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A PROBLEM IN PHASED DEVELOPMENT

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A Problem in Phased Development Solved with the aid of Linear Programming.

In formulating a plan for future development the farmer has certain scope and certain difficulties. This paper outlines a procedure in which linear programming was used to discover the solution to a development problem. The results proved realistic enough to encourage the farmer to adopt one of the two plans which the method produced.

Normally it will be possible for a farmer to specify his objectives over time; indeed unless his objectives are clarified it will be impossible to proceed with rational planning by any method. It is likely that these objectives can be formulated in terms of a level of aspiration, either income or net worth, for some date ahead, before which its achievement is highly unlikely and after which it would hold less attraction, and certain likes and dislikes which to some extent predetermine the range of action and the methods he is prepared to employ. As a result there will often be conflicts of a socio-economic kind to resolve before planning may even begin.

The farmer has a present or foreseeable situation which may consist of first, an array of resources which have certain acquisition and salvage values and which are transformable into products, second, existing production lines in respect to which he has certain knowledge about techniques and past performances, third, expectations about future prices, performances and changing values of resources. Lastly, he has certain skills and sources of information which will influence the course of action he will choose.

Now where the existing or expected situation diverges from the objectives and level of aspiration, the farmer has a problem involving time, and must plan to change. This planning will involve the determination of that combination of activities which will provide the most rapid growth of resource control to reach the plane of functioning visualised. The best combination of activities is dependent upon their expected price ratios and expected marginal rates of technical substitution. Thus a prerequisite for such planning is the prescription of costs and revenues of individual activities, given input-output data.

Moreover, when time is introduced into the analysis there are some additional components to the problem. For example it is necessary to know how to extend control over resources not present in the existing structure, and it is necessary to allow for the possibility of substituting resources for each other over time.

In the past farm economists have handled this kind of problem by means of development budgets. Where the possible alternatives are few, and where the experience and judgement of the farm economist or the farmer are highly developed, then budgets may provide valuable guides to the best courses of action. But when the problem is complex, involving not only competing production activities, but also alternative investment patterns in relation to limited capital, most farm economists would confess to a certain amount of groping in arithmetical gloom. Just as linear programming may remove some of the arbitrary elements from the budgeting of complex production situations in a static sense, so it may serve when the dimensions of the problem are extended to take account of time.

Early in the history of linear programming Dorfman (1951) pointed out that the models may be permitted to vary in several dimensions, including time. He indicated that a genuine dynamic element is imparted to the model when the level of, and limitations to, activities in one period are determined by activities in previous periods. If a production programme is considered as continuing over a number of periods of time, and the levels of resources and production are specified as a function of activities in earlier periods, the framework of a dynamic analysis results when we seek to determine the level of each process in each period.

There have been some previous excursions in this field by agricultural economists. Swanson (1955) introduced time into the analysis of a problem involving the optimum integration of crop and livestock activities. Taking a planning horizon of five years, and allowing transfer of capital between years, he showed that as the plan matured through reinvestment, different optimum organizations resulted.

In meeting the problem of the constraint on operating capital, and the inter-period transfer of funds generated in the system, Swanson was confronted by the problem of seasonality. He attempted to meet this by assuming that all expenses for a given year are met on a particular day, January 1st, and all income is received on a particular day, December 31st. This is an assumption of considerable significance, for if there is marked seasonality of capital movements, then the single capital constraint is likely to produce sub-optimum solutions.*

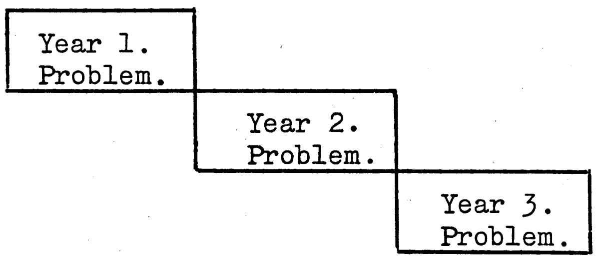
Loftsgard and Heady (1959) presented a dynamic model for deriving optimum farm and home plans for Iowa farms. The basic problem was the element of interdependence between household and farm requirements for operating capital. The model was devised for solving optimum farm plans for a number of years, where productivity of resources in the farm business was related to expenditure needs of the farm family. It was described as

* This problem is discussed by Stewart (1961).

dynamic in that each coefficient was identified with a particular time period. Outputs of one period became inputs of succeeding periods, and successive periods were linked by capital flows. Household expenditure, and other fixed costs were forced into the programme by ascribing fictitiously high revenue values to them.

The model did not include provision for new investment. Added investments were determined exogenously, and were allowed for in the fixed cost deductions. The latter also included household expenditure, machinery depreciation, taxes and insurance.

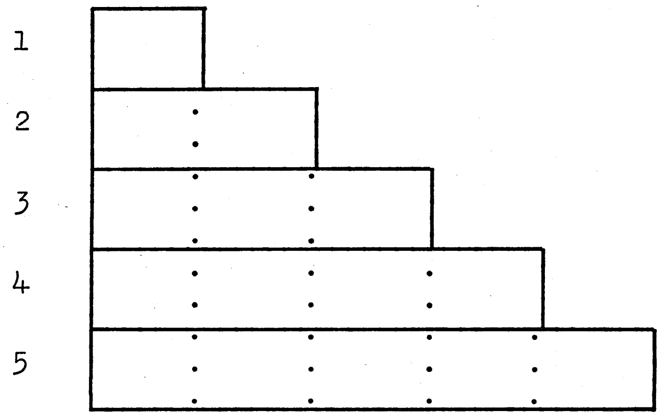
Candler (1960) has pointed out that the Loftsgard-Heady model could be simplified to an ordinary parametric model, allowing operating capital to be the variable parameter, if the only difference between years is the supply of operating capital. Candler suggested that such problems are characterised by block diagonal-type matrices which may be illustrated as follows:-



The matrix elements of these blocks are the same, the only difference being in the supply of operating capital for each period. The problem could therefore be reduced to a matrix incorporating only one year's activities, and allowing capital to be the parameter to be varied. The solution would indicate the optimum programme under various levels of capital availability. The optimum expansion path of the farm could then be derived by relating the funds generated in any one programme to the operating capital requirements of other programmes.

But neither the Loftsgard-Heady model, nor Candler's modification of it are adequate where the motive is to examine the possibility of further investment in fixed resources during the time period being considered, and where this investment is to be endogenously determined. In this case the resource pattern changes in successive periods as investment activities influence it. It seems necessary that investments in a resource in different periods have to be defined as separate activities. Correspondingly, because production activities in any time period are constrained not only by the level of initial resources, but also by previous investments in those resources, successive production activities must be defined independently.

The result is a series of cumulative sub-matrices as shown below:



Each of the five rectangles corresponds to an individual time period. Rectangle 1 relates to period 1, and is a simple matrix, relating the possible production and investment activities in period 1 to the initial constraints. In the left hand sector of rectangle 2, resources may be augmented by the activities of period 1. For example, capital funds will be augmented by production activities of period 1, and physical resources will be augmented by investment activities in period 1. In the right hand sector of rectangle 2, period 2 production and investment activities are introduced. In period 3, the physical resources are augmented by the period 1 investments in the left hand sector, by period 2 investments in the centre sector, and the capital funds by period 2 production in the centre sector. In the right hand sector period 3 activities are defined.*

Clearly, a model of this kind would have to incorporate a number of assumptions as to the timing of investment and production processes. For example if the time periods are defined in years, then it would be necessary to make an assumption, such as, where an investment occurs in year t - 1, then its production capacity becomes available at the beginning of year t. This does not appear to be too unrealistic, especially if the time periods are shorter, as in the example which is described later. It is also necessary to make a similar assumption in respect to production activities, to assume,for example, that capital funds generated by production activities in period t - 1, become available for further production or investment at the beginning of period t.**

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* This kind of construction clearly can lead to very large and sparse matrices, e.g. if there are twelve activities, six constraints and six time periods, the matrix will be 72 x 36 excluding disposal vectors.

** Problems may arise here in respect to the assumptions of additivity.

The Model.

In developing a linear programming model incorporating time, the customary terminology is used.

x_j states the level at which activity is to be carried on, where $j = 1, 2, \ldots\ldots, n$, there being n possible activities.

b_i states the level of availability of the ith resource, where $i = 1, 2, \ldots\ldots, m$, there being m limiting resources.

r_{ij} states the requirement of the jth activity for the ith resource.

c_j denotes the net revenue per unit of the jth activity, where net revenue is defined as the gross revenue per unit less the cost of inputs which are in variable supply.

Introducing time into the model, each period of time may be denoted by the superscript k, where $k = 1,2,\ldots\ldots,t$, there being t time periods in the plan.

Thus x_j^k states the level of the jth activity in kth time period, b_i^k the level of availability of the ith resource in the kth period, r_{ij}^k the requirements in the kth period of the jth activity for the ith resource, and c_j^k the net revenue of the jth activity in the kth time period.

For the first step in the development of the model interperiod transfers of capital and augmentation of resources are omitted. The first relationship for the first period may then be expressed as follows:-

$$(1) \quad b_1^1 \geqslant r_{11}^1 x_1^1 + r_{12}^1 x_2^1 + \ldots\ldots\ldots + r_{1n}^1 x_n^1$$

as all $r_{ij} = o$ for periods 2 to t inclusive. In the general case of the constraint imposed by the ith resource in the kth year, we have

$$(2) \quad b_i^k \geqslant \sum_{j=1}^{n} r_{ij}^k x_j^k \qquad (i = 1,2,\ldots\ldots,m)$$

where all $r_{ij} = o$ for periods other than k.

The second step is to consider interperiod transfers of capital. Let the activities n-2, n-1, and n be production activities producing capital in one period, which becomes available at the beginning of the succeeding period.

The capital externally available in period 1 is designated b_c^1. Then for period 1, the relevant constraint is expressed:-

(3) $$b_C^1 \geqslant \sum_{j=1}^{n} r_{Cj}^1 \, x_j^1$$

Here the r's are positive, indicating the per unit requirements of production activities and investment activities for capital in period 1. The figure for the production activities will be equivalent to the direct costs per unit, and for the investment activities the total initial capital costs per unit.

Then in period 2, the capital available is any which is externally available, plus that which is produced internally. The relationship then takes the following form:-

(4) $$b_C^2 \geqslant \sum_{j=1}^{n} r_{Cj}^2 \, x_j^2 - \sum_{j=n-2}^{n} r_{Cj}^1 \, x_j^1$$

Here the r's are positive in column vectors representing production and investment activities of period 2, and negative in column vectors representing period 1 activities which produce capital funds for period 2. The latter coefficients will be equal to the unit gross revenues of these activities. Subsequently it will be shown that provision may be made for transferring capital not required in one period to the next, by defining an addition vector. This means that unless there is an external source of capital in period 2, the coefficient b_C^2 will be zero.

The general form of (4) where b_C^k refers to the availability of capital in the kth period, and where again the activities which generate capital funds are denoted n-2, n-1 and n, is:-

(5) $$b_C^k \geqslant \sum_{j=1}^{n} r_{Cj}^k \, x_j^k - \sum_{j=n-2}^{n} r_{Cj}^{k-1} \, x_j^{k-1}$$

In (4) all r = o for periods 3 to t inclusive, and in (5) all r = o for periods k + 1 to t inclusive.

The third step is to consider the case of investment activities, which augment the supply of limiting resources.

Let the activity q, the level of which is denoted x_q, augment the supply of resource i, whose initial level of availability is denoted b_i. In period 1 the relevant constraint is:-

(6) $$b_i^1 \geqslant \sum r_{ij}^1 \, x_j^1 \qquad (j = 1, 2, \ldots, q, \ldots, n)$$

For all activities in period 1 which do not require the ith resource, including q, and for all activities where k 1, r_{ij} = o. Then in period 2 the relationship becomes:-

$$(7) \qquad b_i^2 \geqslant \sum_{j=1}^{n} r_{ij}^2 x_j^2 - r_{iq}^1 x_q^1$$

In (7) the term r_{iq}^1 refers to the number of units of the ith resource which are provided for period 2 by one unit of the qth activity in period 1.

The fourth step is to secure a method for introducing additional loan capital into the system. Assuming that the farmer is able to command additional capital in the form of borrowing, additional vectors may be added by which the economics of this may be analysed. There would be a negative entry in the revenue function equivalent to the rate of interest charged for the loan, and a negative coefficient in the row vector corresponding to the capital constraint for the appropriate period. An assumption has to be made in respect to the timing of the availability of borrowed funds. A realistic approach would be to assume that funds are borrowed at the beginning of a time period, and are immediately available.

Thus, if the borrowing activity is designated L, and its possible level in the kth period is denoted x_L^k then the relevant constraint would read:-

$$(8) \qquad b_C^k \geqslant \sum r_{Cj}^k x_j^k - r_{CL}^k x_L^k \qquad (j = 1,2,\ldots,L-1,L+1,\ldots,n)$$

Here the coefficient r_{CL} indicates the number of units of capital supplied by one unit of borrowing. An upper limit on borrowing over the period being programmed may be imposed by an additional constraint such as (8).

$$(9) \qquad b_i \geqslant \sum r_{ij} x_j \qquad (j = L^1, L^2,\ldots\ldots,L^k,\ldots L^t)$$

where b_i states the total permissible borrowings.

If it were wished to consider a discontinuous cost function in respect to borrowed funds - for example if the farmer could secure a second loan but only at a higher rate of interest, then this could be achieved by introducing a second activity for borrowing. It would have a different c_j value in the revenue function corresponding to the higher rate of interest. It is clear that the second loan would not be taken up until the first was fully exploited.

The basic form of the model is now established. Provision has been made for, (a) the dating of inputs and outputs, (b) the interperiod transference of capital funds generated by production activities, (c) the endogenous augmentation of resources by investment activities, and (d) the borrowing of additional capital. But there is one further complication. This is the problem of defining the objective function.

Hicks (1939) has defined the criterion of a dynamic production plan in terms of the capitalised value of a stream of surpluses, and has stated that the problem of maximising the present value of a production plan is formally identical with the problem of maximising the surplus of receipts over costs in the static problem of the firm. "Outputs of different dates are to be regarded as different outputs, inputs of different dates are to be regarded as different inputs, and beyond that there is one little difference . . . . Future costs only enter into the present value of a plan at the discounted values, and the same is true of future receipts. Consequently when we are adopting our static analysis, we must always replace the 'prices' of statics by discounted prices, in order to fit the dynamic problem. With these adjustments the whole static theory of the firm still holds."

In the Loftsgard-Heady model, which, it will be recalled, does not include investment activities, the above criterion has been met by making the transposition:-

$$c_j^k = \bar{c}_j^k \div (1 + \rho)^k \tag{10}$$

where $\bar{c}_j^k$ is the non-discounted net revenue of the jth activity in the kth year, and ρ is the rate of interest.

Conversely, if there are outlays on investment activities, as in the present problem, provision can be made for the compounding of costs by transposing as follows:-

$$- c_j^k = - \bar{c}_j^k (1 + \rho)^{t-k} \tag{11}$$

where $- \bar{c}_j^k$ is the non-compounded cost of the jth activity in the kth year, and where the production plan is for t years.

It is important to be aware that there are two aspects of cost to the investment activities. First there is the initial capital cost of a unit of the investment, which is a once and for all cost. Second there are the annual costs relating to the asset created, such as depreciation and repairs and maintenance. Clearly the appropriate cost for incorporation in the objective function is the annual cost, and this could be compounded in the way illustrated in (11) above. To compound the initial capital cost would require some other expedient.

Similarly where there are livestock purchasing activities, there are both the initial capital cost, and continuing annual costs, such as feed, miscellaneous expenses and depreciation. The same problem of compounding arises in respect to the initial capital cost.

A third complication in the defining of the revenue function has already been mentioned. This is the need to force the "fixed costs" activity for each period into the solution by assigning a fictitiously high net revenue value to it. Fictitious revenue thus generated in the solution, would have to be deducted subsequently. For each period there would have to be an appropriate constraint, where the entry in the limitations column would be equal to the total level of fixed costs. The coefficient in this row, and the fixed costs activity column would be 1, assuming identical units of measurement are adopted.

It appears that this complication may be avoided, as well as the whole discounting and compounding problem, if, (a) the objective is defined as the maximization of current revenue in the t^{th} period, (b) a specific "Capital transfer" activity* is introduced, the function of which is to allow capital not required for production and investment activities to be maintained intact (in the bank) and transferred to the next period.

By following (a) the objective function is expressed as:-

$$f = \sum c_j^t \, x_j^t \tag{12}$$

Where c_j^t states the net revenue of the jth activity in the t^{th} period. In this case all c_j's = o for k < t. Under this definition the sole function of activities in periods preceding t is to generate productive capacity in period t, in order to maximize revenue in that final period. Clearly there is no point in including investment activities in period t, as these would not enter the solution. But any surplus capital in period t may be absorbed by including a banking activity and assigning it a net revenue equal to the rate of interest.

It would then be possible to force the "fixed cost" activity for each period into the solution by imposing a minimum constraint for each period, a constraint of the kind:-

$$b_i^k \leqslant r_{ij}^k \, x_j^k \tag{13}$$

Where b_i^k states the minimum level of fixed costs which must be met in period k, x_j^k states the level of the "fixed costs" activity in the kth year, and r_{ij}^k has the customary meaning. (Normally, where the units of "fixed costs" activity were £1, then the r_{ij} coefficient would be 1). It is clear that when this condition is imposed the "fixed cost" activity would come in only at its minimum level in each period. As it generates no capital, and augments no resource, it is dominated by all other activities, including the "capital transfer" activity.

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* See Candler (1960).

By allowing the "capital transfer" activity to gather interest the need for discounting future revenues and compounding present costs is avoided. In effect such an activity becomes competitive with production and investment activities for capital. For example, assume a rate of interest of 5%. Then one unit of the capital transfer activity in period 1 will provide 1.05 units of capital in period 2, where the periods are years. Thus every unit of capital which is used for production or investment in period 1 sacrifices this return, which is equivalent to compounding its costs. Conversely, the present earnings of the capital transfer activity have the effect of discounting future incomes from production activities.

The scope of the generalized capital constraint:-

$$b_i^k \geqslant \sum r_{ij}^k \; x_j^k \qquad \begin{array}{l} i = 1,2,\ldots\ldots\ldots,m \\ j = 1,2,\ldots\ldots\ldots,n \\ k = 1,2,\ldots\ldots\ldots,t \end{array} \tag{14}$$

may now be extended to account for:-

(a) positive r_{ij}'s indicating the per unit capital requirements of production activities in period k, investments in k, and annual costs resulting from investment in periods 1 to k-1 inclusive,

(b) negative coefficients in vectors representing production activities in period k-1, corresponding to the gross output per unit of these activities,

(c) negative coefficients in vectors representing capital transfer activities of period k-1, which will be equal to 1 + ρ times the unit of this activity where ρ equals the rate of interest being earned,

(d) negative coefficients in vectors relating to borrowing activities in period k, which will be equal to 1 - ρ times the unit of borrowing, where ρ equals the rate of interest being charged for the loan.

The determination of the objective function as the maximisation of current revenue in the t^{th} period clearly cannot be accepted on the grounds of computational convenience alone. It is necessary that it be sustained by consideration of realistic objectives for farmers in particular situations.

A classical dynamic model involves the concept of present decisions resulting in future activities, which, with due allowance for discounting, produce a stream of satisfactions. The objective normally is to maximise this stream of satisfactions. For the problem to be amenable to programming however, it must have a terminating period. But this is not an unrealistic basis on which to construct a farm management model. A common decision making framework for farmers involves a level of aspiration to be fulfilled in a certain time period. Uncertainties and human impatience frequently cause this to be quite short.

In the present example the farmer had an initial planning horizon of two or three years. His objective was to maximise his revenue earning capacity in this time, in relation to the availability of capital, and other physical and managerial constraints.

By excluding the possibility of salvaging fixed assets in the t^{th} period, we ensure that not only is current revenue maximized but also the revenue earning capacity is sustained.

The Example.

A young farmer, after a period as a successful manager of a mixed farm, purchased a 30 acre holding with the intention of developing an intensive pig unit. The initial resources consisted of an amount of capital, the land, three small sheds which could be converted for pig fattening on a deep litter system, and the labour of the farmer himself and one man. The problem was to plan this farm over an initial period of two years, having in mind that in relation to the available resources there were a number of different investment possibilities to consider, such as different types of sow accommodation and fattening houses, a number of different production activities, including pork, bacon, and heavy hogs, and alternative ways of obtaining store pigs, by breeding them or purchasing weaners. It was decided to plan on the basis of six-monthly periods.

However, at the date on which a plan could begin, the farmer had already committed himself to some initial investment in fattening accommodation and pigs, as well as some essential outlays on improvements to the farm house, roads and buildings. Therefore there was an additional complication of accounting for pigs "in the pipeline" at the date on which the programme was to begin.

In order to minimize the matrix size, yet not to omit any possible profitable alternative, some initial budgeting was carried out. A number of activities obviously less efficient in the utilization of limiting resources were then omitted. In period 1, the possible activities finally included in the programme were:- *

Production	1	140 lb. Porkers.
	2	Heavy Hogs.
Purchase of sows	3	Sows (type A).
	4	Sows (type B).
	5	Sows (type C).

* Fuller details of these activities are given in Appendices I to IV. Appendix V is a comment on the problem of making provision for resource substitution during the course of the plan.

Pig Accommodation	6	Sow accommodation (type A).
	7	Sow accommodation (type B).
	8	Sow accommodation (type C).
	9	Fattening accommodation.
Purchase stores	10	Weaners.
Capital Items	11	Fixed costs.
	12	Transfer capital.
	13	Borrow capital.

For periods 2 and 3 the activities are the same, their possible levels being denoted x_j^2 and x_j^3 where j is again 1 to 13. But in period 4 investment in pig accommodation, purchase of sows, and transfer of capital are not included. The only function of these in the model is to augment physical or capital resources in succeeding periods. Thus, whatever number of periods are included in the programme this kind of investment will cease in the penultimate period.

The capital transfer activity is converted to a banking activity, with a positive net revenue equivalent to the rate of interest earned per period. Also, in period 4, the four borrowing activities are gathered together into a loan activity, which has a negative net revenue corresponding to the rate of interest on the loan. The period 4 activities therefore are as follows:-

1 140 lb. Porkers.
2 Heavy Hogs.
3 Purchase weaners.
4 Fixed costs.
5 Borrow capital.
6 Bank.
7 Loan.

The Constraints.

(a) Fixed costs. Constraints 1 to 4 impose the minimum requirement on the level of fixed costs which must be met in each period. These are detailed in appendix IV amounting to £1,250 per period. Thus row 1, which imposes the condition for period 1 is:-

$$x_{11}^1 \geqslant 1250 \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

If it were necessary, the level of drawings or wages could be increased during the currency of the plan, by enlarging the entry in the limitations vector. In the present case, the figure remained at £1,250 for the four periods. This amount includes an allowance for maintenance of the existing buildings. Maintenance costs for new buildings erected as part of the programmed plan are met by deductions in succeeding capital rows.

(b) Capital. The capital available for period 1 included a £2,000 loan which had not been taken up, plus that which was being produced by the pigs being fattened in the existing accommodation. The farmer had decided on the weights at which he would sell these pigs. Estimates of their prices, and of the outlays necessary to take them to this stage were made, and their contribution to the capital supply made accordingly. This figure came to £7,500, giving the initial capital supply of £9,500.

The capital requirements per unit of each activity have been detailed in the appendices to this chapter. In respect to the pork activity the additivity assumption would be violated unless some account was taken of the fact that the capital turnover per porker is more rapid than six-monthly. In fact the turnover is about twice this rate. This was met by defining a unit of the activity as two pigs, and expressing the capital requirement as that for one pig only. The capital produced for the succeeding period then becomes the net revenue for two porkers, not the gross revenue as for heavy hogs.*

The capital constraint for the first period is accordingly expressed:-

$$9{,}500 \geqslant 3.5\, x_1^1 + 9.25\, x_2^1 + 40\, x_3^1 + 40\, x_4^1 + 40 x_5^1 + 309 x_6^1 + 626 x_7^1 + 937\, x_8^1 + 9.75\, x_9^1 + 5.7\, x_{10}^1 + x_{11}^1 + x_{12}^1 - .96\, x_{13}^1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (5)$$

The period 2 capital row will have identical coefficients for all j, but in the period 1 sector, the coefficients will indicate the capital generated per unit of production activities in period 1, or the operating capital required per unit of investment activity of period 1. For example, the coefficient in the x_5^1 (purchase sows) column vector, and the Capital (Period 2) row, will indicate for period 2 the direct costs per sow purchased in period 1. The coefficient in the x_9^1 (Investment in fattening accommodation) vector and the Capital (Period 2) row will indicate the period 2 maintenance costs per unit of fattening accommodation erected in period 1. The full relationship for capital in period 2 is as follows:-

$$0 \geqslant -18\, x_1^1 - 21\, x_2^1 + 26.15\, x_3^1 + 23.85\, x_4^1 + 24.95\, x_5^1 + 24.75\, x_6^1 + 70.25\, x_7^1 + 75\, x_8^1 + 0.61\, x_9^1 - 1.025\, x_{12}^1 + \sum_{j=14}^{26} r_{2j}^2\, x_j^2 \quad \ldots\ldots\ldots\ (6)$$

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* Where Gross revenue per porker = £12.5
direct costs = £3.5
net revenue = £9.0
Then the first turnover produces £9 for the second turnover, which requires £3.5 for direct costs leaving £5.5 + £12.5 = £18 to be transferred through.

where $\sum_{j=14}^{26} r_{2j}^{2} x_{j}^{2}$ repeats the coefficients of equation (5).

Constraints (7) and (8) apply to Capital in periods (3) and (4) respectively.

(c) Borrowing.

Constraint 9 of the model imposes an upper limit of £5,000 on the total amount of borrowing. This was based on the farmer's own judgement of the maximum additional borrowing he wished to contemplate in the short term.

The constraint is expressed as:-

$$5{,}000 \geqslant x_{13}^{1} + x_{13}^{2} + x_{13}^{3} + x_{5}^{4} \quad \ldots\ldots\ldots\ldots(9)$$

The four borrowing activities are brought together in the loan activity of period 4 in the following way:-

$$x_{7}^{4} \geqslant x_{13}^{1} + x_{13}^{2} + x_{13}^{3} + x_{5}^{4}$$

$$0 \geqslant x_{13}^{1} + x_{13}^{2} + x_{13}^{3} + x_{5}^{4} - x_{7}^{4} \quad \ldots\ldots\ldots\ldots(10)$$

(d) Fattening Accommodation.

Constraints 11 to 14 inclusive reconcile the availability of fattening accommodation in each period consecutively with the requirements of production activities. For period 1, at the beginning of which there were already pigs in the course of production, it was necessary to make an allowance for the fattening space which these would occupy until they were disposed of at pre-determined weights. The unit of measurement for fattening accommodation was taken as one standard fattening day, which was equivalent to the space required by one weaner pig (up to 140 lbs. liveweight) for one day. This was 10 square feet per day. The space requirements of larger pigs were:-

baconers	11.5	sq.feet
heavy hogs	16	sq.feet

The average days in the fattening house from the weaner stage to slaughter were:-

140 lb. porkers	74	days
200 lb. baconers	109	days
260 lb. heavy hogs	134	days

Therefore the standard fattening day requirements of the two classes of pig considered in the final programme were calculated as follows:-

$$140 \text{ lb. porkers} \quad = \quad 74 \text{ standard fattening days}$$

$$260 \text{ lb. heavy hogs} \quad = \quad 74 + \frac{11.5}{10}(109-74) + \frac{16}{10}(134-109)$$

$$= \quad 154 \text{ standard fattening days.}$$

The fattening accommodation in existence at the beginning of period 1 was equivalent to 110,640 standard fattening days. The residual after allowing for pigs in the course of production was 81,540 standard fattening days. Thus the relevant constraint for period 1 is expressed:-

$$81{,}540 \geqslant 148\, x_1^1 + 154\, x_2^1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(11)$$

In period 2 a unit of fattening accommodation constructed in period 1, would augment the existing fattening accommodation by 182 standard fattening days (1 unit of fattening accommodation = 10 square feet). Therefore, the relevant constraint for period 2 is as follows:-

$$110{,}640 \geqslant -\,182\, x_9^1 + 148\, x_1^2 + 154\, x_2^2 \quad \ldots\ldots\ldots\ldots(12)$$

Similarly, the 110,640 standard fattening days appear in the limitations vector for the remaining fattening accommodation rows (13) and (14), but it will be cumulatively augmented by investment in fattening accommodation in successive periods. This augmentation is represented by negative coefficients in the x_9^k vectors.

(e) Sow accommodation.

The details concerning the sow accommodation are given in appendix II. There was none in existence at the beginning of period 1, thus there are all zero's in the limitations vector. The number of sows purchased in any period must be related to the amount of sow accommodation built in that period. Therefore the number of sows and the amount of accommodation can be reconciled by a simple relationship for each period. For example in period 1, the purchase of A type sows and A type accommodation are reconciled as follows:-

$$x_3 \leqslant 40\, x_6^1$$

or

$$0 \geqslant x_3^1 - 40\, x_6^1 \quad \ldots\ldots\ldots\ldots\ldots(15)$$

Sow accommodation is defined in terms of 40 sow units. x_6^1 denotes the level of the A type sow accommodation activity in period 1, and x_3^1 the level of purchases of A type sows.

It is possible to express the sow and sow accommodation relationships in this simplified way because of the short time period being considered. If it were longer, then the complication of wastage in sows would occur, and a slightly different approach would be necessary. It would be necessary to accumulate sows and sow accommodation by defining additional activities, and additional rows to reconcile the absolute level of these in each period with the current and previous investment in them. If for example the effective producing life of a sow was taken as three years, then sows purchased in period 1 would augment the number of sows in the additional vector in periods 2,3 and 4, but would disappear in period 5. In calculating the net revenue some account would have to be taken of depreciation in the value of the sow.

Moreover, the accommodation constraint would have to be related cumulatively to the number of sows, rather than independently for each period as in the present model. However these are differences of mechanics, rather than of principles. In every situation mechanical aspects of a linear programming model must be adjusted to suit the particular problems of that situation.

With three different types of sow accommodation, over three periods (there is none in the first period) there are nine rows for this constraint, (15) to (23).

(f) <u>Weaners</u>. There are two sources of weaner pigs, direct purchase and breeding. Sows purchased in period 1, provide weaners in period 2, and so on. It will be recalled that our assumption as to investment activities is that corresponding resources become available on the first day of the period succeeding the actual investment. However purchased weaners are available immediately in the current period.

The three types of sow have slightly different estimates of weaner production, type A produce 17 per annum, type B, 18 and type C, 19. This gives coefficients of 8.5, 9, and 9.5 respectively for the six monthly periods of the model.

It will be recalled that the porker activity is in units of 2 pigs. In period 1, because there are no sows, the weaner constraint is:-

$$x_{11}^{1} \geqslant 2\,x_{1}^{1} + x_{2}^{1}$$

$$\text{or} \quad 0 \geqslant 2\,x_{1}^{1} + x_{2}^{1} - x_{11}^{1} \quad \ldots\ldots\ldots\ldots(24)$$

In period 2, weaners from sows purchased in period 1 become available.

$$8.5\,x_3^1 + 9\,x_4^1 + 9.5\,x_5^1 + x_{11}^2 \geqslant 2\,x_1^2 + x_2^2.$$

or $$0 \geqslant -8.5\,x_3^1 - 9\,x_4^1 - 9.5\,x_5^1 - x_{11}^2 + 2\,x_1^1 + x_2^2 \quad \ldots\ldots(25)$$

For periods 3 and 4, represented by rows (26) and (27), the weaners available are augmented by the production of sows purchased in periods 2 and 3. That is, in period 4, weaners are produced by sows purchased in periods 1, 2 and 3. The same coefficients are used in each period for all sows of a particular type. However, if it were justified, an allowance could be made for the diminishing productivity of sows with increasing age, by having successively lower coefficients. Again, if the length of the plan being programmed exceeded the estimated productive life of the sows, then this would be met by entering zeros in rows corresponding to periods beyond the productive life.

(g) Rate of building new structures.

While the theoretical limitation on the rate at which new buildings may be constructed is the availability of capital, in practice other constraints operate. These are of a more subjective kind but are none-the-less realistic. They include the planning and management limitations imposed by the farmer himself. There is generally a limit to the rate of expansion which he considers is possible to manage efficiently. All farm advisors must have had experiences of farmers over-reaching themselves in the matter of rate of expansion.

In the present case after all relevant factors had been taken into consideration, it was decided to limit the rate of building expansion to that requiring an annual investment of £2,500. This gives a constraint for period 1 as follows:-

$$1250 \geqslant 309\,x_6^1 + 626\,x_7^1 + 937\,x_8^1 + 9.75\,x_9^1 \quad \ldots\ldots\ldots\ldots(28)$$

The coefficients on the right hand side of the relationship represent the per unit capital costs of the three sow accommodation and the fattening accommodation activities.

(h) Land.

The three classes of sow accommodation considered in the programme have different requirements of land, per 40 sow unit. Type A has the heaviest demands, requiring 60 acres per 40 sow unit, type B requires 20 acres per 40 sow unit, and type C, 15 acres per 40 sow unit. These are the only activities included in the programme which use land, therefore the land constraint is expressed in relation to these activities alone, or in respect to their complementary activities, the purchase of sows. One row will suffice to express the total constraint:-

$$30 \geqslant 60\, x_6^k + 20\, x_7^k + 15\, x_8^k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (29)$$

where activities 6, 7 and 8 refer to investment in sow accommodation of the three types, and k refers to the time periods in which this investment may occur.

This completes the description of the mechanics of the dynamic programming model. The omission of labour as a constraint may be queried immediately. Clearly, any attempt to construct detailed labour profiles under the circumstances of this example would be spurious. The only reasonable and realistic approach is to make an overall assessment of the total size of the enterprise which could be operated by the existing labour. In this respect there was some flexibility, as the owner was doing additional managerial work elsewhere, which he was prepared to forego as soon as the work on his own farm required his full-time services. It was therefore considered that the labour constraint was dominated by the limitation on land in the first instance. That is, the most sows which could enter the solution were 80. represented by two units of type C, which require 15 acres per unit. Thus the total number of fattening pigs to be handled per period could not exceed 80 x 9.5 plus purchased weaners. The farmer was quite confident of being able to handle the number of pigs likely to appear in this fairly short term programme. Therefore programming proceeded on the basis of the labour being adequate to manage any feasible programme.

The Solution.

With land limited to 30 acres in the first instance, the following was the programmed solution:-

		Units	Capital requirements (£)	Capital transferred (£)
Period 1				
Production				
2	Heavy hogs	529	4,893	11,109
Purchase of sows				
5	Sows (type C)	53	2,120	
Pig Accommodation				
8	Sow accommodation (type C)	1.33	1,218	
Purchase stores				
10	Weaners	529	3,015	

Period 1 (continued)		Units	Capital requirements (£)	Capital transferred (£)
Capital Items				
11	Fixed costs	1250	1,250	
13	Borrow capital	3177	-3,049	
			9,447	11,109
Period 2				
Production				
2	Heavy hogs	624	5,772	13,104
Purchase of sows				
5	Sows (type C)	19	752	
Pig Accommodation				
8	Sow accommodation (type C)	0.47	440	
9	Fattening accommodation	82.9	808	
Purchase Stores				
10	Weaners	117	667	
Capital Items				
11	Fixed costs	1,250	1,250	
Maintenance				
Sows (C)		53	1,322	
Sow accommodation (C)		1.3	98	
			11,109	13,104*
Period 3				
Production				
2	Heavy hogs	817	7,557	17,157
Purchase of sows				
5	Sows (type C)	8	312	
Pig Accommodation				
8	Sow accommodation (type C)	0.2	187	
9	Fattening accommodation	109.4	1,067	
Purchase stores				
10	Weaners	131	747	

* Rounding errors account for some slight discrepancies between capital required in one period and capital transferred from the previous period.

Period 3 (continued)	Units	Capital requirements (£)	Capital transferred (£)
Capital Items			
11 Fixed costs	1,250	1,250	
Maintenance			
Sows (C)	72	1,791	
Sow accommodation (C)	1.77	133	
Fattening accommodation	82.9	51	
		13,095*	17,157
Period 4			
Production			
1 Porkers	1968	3,444	
Purchase of stores			
10 Weaners	1208	6,886	
Capital Items			
4 Fixed costs	1250	1,250	
6 Bank	3301	3,301	
Maintenance			
Sows (C)	80	1,986	
Sow accommodation (C)	2	148	
Fattening accommodation	192.3	118	
		17,133*	

The programmed net revenue for period 4 was £10,782 derived as follows:-

Sales of 1968 porkers	=	£17,712
less purchase of 1208 weaners		£6,886

In addition a banking activity of £3,301 yielding £83 interest was opposed by a loan activity of £3,177 costing £127. In practice these activities would not occur, of course but the borrower's overdraft would run down. Profit would finally be derived by deducting all maintenance costs for sows and buildings, all fixed costs and sundry costs not accounted for in the definition of activities.

Availability of capital proved not to be the limiting factor, with the assumption that an additional £5,000 would become available at 8% interest. In fact the limit to the rate of expansion was imposed by the constraint on the amount of investment in structures per period which the manager decided

* Rounding errors account for some slight discrepancies between capital required in one period and capital transferred from the previous period.

that he could cope with. This was at its maximum of £1,250 in each of the first three periods. Surplus capital in period 4 was banked and earned 5% This occurred because the loan of £3,000 had to be raised in period 1 in order to exploit the full expansion potential in the early periods, but in later periods, the limits on physical expansion meant that some capital became idle. Of course expansion need not cease in period 4, as depicted in the model and its solution, and some of the money in the bank would in fact be available for further investment in period 4.

The very high profits were a result of several factors. In the first place, the farmer had access to a large amount of credit, in relation to the initial size of the business. At the time that the programme begins the fattening capacity was already very large (over 1,400 heavy hogs per annum) which alone would result in a total turnover of approximately £15,000 per six month period.

Whether the same performances in respect to conversion rates, disease and mortality, labour output, utilization of accommodation, and sow management could be maintained at these high output levels was a matter which required further consideration by the farmer and the adviser. In addition, a programme which involved the purchase of over 2,000 store pigs per annum on the open market, increased the disease risk substantially, and this was a further matter requiring careful consideration. However, what is more important in the present context is not so much that the programmed solution required further assessment from the point of view of its practical interpretation, but that if these practical considerations can be quantitatively formulated, then it is a simple matter to incorporate them in the model. For example, in the preliminary development of this model, the farmer imposed an upper limit of 700 per period on the weaners to be purchased, but later decided to release this constraint.

The changeover from heavy hogs to pork production in the optimum programme was another matter requiring an appraisal from the practical point of view. Examination of the marginal cost figures for excluded production activities showed that in the first three periods very little revenue would be lost by producing porkers rather than heavy hogs. The relevant figures per porker were, for period 1, £0.05, for period 2, £0.04 and for period 3, £0.035. However, in respect to the doubts about buying very large numbers of weaners, the marginal cost of heavy hogs in period 4, was £0.8 per pig. Thus if the farmer persisted with heavy hog production in this period after a build up in pig accommodation corresponding to that planned, the loss in net revenue could be at least £780.* There seemed to be substantial grounds for recommending a changeover at this point. On the other hand, the loss in net revenue if a pork policy was pursued from the outset would not be very great.

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* The total fattening capacity available in period 4 would enable 900 heavy hogs to be produced. The marginal cost would not be less than £0.8 per pig.

The farmer also wished to consider the economics of renting 30 acres of additional land, which he expected to become available.* This was within workable distance of the home farm.

The reprogrammed solution with land increased to 60 acres was as follows:-

Period 1.		Units	Capital requirements (£)	Capital transferred (£)
Production				
2	Heavy hogs	529	4,893	11,109
Purchase of sows				
4	Sows (B)	3	128	
5	Sows (C)	51	2,048	
Pig Accommodation				
7	Sow accommodation (B)	0.08	50	
8	Sow accommodation (C)	1.28	1,199	
Purchase stores				
10	Weaners	529	3,015	
Capital Items				
11	Fixed costs	1,250	1,250	
12	Transfer capital	988	988	1,013
13	Borrow capital	4,250	-4,250	
			9,491	12,122**

Period 2.		Units	Capital requirements (£)	Capital transferred (£)
Production				
2	Heavy hogs	607	5,615	12,747
Purchase of sows				
5	Sows (C)	51	2,048	
Pig Accommodation				
8	Sow accommodation (C)	1.28	1,199	
9	Fattening accommodation	5.3	52	

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* The marginal product figure for land in the original solution was £94.4 per acre.

**Rounding errors account for any discrepancies between capital transferred for one period and used in the next.

Period 2 (continued)		Units	Capital requirements (£)	Capital transferred (£)
Purchase of stores				
10	Weaners	91	519	
Capital Items				
11	Fixed costs	1,250	1,250	
Maintenance				
	Sows	54	1,353	
	Sow accommodation	1.36	102	
			12,138	12,747
Period 3				
Production				
1	Porkers	532	931	4,788
2	Heavy hogs	469	4,338	9,849
Purchase of sows				
5	Sows (C)	54	2,136	
Pig accommodation				
8	Sow accommodation (C)	1.33	1,250	
Capital Items				
11	Fixed costs	1,250	1,250	
Maintenance				
	Sows	105	2,630	
	Sow accommodation	2.64	198	
	Fattening accommodation	5.3	3	
			12,736	14,637
Period 4				
Production				
1	Porkers	1,508	2,639	
Capital Items				
4	Fixed costs	1,250	1,250	
6	Bank	6,487	6,487	
Maintenance				
	Sows	159	3,962	
	Sow accommodation	3.97	300	
	Fattening accommodation	5.3	3	
			14,641	

The programmed net revenue was:-

Porkers	1508	@	£9	=	£13,572
Bank	6487	@	£.025		162
Loan	4250	@	£-.04		-170
					£13,564

The essential difference in this solution was that the additional land was utilized fully by doubling the sow accommodation. This utilized the full investment capacity, apart from a small amount of fattening accommodation which for practical purposes could be neglected. Similarly the very small amount of sow accommodation (type B) in the optimum solution could be neglected in the practical interpretation of the theoretically optimum solution.

The emphasis on investment in sows led to a programme in the fourth period in which all the weaners were bred on the farm. Although the actual number of fat pigs being produced was reduced by over 450, the net revenue was higher. The increase in the programmed net revenue revealed a very high marginal return to land, at least over a range of 30 acres, but it should be kept in mind that this was under special circumstances in respect to labour and capital. It was assumed that the existing labour would handle this programme, and the capital (credit) was again adequate. However, even the employment of an additional full-time man, would still result in a substantial financial advantage in acquiring the extra 30 acres, and concentrating on investment in sows and intensive sow accommodation.

The results of this experiment, therefore, confirm that the present linear programming procedure can, with certain modifications, solve problems of short-term development where the entrepreneur's level of aspiration can be stated in terms of some maximum level of production to be reached at a certain date. It is believed that planning frequently embraces periods of two or three years and there are many reasons why a longer period planned in this way would be unrealistic. The uncertainty of the future is one consideration that has already been referred to; indeed, it bedevils all long term planning. In addition declining efficiency with increasing scale and, parallel to this, the increasing incidence of taxation with increasing profits are factors which it is difficult to incorporate in a logical way. In the experiment already described it was decided that neither of these was likely to upset the planning process. The records available of the manager's previous experience suggested he was capable of highly efficient large scale pig keeping; the incidence of taxation could be safely ignored (a) because the farmer had only been in occupation six months at the starting date of the plan and (b) the sums available for paying tax by the end of period 4 would be very considerable.

APPENDIX I.

Production activities

1. 140 lb. Porkers

Weight of weaner	43 lbs.
Conversion rate	2.8
Days to slaughter	74

	£	£
Gross revenue		12.5
Feed Costs	3.35	
Miscellaneous costs	0.15	3.5
Net revenue		9.0

2. 260 lb. Heavy Hogs

Weight of weaner	43 lbs.
Conversion rate	3.7
Days to slaughter	134

	£	£
Gross revenue		21
Feed costs	9	
Miscellaneous costs	0.25	9.25
Net revenue		11.75

APPENDIX II.

Sow activities

	3. Type A		4. Type B		5. Type C	
Purchase price per sow (£)		40		40		40
Annual costs per sow (£)						
Creep feed	8.9		9.4		10.0	
Sow feed	34.4		29.4		30.3	
Straw	2		1.8		2.8	
Sow depreciation	5.3		5.3		5.3	
Total annual costs per sow (£)		52.3		47.7		49.9
Pigs weaned per annum		17		18		19

APPENDIX III.

Pig Accommodation

6. Sow accommodation (type A)
   Communal farrowing with outdoor rearing.
   Items per 40 sow unit — 40 huts
   7 creeps
   Perimeter fence
   Water installation

   Capital cost per 40 sow unit = £309
   Land. 20 acres per year with three year rotation = 60 acres

7. Sow accommodation (type B)
   Outdoor farrowing and rearing
   Items per 40 sow unit — 6 farrowing huts
   12 suckling huts
   Perimeter fence
   Electric fence
   Water installation
   Dry sow accommodation

   Capital cost per 40 sow unit = £626
   land. 20 acres

cont/

APPENDIX III. (Continued)

Pig accommodation (continued)

8. Sow accommodation (type c) outdoor
   Indoor farrowing, outdoor rearing
   Items per 40 sow unit

   Farrowing house (6 pens)
   12 suckling huts
   Fencing
   Water installation
   Dry sow accommodation

   Capital cost per 40 sow unit = £937
   Land. 15 acres

9. Fattening accommodation
   Covered yard with deep litter and self feeders.
   Capital costs on basis of 40' x 20' structures

Structure	£720	(i.e. 18/- per sq.ft.)
Feeders	60	
	£780	

   Per 10 square feet = £9.75

APPENDIX IV.

Capital Items

11. Fixed costs per annum

	£
Farmer's drawings and man's wages	1,400
Interest on existing loans	680
Machinery costs	250
Sundry overheads	170
	£2,500

12. Transfer of capital.
    Rate of interest earned = 5% per annum.

13. Borrowed capital.
    Rate of interest charged = 8% per annum.

APPENDIX V.

In planning the development of a farm it may be desirable to examine the possibility of resource substitution as the plan proceeds.

In this example for instance, if capital or credit had been more restricted, then the optimum course might have been to install type A sow accommodation at first. This type has low capital requirements per 40 sow unit, £309, as against £626 for type B and £937 for type C. But it is much less efficient in the use of land, requiring 60 acres per 40 sows, against 20 acres for type B and 15 acres for type C. As capital became more plentiful, then the replacement of type A with type C might have been necessary in order to maximize net revenue in the 4th period.

But the model makes no provisions for such substitution. It only permits a choice of one type or another, or any combination of the types, considering the four periods as a whole. Thus an important dynamic element in farm development is not incorporated. In the present example, because of the capital situation, this is not an important deficiency. But it would be an important consideration in much farm development. It is often necessary to "make do" with temporary structures and second hand machinery in the early years of development, and then to dispose of these when it becomes possible to use more efficient and expensive items.

Hildebrand (1959) developed a model which permitted the salvage or acquisition of fixed resources when their marginal value productivities did not lie between their salvage value and their acquisition price. He defined fixed resources as those which it does not pay to vary, i.e. those resources for which acquisition price is greater than or equal to marginal value product, which is in turn, greater than or equal to salvage value. In determining the salvage value and the acquisition price he distinguished between the cost or value of a stock of the asset, and the cost or value of a flow. In the dynamic sense we should be interested only in flows, i.e. the annual cost per unit of a resource will be the annual sum of interest, depreciation, insurances, repairs. The criterion for the purchase of an additional unit of an asset would therefore be that its annual marginal value product must exceed the annual cost of ownership. These annual costs are in effect the marginal fixed costs per unit, so we have the familiar criterion MVP $>$ MFC. If the marginal value productivity of an asset is less than its salvage value, where salvage value is again interpreted as the annual flow of cash resulting from salvage, then the asset should be salvaged.

The solution matrix of a linear programming model imputes values to the limiting resources. These values are the marginal value products of the resources, the amount of revenue which the firm would gain by acquiring an additional unit of the resource, or would lose by disposing of a unit. But the true value of a factor to a firm is never less than its salvage value since the firm could realise at least this amount if it disposed of the factor in the market. Similarly, if the productivity is greater than the cost of acquisition (MFC) the firm would gain by purchasing and using more of the asset. Hildebrand proceeded to detail a programming procedure in which this criterion was applied and resource levels were endogenously determined.

Because of the problem of matrix size, and also because it did not appear possible to ascribe realistic salvage values to the resources of interest, the above approach was not attempted with the present model. However a simplified version of a model in symbolic form, around which a fuller model could be developed is outlined as follows:-

Using the same terminology as before, let an extra activity be added in the kth period, which allows the salvage of units of Sow Accommodation (type A). The level of this salvage activity may be denoted x_S^k, where again $j = 1,2,\ldots,S,\ldots,n$ lists the possible activities, and the kth period may be any of 2,3, ..., t inclusive. (Period 1 is not relevant since there is no sow accommodation present initially).

Let the level of availability of sow accommodation (type A) in period k - 1 be denoted b_A^{k-1}. The relevant constraint is then:-

$$(1) \qquad b_A^{k-1} \geqslant \sum r_{Aj}^{k-1} x_j^{k-1} \qquad (j=1, 2,\ldots,S,\ldots,n)$$

Here the r's will be positive in vectors representing type A sows, zeros elsewhere. Now in the kth period we have:-

$$(2) \qquad b_A^k \geqslant \sum r_{Aj}^k x j \qquad j = 1,2,\ldots,s,\ldots,n$$

Again the r's will be positive in type A sows vectors, but also will be positive in the vector pertaining to the salvage activity S. The coefficient will represent the number of units of the resource, required by one unit of salvage.

In the capital row pertaining to the kth period, provision would have to be made for the release of capital by the salvage of the sow accommodation. This would be equivalent to the cash value of one unit of the asset.

In the land row similar provision would have to be made for the release of land following the salvage of sow accommodation. In both the capital and land rows this release of resources would be obtained in the matrix by appropriate negative coefficients.

The elements c_S^k, that is, the net revenue per unit of S in period k would be equivalent to the annual flow of cash resulting from the salvage.

It is obvious that a full elaboration of a model of this kind would lead to a very large and complex matrix. Nevertheless, without such elaboration, a dynamic model of farm development loses some of its realism, as an important element of development often involves the substitution of one resource for another, whether it be machinery, buildings, or other structures.

It can be expected that future developments in dynamic linear programming will involve refinements of models of this kind, and that the increasing availability and capacity of electronic computers will make these developments practicable.

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