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SEASONALITY, OUTLIERS
AND LINEARITY

Ph.H. FRANSES

REPORT 9050/A

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SEASONALITY, OUTLIERS AND LINEARITY

by

Philip Hans FRANSES

Econometric Institute and Tinbergen Institute
Erasmus University Rotterdam

ABSTRACT

In the present paper a case is made for the simultaneous treatment of seasonality, outliers and nonlinearity in economic time series. It is empirically shown that outlying observations may cause that a regularly applied differencing procedure for monthly data induces a nonlinear time series. A more appropriate handling of the eventual seasonality and nonstationarity yields that linearity can not be rejected.

KEYWORDS

Seasonal Unit Roots; Additive Outliers; Bilinearity

ADDRESS

Econometric Institute, Erasmus University Rotterdam,

P.O.Box 1738, NL-3000 DR Rotterdam, The Netherlands

1. INTRODUCTION

At present there is a vast amount of literature on seasonality, outliers and (non–) linearity in economic time series. Examples are methods for outlier detection in a linear model context (see for recent studies e.g. Bruce and Martin, 1989 and Peña, 1990), and tests for linearity assuming the absence of outlying observations such as e.g. in Chan and Tong (1986). In case such methods, or nonlinear models, are applied to time series consisting of seasonal observations, one usually initially transforms the original series to get rid of seasonal influences, see e.g. Maravall (1983). So, in general, these issues are studied separately while assuming that the others have been appropriately handled.

In the present paper however a case is made for the simultaneous treatment of seasonality, outliers and linearity. More precisely, it is argued that the incorrect transformation of a seasonal time series containing some additive outliers can result in a time series which shows sequences of outliers, and hence, for which a nonlinear model may be more appropriate. To provide an illustration, consider the following simple experiment. For an artificial time series of quarterly data from 1950.I through 1990.IV observations are generated from the model

$$y_t = y_{t-1} + 2 + D_1 - 2D_2 - 1.5D_3 + \varepsilon_t \tag{1.1}$$

where y_0=0, ε_t is drawn from a standard normal distribution, and D_1, D_2 and D_3 are seasonal dummies with a 1 in the corresponding quarter, and 0 in other quarters. Furthermore, construct the series add_t which consists of zeroes, except in 1969.IV, 1970.I, 1979.IV and 1980.I where the values are 6, –4, –15 and 9, repectively. When the series ys_t=y_t+add_t is constructed, it is obvious that this ys_t contains some additive outliers as can be seen from figure 1.

insert figure 1

Note that such additive outliers are by no means uncommon in economic time series, where e.g. a strike at a registration office can induce that part of the registrations have to be made up in the next period. An often applied step in current quarterly time series model building is the transformation of the series into $\Delta_1\Delta_4 ys_t$, where $\Delta_k x_t \equiv x_t - x_{t-k}$. The plot of this series is given in figure 2.

insert figure 2

Suppose one encounters the series $z_t = \Delta_1\Delta_4 ys_t$ without any prior knowledge of the transformation. Fitting a univariate autoregressive moving average (*ARMA*) model to z_t would probably result in two patches of outliers. Alternatively, anyone familiar with the literature on bilinear models, and especially with the graphs of series generated by such models (see e.g. Granger and Andersen, 1978 and Subba Rao and Gabr, 1984), would presumably start fitting a bilinear model to z_t to account for its 'temporary bubbles'. However, in case one would have detected the underlying seasonal process (1.1), a simple linear model could have been fitted and the additive outliers could have been spotted rather easily.

Of course, the major issue is to find the appropriate model for the seasonal pattern. One strategy is to apply the techniques developed in Hylleberg, Engle, Granger and Yoo (1990) (HEGY), which have been applied to quarterly time series. In section 2, a brief account is given of an extension of the HEGY method to monthly time series, which is applied to two empirical monthly series. Recently, a $\Delta_1\Delta_{12}$ transformed version of one of these series has been successfully modeled with a bilinear model. In section 3, a well-known test for linearity is discussed and applied to some transformations of the two series. The final section contains some concluding remarks.

2. TESTING FOR SEASONAL UNIT ROOTS

Consider three simple classes of monthly time series models for modeling seasonality in a time series y_t. The first is a purely deterministic seasonal process, or

$$y_t = \alpha_0 + \sum_{j=1}^{11} \alpha_j D_{jt} \tag{2.1}$$

where the D_{jt} are seasonal dummy variables. The second is a stationary seasonal process, or

$$y_t = \rho y_{t-12} + \varepsilon_t \tag{2.2}$$

where $|\rho| < 1$, and where ε_t denotes a white noise process with $E(\varepsilon_t)=0$, $E(\varepsilon_t^2)=\sigma^2$ and $E(\varepsilon_s\varepsilon_t)=0$ for $s \neq t$. This interpretation for ε_t will be used throughout this paper. The third is an integrated seasonal process which can be written as (2.2) with $\rho = 1$, or

$$y_t = y_{t-12} + \varepsilon_t \tag{2.3}$$

Using the familiar backward shift operator B, to be defined as $B^k y_t \equiv y_{t-k}$, this (2.3) can be rewritten as

$$(1-B^{12})y_t = \varepsilon_t \tag{2.4}$$

The equation $1-B^{12}=0$ has 12 solutions lying on the unit circle which becomes clear from noting that

$$\begin{aligned}1-B^{12} = &(1-B)(1+B)(1-iB)(1+iB) \\ &(1+(\surd 3+i)B/2)(1+(\surd 3-i)B/2)(1-(\surd 3+i)B/2)(1-(\surd 3-i)B/2) \\ &(1+(i\surd 3+1)B/2)(1-(i\surd 3-1)B/2)(1-(i\surd 3+1)B/2)(1+(i\surd 3-1)B/2)\end{aligned} \tag{2.5}$$

where all terms other than $(1-B)$ correspond to seasonal unit roots. Collecting two terms at a time in (2.5) yields

$$\begin{aligned}1-B^{12} &= (1-B^2)(1+B^2)(1+\surd 3B+B^2)(1-\surd 3B+B^2)(1+B+B^2)(1-B+B^2) \\ &= (1-B^4)(1-B^2+B^4)(1+B^2+B^4)\end{aligned} \tag{2.6}$$

The method, as it is developed in HEGY for quarterly time series, makes it possible to test whether all roots are indeed on the unit circle. The extension of the HEGY method to monthly data is straightforward, so only the final test equation will be presented. In Franses (1990) all derivations are given in detail, as well as extensive tables with critical values for several test statistics and some power investigations.

Testing for unit roots in monthly time series is equivalent to testing the significance of the parameters in the auxiliary regression

$$\varphi^*(B)x_{8,t} = \pi_1 x_{1,t-1} + \pi_2 x_{2,t-1} + \pi_3 x_{3,t-1} + \pi_4 x_{3,t-2} + \pi_5 x_{4,t-1}$$

$$+ \pi_6 x_{4,t-2} + \pi_7 x_{5,t-1} + \pi_8 x_{5,t-2} + \pi_9 x_{6,t-1} + \pi_{10} x_{6,t-2}$$

$$+ \pi_{11} x_{7,t-1} + \pi_{12} x_{7,t-2} + \mu_t + \varepsilon_t \qquad (2.7)$$

where $\varphi^*(B)$ is some polynomial function of B, with all roots outside the unit circle, and ε_t is white noise, and with

$$x_{1,t} = (1+B)(1+B^2)(1+B^4+B^8)x_t$$

$$x_{2,t} = -(1-B)(1+B^2)(1+B^4+B^8)x_t$$

$$x_{3,t} = -(1-B^2)(1+B^4+B^8)x_t$$

$$x_{4,t} = -(1-B^4)(1-\sqrt{3}B+2B^2-\sqrt{3}B^3+2B^4-\sqrt{3}B^5+B^6)x_t$$

$$x_{5,t} = -(1-B^4)(1+\sqrt{3}B+2B^2+\sqrt{3}B^3+2B^4+\sqrt{3}B^5+B^6)x_t$$

$$x_{6,t} = -(1-B^4)(1-B+B^3-B^5+B^6)x_t$$

$$x_{7,t} = -(1-B^4)(1+B-B^3+B^5+B^6)x_t$$

$$x_{8,t} = (1-B^{12})x_t$$

and where μ_t covers the determistic part and might consist of a constant, seasonal dummies, or a trend.

Applying ordinary least squares to (2.7), where the order of $\varphi^*(B)$ is chosen in an experimental way to whiten the residuals, gives estimates of the π_i. Because the π_i are zero in case the corresponding roots are on the unit circle, testing the significance of the estimated π_i implies testing for unit roots. There will be no seasonal unit roots if π_2 through π_{12} are significantly different from zero. If π_1=0, then the presence of root 1 can not be rejected. In case all π_i, i=1,..,12 are equal to zero, it is appropriate to apply the Δ_{12} filter, and if they are all unequal to zero, one has encountered a stationary seasonal pattern and one can use seasonal dummies. It should be noted that e.g. roots i and $-i$ are present only if π_3 and π_4 both equal zero, and hence it might be convenient to test them simultaneously in two-sided tests. Tables with critical values for the t-tests for the 12 π's, and the F tests of the restrictions $\pi_3=\pi_4=0$ through $\pi_{11}=\pi_{12}=0$ and also $\pi_3=...=\pi_{12}$ can be found in Franses (1990). Some critical values which will be of relevance in the forthcoming examples are given in the appendix.

The above method to test for seasonal unit roots is applied to two empirical time series, both of which were measured in natural logarithms. The first consists of 348 monthly observations from January 1960 to December 1988 of new truck registrations, $lnqt_t$, in the Netherlands. A graph of this series is given in figure 3.

insert figure 3

From this plot it can be seen that there are several periods which might contain outlying observations, such as e.g. 1975-1977.

The second series considered here consists of 396 monthly unemployment figures for West Germany from January 1948 to December 1980, as they are given in Subba Rao and Gabr (1984) (see figure 4).

insert figure 4

For this series lnu_t a rather deterministic seasonal pattern seems to be present, and the period 1966-1969 seems to be the most likely to contain some outlying observations.

The results of the testing for seasonal unit roots procedure are given in table 1.

insert table 1

For both cases, the μ_t in (2.7) contains a constant and seasonal dummies, because most of their estimated parameters are highly significant. For $lnqt_t$ a trend variable has also been included. Some other experience with the test procedure indicated that its outcomes do not depend too critically on the specification of $\varphi^*(B)$ and on single observations in case of time series as lengthy as those considered here. Furthermore, in Franses (1990) it has been found that the power of the tests increases rather rapidly with the number of observations, and that the F-test for $\pi_3=..=\pi_{12}=0$ obtains high power in most occasions. From table 1 it is obvious that seasonality and nonstationarity in $lnqt_t$ is appropriately modeled by the transformation Δ_1 and 11 seasonal dummies. For lnu_t the inference is somewhat more complicated because of the simultaneous insignificance of (π_7, π_8) and (π_{11}, π_{12}), although some of them individually are highly significant. Taking into account the significance of the F-test for π_3 through π_{12}, it is decided to model seasonality for lnu_t as is done for the other variable. Note that these models of seasonality correspond to (1.1), when it allowed for ε_t to follow a mixed *ARMA* process.

Finally, it is striking that the regularly applied Δ_{12} filter is not found. This corresponds to the results in Osborn (1990) for several quarterly UK macroeconomic variables, and also to those for monthly U.S. data in Beaulieu and Miron (1990). In the latter paper the authors also extend the HEGY procedure to monthly data. Their test equation however is more complicated than, and is essentially different from, that in (2.7) because of distinct definitions of several of the parameters. Furthermore, for some parameters, the authors compute critical values for one sided tests only, and they also do not consider the useful joint F test for the presence of the complex unit roots.

3. TESTING FOR LINEARITY

The test for linearity used in the present paper has been developed in Keenan (1985). It is a general linearity test in the sense that it is not necessary to construct a specific nonlinear alternative model. This can be a drawback, especially compared to tests which test against a certain alternative, and which may possess higher power in some occasions. From power studies such as in Chan and Tong (1986), Tsay (1986), Luukkonen *et al.* (1988) and Lee *et al.* (1989) it emerges that in large samples Keenan's test often obtains reasonably high power in case bilinear models are the data generating processess. Given the experiment in the introduction, it is just this type of models which deserves our special attention here. Furthermore, a conclusion of Lee *et al.* (1989) is that no single test is uniformly superior to others. Combining this with its ease of implementation, it is felt that the Keenan test is useful for our illustration purposes.

In short, the test boils down to three auxiliary regressions. First, an autoregressive model of order M is fitted to the original time series y_t, giving fitted values $\hat{y}_t$ and residuals $\hat{\varepsilon}_t$. Then, $\hat{y}_t^2$ is regressed on 1, y_{t-1}, .., y_{t-M}, which gives the residuals ξ_t. Finally, the $\hat{\varepsilon}_t$ is regressed on ξ_t, yielding the regression coefficient $\hat{\eta}_0$. Denoting RSS_ε and RSS_ξ as the sums of squared residuals for $\hat{\varepsilon}_t$ and ξ_t, respectively, the test statistic equals

$$\hat{F}_{1,n-2M-2} = \frac{\hat{\eta}_0^2 . RSS_{\xi} . (n-2M-2)}{RSS_{\varepsilon} - \hat{\eta}_0^2 . RSS_{\xi}} \tag{3.1}$$

which follows an $F(1,n\text{-}2M\text{-}2)$ distribution under the null hypothesis of linearity. From (3.1) one can observe that the Keenan test can be viewed as a special case of the well-known RESET test. The choice of M does seem to be of importance, especially when it is too small, for in that case low powers and incorrect sizes may occur because the test will then also respond to remaining autocorrelation in the residuals. Hence, in the forthcoming applications M will be set equal to 12, 24 or 36, the choice of which will be based on the usual checks of the first 48 autocorrelations.

One might argue that the Tsay (1986) test should preferably be used because it is an expansion of the Keenan test, and, as expected, obtains often higher power. However, the Tsay test involves a regression equation containing $M(M+1)/2$ variables, i.e. e.g. 300 (!) in case M=24. Even in our large sample case, this does not seem sensible.

The variables $\Delta_1 lnqt_t$ and $\Delta_1 lnu_t$ are regressed on a constant and 11 seasonal dummies. The residuals of these regressions are used for testing for linearity, the results of which are displayed in table 2. In the same table test results are given for the $\Delta_1\Delta_{12}$ transformed variables. Note again that these transformations are common practice in current and traditional seasonal *ARMA* model building, see e.g. Granger and Newbold (1986).

insert table 2

From this table it can be seen that the general result is that, at reasonable significance levels, linearity is accepted for the variables where seasonality is correctly taken into account, and that linearity is rejected for the incorrectly transformed variables. The result for lnu_t confirms the apparent success of detecting the nonlinearity of, and fitting a bilinear model to, $\Delta_1\Delta_{12}lnu_t$ (see Subba Rao and Gabr, 1984). It should be mentioned that the filter for lnu_t indicated by the F-test statistics for pairs of π_i in table 2 gives comparable $\hat{F}$ values. Hence, a linear time series model may be appropriate for lnu_t when suitably transformed, after which e.g. the procedure described in Peña (1990) can be applied to detect eventual outliers.

To more clearly illustrate the test results, it might be instructive to consider the graphs of $\Delta_1\Delta_{12}x_t$ versus $\Delta_1 x_t$, where x_t is $lnqt_t$ and lnu_t in figure 5 and 6, respectively.

insert figures 5,6

From figure 5 one can see that the $\Delta_1 lnqt_t$ already looks like a variable with no obvious pattern apart from seasonality, but that $\Delta_1\Delta_{12}lnqt_t$ shows some bubbling behaviour in the years with the suspected outliers. Moreover, several outliers in 1987 and 1988 seem to have been introduced by the latter filter. Finally, from figure 6 one can imagine that some outliers will be present in $\Delta_1 lnu_t$ series, but their impact on the $\Delta_1\Delta_{12}$ transformed series is really striking. The periods 1955 through 1960 and 1967–1968 probably play an important part in the success of the previously mentioned bilinear model.

4. SOME CONCLUDING REMARKS

In the present paper it has been demonstrated that an appropriate transformation of a monthly time series, necessary to account for the eventual presence of seasonality and nonstationarity, can lead to the acceptance of linearity. Furthermore, it is shown that incorrect filtering of the data can result in rejection of linearity, which can cause subsequent modeling to involve unnecessary, and computationally cumbersome, steps, such as e.g. the fitting of bilinear models. Now, it would also be interesting to investigate whether the success of fitting a bilinear model to the residuals of the linear model for the approximately $\Delta_1\Delta_{12}$ transformed variable in Maravall (1983) is also due to this transformation.

In case one however decides to fit a linear model, one would probably encounter a large amount of outliers. This corresponds e.g. to the recent findings of Bruce and Martin (1989), where patches of outliers emerged for the $\Delta_1\Delta_{12}$ transformed variable in their example 6. In summary, it is proposed to first test for the presence of seasonal unit roots with the simple procedure described in section 2. Depending on the outcome of this test, some outlier issues may also probably become more easy to handle.

Some additional comments are in order. The first is that it is of course not true that an appropriate filter for monthly data automatically implies linearity. Some experience with several other unemployment series mostly supports the above findings, but also reveals that if a correctly transformed variable shows some nonlinearity, then the $\Delta_1\Delta_{12}$ transformed does too. Secondly, the Keenan test is not entirely beyond discussion, and more research into its properties is needed. Furthermore, in case the test statistic indicates the presence of nonlinearity it remains unclear exactly which nonlinear model is most appropriate, although the available power studies suggest that a bilinear model may do.

On the whole, however, it is felt that strong arguments are provided for the simultaneous treatment of seasonal processes, outliers and linearity. Future empirical, as well as theoretical, research may shed additional light on how this treatment should proceed.

ACKNOWLEDGEMENTS

I wish to thank Lourens Broersma for making some supporting calculations. Furthermore, I am grateful to Teun Kloek, Denise Osborn and Timo Teräsvirta for their helpful suggestions.

APPENDIX

Some critical values for testing for seasonal unit roots in monthly data
Based on 5000 Monte Carlo simulations, DGP: $y=y(-12)+\varepsilon$, $\varepsilon\sim N(0,1)$
Number of observations is 240

	Auxiliary regression							
	constant, dummies and trend				constant, dummies and no trend			
t-statistics		0.05	0.10			0.05	0.10	
π_1		-3.30	-3.02			-2.75	-2.45	
π_2		-2.79	-2.49			-2.79	-2.49	
t-statistics	0.025	0.05	0.95	0.975	0.025	0.05	0.95	0.975
π_3	-2.15	-1.82	1.87	2.18	-2.17	-1.83	1.87	2.19
π_4	-3.51	-3.29	-0.49	-0.22	-3.51	-3.29	-0.49	-0.20
π_5	-3.34	-3.09	-0.09	0.19	-3.33	-3.10	-0.09	0.21
π_6	-3.46	-3.21	-0.47	-0.19	-3.46	-3.21	-0.47	-0.18
π_7	-0.17	0.11	3.12	3.39	-0.22	0.07	3.14	3.41
π_8	-3.50	-3.23	-0.48	-0.18	-3.50	-3.22	-0.48	-0.16
π_9	-2.91	-2.61	0.77	1.13	-2.92	-2.62	0.78	1.13
π_{10}	-3.52	-3.25	-0.42	-0.10	-3.50	-3.25	-0.41	-0.10
π_{11}	-1.19	-0.84	2.61	2.91	-1.21	-0.87	2.62	2.92
π_{12}	-3.45	-3.20	-0.50	-0.21	-3.44	-3.20	-0.50	-0.20
F-statistics		0.90	0.95			0.90	0.95	
π_3,π_4		5.35	6.31			5.33	6.36	
π_5,π_6		5.15	6.05			5.16	6.05	
π_7,π_8		5.30	6.22			5.29	6.23	
π_9,π_{10}		5.19	6.14			5.21	6.16	
π_{11},π_{12}		5.14	6.04			5.15	6.03	
$\pi_3,..,\pi_{12}$		4.08	4.48			4.09	4.48	

Source: Franses (1990,pp.12-18). Note that the tests for π_1 and π_2 are one-sided tests, while the other tests are two-sided.

TABLES

TABLE 1. Testing for seasonal unit roots

	Variable	
t-statistics	$lnqt^{(1)}$	$lnu^{(2)}$
π_1	-2.766	-1.586
π_2	-3.232**	-4.186**
π_3	-0.727	-2.137*
π_4	-5.008**	-3.245*
π_5	-3.397**	-4.639**
π_6	-4.256**	-4.374**
π_7	3.852**	-0.529**
π_8	-6.372**	-0.547
π_9	-2.320	-3.341**
π_{10}	-4.324**	-4.003**
π_{11}	1.920	-1.389**
π_{12}	-6.460**	-1.282
F-statistics		
π_3,π_4	12.806**	7.332**
π_5,π_6	9.286**	10.977**
π_7,π_8	29.587**	2.129
π_9,π_{10}	9.350**	9.036**
π_{11},π_{12}	22.502**	3.482
$\pi_3,..,\pi_{12}$	19.227**	5.466**

** Significant at the 5% level
* Significant at the 10% level
(1) The auxiliary regression contains constant, trend and seasonal dummies. The polynomial $\varphi^*(B)$ is $(1\text{-}\varphi_1 B)$, so the number of observations equals 335.
(2) The auxiliary regression contains constant and seasonal dummies. The $\varphi^*(B)$ is $(1\text{-}\varphi_1 B\text{-}\varphi_{12}B^{12}\text{-}\varphi_{13}B^{13})$, and the number of observations equals 371.

TABLE 2. Testing the null hypothesis of linearity with the Keenan test

	M	Test statistic $\hat{F}$
Correctly transformed variables		
$\Delta_1 lnqt$	12	0.093
$\Delta_1 lnu$	24	0.479
Incorrectly transformed variables		
$\Delta_1 \Delta_{12} lnqt$	24	7.566**
$\Delta_1 \Delta_{12} lnu$	36	9.721**

** Significant at the 5% level

FIGURES

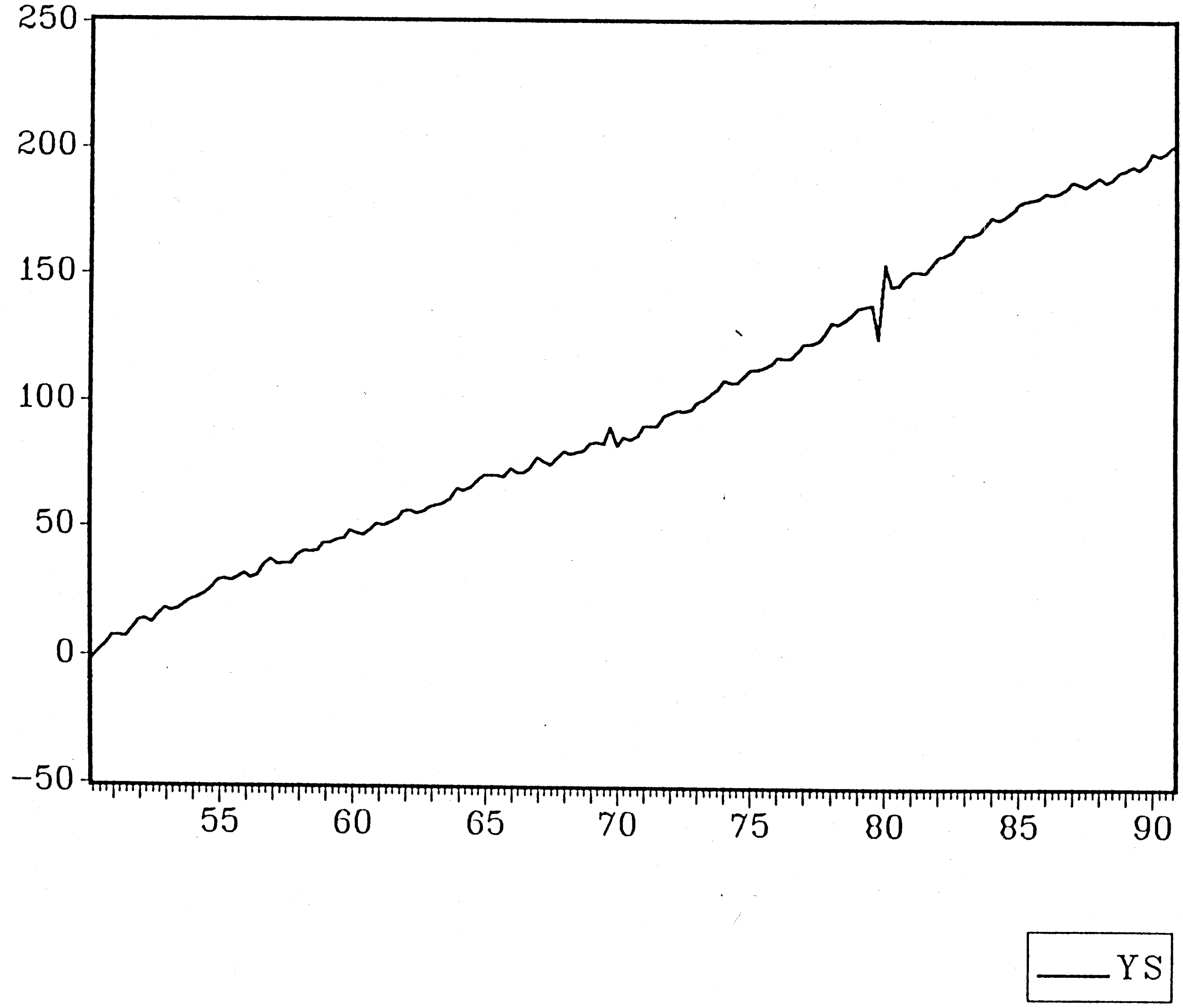

FIGURE 1

Artificial quarterly time series:

A random walk with deterministic seasonal and four additive outliers

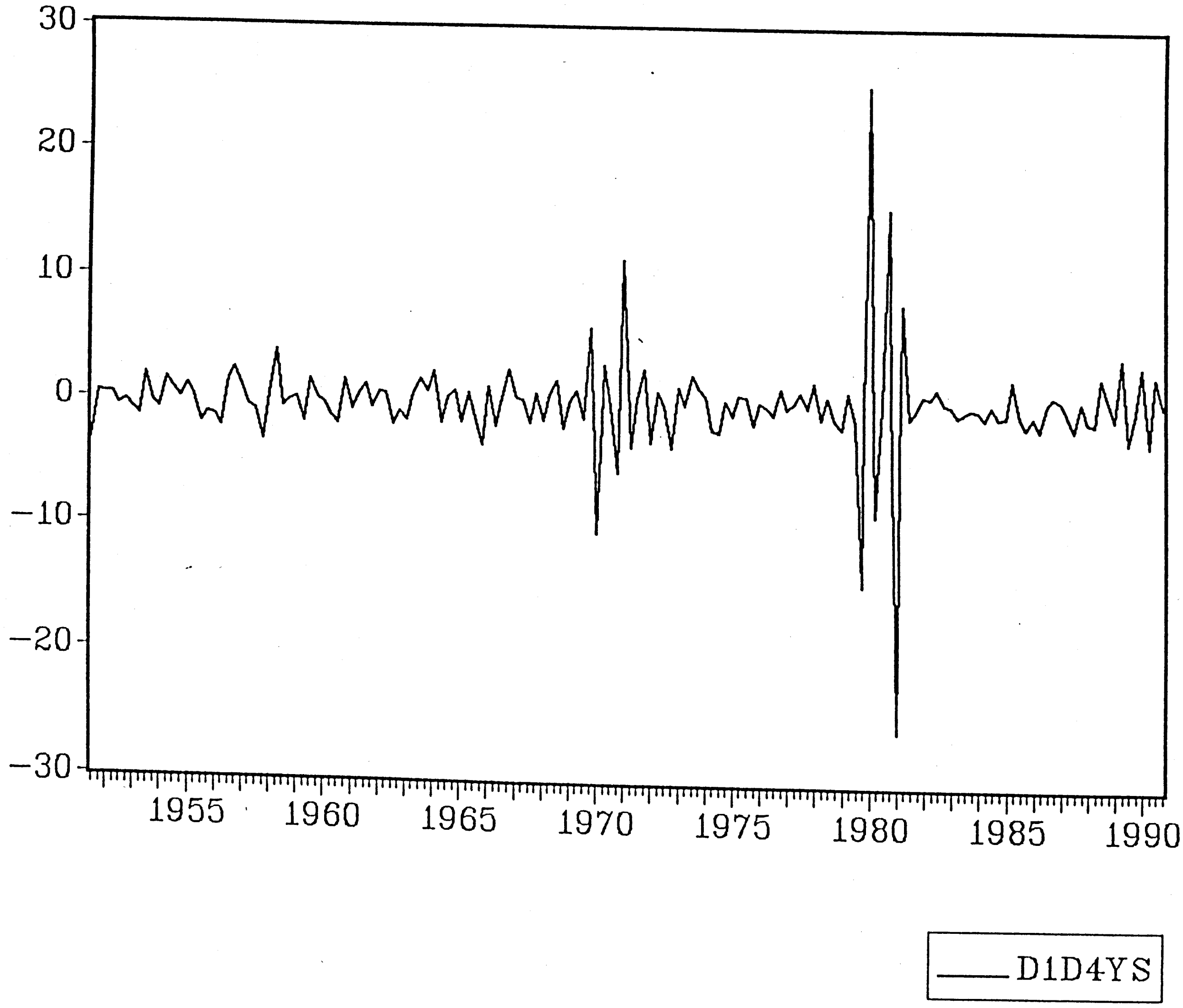

FIGURE 2

Artificial quarterly time series transformed with $\Delta_1\Delta_4$ filter

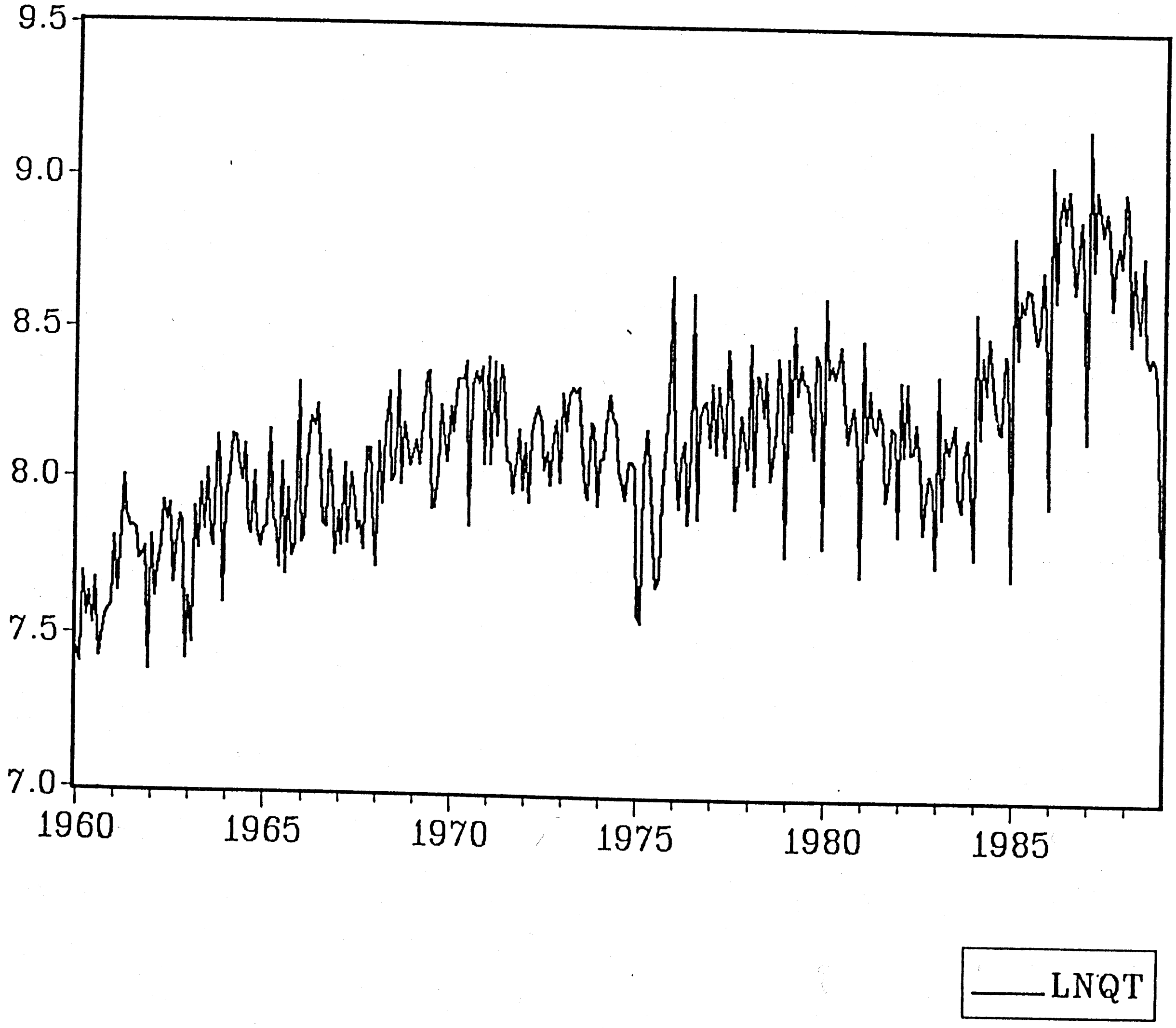

FIGURE 3

Monthly new truck registrations in the Netherlands, 1960.01–1988.12

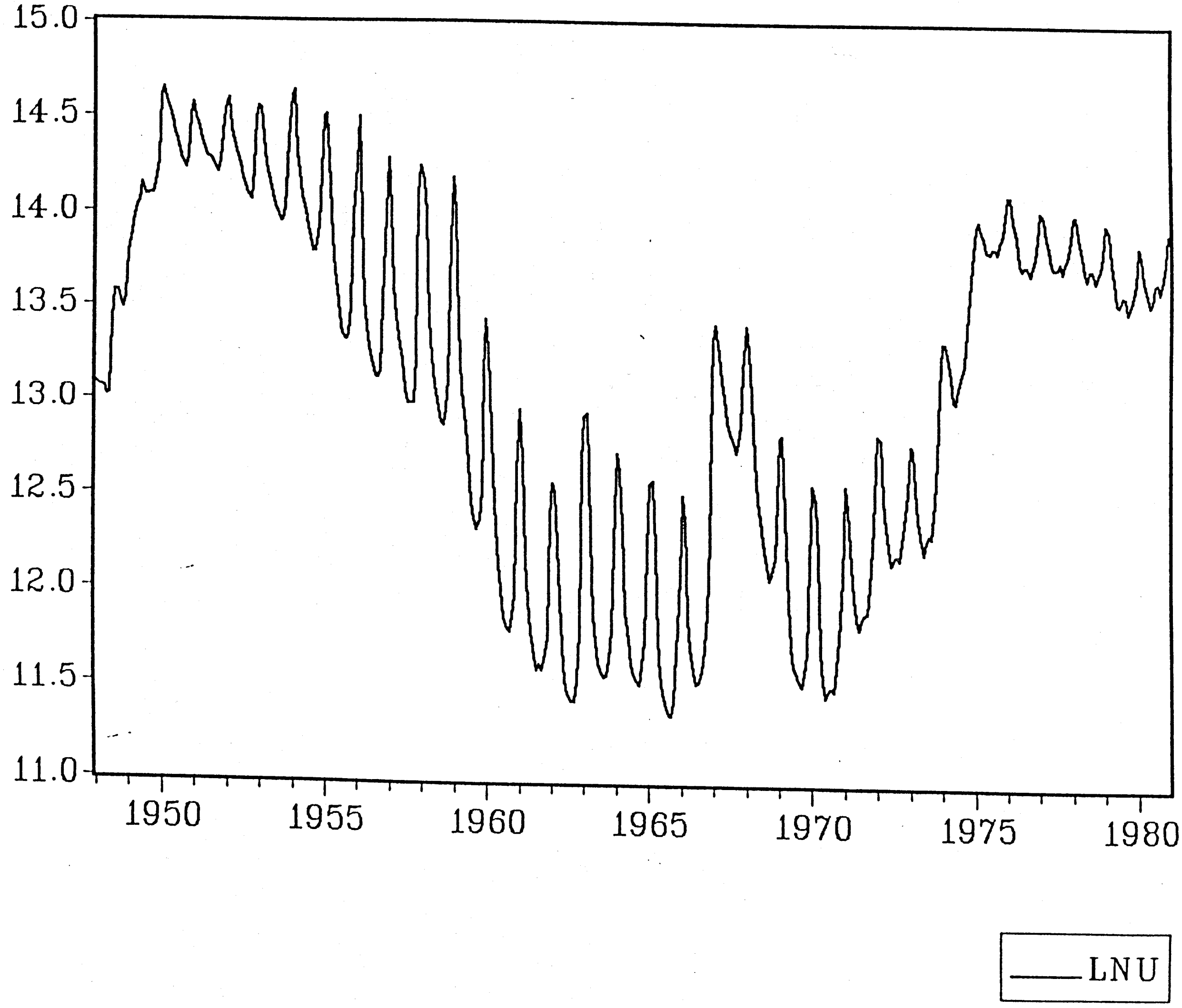

FIGURE 4

Monthly number of unemployed in West Germany, 1948.01–1980.12

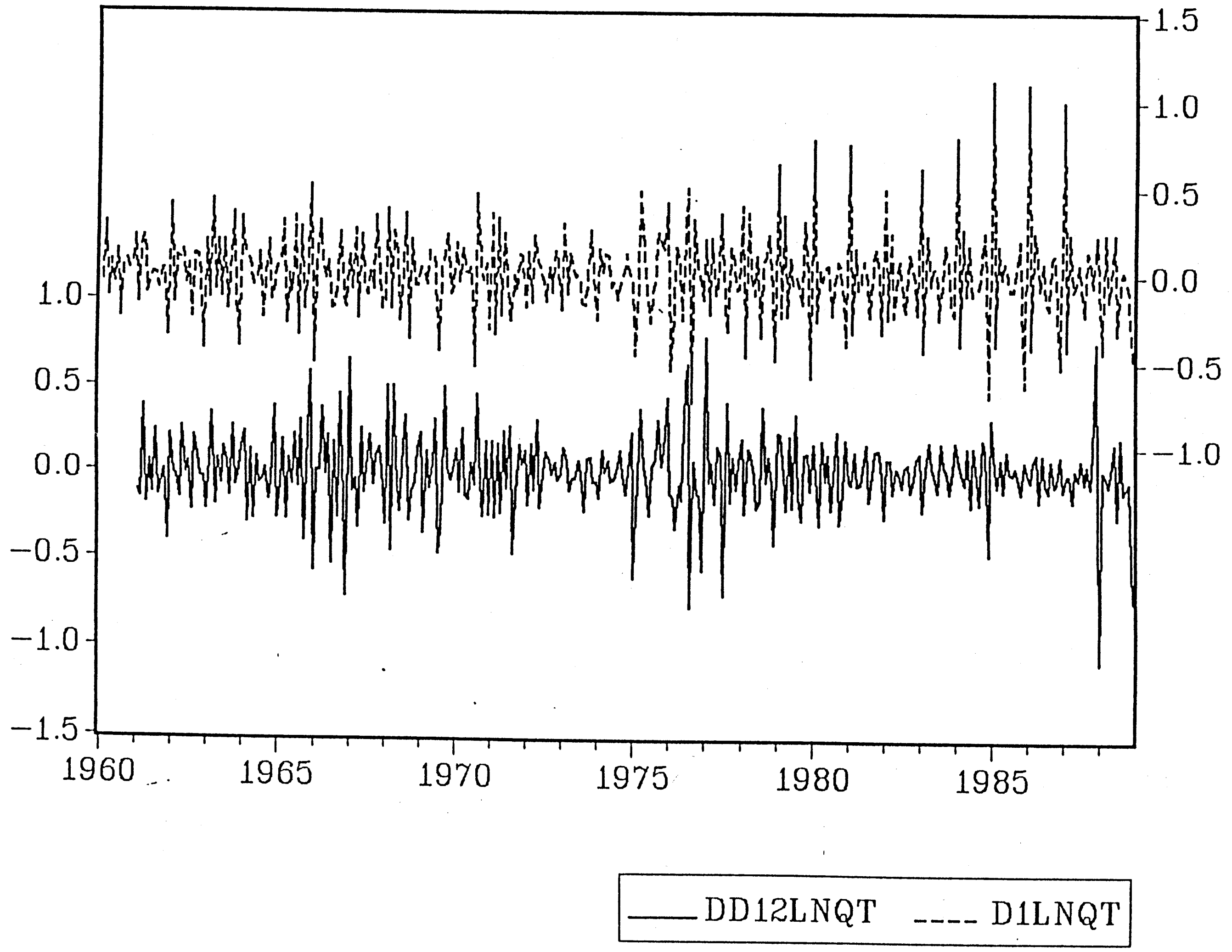

FIGURE 5

Transformed monthly new truck registrations:

$\Delta_1 \Delta_{12} lnqt$ **and** $\Delta_1 lnqt$

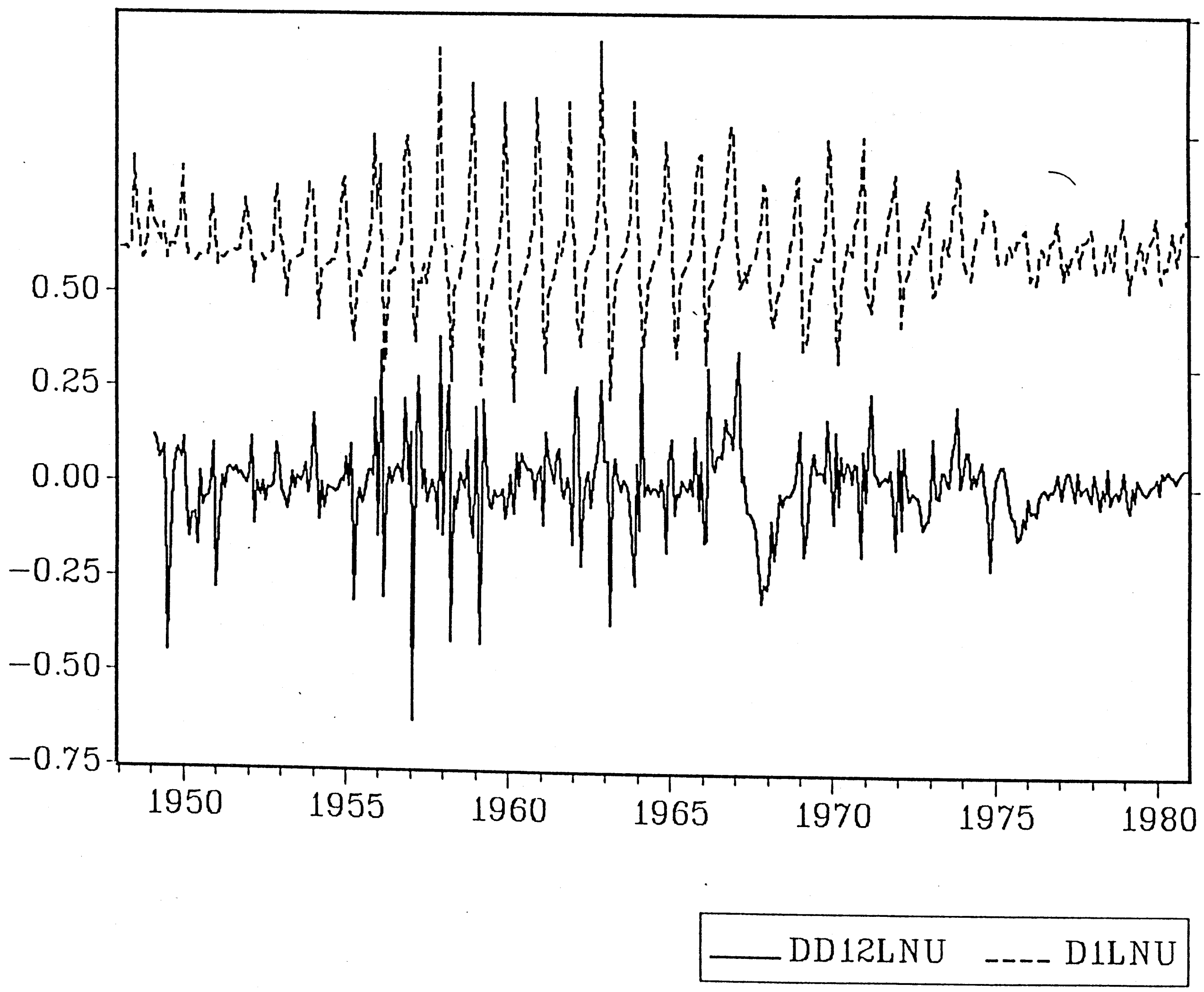

FIGURE 6

Transformed unemployment figures:

$\Delta_1\Delta_{12}lnu$ and $\Delta_1 lnu$

REFERENCES

Beaulieu, J.J. and J.A. Miron (1990), "Seasonal unit roots and deterministic seasonals in aggregate U.S. data", *Journal of Econometrics*, forthcoming.

Bruce, A.G. and R.D. Martin (1989), "Leave-k-out diagnostics for time series", *Journal of the Royal Statistical Society*, *B*, 51, 363-424.

Chan, W.S. and H. Tong (1986), "On tests for non-linearity in time series analysis", *Journal of Forecasting*, 5, 217-228.

Franses, P.H. (1990), "Testing for seasonal unit roots in monthly data", Econometric Institute Report #9032/A, Erasmus University Rotterdam.

Granger, C.W.J. and A.P. Andersen (1978), *An introduction to bilinear time series models*, Göttingen: Vandenhoeck & Ruprecht.

Granger, C.W.J. and P. Newbold (1986), *Forecasting Economic Time Series*, *Second Edition*, San Diego: Academic Press.

Hylleberg, S., R.F. Engle, C.W.J. Granger and B.S. Yoo (1990), "Seasonal integration and cointegration", *Journal of Econometrics*, 44, 215-238.

Keenan, D.M. (1985), "A Tukey nonadditivity-type test for time series nonlinearity", *Biometrika*, 72, 39-44.

Lee, T.-H., H. White and C.W.J. Granger (1989), "Testing for neglected nonlinearity in time series models: a comparison of neural network methods and alternative tests", Economics Department Discussion paper #89-42, University of California, San Diego.

Luukkonen, R., P. Saikkonen and T. Teräsvirta (1988), "Testing linearity in univariate time series models", *Scandinavian Journal of Statistics*, 15, 161-175.

Maravall, A. (1983), "An application of nonlinear time series forecasting", *Journal of Business & Economics Statistics*, 1, 66-74.

Osborn, D.R. (1990), "A survey of seasonality in UK macroeconomic variables", *International Journal of Forecasting*, forthcoming.

Peña, D. (1990), "Influential observations in time series", *Journal of Business & Economic Statistics*, 8, 235-241.

Subba Rao, T. and M.M. Gabr (1984), *An introduction to bispectral analysis and bilinear time series models*, New York: Springer-Verlag.

Tsay, R.S. (1986), "Nonlinearity tests for time series", *Biometrika*, 73, 461- 466.