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TESTING FOR SEASONAL UNIT ROOTS IN MONTHLY DATA

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TESTING FOR SEASONAL UNIT ROOTS IN MONTHLY DATA

by

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ABSTRACT

In Hylleberg, Engle, Granger and Yoo (1988) a method is proposed to test for seasonal unit roots in the presence of other unit roots and seasonal processes. This method is applied to quarterly time series. In the present paper the application of the method is extended to time series consisting of monthly observations. Tables with critical values for several test statistics are provided. A small empirical power investigation is also conducted.

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INTRODUCTION

In Hylleberg, Engle, Granger and Yoo (1988) (HEGY) a method is proposed to test whether there are seasonal unit roots in the presence of other unit roots and seasonal processes in a given time series. The method is illustrated for and applied to time series consisting of quarterly observations. In the present paper their approach is extended to time series consisting of monthly observations.

In section 1, the HEGY method for monthly time series is briefly described. Several test statistics are derived, of which tables with critical values will be displayed in the appendix. The testing procedure is also applied to an empirical monthly time series for illustrative purposes. In section 2, a small power investigation is carried out. The final section contains some concluding remarks.

1. THE TESTING PROCEDURE

Consider three simple classes of monthly time series models for modeling seasonality in a time series y_t. The first is a purely deterministic seasonal process, or

$$y_t = \alpha_0 + \sum_{j=1}^{11} \alpha_j D_{jt} \tag{1.1}$$

where the D_{jt} are seasonal dummy variables. The second is a stationary seasonal process, or

$$y_t = \rho y_{t-12} + \varepsilon_t \tag{1.2}$$

where $|\rho| < 1$, and where ε_t denotes a white noise process with $E(\varepsilon_t)=0$, $E(\varepsilon_t^2)=\sigma^2$ and $E(\varepsilon_s \varepsilon_t)=0$ for $s \neq t$. This interpretation for ε_t will be used throughout this paper. The third is an integrated seasonal process which can be written as (1.2) with $\rho = 1$, or

$$y_t = y_{t-12} + \varepsilon_t \tag{1.3}$$

(see also Engle, Granger and Hallman (1989) for several notational issues). Using the familiar backward shift operator B, to be defined as $B^k y_t \equiv y_{t-k}$, this (1.3) can be rewritten as

$$(1-B^{12})y_t = \varepsilon_t \tag{1.4}$$

The equation $1-B^{12}=0$ has 12 solutions lying on the unit circle which becomes clear from noting that

$$\begin{aligned} 1-B^{12} = & (1-B)(1+B)(1-iB)(1+iB) \\ & (1+(\sqrt{3}+i)B/2)(1+(\sqrt{3}-i)B/2)(1-(\sqrt{3}+i)B/2)(1-(\sqrt{3}-i)B/2) \\ & (1+(i\sqrt{3}+1)B/2)(1-(i\sqrt{3}-1)B/2)(1-(i\sqrt{3}+1)B/2)(1+(i\sqrt{3}-1)B/2) \end{aligned} \tag{1.5}$$

where all terms other than $(1-B)$ correspond to seasonal unit roots. Collecting two terms at a time in (1.5) yields

$$\begin{aligned} 1-B^{12} &= (1-B^2)(1+B^2)(1+\sqrt{3}B+B^2)(1-\sqrt{3}B+B^2)(1+B+B^2)(1-B+B^2) \\ &= (1-B^4)(1-B^2+B^4)(1+B^2+B^4) \end{aligned} \tag{1.6}$$

which will be useful in the forthcoming test equation.

From (1.5) it can be seen that transforming the monthly time series with a $(1-B^{12})$ filter is appropriate only in the case of the simultaneous presence of 12 unit roots. However, in case only one unit root is present such that e.g. applying the $(1-B)$ filter is sufficient to make the series stationary and that seasonality can be modeled with the inclusion of seasonal dummies as in (1.1), then transforming the series with $(1-B^{12})$ yields an overdifferenced series. This may cause serious trouble for the construction of e.g. time series models because the (partial) autocorrelation patterns may become hard to interpret. Furthermore, one may expect estimation problems because of the introduction of moving average polynomials with roots close to the unit circle. Underdifferenced series may yield unit roots in their autoregressive parts, and so classical arguments for time series containing neglected unit roots apply. In summary, it is important to test for the presence of (seasonal) unit roots in monthly data.

The crucial proposition, which makes the testing procedure relatively simple, is given in HEGY (page 10/11). For completeness this proposition is given here again literally (for the proof the reader should consider HEGY).

Proposition:

Any (possibly infinite or rational) polynomial, $\varphi(B)$, which is finite valued at the distinct, non-zero, possibly complex points, $\theta_1, \ldots \theta_p$, can be expressed in terms of elementary polynomials and a remainder as follows:

$$\varphi(B) = \sum_{k=1}^{p} \lambda_k \Delta(B)/\delta_k(B) + \Delta(B)\varphi^{**}(B) \tag{1.7}$$

where the λ_k are a set of constants, $\varphi^{**}(B)$ is a (possibly infinite or rational) polynomial and

$$\delta_k(B) = 1 - (1/\theta_k)B \tag{1.8}$$

$$\Delta(B) = \prod_{k=1}^{p} \delta_k(B) \tag{1.9}$$

An alternative form of (1.7), and which will be used in the sequel, is

$$\varphi(B) = \sum_{k=1}^{p} \lambda_k \Delta(B)(1-\delta_k(B))/\delta_k(B) + \Delta(B)\varphi^{*}(B) \tag{1.10}$$

where $\varphi^*(B) = \varphi^{**}(B) + \sum\lambda_k$. From (1.10) it can be seen that the polynomial $\varphi(B)$ has unit roots at θ_k if and only if the corresponding λ_k equal zero.

Application of this proposition to the case where $\Delta(B) = 1-B^{12}$ (and to be decomposed as in (1.5)) gives

$$\begin{aligned} \varphi(B) = {} & \lambda_1 B\varphi_1(B) + \lambda_2(-B)\varphi_2(B) + \lambda_3(i-B)B\varphi_3(B) + \lambda_4(-i-B)B\varphi_3(B) \\ & + \lambda_5(-(\sqrt{3}+i)/2-B)B\varphi_4(B) + \lambda_6(-(\sqrt{3}-i)/2-B)B\varphi_4(B) \\ & + \lambda_7((\sqrt{3}+i)/2-B)B\varphi_5(B) + \lambda_8((\sqrt{3}-i)/2-B)B\varphi_5(B) \\ & + \lambda_9(-(i\sqrt{3}+1)/2-B)B\varphi_6(B) + \lambda_{10}((i\sqrt{3}-1)/2-B)B\varphi_6(B) \\ & + \lambda_{11}((i\sqrt{3}+1)/2-B)B\varphi_7(B) + \lambda_{12}(-(i\sqrt{3}-1)/2-B)B\varphi_7(B) \\ & + \varphi^*(B)\varphi_8(B) \end{aligned} \tag{1.11}$$

where

$$\begin{aligned}
\varphi_1(B) &= (1+B)(1+B^2)(1+B^4+B^8)=(1+\sum_{j=1}^{11} B^j) \\
\varphi_2(B) &= (1-B)(1+B^2)(1+B^4+B^8) \\
\varphi_3(B) &= (1-B^2)(1+B^4+B^8) \\
\varphi_4(B) &= (1-B^4)(1-\sqrt{3}B+B^2)(1+B^2+B^4) \\
\varphi_5(B) &= (1-B^4)(1+\sqrt{3}B+B^2)(1+B^2+B^4) \\
\varphi_6(B) &= (1-B^4)(1-B^2+B^4)(1-B+B^2) \\
\varphi_7(B) &= (1-B^4)(1-B^2+B^4)(1+B+B^2) \\
\varphi_8(B) &= (1-B^{12})
\end{aligned} \tag{1.12}$$

To get rid of the complex terms in (1.11), it is suitable to define

$$\begin{aligned}
\lambda_1 &= -\pi_1 \\
\lambda_2 &= -\pi_2 \\
\lambda_3 &= -(i\pi_3+\pi_4)/2 \\
\lambda_4 &= -(-i\pi_3+\pi_4)/2 \\
\lambda_5 &= i\pi_5 - ((1+i\sqrt{3})/2)\pi_6 \\
\lambda_6 &= -i\pi_5 - ((1-i\sqrt{3})/2)\pi_6 \\
\lambda_7 &= -i\pi_7 - ((1+i\sqrt{3})/2)\pi_8 \\
\lambda_8 &= i\pi_7 - ((1-i\sqrt{3})/2)\pi_8 \\
\lambda_9 &= i\sqrt{3}\pi_9/3 - ((1+(1/3)i\sqrt{3})/2)\pi_{10} \\
\lambda_{10} &= -i\sqrt{3}\pi_9/3 - ((1-(1/3)i\sqrt{3})/2)\pi_{10} \\
\lambda_{11} &= -i\sqrt{3}\pi_{11}/3 - ((1+(1/3)i\sqrt{3})/2)\pi_{12} \\
\lambda_{12} &= i\sqrt{3}\pi_{11}/3 - ((1-(1/3)i\sqrt{3})/2)\pi_{12}
\end{aligned} \tag{1.13}$$

Substituting (1.13) into (1.11) gives

$$\begin{aligned}
\varphi(B) = {} & -\pi_1 B\varphi_1(B) + \pi_2 B\varphi_2(B) + (\pi_3+\pi_4 B)B\varphi_3(B) + (\pi_5+\pi_6 B)B\varphi_4(B) \\
& + (\pi_7+\pi_8 B)B\varphi_5(B) + (\pi_9+\pi_{10}B)B\varphi_6(B) + (\pi_{11}+\pi_{12}B)B\varphi_7(B) \\
& + \varphi^*(B)\varphi_8(B)
\end{aligned} \tag{1.14}$$

Assuming that the monthly data at hand are generated by the autoregression

$$\varphi(B)y_t = \varepsilon_t + \mu_t \tag{1.15}$$

the test equation for the presence of (seasonal) unit roots becomes

$$\begin{aligned}\varphi^*(B)y_{8,t} = {} & \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-1} + \pi_4 y_{3,t-2} + \pi_5 y_{4,t-1} \\ & + \pi_6 y_{4,t-2} + \pi_7 y_{5,t-1} + \pi_8 y_{5,t-2} + \pi_9 y_{6,t-1} + \pi_{10} y_{6,t-2} \\ & + \pi_{11} y_{7,t-1} + \pi_{12} y_{7,t-2} + \mu_t + \varepsilon_t \end{aligned} \tag{1.16}$$

where $\varphi^*(B)$ is some polynomial function of B with all roots outside the unit circle, and with

$$y_{it} = \varphi_i(B)y_t \qquad \text{for } i=1,8 \tag{1.17}$$

and

$$y_{it} = -\varphi_i(B)y_t \qquad \text{for } i=2,\dots,7$$

where the φ_i are given in (1.12). The μ_t covers the deterministic part of the time series, and might consist of a constant, seasonal dummies, or a trend.

Applying OLS to (1.16), where the order of $\varphi^*(B)$ is chosen in an experimental way to whiten the residuals, gives estimates of the π_i. Because the π_i are zero in case the corresponding roots are on the unit circle, testing the significance of the estimated π_i implies testing for unit roots. There will be no seasonal unit roots if π_2 through π_{12} are significantly different from zero. If $\pi_1=0$, then the presence of root 1 can not be rejected. In case all π_i, $i=1,..,12$ are equal to zero, it is appropriate to apply the $(1-B^{12})$ filter, and if they are all unequal to zero, one has encountered a stationary seasonal pattern and one can use seasonal dummies.

Tables for the critical t values of the individual π_i are given in the appendix (part 1) for 5 types of μ_t. The μ_t can contain combinations of trend, seasonal dummies, and a constant. The tables have been generated by assuming a true data generating process as in (1.3), where ε_t has been drawn from the standard normal distribution. The number of replications is 5000. Note that the tests for π_1 and π_2 are one-sided because the alternative hypothesis is as in (1.2) (see also the discussion in HEGY), while the tests for the other π_i are two-sided tests. From (1.13) it can be seen that e.g. roots i and $-i$ are present only if π_3 and π_4 both equal zero, and hence it might be more convenient to test them simultaneously. Tables for F tests of $\pi_3=\pi_4=0$ through $\pi_{11}=\pi_{12}=0$ have also been generated and are displayed in the second part of the appendix. Finally, a table for the F test for the restriction $\pi_3=..=\pi_{12}=0$ is given, also to facilitate inference in the forthcoming power experiments.

To illustrate the above described procedure for testing for seasonal unit roots, it has been applied to 348 monthly new car registrations (in logs) in the Netherlands for the period 1960.01 to 1988.12. For completeness all 5 auxiliary regressions have been carried out, although the one with trend, constant and seasonal dummies is most important (see also the figure). The test results are displayed below in table 1.

TABLE 1

Testing for seasonal unit roots in monthly new car sales 1960.01–1988.12

	Auxiliary regression[(1)(2)]				
t–statistics	nc,nd,nt	c,nd,nt	c,nd,t	c,d,nt	c,d,t
π_1	2.065	-3.900*	-1.947	-3.161*	-1.844
π_2	-0.105	-0.114	-0.122	-1.310	-1.311
π_3	0.404	0.441	0.431	1.248	1.239
π_4	-2.144	-2.048	-2.026	-4.721*	-4.690*
π_5	-0.969	-0.030	-0.028	-1.103	-1.102
π_6	-0.685	-0.627	-0.623	-3.175	-3.166
π_7	1.186	1.271	1.266	3.063	3.052
π_8	-2.029	-1.981	-1.960	-4.541*	-4.506*
π_9	-1.509	-1.472	-1.461	-3.378*	-3.364*
π_{10}	-0.715	-0.628	-0.616	-0.564	-0.563
π_{11}	-0.513	-0.503	-0.504	-1.822*	-1.817*
π_{12}	-0.537	-0.508	-0.512	0.480	0.471
F–statistics					
$\pi_3 \cap \pi_4$	2.298	2.100	2.054	11.329*	11.176*
$\pi_5 \cap \pi_6$	0.613	0.605	0.601	8.003*	7.980*
$\pi_7 \cap \pi_8$	2.760	2.386	2.320	12.550*	12.281*
$\pi_9 \cap \pi_{10}$	1.156	1.090	1.007	5.728	5.681
$\pi_{11} \cap \pi_{12}$	0.599	0.554	0.561	1.885	1.881

* Significant at the 5% level.

(1) The polynomial $\varphi^*(B)$ is $(1-\varphi_1 B-\varphi_2 B^9-\varphi_3 B^{12})$ for all columns. The estimated parameters φ_i are all highly significant.

(2) The auxiliary regression can contain (no) constant (n(c)), (no) seasonal dummies (n(s)) amd (no) trend (n(t)).

From the results in the last column of table 1 it can be concluded that $\pi_i=0$ for $i=1,2,9,10,11,12$. This implies that the filter necessary to transform the monthly new car registrations to provide stationarity equals $(1-B^6)$, because

$$(1-B^2)(1+B+B^2)(1-B+B^2) = (1-B^6) \tag{1.18}$$

and that seasonal dummies are included in future models. Finally, note that the polynomial in footnote 1 in table 1 gives some indication for the eventual models to be built after transforming the y_t.

2. EMPIRICAL POWER INVESTIGATION

The size of the test has now been established. It is however also of interest to investigate the power of the testing procedure. Often, the powers of tests for unit roots are not high, so such results can also be expected in the present case. Attention will be given to the comparison of the five types of auxiliary regressions, for they may give conflicting results.

The roots of the polynomial $1-B^{12}=0$ all lie on the unit circle, and hence are equidistant to the origin. However, the roots of $1-\rho B^{12}=0$ with $0<\rho<1$ have distinct distances to the origin, i.e. those not lying on the axes of the complex space are farther away. In table 2 the norms of the roots for some values for ρ are displayed.

TABLE 2

Roots of the polynomial $1-\rho B^{12}=0$, $0<\rho<1$ (lower half of first quadrant only)

ρ	roots	norms
0.9	1.00881,	1.00881
	0.8737 + 0.5044i	1.01771
0.5	1.05946,	1.05946
	0.9175 + 0.5297i	1.12246
0.2	1.14352,	1.14352
	0.9903 + 0.5717i	1.30766

This implies that it might be expected that the power of the t tests on π_1 and π_2 (in fact also π_3 and π_4, but I will not consider these here) will be less than the power of the F test on the other π's. Furthermore, the choice of ρ to be positive induces that the power of the t test for π_2 will be larger. The results, as displayed in table 3 and 4 for ρ equal to 0.9 and 0.5, respectively, confirm these expectations. In table 5 the results are given in case a deterministic type of seasonality is present. The data generating process is now

$$y_t = \sum_{i=1}^{12} \alpha_i D_{it} + \varepsilon_t \quad (2.1)$$

with the α_1 through α_{12} set equal to -1,1,2,3,5,6,8,6,4,2,1,-2, and where ε_t is again drawn from the standard normal distribution. The zero power for π_1 in table 5 (first two rows) is caused by the occurence that the regression line of two reasonably constant variables on each other is forced through the origin. This implies that the estimated π_1 is expected to be zero.

TABLE 3

Empirical powers of test statistics for seasonal unit roots in monthly data based on 1000 Monte Carlo replications.

DGP: $y_t = 0.9y_{t-12} + \varepsilon_t$, $\varepsilon_t \sim$ N(0,1), size 0.05

Auxiliary regression	T	$t{:}\pi_1$	$t{:}\pi_2$	$F{:}\pi_3,..\pi_{12}$
nc,nd,nt	120	0.091	0.129	0.191
	240	0.156	0.162	0.494
c,nd,nt	120	0.065	0.129	0.196
	240	0.072	0.164	0.505
c,d,nt	120	0.067	0.051	0.093
	240	0.076	0.063	0.185
c,nd,t	120	0.059	0.130	0.210
	240	0.048	0.166	0.509
c,d,t	120	0.061	0.049	0.092
	240	0.056	0.065	0.185

TABLE 4

Empirical powers of test statistics for seasonal unit roots in monthly data based on 1000 Monte Carlo replications.

DGP: $y_t = 0.5y_{t-12} + \varepsilon_t$, $\varepsilon_t \sim N(0,1)$, size 0.05

Auxiliary regression	T	$t{:}\pi_1$	$t{:}\pi_2$	$F{:}\pi_3,..\pi_{12}$
nc,nd,nt	120	0.500	0.564	0.999
	240	0.928	0.926	1.000
c,nd,nt	120	0.198	0.560	0.999
	240	0.490	0.934	1.000
c,d,nt	120	0.207	0.213	0.809
	240	0.523	0.515	1.000
c,nd,t	120	0.127	0.583	0.999
	240	0.289	0.933	1.000
c,d,t	120	0.119	0.214	0.801
	240	0.312	0.515	1.000

TABLE 5

Empirical powers of test statistics for seasonal unit roots in monthly data based on 1000 Monte Carlo replications.

DGP: (2.1), size 0.05

Auxiliary regression	T	$t{:}\pi_1$	$t{:}\pi_2$	$F{:}\pi_3,..\pi_{12}$
nc,nd,nt	120	0.000	0.595	0.999
	240	0.000	0.976	1.000
c,nd,nt	120	0.508	0.652	1.000
	240	0.985	0.984	1.000
c,d,nt	120	0.823	0.831	1.000
	240	0.997	1.000	1.000
c,nd,t	120	0.252	0.664	1.000
	240	0.842	0.984	1.000
c,d,t	120	0.581	0.828	1.000
	240	0.984	1.000	1.000

From these power investigations several conclusions can be drawn. The first is that it is difficult to distinguish between stationary and integrated seasonality, or model (1.2) versus (1.3), even if the ρ in (1.2) equals 0.5. Fortunately, these difficulties mainly concern the test for the presence of the first unit root. With respect to the detection of the seasonal unit roots the test procedure does seem to have power. Secondly, it can be seen that a clear recognition of the alternative hypothesis, i.e. the μ_t, does have a significant impact on the power. Finally, the power of the tests often increases rather rapidly with the number of observations.

3. CONCLUDING REMARKS

The procedure for testing for (seasonal) unit roots, as is developed in Hylleberg, Engle, Granger and Yoo (1988), has been extended to time series consisting of monthly observations. The method is applied to monthly new car registrations. From this illustration it can be seen that a clear idea of the alternative hypothesis, i.e. determistic seasonality or a trend, is indispensable for conducting appropriate inference. This result also emerges from some small power experiments, where, as expected, higher power is ob- tained in the cases where the alternative hypothesis is correctly formula- ted. Additionally, from the same experiments it can be seen that the test for a unit root at the zero frequency may lack power, but that the tests for the eventual presence of seasonal unit roots often obtain high power, notably in large samples.

APPENDIX
CRITICAL VALUES FOR TESTING FOR SEASONAL UNIT ROOTS

PART ONE

Critical t-values based on 5000 Monte Carlo simulations

DGP: $y_t = y_{t-12} + \varepsilon_t,\ \varepsilon_t \sim N(0,1)$

Legenda: (n)c: (no) constant term in auxiliary regression
(n)d: (no) seasonal dummies
(n)t: (no) trend
T : number of observations

Table: π_1 and π_2

		π_1				π_2			
Regression	T	.01	.025	.05	.10	.01	.025	.05	.10
nc,nd,nt	120	-2.49	-2.20	-1.85	-1.51	-2.37	-2.03	-1.78	-1.46
	240	-2.54	-2.20	-1.88	-1.56	-2.58	-2.22	-1.90	-1.58
c,nd,nt	120	-3.30	-2.99	-2.69	-2.40	-2.36	-2.02	-1.77	-1.46
	240	-3.35	-3.05	-2.80	-2.49	-2.55	-2.23	-1.89	-1.58
c,nd,t	120	-3.82	-3.54	-3.26	-2.96	-2.35	-2.01	-1.76	-1.46
	240	-3.85	-3.60	-3.35	-3.05	-2.54	-2.22	-1.89	-1.58
c,d,nt	120	-3.30	-2.93	-2.63	-2.35	-3.25	-2.93	-2.65	-2.40
	240	-3.24	-3.02	-2.75	-2.45	-3.27	-3.03	-2.79	-2.49
c,d,t	120	-3.73	-3.47	-3.24	-2.92	-3.23	-2.91	-2.65	-2.39
	240	-3.82	-3.53	-3.30	-3.02	-3.29	-3.03	-2.79	-2.49

Table: π_3

Regression	T	.01	.025	.05	.10	.90	.95	.975	.99
nc,nd,nt	120	−2.35	−1.96	−1.66	−1.26	1.31	1.67	1.94	2.24
	240	−2.23	−1.93	−1.64	−1.25	1.24	1.61	1.93	2.22
c,nd,nt	120	−2.33	−1.95	−1.64	−1.25	1.29	1.65	1.92	2.22
	240	−2.25	−1.93	−1.64	−1.24	1.23	1.60	1.92	2.23
c,nd,t	120	−2.29	−1.93	−1.63	−1.24	1.27	1.63	1.92	2.20
	240	−2.25	−1.92	−1.63	−1.24	1.22	1.59	1.91	2.20
c,d,nt	120	−2.45	−2.11	−1.76	−1.39	1.37	1.74	2.11	2.54
	240	−2.55	−2.17	−1.83	−1.42	1.47	1.87	2.19	2.62
c,d,t	120	−2.41	−2.05	−1.71	−1.36	1.36	1.72	2.10	2.52
	240	−2.54	−2.15	−1.82	−1.41	1.47	1.87	2.18	2.61

Table: π_4

Regression	T	.01	.025	.05	.10	.90	.95	.975	.99
nc,nd,nt	120	−2.49	−2.15	−1.80	−1.46	1.15	1.48	1.75	2.12
	240	−2.48	−2.17	−1.87	−1.54	1.06	1.40	1.68	1.99
c,nd,nt	120	−2.48	−2.14	−1.81	−1.45	1.12	1.46	1.72	2.12
	240	−2.47	−2.17	−1.87	−1.55	1.05	1.39	1.67	1.98
c,nd,t	120	−2.45	−2.12	−1.81	−1.45	1.11	1.44	1.71	2.09
	240	−2.46	−2.18	−1.87	−1.54	1.04	1.38	1.65	1.98
c,d,nt	120	−3.65	−3.34	−3.12	−2.83	−0.73	−0.44	−0.14	0.12
	240	−3.78	−3.51	−3.29	−2.97	−0.82	−0.49	−0.20	0.12
c,d,t	120	−3.64	−3.34	−3.12	−2.82	−0.74	−0.45	−0.15	0.11
	240	−3.79	−3.51	−3.29	−2.97	−0.82	−0.49	−0.22	0.11

Table: π_5

Regression	T	.01	.025	.05	.10	.90	.95	.975	.99
nc,nd,nt	120	−2.51	−2.14	−1.81	−1.43	1.11	1.48	1.79	2.18
	240	−2.47	−2.13	−1.83	−1.47	1.04	1.40	1.71	2.11
c,nd,nt	120	−2.50	−2.13	−1.81	−1.43	1.10	1.46	1.77	2.11
	240	−2.47	−2.13	−1.83	−1.47	1.03	1.39	1.70	2.10
c,nd,t	120	−2.47	−2.12	−1.80	−1.42	1.09	1.44	1.73	2.20
	240	−2.47	−2.12	−1.83	−1.47	1.02	1.37	1.69	2.08
c,d,nt	120	−3.66	−3.29	−3.00	−2.71	−0.42	−0.05	0.25	0.61
	240	−3.62	−3.33	−3.10	−2.77	−0.42	−0.09	0.21	0.54
c,d,t	120	−3.65	−3.29	−2.99	−2.70	−0.43	−0.06	0.24	0.61
	240	−3.61	−3.34	−3.09	−2.77	−0.43	−0.09	0.19	0.53

Table: π_6

Regression	T	.01	.025	.05	.10	.90	.95	.975	.99
nc,nd,nt	120	−2.45	−2.08	−1.79	−1.42	1.10	1.47	1.79	2.18
	240	−2.40	−2.11	−1.82	−1.51	1.04	1.41	1.76	2.12
c,nd,nt	120	−2.45	−2.07	−1.77	−1.42	1.09	1.46	1.77	2.15
	240	−2.40	−2.10	−1.82	−1.51	1.04	1.41	1.75	2.10
c,nd,t	120	−2.43	−2.05	−1.76	−1.42	1.07	1.43	1.74	2.11
	240	−2.41	−2.10	−1.81	−1.51	1.03	1.40	1.73	2.09
c,d,nt	120	−3.76	−3.39	−3.12	−2.85	−0.73	−0.42	−0.09	0.31
	240	−3.76	−3.46	−3.21	−2.93	−0.79	−0.47	−0.18	0.19
c,d,t	120	−3.77	−3.38	−3.12	−2.84	−0.75	−0.44	−0.11	0.26
	240	−3.75	−3.46	−3.21	−2.93	−0.79	−0.47	−0.19	0.17

Table: π_7

Regression	T	.01	.025	.05	.10	.90	.95	.975	.99
nc,nd,nt	120	-2.15	-1.82	-1.48	-1.11	1.45	1.78	2.08	2.44
	240	-2.14	-1.76	-1.42	-1.03	1.52	1.88	2.22	2.53
c,nd,nt	120	-2.13	-1.80	-1.48	-1.09	1.44	1.77	2.05	2.40
	240	-2.12	-1.76	-1.41	-1.02	1.50	1.87	2.21	2.53
c,nd,t	120	-2.16	-1.80	-1.46	-1.09	1.45	1.77	2.07	2.38
	240	-2.11	-1.75	-1.42	-1.02	1.50	1.85	2.20	2.49
c,d,nt	120	-0.57	-0.27	0.05	0.41	2.68	3.00	3.31	3.67
	240	-0.58	-0.22	0.07	0.44	2.81	3.14	3.41	3.72
c,d,t	120	-0.49	-0.18	0.12	0.47	2.66	2.98	3.28	3.59
	240	-0.54	-0.17	0.11	0.46	2.81	3.12	3.39	3.71

Table: π_8

Regression	T	.01	.025	.05	.10	.90	.95	.975	.99
nc,nd,nt	120	-2.42	-2.10	-1.85	-1.52	1.08	1.46	1.76	2.15
	240	-2.52	-2.20	-1.89	-1.56	0.98	1.38	1.70	2.07
c,nd,nt	120	-2.44	-2.13	-1.84	-1.52	1.07	1.43	1.73	2.09
	240	-2.54	-2.18	-1.90	-1.56	0.98	1.37	1.71	2.05
c,nd,t	120	-2.51	-2.15	-1.84	-1.53	1.07	1.42	1.77	2.07
	240	-2.53	-2.17	-1.91	-1.56	0.98	1.37	1.71	2.04
c,d,nt	120	-3.73	-3.39	-3.14	-2.84	-0.72	-0.42	-0.18	0.20
	240	-3.78	-3.50	-3.22	-2.94	-0.81	-0.48	-0.16	0.11
c,d,t	120	-3.74	-3.40	-3.15	-2.85	-0.71	-0.43	-0.17	0.21
	240	-3.78	-3.50	-3.23	-2.95	-0.81	-0.48	-0.18	0.12

Table: π_9

Regression	T	.01	.025	.05	.10	.90	.95	.975	.99
nc,nd,nt	120	−2.38	−1.99	−1.68	−1.32	1.18	1.53	1.84	2.21
	240	−2.46	−2.04	−1.75	−1.40	1.15	1.49	1.80	2.16
c,nd,nt	120	−2.35	−1.98	−1.68	−1.31	1.16	1.51	1.80	2.18
	240	−2.45	−2.03	−1.75	−1.40	1.14	1.47	1.79	2.14
c,nd,t	120	−2.35	−1.97	−1.66	−1.30	1.15	1.49	1.79	2.16
	240	−2.44	−2.03	−1.74	−1.40	1.14	1.46	1.77	2.12
c,d,nt	120	−3.19	−2.87	−2.54	−2.20	0.44	0.82	1.13	1.51
	240	−3.31	−2.92	−2.62	−2.28	0.38	0.78	1.13	1.57
c,d,t	120	−3.21	−2.86	−2.54	−2.19	0.43	0.81	1.12	1.46
	240	−3.30	−2.91	−2.61	−2.27	0.38	0.77	1.13	1.54

Table: π_{10}

Regression	T	.01	.025	.05	.10	.90	.95	.975	.99
nc,nd,nt	120	−2.41	−2.02	−1.77	−1.46	1.13	1.56	1.87	2.24
	240	−2.46	−2.20	−1.88	−1.55	1.03	1.44	1.80	2.20
c,nd,nt	120	−2.41	−2.01	−1.77	−1.46	1.12	1.53	1.84	2.23
	240	−2.47	−2.20	−1.87	−1.55	1.02	1.43	1.79	2.19
c,nd,t	120	−2.39	−2.01	−1.76	−1.45	1.09	1.51	1.83	2.17
	240	−2.46	−2.20	−1.87	−1.55	1.01	1.42	1.78	2.18
c,d,nt	120	−3.71	−3.37	−3.07	−2.79	−0.71	−0.39	−0.07	0.30
	240	−3.87	−3.50	−3.25	−2.94	−0.78	−0.41	−0.10	0.17
c,d,t	120	−3.70	−3.36	−3.07	−2.79	−0.72	−0.40	−0.09	0.28
	240	−3.87	−3.52	−3.25	−2.94	−0.78	−0.42	−0.10	0.17

Table: π_{11}

Regression	T	.01	.025	.05	.10	.90	.95	.975	.99
nc,nd,nt	120	−2.30	−1.85	−1.56	−1.21	1.38	1.75	2.03	2.41
	240	−2.21	−1.79	−1.47	−1.16	1.42	1.76	2.06	2.38
c,nd,nt	120	−2.28	−1.82	−1.56	−1.19	1.36	1.72	2.00	2.40
	240	−2.17	−1.79	−1.46	−1.15	1.42	1.75	2.06	2.37
c,nd,t	120	−2.27	−1.84	−1.55	−1.17	1.34	1.71	1.98	2.38
	240	−2.15	−1.77	−1.46	−1.14	1.40	1.74	2.04	2.35
c,d,nt	120	−1.48	−1.11	−0.78	−0.43	2.19	2.56	2.83	3.22
	240	−1.67	−1.21	−0.87	−0.44	2.28	2.62	2.92	3.29
c,d,t	120	−1.43	−1.08	−0.73	−0.39	2.19	2.55	2.80	3.18
	240	−1.61	−1.19	−0.84	−0.42	2.29	2.61	2.91	3.26

Table: π_{12}

Regression	T	.01	.025	.05	.10	.90	.95	.975	.99
nc,nd,nt	120	−2.49	−2.16	−1.85	−1.49	1.09	1.42	1.75	2.18
	240	−2.48	−2.14	−1.85	−1.51	1.02	1.40	1.71	2.05
c,nd,nt	120	−2.47	−2.15	−1.83	−1.49	1.07	1.41	1.74	2.16
	240	−2.47	−2.15	−1.84	−1.51	1.01	1.39	1.71	2.06
c,nd,t	120	−2.47	−2.15	−1.83	−1.50	1.04	1.39	1.72	2.11
	240	−2.47	−2.14	−1.84	−1.50	0.99	1.39	1.69	2.05
c,d,nt	120	−3.71	−3.43	−3.16	−2.85	−0.73	−0.42	−0.14	0.25
	240	−3.66	−3.44	−3.20	−2.90	−0.79	−0.50	−0.20	0.05
c,d,t	120	−3.75	−3.42	−3.16	−2.83	−0.73	−0.44	−0.17	0.25
	240	−3.66	−3.45	−3.20	−2.91	−0.80	−0.50	−0.21	0.03

PART TWO

Critical F-values based on 5000 Monte Carlo simulations

DGP: $y_t = y_{t-12} + \varepsilon_t$, $\varepsilon_t \sim N(0,1)$

Table: F-tests for $\pi_3=\pi_4=0$, $\pi_5=\pi_6=0$, $\pi_7=\pi_8=0$

		$\pi_3=\pi_4=0$			$\pi_5=\pi_6=0$			$\pi_7=\pi_8=0$		
Regression	T	.90	.95	.99	.90	.95	.99	.90	.95	.99
nc,nd,nt	120	2.37	3.04	4.71	2.38	3.09	4.69	2.31	2.98	4.66
	240	2.33	2.94	4.50	2.32	2.96	4.34	2.40	3.11	4.92
c,nd,nt	120	2.32	3.00	4.68	2.32	3.04	4.59	2.27	2.97	4.67
	240	2.31	2.93	4.51	2.30	2.93	4.33	2.41	3.12	4.91
c,nd,t	120	2.30	2.94	4.54	2.29	2.96	4.54	2.30	3.00	4.67
	240	2.30	2.91	4.46	2.29	2.90	4.33	2.39	3.09	4.92
c,d,nt	120	4.83	5.62	7.86	4.89	5.86	8.07	4.94	5.86	8.24
	240	5.33	6.36	8.46	5.16	6.05	8.01	5.29	6.23	8.42
c,d,t	120	4.81	5.63	7.74	4.86	5.84	8.03	4.94	5.90	8.27
	240	5.35	6.31	8.38	5.15	6.05	7.98	5.30	6.22	8.18

Table: F-tests for $\pi_9=\pi_{10}=0$, $\pi_{11}=\pi_{12}=0$, $\pi_3=..=\pi_{12}=0$

		$\pi_9=\pi_{10}=0$			$\pi_{11}=\pi_{12}=0$			$\pi_3=..=\pi_{12}=0$		
Regression	T	.90	.95	.99	.90	.95	.99	.90	.95	.99
nc,nd,nt	120	2.29	2.98	4.53	2.33	3.08	4.98	1.68	1.95	2.46
	240	2.38	3.08	4.62	2.32	2.93	4.54	1.65	1.91	2.47
c,nd,nt	120	2.26	2.92	4.39	2.31	3.03	4.88	1.65	1.92	2.42
	240	2.35	3.05	4.57	2.31	2.92	4.56	1.65	1.90	2.45
c,nd,t	120	2.21	2.86	4.39	2.27	3.00	4.85	1.62	1.89	2.39
	240	2.34	3.02	4.54	2.30	2.91	4.54	1.64	1.88	2.43
c,d,nt	120	4.79	5.75	7.76	4.94	5.89	7.84	4.00	4.46	5.53
	240	5.21	6.16	8.43	5.15	6.03	7.85	4.09	4.48	5.41
c,d,t	120	4.76	5.71	7.68	4.92	5.84	7.88	4.00	4.45	5.51
	240	5.19	6.14	8.47	5.14	6.04	7.82	4.08	4.48	5.37

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FIGURE

Monthly new car registrations in the Netherlands, 1960.01–1988.12