of exchange rate on prices and on earnings. Throughout, we are concerned with prices and earnings both in terms of the foreign currency and in terms of the local currency. Sections 4 and 5 moves on to an analysis of the effects of exchange rate instability on the trend and instability of prices and earnings. In this part of the paper supply shocks are suppressed in order to focus attention on the exchange rate. But in Section 6 we analyze the interesting case of fluctuations in both supply and exchange rate. As might be expected, the correlation between these two turns out to be an important determinant of price and earnings instability. Section 7 traces out, briefly, some implications of the foregoing analysis for commodity price stabilization schemes and their impact on earnings instability in the presence of exchange rate fluctuations. Section 8 concludes the paper with suggestions for further research.

The main results of the paper are stated in terms of a series of propositions. While the mathematical and intuitive reasoning behind them, and the context of the specific model from which they are derived, is detailed in the text, we draw together here the propositions in summary form for easy reference:

Proposition 1: An appreciation (depreciation) of the dollar relative to the local currency will lower (raise) the world dollar price of the commodity.

Proposition 2: An appreciation (depreciation) of the dollar relative to the local currency will raise (lower) the local currency price of the commodity.

Proposition 3: An appreciation of the dollar relative to the local currency will raise (lower) dollar earnings from the commodity if elasticity of demand is greater than (less than) unity.

Proposition 4: An appreciation (depreciation) of the dollar raises (lowers) the earnings from the commodity in terms of the local currency.

Proposition 5: Increased variability in the local currency/dollar exchange rate will increase the trend dollar price of the commodity.

Proposition 6: Increased variability in the local currency/dollar exchange rate will decrease the trend price of the commodity in domestic currency terms.

Proposition 7: If the elasticity of demand is less than unity then increased variability in the domestic currency/dollar exchange rate will increase the trend value of dollar earnings.

Proposition 8: If the elasticity of demand is less than unity then increased variability in the domestic currency/dollar exchange rate will decrease the trend value of commodity earnings in domestic currency terms.

Proposition 9: Increased (decreased) variability in the exchange rate leads to increased (decreased) variability in farmers' earnings in domestic currency terms.

Proposition 10: Increased (decreased) exchange rate variability leads to increased (decreased) dollar earnings variability.

Proposition 11: Increased (decreased) exchange rate variability leads to increased (decreased) variability in the dollar price of the commodity.

Proposition 12: Increased (decreased) exchange rate variability leads to increased (decreased) variability in the domestic currency price of the commodity.

Proposition 13: A positive (negative) correlation between supply shocks and fluctuations in the domestic currency/dollar exchange rate leads, ceteris paribus, to greater (lesser) instability in the world dollar price of the commodity.

Proposition 14: A positive (negative) correlation between supply shocks and fluctuations in the domestic currency/dollar exchange rate leads, ceteris paribus, to lesser (greater) instability in the domestic currency price of the commodity.

Proposition 15: A positive (negative) correlation between supply shocks and domestic currency/dollar exchange rate shocks will, ceteris paribus, increase (decrease) the variability of dollar earnings.

Proposition 16: If the elasticity of demand is less than unity then positive (negative) correlation between supply shocks and the domestic currency/dollar exchange rate movements will, ceteris paribus, reduce (increase) domestic currency earnings instability.

Proposition 17: If the correlation between the domestic currency/dollar exchange rate and supply shocks is zero or positive, then  $\epsilon > \frac{1-\gamma}{2}$  is no longer a necessary condition for price stabilization to stabilize earnings. A much weaker condition then holds.



## 2. The Basic Model

As noted in the introduction, we have in mind a partial equilibrium setting in which demand for a commodity is determined by its dollar price in world markets, but supply is determined by local currency price. What is relevant to local farmers is not their dollar earnings but their earnings in local currency. The assumption is thus that farmers' consumption is primarily of goods whose price is given in local currency terms. In a West African context, for example, the local currency is often pegged to the French Franc. Changes in the Dollar/Franc rate, for example, will change the Dollar/local currency rate. Put another way, if most of the purchases of the producing country are in terms of Francs, then a change in the Dollar/Franc rate will be like a change in its terms of trade.

Let demand be a function of commodity price in dollars as follows

$$Q^D = p_{\$}^{-\varepsilon} \quad (2.1)$$

where  $Q^D$  is demand,  $p_{\$}$  is price in dollars and  $\varepsilon$  is the elasticity of demand. The demand function is assumed to be stable throughout this paper, although extensions to the case of demand side instability would not prove too difficult. Supply is given by

$$Q^S = \theta p^{\gamma} \quad (2.2)$$

where  $p$  is the price in local currency,  $\gamma$  is the elasticity of supply and  $\theta$  is a supply side shock which will generate instability in the commodity market. The dollar price and domestic price are connected by

$$p = rp_{\$} \quad (2.3)$$

where  $r$  is the exchange rate giving units of local currency per dollar. In later sections some of the instability in the market will be caused by instability in  $r$ . In this simple model we are assuming no government taxes on the price of the commodity - or, rather, we are assuming these to be proportional to  $p$  and to be constant. Then it should be clear that the presence of such taxes will not affect the analysis. However, an interesting question for further research (see section 8) is the extent to which such taxes could be used to insulate farmers from global exchange rate fluctuations.

Supply and demand equilibrium in this commodity market is given by

$$p_{\$}^{-\varepsilon} = \theta(rp_{\$})^{\gamma} \quad (2.4)$$

Solving (2.4) for  $p_{\$}$  gives us

$$p_{\$} = r \frac{-\gamma}{\gamma+\varepsilon} \theta^{-\frac{1}{\gamma+\varepsilon}} \quad (2.5)$$

and

$$p_{\$} = rp = r \frac{\varepsilon}{\gamma+\varepsilon} \theta^{-\frac{1}{\gamma+\varepsilon}} \quad (2.6)$$

The quantity traded is

$$Q = r \frac{\gamma\varepsilon}{\gamma+\varepsilon} \theta^{\frac{\varepsilon}{\gamma+\varepsilon}} \quad (2.7)$$

and farmers' earnings in dollars are thus

$$y_{\$} = p_{\$}Q = r \frac{Y(\varepsilon-1)}{Y+\varepsilon} \theta \frac{\varepsilon-1}{Y+\varepsilon} \quad (2.8)$$

However, our assumption is that farmers are interested not in the dollar value of their earnings, but in the local currency value of income. This is given by

$$y = ry_{\$} = r \frac{\varepsilon(Y+1)}{Y+\varepsilon} \theta \frac{\varepsilon-1}{Y+\varepsilon} \quad (2.9)$$

This completes the statement of the basic model and the description of equilibrium. As can be seen, in equilibrium prices and earnings depend on the exchange rate. The next section will begin the investigation of this relationship.

### 3. Exchange Rate Changes, Prices and Earnings

For the moment, let us ignore the supply shock  $\theta$  (which is taken up in Section 6), and ask the question: how does a change in  $r$  affect prices and earnings. From (2.5) we see that  $p_{\$}$  is a decreasing function of  $r$ , and we have our first proposition:

Proposition 1: An appreciation (depreciation) of the dollar relative to the local currency will lower (raise) the world dollar price of the commodity.

From (2.6), however, we see that  $p$  is an increasing function of  $r$ :

Proposition 2: An appreciation (depreciation) of the dollar relative to the local currency will raise (lower) the local currency price of the commodity.

Both propositions rely of course on strong ceteris paribus assumptions. But given these the intuitions behind the propositions are clear. At the old equilibrium price an appreciation of the dollar will increase price in domestic currency terms and hence will induce a movement along the supply curve in  $(p,Q)$  space. But, as shown in Figure 3.1, this constitutes a rightward shift of the supply curve in  $(p_s,Q)$  space. This will lower the dollar price on world markets, but, given the constant elasticity formulation, not by so much as to overturn the initial effect on domestic currency price. Thus the dollar price will fall while the domestic currency price will rise.

What about the effect on earnings? It is well known in the standard literature on commodity price stabilization that the effect of an increase in supply on earnings will depend on the elasticity of demand, in particular whether it is greater or lesser than unity. Equation (2.8) tells us that a similar proposition is true for the effect of exchange rate changes on dollar earnings from the commodity:

Proposition 3: An appreciation of the dollar relative to the local currency will raise (lower) dollar earnings from the commodity if elasticity of demand is greater than (less than) unity.

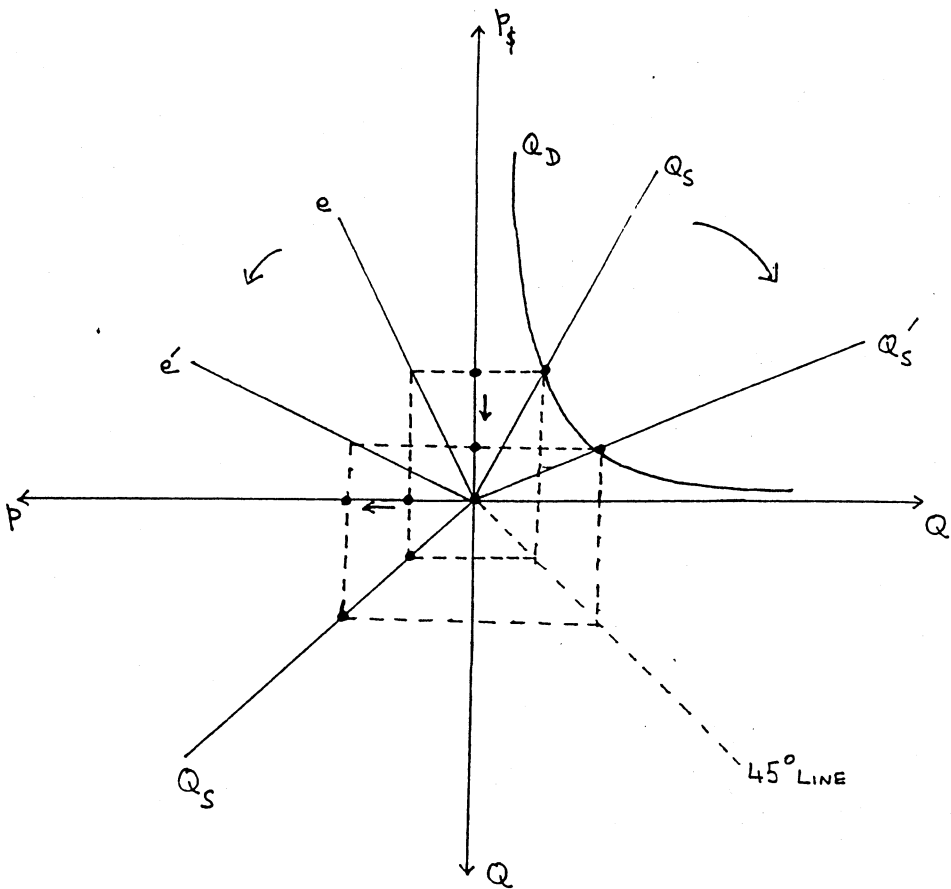
The converse holds true for a depreciation of the dollar. Hence, exchange rate changes act like supply shocks so far as the dollar earnings from the commodity are concerned. However, (2.9) shows that

there is always a positive relationship between  $r$  and  $y$ , the domestic currency value of dollar earnings:

Proposition 4: An appreciation (depreciation) of the dollar raises (lowers) the earnings from the commodity in terms of the local currency.

An appreciation of the dollar increases supply and this will lower the dollar price. Dollar earnings will increase if demand is elastic, and the effect on domestic currency earnings is reinforced by dollar appreciation. Dollar earnings will fall if demand is inelastic (as is the case for most agricultural commodities) but always by proportionately less than the dollar appreciation - hence the increase in earnings in domestic currency terms.

Figure 3.1



#### 4. Exchange Rate Instability and the Trend of Prices and Earnings

So far we have only considered deterministic changes in the exchange rate  $r$ . This section begins the extension of the analysis to the case where the exchange rate is unstable. We will ask how changes in the extent of instability affect the trend (average value) of prices and earnings, holding other things constant. Joint instability of supply shocks and exchange rate is taken up in section 6. In the first part of this section we will consider instability  $r$ , the value of the domestic currency in terms of the dollar. The relationship of this to the exchange rate defined the other way round, *i.e.*, dollars in terms of domestic currency, is considered later on in this section.

The relationship between  $p_s$  and  $r$  in (2.5) is depicted in Figure 4.1. As can be seen,  $p_s$  is a convex function of  $r$  - as  $r$  increases  $p_s$  falls, but at a slower and slower rate. Consider now an unstable  $r$  with two values  $r^H$  and  $r^L$  satisfying

$$\bar{r} = \frac{1}{2} r^H + \frac{1}{2} r^L \quad (4.1)$$

We can interpret the situation as being one where the high value  $r^H$  occurs with probability one half and the low value  $r^L$  occurs with probability one half, versus a situation where  $r$  stays fixed at  $\bar{r}$ . What impact will the increased instability of exchange rate have on the dollar price of this commodity?

As can be seen from Figure 4.1, the values of  $p_s$  when  $r$  is fixed at  $\bar{r}$ ,  $\bar{p}_s$  is lower than the average value when  $r$  fluctuates,  $\bar{p}_s$ . Mathematically,  $p_s$  is a convex function of  $r$  and a mean preserving spread in  $r$  will increase the mean of  $p_s$ :

Proposition 5: Increased variability in the local currency/dollar exchange rate will increase the trend dollar price of the commodity.

However, from (2.6) and as depicted in Figure 4.2,  $p$  is a concave function of  $r$ , and a mean preserving spread in  $r$  will decrease the mean of  $p$ :

Proposition 6: Increased variability in the local currency/dollar exchange rate will decrease the trend price of the commodity in domestic currency terms.

The intuition behind these propositions is as follows. An appreciation in the dollar increases domestic supply and hence reduces the world dollar price. But further increases in supply reduce the price by less and less, because of the (convex) shape of the demand curve. Hence successive appreciations reduce the dollar price less and less, with the result that a combination of high  $r$  followed by low  $r$  gives a higher average value of  $p_s$  than if the  $r$  had stayed fixed at an intermediate value. However, so far as  $p$  is concerned the higher value of domestic currency has to be balanced against the lower dollar price. If supply was inelastic ( $\gamma = 0$ ) then the latter factor is suppressed and all the gains from dollar appreciation are translated into domestic currency terms. To the extent that supply is elastic, this effect is cancelled out, but by more and more for successive appreciations of the dollar. Hence the result.

Turning now to the effects of exchange rate instability on the trend value of earnings, from (2.8) and (2.9) we can show the following:



Proposition 7: If the elasticity of demand is less than unity then increased variability in the domestic currency/dollar exchange rate will increase the trend value of dollar earnings.

Proposition 8: If the elasticity of demand is less than unity then increased variability in the domestic currency/dollar exchange rate will decrease the trend value of commodity earnings in domestic currency terms.

If (implausibly) the elasticity of demand for our agricultural commodity is less than one, then the above two propositions would be reversed. The mathematical basis of the two propositions again lies in the fact that while  $y_\$$  is a convex function of  $r$ ,  $y$  is a concave function of  $r$ , and the effects of a mean preserving spread in  $r$  on the mean values of  $y$  and  $y_\$$  therefore have opposite signs.

We should now point to a seeming paradox. If instead of the domestic currency/dollar exchange rate we took its inverse i.e. the dollar/domestic currency exchange rate, the above propositions would be reversed. Let

$$r_* = \frac{1}{r} \quad (4.2)$$

Then (2.5), (2.6), (2.8) and (2.9) become, respectively,

$$p_\$ = r_* \frac{\frac{y}{y+\varepsilon}}{\theta} - \frac{1}{y+\varepsilon} \quad (4.3)$$

$$p = r_* \frac{-\frac{\varepsilon}{y+\varepsilon}}{\theta} - \frac{1}{y+\varepsilon} \quad (4.4)$$

$$y_{\xi} = r_{*} \quad - \frac{\gamma(\varepsilon-1)}{\gamma+\varepsilon} \quad - \frac{(\varepsilon-1)}{\gamma+\varepsilon} \quad \theta \quad (4.5)$$

$$y = r_{*} \quad - \frac{\varepsilon(\gamma+1)}{\gamma+\varepsilon} \quad - \frac{\varepsilon-1}{\gamma-\varepsilon} \quad \theta \quad (4.6)$$

Since  $r_{*}$  is the inverse of  $r$ ,  $p_{\xi}$  and  $y_{\xi}$ , which are convex in  $r$ , are concave in  $r_{*}$ . Similarly,  $p$  and  $y$ , which are concave in  $r$ , are convex in  $r_{*}$ . Thus propositions 5-8 would be reversed if we are talking about increased variability in  $r_{*}$ .

The "problem" arises because there is not a unique way of measuring the variability of the exchange rate - we could use a mean preserving spread in  $r$  or in  $r^{*}$ . But since  $r_{*}$  is itself a convex function of  $r$  ( $r_{*} = \frac{1}{r}$ ), and since  $r$  is a convex function of  $r_{*}$  ( $r = \frac{1}{r_{*}}$ ), a mean preserving spread in one is not a mean preserving spread in the other. In fact, a mean preserving spread in one is bound to increase the mean of the other, given the convexity of the relationship between them. This fact was pointed out by Diamond and Stiglitz (1974). The propositions as stated are correct, but we have to be careful to distinguish between increased variability in  $r$  and  $r_{*}$ .

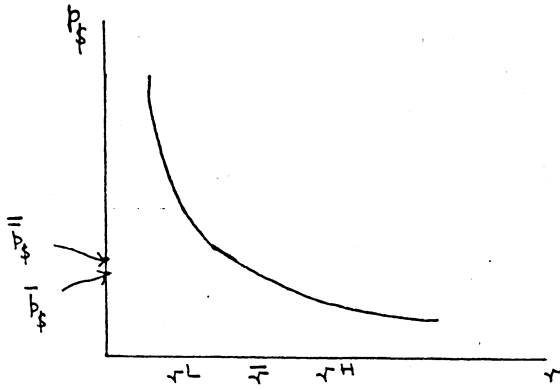


Figure 4.1

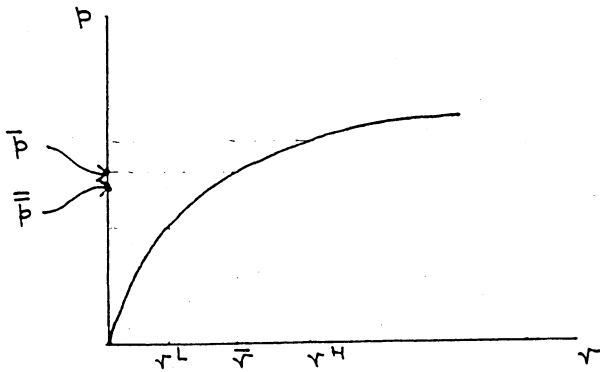


Figure 4.2

## 5. Exchange Rate Instability and the Instability of Prices and Earnings

If two economic variables are linked by a functional relationship, then we can, in principle, measure the impact of instability in one variable on instability in the other variable. An immediate question is the measure of instability we are to use. In general a mean preserving spread in a variable will change the mean of the other variable (unless the relationship between them is linear) and we will have to take account of this change in measuring the induced change in instability. Moreover, linking the distribution of one variable to the distribution of another is in general a difficult task, depending both on the functional relationship between the two variables and on the exact distribution of the original disturbance. However, matters are simplified considerably in the so called "lognormal model" where the originating distribution of disturbances is lognormal, and functional relationships are of the constant elasticity form. Before proceeding to apply this lognormal model to our problem, we will first of all recall some extremely useful properties of the lognormal distribution.

If a random variable  $X$  is such that its logarithm,  $\log X$ , is normally distributed with mean  $\mu$  and variance  $\sigma^2$ , then we say that  $X$  is lognormally distributed with parameters  $\mu$  and  $\sigma^2$ . This is sometimes written (see Aitchison and Brown, 1957):

$$X \sim \Lambda (\mu, \sigma^2) \quad (5.1)$$

The most interesting and important property of the lognormal distribution from our point of view is that if  $X$  is lognormal with parameters  $\mu$  and  $\sigma^2$ , then  $aX^b$  is also lognormal, with parameters  $\mu + \log a$  and  $b^2\sigma^2$ :

$$aX^b \sim \Lambda (b\mu + \log a, b^2\sigma^2) \quad (5.2)$$

The property follows directly by noting that  $\log (aX^b) = \log a + b \log X$ , and from the property of the normal distribution we get that if  $\log X$  is normal with mean  $\mu$  and variance  $\sigma^2$ , then  $\log a + b \log X$  is normal with mean  $\log a + b\mu$  and variance  $b^2\sigma^2$ .

Now  $\mu$  and  $\sigma^2$  are not, respectively, the mean and variance of  $X$ . It can be shown that (see Aitchison and Brown, 1957)

$$E(X) = e^{\mu + \frac{1}{2}\sigma^2} \quad (5.3)$$

Thus to find the mean of a lognormal variable we add the first parameter to half the second, and raise the exponential factor  $e$  to this power. In particular, therefore, a change in  $\sigma^2$  will change  $E(X)$ . What about the variance of  $X$ ? For this we use, from (5.2),

$$X^2 \sim \Lambda (2\mu, 4\sigma^2) \quad (5.4)$$

so that

$$E(X^2) = e^{2\mu + 2\sigma^2} \quad (5.5)$$

and

$$V(X) = E(X^2) - (E(X))^2 = e^{2\mu+2\sigma^2} - e^{2\mu+\sigma^2}$$

Now the square of the coefficient of variation of X is one candidate for a mean independent measure of variability:

$$c_x^2 = \frac{V(X)}{(E(X))^2} = e^{\sigma^2-1} \quad (5.6)$$

It follows, therefore, that in the lognormal case  $\sigma^2$  and  $c_x^2$  are monotonically related. In fact, using the fact that

$$e^{\sigma^2} = 1 + \sigma^2 + \frac{(\sigma^2)^2}{2!} + \frac{(\sigma^2)^3}{3!} + \dots$$

and ignoring terms in  $(\sigma^2)^2$  and above, we get that approximately

$$c_x^2 \sim \sigma^2 \quad (5.7)$$

The relationship in (5.6) and the approximation in (5.7) will be our justification for using, in what follows,  $\sigma^2$  as a measure of variability in the lognormal framework.

Let us start, then, with an investigation of the effects of exchange rate variability on earnings variability in domestic currency terms. For the moment we continue to suppress supply side shocks, whose interaction with exchange rate instability is taken up in the next section. In order to keep the manipulations simple, we set  $\theta = 1$ . Then from (2.9) it is seen that if

$$r \sim \Lambda(\mu_r, \sigma_r^2) \quad (5.8)$$

then

$$y \sim \Lambda (\mu_y, \sigma_y^2) \quad (5.9)$$

where

$$\mu_y = \frac{\varepsilon(\gamma+1)}{\gamma+\varepsilon} \mu_r \quad (5.10)$$

$$\sigma_y^2 = \frac{\varepsilon^2(\gamma+1)^2}{(\gamma+\varepsilon)^2} \sigma_r^2 \quad (5.11)$$

From (5.11) it is clear that

$$\frac{d\sigma_y^2}{d\sigma_r^2} = \frac{\varepsilon^2(\gamma+1)^2}{(\gamma+\varepsilon)^2} > 0 \quad (5.12)$$

and this provides us with the first proposition of this section:

Proposition 9: Increased (decreased) variability in the exchange rate leads to increased (decreased) variability in farmers' earnings in domestic currency terms.

This is an important proposition, establishing directly the link between a measure of exchange rate instability and a measure of income instability. The intuition behind it is that in this model exchange rate fluctuations act rather like supply shocks. Greater variability here will thus feed into greater variability in earnings. In fact, it can be shown that Proposition 9 is immune from measuring the exchange rate by  $r_x$  rather than  $r$ . Since

$$r_* = r^{-1}$$

$$r_* \sim \Lambda (\mu_{r_*}, \sigma_{r_*}^2)$$

where  $\mu_{r_*} = -\mu_r$

$$\sigma_{r_*}^2 = \sigma_r^2 \quad (5.13)$$

so that if  $\sigma^2$  is used the measure of instability then Proposition 9 holds for instability in  $r_*$  as well as  $r$ .

However, it could be argued that we should look not at  $\sigma_y^2$  but directly at farmers' welfare via a risk averse utility function. If we let

$$U(y) = \begin{cases} \frac{y^{1-R}}{1-R} & ; \quad R \neq 1 \\ \log y & ; \quad R = 1 \end{cases} \quad (5.14)$$

be the representative farmer's von-Neumann/Morgenstern utility function, then

$$R = - \frac{yU''(y)}{U'(y)} \quad (5.15)$$

is the Arrow-Pratt measure of relative risk aversion. To calculate expected utility we use the fact that

$$\frac{y^{1-R}}{1-R} \sim \Lambda ((1-R)\mu_y + \log(1-R) ; (1-R)^2 \sigma_y^2) \quad (5.16)$$



and that

$$\log y \sim N(\mu_y, \sigma_y^2) \quad (5.17)$$

where  $N$  represents the normal distribution. Thus

$$W_1 = E(\log y) = \mu_y = \frac{\varepsilon(\gamma+1)}{\gamma+\varepsilon} \mu_r \quad (5.18)$$

and this does not depend on  $\sigma_r^2$  at all. Thus if relative risk aversion is unity ( $R = 1$ ), then changes in  $\sigma_r^2$  do not affect welfare but they do affect instability of income in domestic currency as measured by  $\sigma_y^2$ .

If  $R \neq 1$ , then

$$\begin{aligned} W_R &= E\left(\frac{y^{1-R}}{1-R}\right) = \exp[(1-R)\mu_y + \log(1-R) + \frac{1}{2}(1-R)^2\sigma_y^2] \\ &= \exp[(1-R)\frac{\varepsilon(\gamma+1)}{\gamma+\varepsilon}\mu_r + \log(1-R) + \frac{1}{2}(1-R)^2\frac{\varepsilon^2(\gamma+1)^2}{(\gamma+\varepsilon)}\sigma_r^2] \end{aligned} \quad (5.19)$$

In this case there is a positive relationship between welfare and  $\sigma_r^2$ . Although the higher  $\sigma_r^2$  induces higher instability in  $y$ , it also increases the mean of  $r$  and hence  $y$ , and by so much that welfare ends up being higher.

To suppress the "mean" effect, consider a change in  $\sigma_r^2$  which leaves the mean of  $r$  constant. Since

$$E(r) = e^{\mu_r + \frac{1}{2}\sigma_r^2}$$

what we need is that when  $\sigma_r^2$  changes,  $\mu_r$  change by

$$\frac{d\mu_r}{d\sigma_r^2} = -\frac{1}{2}$$

Using this and differentiating (5.19) will give us that the sign of

$$\frac{dW_R}{d\sigma_r^2}$$

is the same as the sign of

$$\begin{aligned} & (1-R) \varepsilon \frac{(\gamma+1)}{\gamma+\varepsilon} \left( -\frac{1}{2} \right) + \frac{1}{2} (1-R)^2 \varepsilon \frac{2(\gamma+1)^2}{(\gamma+\varepsilon)^2} \\ &= \frac{1}{2} (1-R) \frac{\varepsilon(\gamma+1)}{\gamma+\varepsilon} \left[ (1-R) \frac{\varepsilon(\gamma+1)}{\gamma+\varepsilon} - 1 \right] \end{aligned} \quad (5.20)$$

Thus if  $R$  is greater than one,  $\frac{dW_R}{d\sigma_r^2} > 0$  and welfare increases. If

$0 < R < 1$ , then if  $\varepsilon < 1$ ,  $\frac{dW_R}{d\sigma_r^2} < 0$  and welfare will decrease.

In fact, for welfare evaluation with a mean preserving spread, it matters whether  $r$  or  $r_*$  is used. From (5.13)

$$E(r_*) = e^{-\mu_r + \frac{1}{2} \sigma_r^2}$$

so that the condition for a mean preserving increase in the variability of  $r_*$  is that

$$\frac{d\mu_r}{d\sigma_r^2} = \frac{1}{2}$$

Using this we get that the sign of  $\frac{dW_R}{d\sigma_r^2}$  is the same as the sign of

$$\frac{1}{2} (1-R) \frac{\varepsilon(\gamma+1)}{\gamma+\varepsilon} \left[ (1-R) \frac{\varepsilon(\gamma+1)}{\gamma+\varepsilon} + 1 \right] \quad (5.21)$$

Thus if  $0 < R < 1$  then  $\frac{dW_R}{d\sigma_r^2} > 0$  and welfare increases when  $\sigma_r^2$  increases.

If  $1 < R < 1 + \frac{\gamma+\varepsilon}{\varepsilon(\gamma+1)}$  then  $\frac{dW_R}{d\sigma_r^2} < 0$  and welfare decreases when  $\sigma_r^2$

decreases. If  $R > 1 + \frac{\gamma+\varepsilon}{\varepsilon(\gamma+1)}$  then  $\frac{dW_R}{d\sigma_r^2} > 0$  again, and welfare increases

when  $\sigma_r^2$  increases.

Thus although Proposition 9 is useful as a benchmark exercise, it is worth pointing out that the relationship between welfare and exchange rate variability is a more complicated matter. Similar complications would arise in the analysis of  $y_\S$ , although here the rationale behind use of the expected utility welfare criterion is not clear - since dollar earnings are not the relevant magnitude for farmers. Focusing purely on instability, then, from (2.8), with  $\theta = 1$ , we get

$$y_\S \sim \Lambda(\mu_{y_\S}, \sigma_{y_\S}^2)$$

where

$$\begin{aligned}\mu_{y_r} &= \frac{\gamma(\varepsilon-1)}{\gamma+\varepsilon} \mu_r \\ \sigma_{y_\$}^2 &= \frac{\gamma^2(\varepsilon-1)^2}{(\gamma+\varepsilon)^2} \sigma_r^2\end{aligned}\quad (5.22)$$

from where it follows that

$$\frac{d\sigma_{y_\$}^2}{d\sigma_r^2} = \frac{\gamma^2(\varepsilon-1)^2}{(\gamma+\varepsilon)^2} > 0 \quad (5.23)$$

Proposition 10: Increased (decreased) exchange rate variability leads to increased (decreased) dollar earnings variability.

Similarly, from (2.5) and (2.6) we can show that

$$p_\$ \sim \Lambda(\mu_{p_\$}, \sigma_{b_\$}^2)$$

where

$$\text{where } \mu_{p_\$} = -\frac{\gamma}{\gamma+\varepsilon} \mu_r \quad ; \quad \sigma_{p_\$}^2 = \frac{\gamma^2}{(\gamma+\varepsilon)^2} \sigma_r^2 \quad (5.24)$$

$$p \sim \Lambda(\mu_p, \sigma_p^2)$$

$$\text{where } \mu_p = \frac{\varepsilon}{\gamma+\varepsilon} \mu_r \quad ; \quad \sigma_p^2 = \frac{\varepsilon^2}{(\gamma+\varepsilon)^2} \sigma_r^2 \quad (5.25)$$

Thus

$$\frac{d\sigma_{p_s}^2}{d\sigma_r^2} = \frac{\gamma^2}{(\gamma+\epsilon)^2} > 0 \quad ; \quad \frac{d\sigma_p^2}{d\sigma_r^2} = \frac{\epsilon^2}{(\gamma+\epsilon)^2} > 0 \quad (5.26)$$

and we have the following Propositions:

Proposition 11: Increased (decreased) exchange rate variability leads to increased (decreased) variability in the dollar price of the commodity.

Proposition 12: Increased (decreased) exchange rate variability leads to increased (decreased) variability in the domestic currency price of the commodity.

#### 6. Joint Instability of Exchange Rate and Supply

Up to now we have focused solely on the consequences of exchange rate fluctuations. It has been seen that by themselves such fluctuations can impart instability to commodity prices and to earnings. However, in reality exchange rate fluctuations are found in conjunction with other types of instability. In particular, fluctuations in supply (because of weather conditions, for example) have always been the primary instability of interest in the existing literature. In this section we will attempt to advance the literature by considering the effects of joint instability in exchange rate and supply.

Supply instability is modeled as fluctuations in  $\theta$  in equation (2.2). Following our lognormal model, we assume that  $\theta$  and  $r$  are jointly lognormally distributed, which means that  $\log \theta$  and  $\log r$  are jointly normally distributed:

$$\log \theta, \log r \sim N(\mu_\theta, \mu_r, \sigma_\theta^2, \sigma_r^2, \rho) \quad (6.1)$$

where  $\rho$  is the correlation coefficient between  $\log \theta$  and  $\log r$ . Before proceeding with the analysis let us recall the basic properties of a bivariate normal distribution. If

$$X, Y \sim N(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$$

then

$$aX + bY \sim N(a\mu_x + b\mu_y ; a^2\sigma_x^2 + 2ab\rho\sigma_x\sigma_y + b^2\sigma_y^2) \quad (6.2)$$

In other words, a linear sum of bivariate normal variables is itself normal, with mean and variance depending on the underlying parameters of the bivariate normal distribution.

Let us start with an analysis of the impact on the world dollar price of our commodity of joint instability in supply and exchange rate. From (2.5),

$$\log p_\$ = -\frac{Y}{Y+\epsilon} \log r - \frac{1}{Y+\epsilon} \log \theta \quad (6.3)$$

Thus, using (6.1) and (6.2), we get

$$\log p_\$ \sim N\left(-\frac{Y}{Y+\epsilon} \mu_r - \frac{1}{Y+\epsilon} \mu_\theta ; \frac{Y^2}{(Y+\epsilon)^2} \sigma_r^2 + \frac{2Y}{(Y+\epsilon)^2} \rho\sigma_r\sigma_\theta + \frac{1}{(Y+\epsilon)^2} \sigma_\theta^2\right) \quad (6.4)$$

Thus we get that

$$p_{\S} \sim \Lambda (\mu_{p_{\S}}, \sigma_{p_{\S}}^2)$$

$$\text{where } \mu_{p_{\S}} = -\frac{\gamma}{\gamma+\varepsilon} \mu_r - \frac{1}{\gamma+\varepsilon} \mu_{\theta}$$

$$\sigma_{p_{\S}}^2 = \frac{\gamma^2}{(\gamma+\varepsilon)^2} \sigma_r^2 + 2 \frac{\gamma}{(\gamma+\varepsilon)^2} \rho \sigma_r \sigma_{\theta} + \frac{1}{(\gamma+\varepsilon)^2} \sigma_{\theta}^2 \quad (6.5)$$

Equation (6.5) shows us that, the correlation coefficient  $\rho$  is an important determinant of the degree of instability in  $p_{\S}$ . In fact it can be seen that:

Proposition 13: A positive (negative) correlation between supply shocks and fluctuations in the domestic currency/dollar exchange rate leads, ceteris paribus, to greater (lesser) instability in the world dollar price of the commodity.

The reasoning behind this proposition is straightforward. A positive supply shock lowers the world dollar price. Similarly, an increase in  $r$  will also shift out the supply curve in  $(p_{\S}, Q)$  space and reinforce the supply shock effect. Thus in order for these effects to mitigate each other what is needed is negative correlation between  $r$  and  $\theta$ .

Consider now the impact of greater supply instability (higher  $\sigma_{\theta}^2$ ) and greater exchange rate instability (higher  $\sigma_r^2$ ) on instability of the world dollar price ( $\sigma_{p_{\S}}^2$ ):

$$\frac{d\sigma_{p_{\S}}^2}{d\sigma_{\theta}^2} = \frac{\gamma}{(\gamma+\varepsilon)^2} \frac{\rho \sigma_r}{\sigma_{\theta}} + \frac{1}{(\gamma+\varepsilon)^2} \quad (6.6)$$

$$\frac{d\sigma_{p_s}^2}{d\sigma_r^2} = \frac{\gamma^2}{(\gamma+\epsilon)^2} + \frac{\gamma}{(\gamma+\epsilon)^2} \frac{\rho\sigma_\theta}{\sigma_r} \quad (6.7)$$

The first point to note from (6.6) and (6.7) is that if  $p = 0$  i.e.  $\log \theta$  and  $\log r$  are uncorrelated, then greater instability of either  $\theta$  or  $r$  will translate itself into greater instability of  $p_s$ . If  $\log \theta$  and  $\log r$  are positively correlated, then the effect is even stronger. However, with negative correlation between  $\log \theta$  and  $\log r$ , some interesting effects begin to happen. We have already noted that with such a negative correlation, exchange rate movements and supply shocks essentially compensate for each other to some extent. Moreover, with a sufficiently large negative correlation between  $\log \theta$  and  $\log r$ , (6.6) and (6.7) tell us that increases in supply or exchange rate instability might actually reduce  $p_s$  instability:

$$\frac{d\sigma_{p_s}^2}{d\sigma_\theta^2} \geq 0 \quad \Leftrightarrow \quad \frac{\rho\sigma_r}{\sigma_\theta} \geq -\frac{1}{\gamma} \quad (6.8)$$

$$\frac{d\sigma_{p_s}^2}{d\sigma_r^2} \geq 0 \quad \Leftrightarrow \quad \frac{\rho\sigma_\theta}{\sigma_r} \geq -\gamma \quad (6.9)$$

In fact  $\frac{\rho\sigma_r}{\sigma_\theta}$  is nothing other than the regression coefficient  $\beta$  in the regression

$$r = \alpha + \beta \theta$$



of  $r$  on  $\theta$ . Thus if  $\beta$  is negative and less than minus one over the supply elasticity, the condition in (6.8) is satisfied. A similar regression interpretation can be provided for (6.9).

Let us turn now to instability in  $p$ . From (2.6),

$$p \sim \Lambda (\mu_p, \sigma_p^2)$$

$$\text{where } \mu_p = \frac{\varepsilon}{\gamma + \varepsilon} \mu - \frac{1}{\gamma + \varepsilon} \mu$$

$$\sigma_p^2 = \frac{\varepsilon^2}{(\gamma + \varepsilon)^2} \sigma_r^2 - 2 \frac{\varepsilon}{(\gamma + \varepsilon)^2} \rho \sigma_r \sigma_\theta + \frac{1}{(\gamma + \varepsilon)^2} \sigma_\theta^2 \quad (6.10)$$

As before, if  $\rho = 0$  then greater instability in  $r$  or  $\theta$  will induce greater instability in  $p$ . However, the effects of correlation between  $r$  and  $\theta$  are exactly the opposite. For  $p_s$ , positive correlation between  $r$  and  $\theta$  led to a magnification of instability. For  $p$ , positive correlation between  $r$  and  $\theta$  leads to a reduction in instability.

Proposition 14: A positive (negative) correlation between supply shocks and fluctuations in the domestic currency/dollar exchange rate leads, ceteris paribus, to lesser (greater) instability in the domestic currency price of the commodity.

As before, a positive supply shock will reduce the dollar price and hence, at fixed exchange rate, the domestic currency price. What is needed to reduce earnings instability is an exchange rate shock to mitigate this fall. An increase in the domestic currency/dollar exchange rate will further lower the dollar price but, as shown in Section 3, it

will increase the domestic currency price - thereby compensating for the effects of the supply shock. Hence the proposition.

Similarly, one can derive conditions for greater instability in  $r$  and  $\theta$  to increase or decrease instability in  $p$ :

$$\frac{d\sigma_p^2}{d\sigma_\theta^2} = -\frac{\varepsilon}{(\gamma+\varepsilon)^2} \frac{\rho\sigma_r}{\sigma_\theta} + \frac{1}{(\gamma+\varepsilon)^2}$$

$$\geq 0 \iff \frac{1}{\varepsilon} \geq \frac{\rho\sigma_r}{\sigma_\theta} \quad (6.11)$$

$$\frac{d\sigma_p^2}{d\sigma_r^2} = \frac{\varepsilon^2}{(\gamma+\varepsilon)^2} - \frac{\varepsilon}{(\gamma+\varepsilon)^2} \frac{\rho\sigma_\theta}{\sigma_r}$$

$$\geq 0 \iff \varepsilon \geq \frac{\rho\sigma_\theta}{\sigma_r} \quad (6.12)$$

Here again, the conclusions are somewhat different from  $p_\$$ . If  $\rho \leq 0$ , then an increase in instability in  $\theta$  or  $r$  will increase instability in  $p$ . If  $\rho > 0$ , then for large enough  $\rho$  an increase in  $\theta$  or  $r$  instability may decrease instability in  $p$  - the exact conditions under which this happens being given in (6.11) and (6.12).

Let us now turn to variability in dollar earnings. From (2.8),

$$y_\$ \sim \Lambda(\mu_{y_\$}, \sigma_{y_\$}^2)$$

where

$$\mu_{y_{\xi}} = \frac{\gamma(\varepsilon-1)}{\gamma+\varepsilon} \mu_r + \frac{(\varepsilon-1)}{\gamma+\varepsilon} \mu_{\theta}$$

$$\sigma_{y_{\xi}}^2 = \frac{\gamma^2(\varepsilon+1)^2}{(\gamma+\varepsilon)^2} \sigma_r^2 + 2 \frac{\gamma(\varepsilon-1)^2}{(\gamma+\varepsilon)^2} \rho \sigma_r \sigma_{\theta} + \frac{(\varepsilon-1)^2}{(\gamma+\varepsilon)^2} \sigma_{\theta}^2 \quad (6.13)$$

From (6.13) we get the following Proposition:

Proposition 15: A positive (negative) correlation between supply shocks and domestic currency/dollar exchange rate shocks will, ceteris paribus, increase (decrease) the variability of dollar earnings.

The argument here is that in  $(p_{\xi}, Q)$  space, exchange rate shocks are rather like supply side shocks. An upward movement in the domestic currency/dollar exchange rate shifts the supply curve outwards in  $(p_{\xi}, Q)$  space, rather like a positive supply shock. Whatever the effects of one on dollar earnings (the sign of the effect will depend on the elasticity of demand), the effect of the other is to reinforce the movement, thereby leading to greater instability.

In fact, what is relevant for farmers is not so much instability in dollar earnings but instability in domestic currency earnings. From (2.9),

$$y \sim \Lambda (\mu_y, \sigma_y^2)$$

$$\text{where } \mu_y = \frac{\varepsilon(\gamma+1)}{\gamma+\varepsilon} \mu_r + \frac{\varepsilon-1}{\gamma+\varepsilon} \mu_\theta$$

$$\sigma_y^2 = \frac{\varepsilon^2(\gamma+1)^2}{(\gamma+\varepsilon)^2} \sigma_r^2 + 2 \frac{\varepsilon(\varepsilon-1)(\gamma+1)}{(\gamma+\varepsilon)^2} \rho \sigma_r \sigma_\theta + \frac{(\varepsilon-1)^2}{(\gamma+\varepsilon)^2} \sigma_\theta^2 \quad (6.14)$$

In our analysis of (6.13), we assume that the elasticity of demand is less than unity ( $\varepsilon < 1$ ). Then if we use  $\sigma_y^2$  as our measure of instability we have the following proposition.

Proposition 16: If the elasticity of demand is less than unity then positive (negative) correlation between supply shocks and the domestic currency/dollar exchange rate movements will, ceteris paribus, reduce (increase) domestic currency earnings instability.

The reasoning behind this is as follows. An increase in supply, given the demand elasticity assumption, will reduce earnings in domestic currency terms. In order to compensate for this what is needed is an appreciation in the dollar i.e. an increase in  $r$ . Hence positively related movements of  $\theta$  and  $r$  mitigate each other's impact on earnings, thereby reducing earnings instability to a level below what it would have been without such a relationship.

## 7. Implications for Commodity Price Stabilization

What implications do exchange rate fluctuations have for commodity price stabilization schemes? In order to answer this question we will first of all briefly review some of the results of the commodity price stabilization literature with only supply shocks (for simplicity we set  $r = 1$ ). If

$$\theta \sim \Lambda (\mu_r, \sigma_\theta^2) \quad (7.1)$$

then from (2.5),

$$p_\S \sim \Lambda \left( -\frac{1}{\gamma+\varepsilon} \mu_\theta ; \frac{1}{(\gamma+\varepsilon)^2} \sigma_\theta^2 \right) \quad (7.2)$$

and thus

$$\bar{p}_\S = E(p_\S) = e^{-\frac{1}{\gamma+\varepsilon} \mu_\theta + \frac{1}{2(\gamma+\varepsilon)^2} \sigma_\theta^2} \quad (7.3)$$

If price is stabilized at  $E(p_\S)$  - by a buffer stock - then demand will be

$$Q^D = (\bar{p}_\S)^{-\varepsilon} = e^{\frac{\varepsilon}{\gamma+\varepsilon} \mu_\theta - \frac{\varepsilon}{2(\gamma+\varepsilon)^2} \sigma_\theta^2}$$

while average supply will be

$$\begin{aligned} E(Q_\S^S) &= (\bar{p})^\gamma E(\theta) \\ &= \left\{ e^{-\frac{\gamma}{\gamma+\varepsilon} \mu_\theta + \frac{\gamma}{2(\gamma+\varepsilon)^2} \sigma_\theta^2} \right\} \left\{ e^{\mu_\theta + \frac{1}{2} \sigma_\theta^2} \right\} \end{aligned}$$

Now

$$E\left(\frac{Q_\S^S}{Q^D}\right) = \exp\left[\frac{1}{2} \sigma_\theta^2 \left(1 + \frac{1}{\gamma+\varepsilon}\right)\right] > 1 \quad (7.4)$$

Thus, as shown in Kanbur (1986), stabilizing price at its mean value will lead to average supply exceeding demand at the stabilized price, an

unsustainable situation in the medium to long term, if financial resources are finite. In fact it can be shown that for average demand to equal average supply, the world dollar price will have to be stabilized at

$$p = e^{-\frac{1}{\gamma+\varepsilon} \mu_\theta - \frac{1}{2(\gamma+\varepsilon)} \sigma_\theta^2} < \bar{p}_\$ \quad (7.5)$$

What is the effect of price stabilization on earnings stability?

From (2.9), with  $r = 1$ , in the free market case,

$$y \sim \Lambda(\mu_y, \sigma_y^2)$$

$$\text{where } \mu_y = \frac{\varepsilon-1}{\gamma+\varepsilon} \mu_\theta \quad ; \quad \sigma_y^2 = \frac{(\varepsilon-1)^2}{(\gamma+\varepsilon)^2} \sigma_\theta^2 \quad (7.6)$$

With price stabilized at some value, earnings variability is simply determined by supply variability - in other words, it is  $\sigma_\theta^2$ . Then

$$\begin{aligned} \sigma_\theta^2 - \frac{(\varepsilon-1)^2}{(\gamma+\varepsilon)^2} \sigma_\theta^2 \\ = \sigma_\theta^2 \left\{ \frac{(\gamma+1)(\gamma+2\varepsilon-1)}{(\gamma+\varepsilon)^2} \right\} \geq 0 \end{aligned}$$

$$\Leftrightarrow \varepsilon \geq \frac{1-\gamma}{2} \quad (7.7)$$

With  $\gamma = 0$  this gives us the well known result that price stabilization will reduce earnings stability if the elasticity of demand is greater than a half.

How are these well known results in the standard analysis of commodity price stabilization affected by the presence of exchange rate instability as well as supply instability? In answering this question, some of the expressions derived in the previous section will be useful. From (6.5), with  $r$  and  $\theta$  distributed jointly lognormally,

$$\bar{p}_S = E(p_S) = \exp\left[\mu_{p_S} + \frac{1}{2} \sigma_{p_S}^2\right]$$

and demand is thus

$$Q^D = (\bar{p}_S)^{-\varepsilon} = \exp\left[-\varepsilon \mu_{p_S} - \frac{1}{2} \varepsilon^2 \sigma_{p_S}^2\right]$$

To find average quantity supplied, notice that

$$\theta r^\gamma \sim \Lambda(\mu_\theta + \gamma \mu_r; \sigma_\theta^2 + 2\gamma \rho \sigma_\theta \sigma_r + \gamma^2 \sigma_r^2)$$

so that

$$\begin{aligned} E(Q^S) &= E((r \bar{p}_S)^\gamma \theta) \\ &= \exp\left[\mu_\theta + \gamma \mu_r + \frac{1}{2} \{\sigma_\theta^2 + 2\gamma \rho \sigma_\theta \sigma_r + \gamma^2 \sigma_r^2\}\right] \end{aligned}$$

Hence

$$\frac{E(Q^S)}{Q^D} = \exp\left\{\frac{1}{2}\left(1+\frac{1}{\gamma+\varepsilon}\right)[\gamma^2\sigma_r^2+2\rho\gamma\sigma_r\sigma_\theta+\sigma_\theta^2]\right\}$$

$> 0$  (7.7)

Thus if the correlation between the domestic currency/dollar exchange rate and supply shocks is non-negative, the earlier result with regard to gradual build up of stocks will hold. To the extent that  $\rho$  is negative, this effect will be mitigated, but it will never dominate, as can be seen by setting  $\rho = -1$  and showing that the argument of the exponential function in (7.7) is still positive. The price  $p_\$^{**}$  which will equate average supply and demand can be shown to be

$$p_\$^{**} = \exp\left\{-\frac{1}{\gamma+\varepsilon}[\mu_\theta+\gamma\mu_r+\frac{1}{2}(\sigma_\theta^2+2\gamma\rho\sigma_\theta\sigma_r+\gamma^2\sigma_r^2)]\right\}$$

$< \bar{p}_\$$  (7.8)

The inequality in (7.8) is similar to that in (7.5) - price will have to be stabilized at a level lower than its pre intervention mean if unlimited accumulation of stocks is to be avoided. But by how much lower? With exchange rate fluctuation we have

$$\frac{\bar{p}_\$}{p_\$^{**}} = \exp\left\{\frac{1}{2(\gamma+\varepsilon)}\left(1+\frac{1}{\gamma+\varepsilon}\right)[\gamma^2\sigma_r^2+2\gamma\rho\sigma_\theta\sigma_r+\sigma_\theta^2]\right\}$$

Without exchange rate fluctuations the ratio is



$$\frac{\bar{p}_\S}{p_\S^{**}} \exp\left\{\frac{1}{2(\gamma+\varepsilon)}\left(1+\frac{1}{\gamma+\varepsilon}\right)[\sigma_\theta^2]\right\}$$

Thus if  $\rho = 0$  then the shortfall of the stabilized price from mean price will have to be greater with exchange rate fluctuations than without.

Turning now to domestic currency earnings instability, from (6.14) we know the formula for this with exchange rate fluctuations but without commodity price stabilization. With dollar price stabilized at some value (say  $p_\S^{**}$ ), we have

$$\begin{aligned} y &= r(p_\S^{**})Q^S = (p_\S^{**})(p_\S^{**})^\gamma r^{(1+\gamma)\theta} \\ &= (p_\S^{**})^\gamma r^{(1+\gamma)\theta} \end{aligned}$$

Thus with stabilization

$$\sigma_y^2 = (\gamma+1)^2 \sigma_r^2 + 2(\gamma+1)\rho\sigma_\theta\sigma_r + \sigma_\theta^2$$

The difference between instability in the two cases is

$$\begin{aligned}
 & (\gamma+1)^2 \sigma_r^2 + 2(\gamma+1) \rho \sigma_r \sigma_\theta + \sigma_\theta^2 - \left\{ \frac{\varepsilon^2 (\gamma+1)^2}{(\gamma+\varepsilon)^2} \sigma_r^2 + 2 \frac{\varepsilon(\varepsilon-1)(\gamma+1)}{(\gamma+\varepsilon)^2} \rho \sigma_r \sigma_\theta \right. \\
 & \quad \left. + \frac{(\varepsilon-1)^2}{(\gamma+\varepsilon)^2} \sigma_\theta^2 \right\} \\
 & = \sigma_r^2 \frac{\gamma(\gamma+1)^2(\gamma+2\varepsilon)}{(\gamma+\varepsilon)^2} + 2 \frac{(\gamma+1) \rho \sigma_r \sigma_\theta (\gamma^2 + 2\gamma\varepsilon + \varepsilon)}{(\gamma+\varepsilon)^2} \\
 & \quad + \sigma_\theta^2 \frac{(\gamma+1)(\gamma+2\varepsilon-1)}{(\gamma+\varepsilon)^2} \tag{7.9}
 \end{aligned}$$

While complicated, it is interesting to note that when  $\sigma_r^2 = 0$  i.e. no exchange rate instability, (7.9) collapses to (7.7). In the general case, if  $\rho \geq 0$  then price stabilization will tend even more strongly to stabilize earnings. We record this finding as a proposition:

Proposition 17: If the correlation between the domestic currency/dollar exchange rate and supply shocks is zero or positive, then  $\varepsilon > \frac{1-\gamma}{2}$  is no longer a necessary condition for price stabilization to stabilize earnings. A much weaker condition then holds.

## 8. Conclusion and Further Research

As noted in the Introduction, the object of this paper has been to broach the issue of exchange rate fluctuations and commodity price instability. Given the importance of the topic, very little work seems to have been done in attempting to link the two phenomena. In this paper we have taken an extremely simple approach in the context of a lognormal model. We have derived various propositions on the impact of exchange

rate instability on price and earnings instability in the producing countries. However, this is only a first step and the whole area is one which is wide open for research. We end with some suggestions of our own:

(1) More work is needed on drawing out the implications of exchange rate instability for commodity price stabilization schemes. For example, how are the welfare results of Newbery and Stiglitz (1981) modified by the presence of exchange rate fluctuations? Also, what are the financial implications of such instability for a stabilization scheme designed on the assumption of stable exchange rates?

(2) We have touched on the welfare implications of exchange rate instability in Section 5. A more detailed analysis of this is warranted, to delineate when stability of earnings and welfare can move in opposite directions.

(3) On the empirical side, we need a feel for the extent to which exchange rate fluctuations and supply shocks are correlated. We have formulated various of our conditions in terms of the coefficient of a regression between the two. Empirical analysis for specific commodities can shed light on whether or not these conditions are satisfied in practice.

(4) Although this may prove difficult, and the lognormal model may be the most tractable formulation available given the complexities involved, there is nevertheless a case for seeing how sensitive our propositions are to relaxation of lognormality.

(5) Our framework of analysis has been essentially one of seeing exchange rate fluctuations rather like an extra supply shock. There may well be other channels through which exchange rate instability might

affect commodity markets. For example, Grilli and Yang (1984), argue that exchange rate instability might influence the demand for some commodities as a hedge against financial risk. This alternative view needs to be explored further and integrated into our supply oriented analysis.

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