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MARKET STRUCTURE, DUAL PRICING AND TAXES

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I Introduction

Dual pricing is a form of price-control and rationing that is, or has been, applied across a wide variety of goods in India. There are a number of variants but the central feature is usually that a specified proportion of the output from firms must be sold to the government at a retention price which is fixed by the government. Much of the requisitioned output is then sold through ration shops at a price (the issue price) which is closely related to the retention price. The scheme is often regarded by governments as an attractive way of controlling the markets for "essential" goods yet the effects on prices, profits and consumer welfare and the relations of these effects with market structure have not been extensively analysed. The initial motivation of the paper was to provide this analysis.

The Indian scheme has many close cousins. For example, it is quite common in housing developments in a number of countries for a government to insist that a certain proportion of the dwellings be let at regulated prices (often to low income groups). Similarly a health authority might leave a doctor free to practice privately for a certain proportion of the time provided the remainder is allocated to the public sector. We shall also see that there are significant analogies with forms of profits taxation and with a combination of sales tax and production subsidy. And black markets generally provide examples of the coexistence of a rationed sector where prices are fixed and a free market. Thus the analysis of markets where there are two prices is of considerable importance.

The effects of the scheme will depend on market structure and thus we shall be investigating a class of models from monopoly and oligopoly to monopolistic and perfect competition. A second purpose of the paper is to assemble these models in a succinct way which allows simple comparative static analysis. We concentrate on partial equilibrium. The third objective is to examine the effects of specific taxes on prices and profits in these models. This will be useful in interpreting the consequences of dual pricing and we shall see that the answers can be very different from the text-book analysis of perfect competition.

In the next section we examine the basic model without dual pricing and investigate the effects of specific taxes. Dual pricing is presented formally in section III where we examine its effects on prices, profits and the number of firms; in section IV we look at effects on household welfare. We discuss in section V an alternative version of the scheme where the retention quantity, rather than proportion is fixed. The penultimate section contains a discussion of possible extensions and the final section some concluding remarks.

Whilst the literature on the theory of quantity and points rationing is fairly extensive, see for example Neary and Roberts (1980) and Tobin (1952) there appears to be rather little on dual pricing. In the Indian context see Mukherji, Pattanaik and Sundram (1980), and an interesting programme of work by Professor V.K. Chetty entitled "Project on Price and Distribution Controls in India" at the Indian Statistical Institute, Delhi.

II The model without dual pricing

We shall make extensive use of an approach to the theory of oligopoly which, following Stigler (1964) and Cowling and Waterson (1976) incorporates an explicit model of the conduct of firms based on conjectural variation. This model has been thoroughly investigated in an important series of papers by Seade (1980a, 1980b and 1983). The treatment here is based on Dixit and Stern (1982). We shall extend the model to one of monopolistic competition by introducing free entry.

Output of firm i is x_i, market share s_i, costs $K_i + c_i x_i$. Total output is X and price $p = \phi(X)$. A firm i conjectures that the reaction of firm j to a small change in its output satisfies

$$\frac{\partial x_i}{\partial x_j} = \alpha \frac{x_i}{x_j} \qquad (1)$$

A necessary condition for the maximisation of profits by the i^{th} firm, given its conjectures, is that perceived marginal revenue be equal to marginal cost. Using (1) this may be written

$$p \quad 1 - \left\{\frac{(\alpha + (1-\alpha)\ s_i)}{\varepsilon}\right\} \quad - c_i = 0 \qquad (2)$$

Adding and dividing by the number of firms, n,

$$p\ (1 - \frac{\gamma}{\varepsilon}) - \bar{c} = 0 \qquad (3)$$

where $\bar{c} = \frac{\sum_i c_i}{n}$ and $\gamma = \alpha + \frac{(1-\alpha)}{n}$.

Where n is fixed we have a "generalised-Cournot" equilibrium. To ease notation we replace $\bar{c}$ by c and assume identical marginal costs but the results for this case extend readily on replacing c by by $\bar{c}$.

When free entry is considered we need to assume firms are identical and we have in addition to (3) the zero-profit condition

$$(p-c)X - Kn = 0 \qquad (4)$$

We can interpret this condition as one of perfect equilibrium where entering firms pay an entrance feee (or fixed cost) K, foreseeing accurately the conduct of the game which will take place after entry. Conditions (3) and (4) give the equilibrium in monopolistic competition. They are the basic equations of the model and should be very familiar as "marginal revenue equals marginal cost" and "average revenue equals average cost".

The existence of a solution to (3) with positive price requires

$$\varepsilon > \gamma \qquad (5)$$

Stability of the generalised-Cournot equilibrium in the sense of Seade (1980a) requires

$$F > 1 - \frac{\varepsilon}{\gamma} \qquad (6)$$

where $F = \frac{p\,\varepsilon'}{\varepsilon}$ and ε is the elasticity of demand, $\varepsilon = -pX'/X$, where the prime denotes the derivative with respect to price . In the adjustment process firms are assumed to move their output towards the level which would satisfy (2) given the output of the other firms. Given (5) we know that (6) is satisfied for $F \geq 0$ (including the isoelastic case $F = 0$). It should be clear that the role of F is important since, for example, if F were negative and large in magnitude then an increase in output (and fall in price) could increase ε and lead firms to raise output still further. The second-order condition around the equilibrium (where $s_i = \frac{1}{n}$) for identical firms) is

$$F/\varepsilon > (\frac{1}{\varepsilon} - \frac{1}{\gamma}) + \frac{(1-\gamma\ (\alpha-\gamma)}{\gamma^2} \qquad (7)$$

Monopoly is the special case of generalised-Cournot with $n = 1$ and perfect competition is a special case of monopolistic competition with $K = 0$, $\alpha = 0$, $\nu = \infty$. Hence the two equations (3) and (4), either singly using (3), or as a pair, cover in a convenient way a wide range of market structures.

Writing the l.h.s. of (3) as $f(c,p,n)$ and the r.h.s. of (4) as $g(c,p,n)$ the comparative statics are derived from

Generalised Cournot:
$$\frac{\partial p}{\partial c} = - f_c/f_p \qquad (8)$$

$$\frac{\partial \Pi}{\partial c} = g_p \frac{\partial p}{\partial c} + g_c \qquad (9)$$

where Π are industry profits

Monopolistic Competition: $$\frac{\partial p}{\partial c} = \frac{f_c\, g_n - f_n\, g_c}{f_n\, g_p - f_p\, g_n} \qquad (10)$$

$$\frac{\partial n}{\partial c} = \frac{f_p\, g_c - f_c\, g_p}{f_n\, g_p - g_p\, g_n} \qquad (11)$$

In generalised-Cournot n is fixed and in monopolistic competition Π is zero. We treat n as a continuous variable. Results are displayed in Table 1.

The stability condition (6) may be rewritten as $f_p \equiv 1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon} > 0$. This tells us that the graph of f against p intersects c from below or, more familiarly we have the analogue in this model of the result in monopoly that the marginal revenue curve should intersect the marginal cost curve from above (and, with monopoly, conditions (6) and (7) are identical). Since f_n and g_p are positive and g_n is negative, the denominator in (10) and (11) is is negative. Thus from (8), (10), (11) the sign of $\frac{\partial \Pi}{\partial c}$ and $\frac{\partial n}{\partial c}$ is the same i.e. an increase in marginal cost increases the number of firms in monopolistic competition if and only if it increases profits in generalised Cournot.

It is also clear that a decrease in the number of firms in oligopoly (generalised-Cournot) will raise the price (f_p and f_n are both positive). Hence $\frac{\partial p}{\partial c}$ under generalised Cournot is lower than $\frac{\partial p}{\partial c}$ in monopolistic competition if and only if an increase in costs decreases profits in generalised Cournot.

For the most part we shall be dealing with a general demand curve. There are two examples we shall use at a number of points: the isoelastic case (constant ε) and the linear case (constant slope of the demand curve). Results for the former are given by putting $F = 0$ in the formulae which follow. For the linear case, $X = a - bp \equiv b(p^*-p)$, where $p^* = a/b$ is the price which gives zero demand, we have $\varepsilon = p/(p^*-p)$ and $F = p^*/p^*-p)$; thus ε goes from zero to infinity as p goes from 0 to p^* and F goes from 1 to infinity. For the linear case (3) takes the convenient form

$$p = \frac{1}{1+\gamma} c + \frac{\gamma}{(1+\gamma)} p^* \tag{3a}$$

i.e. price is a weighted average of the marginal cost (or $\bar{c}$ more generally) and the price at which demand is zero, p^*. We shall assume $p^* > c$. Where $n = 1$, γ is unity and the weights are equal. In the linear case profits must fall in generalised Cournot when c increases, hence in monopolistic competition, the number of firms decreases and the effects of a marginal cost increase on price is higher than for generalised Cournot.

Table 1

Generalised Cournot $\frac{\partial p}{\partial c} = \frac{1}{(1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon})} \gtrless 1 \text{ as } F \lessgtr 1$

It is positive from (6).

$$\frac{\partial \Pi}{\partial c} = \frac{-X\gamma\left[1 - \frac{1}{\varepsilon} + \frac{F}{\varepsilon}\right]}{1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon}}$$

Thus $\frac{\partial \Pi}{\partial c} < 0$ if and only if $F > 1 - \varepsilon$

Also $\frac{\partial(\Pi/n)}{\partial n} < 0$ (using (7))

Monopolistic Competition $\Delta \frac{\partial p}{\partial c} = \frac{pX(1-\alpha)}{\varepsilon n^2} + K \quad (> 0)$

$$\Delta \frac{\partial n}{\partial c} = X(1-\gamma) - X(1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon})$$

where $\Delta = \frac{pX(1-\alpha)\,(1-\gamma)}{\varepsilon n^2} + K(1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon})$ and is positive

$$\frac{\partial p}{\partial c} > \frac{1}{1-\gamma} \text{ if and only if } F < 1-\varepsilon$$

$$\frac{\partial n}{\partial c} < 0 \quad \text{if and only if } F > 1-\varepsilon$$

Table 1 (continued/...)

In the special case of linear demand curve:

<u>Generalised Cournot</u> $\frac{\partial p}{\partial c} = \frac{1}{1+\gamma} < 1$

$$\frac{\partial \Pi}{\partial c} = \frac{-2\gamma X}{1+\gamma} < 0$$

<u>Monopolistic Competition</u> $\frac{1}{1-\gamma} \geq \frac{\partial p}{\partial c} \geq \frac{1}{1+\gamma}$

with equality on the l.h.s. when $\alpha = 1$ (note that when $n \to \infty$ then $\alpha \to 0$ and $\frac{\partial p}{\partial c} \to 1$.

$$\frac{\partial n}{\partial c} < 0.$$

The results on $\frac{\partial p}{\partial c}$ and $\frac{\partial \Pi}{\partial c}$ for the generalised-Cournot model are special cases of those in Seade (1983), where we assume here that all firms are identical. The derivation is particularly straightforward for this case in contrast to that of heterogeneous firms when some subtlety is necessary (Seade, 1983). The result on $\frac{\partial(\Pi/n)}{\partial n}$ is contained in Seade (1980b).

We can interpret an increase in c as an increase in a specific tax on the commodity. As we can see the effect on price of a unit tax increase can be greater or smaller than one. If F is high (see linear case) then an increase in the tax has a small effect on the price because the increase in price, increases the elasticity which dampens (through (3)) the effect of the price increase. On the other hand where F is low the increase in price can be greater than one (and if low enough, i.e. $< 1-\varepsilon$, profits increase). These results are in sharp contrast to the standard analysis of the competitive case where the effect of a unit tax increase on consumer price is $\frac{\eta}{\varepsilon+\eta}$ and $0 \leq \frac{\eta}{\varepsilon+\eta} \leq 1$ where η is the elasticity of supply. Thus where markets are not competitive "full tax shifting" or $\frac{\partial p}{\partial c} = 1$ may be a sensible middle choice for applied work and not a polar case.

III Dual Pricing

Dual pricing involves the compulsory sale of a fixed proportion $(1-\theta)$ of the output of each firm to the government at a "retention price" p_R. We shall examine in this section the effects on prices and profits of introducing, or varying, such a scheme in the different market structures described above. We suppose that all the quantity sold by firms to the government is made available to consumers through a rationing system at the retention price p_R. We shall assume all consumers are identical. In section VI we consider heterogeneous consumers and the possibility that the issue price may differ from the retention price.

To keep things simple, we shall assume that total market demand, i.e. purchases from ration shops plus those from the open market, is a function of the market price, p, only. In order to justify this we must first assume that the effect on demand of the effective increase in lump-sum income associated with the ration is negligible. This effective increase is $\bar{x}(p - p_R)$ where $\bar{x}$ is the ration and $(p - p_R)$ the difference between open market and retention price. Secondly we assume either that the ration $\bar{x}$ is resaleable at p or that each household consumes more than its ration i.e. it makes open-market as well as ration-shop purchases: in each case the opportunity cost to the household of the marginal unit is the open market price p.

The conjectural variation by the firms when the scheme is operating is assumed to be the same as without i.e. given by (1) where x_i is the total output for firm i (including that sold to the government). If each firm knows that all the other firms are being forced to sell the fixed proportion to the government in the same manner as itself, then this seems the natural assumption.

The equilibrium conditions (3) and (4) are replaced by

$$f(\theta, p_R, c, p, n) \equiv p\theta(1 - \frac{\gamma}{\varepsilon}) - c + (1-\theta)p_R = 0 \qquad (12)$$

$$g(\theta, p_R, c, p, n) \equiv (p\theta - c + (1-\theta)p_R) X - K_n = 0 \qquad (13)$$

The effects of the scheme can be seen as a sales tax at rate $(1-\theta)$ together with a per unit production subsidy of $(1-\theta)p_R$. Alternatively the tax liability of the firm is $T_i = (1-\theta)(p-p_R)x_i$. This is like a tax on profits before fixed cost with a marginal cost decreed by the government to be p_R. Given that costs are often unobservable this may not be far away from some profits taxes in practice.

As we suggested in section 1 there are a number of further possible interpretations of the system. We could think of medical doctors being allowed to work a proportion θ of their time in private practice if the remaining $(1-\theta)$ is available to the state at a fixed price. Or one could imagine a market price controlled at p_R but where a proportion θ of sales are on the black market - θ then could be influenced by enforcement. One could think of θ as the black market proportion for each firm, or the probability of any given firm being 'raided' and forced to sell its entire output to the government at the requisition price. From the perspective of these interpretations examples of the scheme are fairly common.

The comparative statics of the scheme can be derived in an analogous manner to (8)-(11). The results for price, profits and the number of firms are given in Table 2. The profits Π are now the l.h.s. of (13). Notice that (12) can be written as

$$p(1 - \frac{\gamma}{\varepsilon}) - \hat{c} = 0 \qquad (12a)$$

where $\hat{c}$ is $p_R + \frac{(c - p_R)}{\theta}$, which is analogous to (3). Hence in generalised Cournot the effects on price of changes in θ and p_R follow straightforwardly from $\frac{\partial p}{\partial c}$ in Table 1 on multiplying by $\frac{\partial \hat{c}}{\partial \theta}$ and $\frac{\partial \hat{c}}{\partial p_R}$.

As we have noted the scheme acts in part like a cost subsidy $(1-\theta)p_R$, and therefore an increase in p_R will lower the price in each of generalised Cournot and monopolistic competition. As before the effect will be greater in magnitude in monopolistic competition if and only if a decrease in costs (increase in p_R) increases profits and the number of firms. An increase in θ increases price if $p_R > c$ under generalised Cournot, as should be clear from (12a) since it increases $\hat{c}$. On the other hand it will generally increase profits and the number of firms hence any price increase from an increase in θ in the case $p_R > c$ will be lower in monopolistic competition.

Table 2

Generalised Cournot

$$\frac{\partial p}{\partial \theta} = \frac{1}{\theta^2} \frac{(p_R - c)}{(1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon})} \qquad \frac{\partial p}{\partial p_R} = \frac{-(1-\theta)}{\theta} \frac{1}{(1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon})}$$

$$\frac{\partial \Pi}{\partial \theta} = \frac{\gamma X(p - p_R) \quad (F - \sigma)}{\varepsilon(1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon})} \qquad \frac{\partial \Pi}{\partial p_R} = \frac{\gamma(1-\theta)X}{(1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon})} (1 - \frac{1}{\varepsilon} + \frac{F}{\varepsilon})$$

where $\sigma = (1-\varepsilon) - \frac{(1-\gamma}{(1 - \frac{p_R}{p})} < 0$

$\frac{\partial p}{\partial \theta} > 0$ if and only if $p_R > c$ $\qquad \frac{\partial p}{\partial p_R} < 0$

$\frac{\partial \Pi}{\partial \theta} > 0$ if $F > \sigma$ $\qquad \frac{\partial \Pi}{\partial p_R} > 0$ if and only if $F > 1 - \varepsilon$

$\frac{\partial \Pi}{\partial \theta} > 0$ if $p_R > c$

Monopolistic Competition

$$\Delta' \frac{\partial p}{\partial \theta} = K(\frac{p_R - c}{\theta}) - (p - p_R) X.p \frac{\theta(1 - \alpha)}{\varepsilon n^2} \qquad \Delta' \frac{\partial p}{\partial p_R} = -(1-\theta)K - (1-\theta)K - (1-\theta) X.p \frac{\theta(1-\alpha)}{\varepsilon n^2}$$

$$\Delta' \frac{\partial n}{\partial \theta} = (p - p_R) X\theta (1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon}) + (p_R - c) X(1 - \gamma) \qquad \Delta' \frac{\partial n}{\partial p_R} = \theta (1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon}) (1 - \theta)X - \theta(1 - \gamma) (1 - \theta)X$$

where $\Delta' = \frac{p\theta(1 - \alpha)}{\varepsilon n^2} . \theta X(1 - \gamma) + \theta K(1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon}) > 0$

Table 2 (continued/...)

Hence

$\frac{\partial p}{\partial \theta} > 0$ at $p_R = p$

< 0 for $p_R \leq c$

$\frac{\partial p}{\partial p_R} < 0$

$\frac{\partial n}{\partial \theta} > 0$ if $F > \sigma$

$\frac{\partial n}{\partial \theta} > 0$ if $p_R > c$

$\frac{\partial n}{\partial p_R} > 0$ if and only if $F > 1 - \varepsilon$

IV Effects on Welfare

The introduction of the scheme will affect the welfare of consumers through the open market price, p, and the value of the lump-sum transfers $(1 - \theta)\, X(p - p_R)$. We assume initially that consumers are identical, and recall that government revenue from the scheme is zero since retention price and issue price are equal. Thus our discussion of welfare is in terms of the levels of profits and consumer surplus (generalised Cournot) and the level of consumer surplus (monopolistic competition). In section V we comment on distribution amongst consumers and the case where issue and retention price may differ.

Changes in consumer surplus dW may be written as

$$dW = d[(1 - \theta)\, X(p - p_R)] - Xdp \qquad (14)$$

$$= X(p - p_R)d\theta - [\theta X - (p - p_R)\,(1 - \theta)X']dp - (1 - \theta)Xdp_R \qquad (15)$$

We look at the effects of changes in θ and p_R for the generalised Cournot case in Table 3. In interpreting the results in the table we should note that if p_R is set so that $p = p_R$ then

$$p = p_R = \frac{c}{1 - \frac{\theta\gamma}{\varepsilon}} \qquad (16)$$

Note that a retention price equal to market price is not the same as abolishing the scheme (compare (16) and (3)) but rather results in lower prices than without the scheme - in effect it raises the elasticity of demand. With p_R chosen for any given θ so that $p = p_R$ the government essentially controls the market price through θ and can choose any price above c. In the limit as θ tends to zero, price tends to marginal cost. This obviously involves negative profits but in the usual way maximises $\Pi + W$ the sum of producer and consumer surplus.

Table 3

Welfare and Profits: Generalised Cournot

$1 - \varepsilon < F < 1 + \delta$	$\frac{\partial \Pi}{\partial p_R} > 0, \quad \frac{\partial W}{\partial p_R} > 0$
$F > 1 + \delta$	$\frac{\partial \Pi}{\partial p_R} > 0, \quad \frac{\partial W}{\partial p_R} < 0$
$F < 1 - \varepsilon$	$\frac{\partial \Pi}{\partial p_R} < 0, \quad \frac{\partial W}{\partial p_R} > 0$

where $\delta = (\theta\gamma)^{-1}(1 - \frac{p_R}{p})(1 - \theta)\varepsilon^2 > 0.$

$F > \sigma$	$\frac{\partial \Pi}{\partial \theta} > 0$

where $\sigma < 0$ (see Table 2).

If $c < p_R$	$\frac{\partial W}{\partial \theta} < 0$

If $c > p_R$ then $\frac{\partial W}{\partial \theta} < 0$ at $\theta = 1$ and $\frac{\partial W}{\partial \theta} > 0$ at $\theta = 0$ (for $F \geq 0$).

Linear Demand

$F = \frac{p^*}{p^* - p}$; $\varepsilon = \frac{p}{p^* - p}$ where p^* is a/b the price at which demand becomes zero. Thus $F > 1$ and at $p = p_R$ (i.e. $\delta = 0$) we have $\frac{\partial W}{\partial p_R} < 0$ and $\left(\frac{\partial \Pi}{\partial p_R} > 0\right)$. As $p \to p^*$ then ε and $F \to \infty$ and $F/\varepsilon \to 1$. Hence for p close to p^*, $F < 1 + \delta$ and $\frac{\partial W}{\partial p_R} > 0$. Thus for a given θ, there exists a p_R which makes W a maximum; and $p_1 < p_R < p_2$ with p_2 given by (16) and p_1 satisfying $\theta p^* + (1-\theta)p_1 = c$, so that if $p_R = p_1$ then $p = p^*$ (see (12)). Note that $p_1 < c < p_2$.

From Table 3 we see that if $F < 1 + \delta$ then raising the retention (and ration) price at given θ yields a fall in market price p sufficient to compensate the consumers for the higher ration price. If F is also above $(1 - \varepsilon)$ then profits rise too with an increase in p_R. On the other hand if $F > 1 + \delta$ then (locally) raising the retention price to the market price (at fixed θ) lowers welfare and increases profits (the case of linear demand provides an example). Welfare rises and profits fall when θ is lowered (for $F > 0$ and $p_R > c$).

Hence although maximisation of $\Pi + W$ requires the raising of p_R to p and the lowering of θ to 0, giving $\Pi < 0$ eventually, this does not by itself tell us that consumer welfare will increase monotonically en route. For example, suppose we start with $p_R < p$ and $\theta > 0$ and first increase p_R to p for given θ and then let θ tend to zero (adjusting p_R to keep it equal to p). Then in the linear case we have consumer welfare falling (if $p_R > \hat{p}_R$, see Table 3) and profits rising as p_R is increased to p followed by movements in the opposite direction as θ is lowered.

The linear case (Table 3) shows that for given θ there is a p_R which maximises W and which is strictly less than p_R. This is a result of some importance since θ may be set exogenously by convention, statute or limitations on enforcement and the government may have control only over the retention/ration price. In such a case, we would, in the linear example, definitely want the scheme from the point of view of consumer welfare and would set p_R at p_R, below the market price. On the other hand in the isoelastic case $F = 0$ we would raise p_R to p to increase consumer welfare.

We also see from Table 3 that if $c > p_R$ then there is a θ between zero and one which maximises W for given p_R. One might therefore ask whether there is an interior maximum (with $p < p_R$ and $0 < \theta < 1$) for W. The answer is negative: it is straightforward to show (see Appendix) that $\frac{\partial W}{\partial p_R} \leq 0$ implies $\frac{\partial W}{\partial \theta} < 0$.

Intuitively the explanation is as follows. The policy variables p_R and θ enter the model only through their effect on the price per unit $p\theta + (1-\theta)p_R$ with the exception that θ affects in addition the mark-up over marginal cost. Thus if p_R is set optimally there is a potential gain from reducing θ and lowering this mark-up. The argument is analogous to one in the theory of policy towards crime where in certain models the optimum selection of penalty will imply that (costly) enforcement should be reduced - p_R and θ play similar roles to (the opposite of) penalties and enforcement (see Stern, 1978).

Since $\frac{\partial W}{\partial \theta} < 0$ at $\theta = 1$ and $p = p_R$, and there is no interior solution, the relevant boundary is $\theta = 0$. Hence the maximum for W is given by $\theta = 0$ and $p_R = c$ (see (12)). Thus, in the model, consumer welfare is maximised by the government taking full control of the market and setting the ration price at marginal cost (the limit of (12) as θ tends to 0 acts as a floor preventing a lower ration price). As in the maximisation of $\Pi + W$ firms make a loss equal to the fixed cost.

Negative profits are ruled out in monopolistic competition, for which results are presented in Table 4. For low F (< 1) we can see that the optimum policy is to raise p_R to p and lower θ to the minimum consistent with zero profits. Since $\frac{\partial p}{\partial \theta}$ and $\frac{\partial n}{\partial \theta}$ are positive for $p = p_R$ this choice of θ (given the policy for p_R) has the effect of minimising the price and the number of firms.

For higher F (e.g. linear demand) we would wish (for any given θ) to lower p_R if it were equal to p, thus introducing the scheme. However one can show as before (see Appendix) that $\frac{\partial W}{\partial p_R} \leq 0$ implies $\frac{\partial W}{\partial \theta} < 0$ so that there is no optimum which is interior to the constraint $n \geq 1$.

The role of F in these results is through the magnitude of the price response following a change in p_R or θ. Consider, for example, an increase in p_R for given θ. The effect of this is to lower the price and this contributes an increase in W through the second term on the r.h.s. of (14). However the decrease in $(p - p_R)$ lowers the value of the lump-sum transfers and the net increase in welfare is positive only if the fall in p is large enough. But, the larger is F the greater the reduction in the elasticity of demand following a fall in price. And a reduction of the elasticity dampens the price fall. Hence the higher is F the less likely is the first effect to dominate and the less attractive is the scheme from the point of view of consumer welfare.

Table 4

Welfare: Monopolistic Competition

$$\frac{\partial W}{\partial p_R} > 0 \text{ if and only if } -\frac{\partial p}{\partial p_R} > \frac{1-\theta}{\theta + (1 - \frac{p_R}{p})(1-\theta)\varepsilon}$$

where

$$-\frac{\partial p}{\partial p_R} = \frac{(1-\theta)}{\theta} \cdot \frac{K + p\theta \frac{(1-\alpha)}{n^2\varepsilon} X}{K(1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon}) + \frac{p\theta(1-\alpha)}{n^2\varepsilon} X(1-\gamma)}$$

A sufficient condition for $-\frac{\partial p}{\partial p_R} > \frac{(1-\theta)}{\theta}$, and hence $\frac{\partial W}{\partial p_R} > 0$, is

$1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon} < 1$ or $F < 1$.

$$\frac{\partial W}{\partial \theta} > 0 \text{ if and only if } \frac{-\theta}{(p - p_R)} \cdot \frac{\partial p}{\partial \theta} > \frac{\theta}{\theta + \left(1 - \frac{p_R}{p}\right)(1-\theta)\varepsilon}$$

where

$$-\frac{\theta}{(p-p_R)} \frac{\partial p}{\partial \theta} = \frac{K\left(1 - \frac{\gamma}{\varepsilon\left(1 - \frac{p_R}{p}\right)}\right) + p\theta \frac{(1-\alpha)}{n^2\varepsilon} X}{K\left(1 - \frac{\gamma}{\varepsilon} + \frac{F\gamma}{\varepsilon}\right) + p\theta \frac{(1-\alpha)}{n^2\varepsilon} X(1-\gamma)}$$

At $p = p_R$, $\frac{\partial W}{\partial \theta} = -\theta X \frac{\partial p}{\partial \theta}$. But $\frac{\partial p}{\partial \theta} > 0$ for $p_R > c$ (Table 2) so that $\frac{\partial W}{\partial \theta} < 0$ at $p = p_R$. Hence for $F < 1$ (e.g. isoelastic case where $F = 0$) we would raise p_R to p and lower θ to the minimum consistent with zero profits. Since $\frac{\partial n}{\partial \theta} > 0$ for $p_R > c$ this involves lowering n to one. The optimum is therefore given by $n = 1$ with θ from $p = \frac{c}{\left(1 - \frac{\theta\gamma}{\varepsilon}\right)}$ and $(p - c) X = K$.

Table 4 (continued/...)

Linear Demand

$$\frac{\partial W}{\partial p_R} < 0 \quad \text{at} \quad p = p_R \quad \text{for given} \quad \theta.$$

But $\frac{\partial p}{\partial p_R} < 0$ (Table 2) hence we refrain from pushing prices to the minimum possible and for given θ we do have the scheme in a non-trivial form. But $\frac{\partial W}{\partial p_R} \leq 0 \Rightarrow \frac{\partial W}{\partial \theta} < 0$ (see Appendix) so there is no optimum for θ and p_R which is interior to the constraint $n \geq 1$.

V Fixed Retention Quantity

An alternative version of the scheme which is also of practical relevance is where the quantity requisitioned per firm is fixed. One can analyse this case along the same lines as that of fixed retention proportion. Space constraints do not permit us to provide full detail and we shall simply indicate how the analysis can proceed and some results.

If we suppose that the total amount requisitioned, X_R, is spread equally across firms then for a given firm the requisition quantity , x_R, is X_R/n. Writing X_F for the free-market quantity we we have X equal to $X_F + X_R$, and X_F is the sum of x_i^F which is defined as $x_i - x_R$. We assume that conjectures concern the free-market quantity so that (1) is replaced by

$$\frac{\partial x_j^F}{\partial x_i^F} = \alpha \frac{x_j^F}{x_i^F} \tag{17}$$

and the perceived marginal revenue MR_i is given by (18)

$$MR_i = p\left(1 - \frac{1}{\varepsilon}\left[(1 - \alpha)\ s_i^F + \alpha\ \theta\right]\ \right) \tag{18}$$

where s_i^F is x_i^F/X. Adding across firms we have, where $\theta = X_F/X$,

$$p\left(1 - \frac{\theta\Upsilon}{\varepsilon}\right) = \bar{c} \tag{19}$$

which replaces (12). If we interpret θ as the choice variable in place of x_R, then we can conduct the analysis as before using the pair of equations (19) and (13) instead of (12) and (13).

Notice that p_R does not enter (19) and thus does not affect the price in the generalised-Cournot case - a reduction in p_R acts, in this case, just like a lump-sum tax on profits. This feature provides the main difference between the system with fixed retention proportions and fixed retention quantities. The existence and stability conditions become $\varepsilon > \theta\Upsilon$ and $F > 1 - \frac{\varepsilon}{\theta\Upsilon}$ respectively and it is straightforward to construct for this case the tables corresponding to Tables 2, 3 and 4 on prices, profits and welfare.

In the generalised Cournot case $\frac{\partial W}{\partial p_R} = - \frac{\partial \Pi}{\partial p_R} = (1 - \theta)X$ since increasing p_R acts simply like a lump-sum transfer from consumers to producers. Raising θ raises prices (see (19) and the stability condition) and lowers consumer welfare. Thus consumers would always want the government to lower both θ and p_R. With monopolistic competition one can again show that $\frac{\partial W}{\partial p_R} \leq 0$ implies $\frac{\partial W}{\partial \theta} < 0$ so that there is no optimum for θ and p_R which is interior to the constraint $n \geq 1$.

VI Some Extensions

(i) Income distribution

If we introduce different consumers, indexed by h, with welfare weights β^h then we can write

$$dW = \sum_h \beta^h \, d[(1 - \theta) \frac{X}{H} (p - p_R) - \sum_h \beta^h x^h \, dp \qquad (20)$$

if everyone has an equal ration. This can be approximated, where $\nu < 1$, by

$$d[(1 - \theta) X (p - p_R)] - \nu X dp \qquad (21)$$

in certain circumstances (where preferences are homothetic and β_h has an isoelastic relation with income then it is exact).

Hence one way of considering income distribution is to place a greater weight on the lump-sum transfer term than on the consumer surplus term. Since we have a zero lump-sum transfer at $p = p_R$ it is clear that the weight on this term could be sufficiently high to guarantee the attractiveness of introducing the scheme.

One could also consider a criterion which was $\lambda \Pi + \mu W$ where the weights λ and μ may differ. In some cases (see e.g. Table 3 and p_R) Π and W move in the same direction but in others (e.g. linear case with p_R to be chosen and θ fixed) trade-offs can arise. Usually an increase in θ will involve increasing profits but reducing consumer welfare.

(ii) Issue price different from retention price

It is straightforward to make the issue price for the ration to households p_I, different from the retention price p_R. The equilibrium price depends only on p_R. The expression for welfare now contains p_I in place of p_R. Since p is independent of p_I we have, for example that $\frac{\partial W}{\partial p_I} = (1 - \theta) X < 0$. If $p_I > p_R$ the scheme makes a profit.

(iii) Effects of ration on demand

It is possible to examine different assumptions concerning the effect of the ration on demand. If the ration is not resaleable and is greater than the amount desired (at the ruling open market price) then total demand will depend on p, p_R and the ration.

VII Concluding Comments

Our three main purposes in this paper have been (i) to examine the effects on prices, profits and welfare of a dual pricing scheme in different market structures (ii) to present a succinct way of summarising these different structures and (iii) to study the effects of specific taxes in these markets not only for its inherent interest but to help us understand dual pricing.

The second and third of these were the subject matter of section II. We saw that a broad range of market structures can be captured in a convenient form within a simple and tractable model. This consists of just one familiar equation, which expresses the equality between marginal revenue and marginal cost, in the case of generalised Cournot with conjectural variations, or two equations in monopolistic competition, where we add the zero profit condition.

Using this pair of equations we examined the effect of specific taxes and found that the proportion shifted may lie above or below one. Thus 100% shifting is certainly not the polar case which it would appear to be in simple models of perfect competition. The proportion shifted is lower the higher is F, the elasticity of the elasticity of demand since with higher F an increase in market price causes a higher rise in the elasticity thus dampening the effect more strongly. It is higher in monopolistic competition than in generalised Cournot provided the number of firms decreases with the tax (this will be the case if profits decrease for a fixed number of firms and this requires F to be sufficiently high).

The dependence of these results, and those concerning dual pricing, on the elasticity of the elasticity indicates that one must be careful in choosing functional form for demand functions in policy analysis (a cautionary tale familiar in other contexts, see e.g. Atkinson and Stiglitz, 1980, Chapter 14). We would, in general, congratulate an econometrician who could produce a reliable estimate of a demand elasticity and here we find ourselves asking for an estimate of the elasticity of the elasticity. Functional forms which might be useful in practice are, for $F \neq 0$,

$$\log X = B - A p^F \tag{22}$$

which has constant elasticity of the elasticity (for $F = 0$ we have the familiar isoelastic form $\log X = B - A \log P$) or for $E \neq 1$,

$$p = B - AX^{1-E} \tag{23}$$

which has constant elasticity E with respect to X of the slope $p' = \frac{dp}{dX}$, of the inverse demand curve (see Seade, 1980a and b and 1983 on the role of E; note $E = -\frac{Xp''}{p'}$ and $F = \frac{p\varepsilon'}{\varepsilon}$ so that $F = (1 + \varepsilon - \varepsilon E)$).

We conclude with a brief summary of the findings concerning dual pricing. First, examples of the scheme are very common in practice. They range from the Indian example which motivated the work, to black markets, to forms of profits taxation and to public contracts for housing or medicine where a certain proportion of the output must be sold at a fixed price.

Secondly, if both the retention price and the open market proportion can be chosen then the optimum policy for all market structures and versions of the scheme is to set the open market proportion to the minimum possible: zero in generalised Cournot and the lowest consistent with zero profits in monopolistic competition. The government essentially takes full control of the market. The limiting values of both p and p_R is the marginal cost c in the case of generalised Cournot.

Thirdly, the open market proportion may be fixed e.g. by convention, legislation or considerations of enforcement. In this case the optimum retention price, from the point of view of consumers, depends on the curvature of the demand curve and on the market structure. If the elasticity of the elasticity (F) is sufficiently low then one would increase the retention price to the market price. However, where F is higher there will be an optimum retention price below the market price (where F is higher there are lower gains to consumers from a reduction in market price following an increase in p_R, which itself acts like a tax or cost reduction). Note that this means that there is an efficiency gain from the scheme in the case of monopolistic competition, since then both profits and government revenue are zero so that consumer surplus is the relevant efficiency criterion.

Fourthly, the versions of the scheme with fixed retention proportion and fixed retention quantity can be analysed in similar ways with broadly similar results. In the latter case the retention price has no effect on open market price in the generalised-Cournot model and consumers unambiguously prefer decreases in both p_R and θ. With monopolistic competition and fixed θ interior solutions for p_R are possible.

Finally, there will in general be distributional arguments in favour of the scheme. With a sufficiently high weight on the lump-sum transfer we would never eliminate it by raising the ration price to the market price.

References

Atkinson, A.B. and Stiglitz J.E. (1980), Lectures on public economics, McGraw-Hill

Cowling, K.G. and Waterson M. (1976), Price-cost margins and market structures, Economica, 43, pp. 267-74.

Dixit, A.K. and Stern N.H. (1982), Oligopoly and welfare: a unified presentation with applications to trade and development, European Economic Review, 19, pp 123-143.

Heineke, J.M. (1978), Economic models of criminal behaviour, North-Holland

Mukherji, B., Pattanaik, P.K. and Sundrum R.M. (1980), Rationing, price control and black marketing, Indian Economic Review, XV, pp 99-118

Neary, J.P. and Roberts K.W.S. (1980), The theory of household behaviour under rationing, European Economic Review, 13, pp 25-42

Seade, J. (1980a), The stability of Cournot revisted, Journal of Economic Theory, 23, pp 15-27

Seade, J. (1980b), On the effects of entry, Econometrica, 48, pp 479-489

Seade, J. (1983), Prices, profits and taxes in oligopoly, mimeo, University of Warwick

Stern, N.H. (1978), On the economic theory of policy towards crime, Ch 4 in ed Heineke (1978)

Stigler, G.J. (1964), A theory of oligopoly, Journal of Political Economy, 72, pp 44-61

Tobin, J. (1952), A survey of the theory of rationing, Econometrica, 20, pp 521-553.

Appendix

It is shown here that in the models examined in this paper there is no optimum for consumer welfare, if both p_R and θ are choice variables, which is interior to the inequality constraints in the problem. We consider the problem

$$\underset{\theta,\ p_R}{\text{Maximise}} \quad W$$

subject to

$$f(\theta,\ p_R, c, p, n) \equiv p\theta(1 - \frac{\gamma}{\varepsilon}) - c + (1 - \theta)\ p_R = 0 \qquad (12)$$

$$g(\theta,\ p_R, c, p, n) \equiv \left(p\theta - c + (1 - \theta)\ p_R\right)X - Kn = 0 \qquad (13)$$

and

$$p \geq p_R \geq 0; \quad 0 \leq \theta \leq 1; \quad X(p) \geq 0; \quad n \geq 1$$

in the case of monopolistic competition. The derivatives of W are given by (15).For generalised Cournot we drop (13) and $n \geq 1$ (since n is fixed).

From (15) we have

$$\frac{\partial W}{\partial \theta} = -\ X(p - p_R) - \left[\theta X - (p - p_R)\ (1 - \theta)X'\right] \frac{\partial p}{\partial \theta} \qquad (22)$$

$$\frac{\partial W}{\partial p_R} = -(1-\theta)X - \left[\theta X - (p - p_R)(1-\theta)X'\right]\frac{\partial p}{\partial p_R} \qquad (23)$$

Further

$$\frac{\partial p}{\partial p_R} = \frac{-f_{p_R}}{f_p} \quad \text{and} \quad \frac{\partial p}{\partial \theta} = \frac{-f_\theta}{f_p} \qquad (24)$$

in the case of generalised Cournot and

$$\Delta' \frac{\partial p}{\partial \theta} = f_{p_R}\, g_n - f_n\, g_{p_R} \qquad \Delta' \frac{\partial p}{\partial p_R} = f_\theta\, g_n - f_n\, g_\theta \qquad (25)$$

where $\Delta' = f_n\, g_p - f_p\, g_n$ in the case of monopolistic competition.

From (22) and (23)

$$(1-\theta)\frac{\partial W}{\partial \theta} - (p - p_R)\frac{\partial W}{\partial p_R} = -M\left[(1-\theta)\frac{\partial p}{\partial \theta} - (p - p_R)\frac{\partial p}{\partial p_R}\right] \qquad (26)$$

where M is the expression in square brackets in (22) and (23).

For generalised Cournot we then have the r.h.s. of (26) as

$$-\frac{M}{f_p}(1-\theta)\frac{p\gamma}{\varepsilon} \qquad (27)$$

Both M and f_p are positive (the latter by stability see (6)) so that (27) is positive and

$$\frac{\partial W}{\partial p_R} \leq 0 \quad \text{implies} \quad \frac{\partial W}{\partial \theta} < 0 \quad \text{for } \theta < 1 \tag{28}$$

It is easily checked using (25) that (28) applies to the case of monopolistic competition, and using (12) and (19) to the case of fixed retention quantity and monopolistic competition. Hence in all these cases we cannot simultaneously have $\frac{\partial W}{\partial p_R} = 0$ and $\frac{\partial W}{\partial \theta} = 0$ (for $\theta < 1$) and there is no interior solution. We can use (28) and the information in Tables 3 and 4 to check the boundary at which the optimum will occur. This will involve $\theta = 0$ in generalised Cournot and $n = 1$ in monopolistic competition. In the former case this implies $p_R = c$ (if p is bounded).