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The Posterior Distribution of Roots in Multivariate Autoregressions

Nonstationarity or near nonstationarity in time series has received substantial attention from statisticians in recent years. There are at least two reasons for this interest. First, the asymptotic sampling distributions of estimators and test statistics is complicated by nonstationarity, and the finite sampling distributions are complicated by near nonstationarity in samples of the sizes typically used in macroeconomics; on the implications for multiple time series see, for example, Solo (1984), Said and Dickey (1985), and Sims, Stock, and Watson (1987). Second, equilibrium relationships have been interpreted as restricting linear combinations of time series to be stationary, thus placing restrictions on multiple, possibly nonstationary time series, without specifying disequilibrium dynamics (Davidson, et. al., 1978; Engle and Granger, 1987; Hylleberg and Mizon, 1988). The second reason provides the motivation for an investigation of Bayesian interpretation and treatment of nonstationarity and near nonstationarity. Results from early stages of this investigation are reported here.

This project began with three objectives. The first was to illustrate that Bayesian inference about the roots of a multiple time series can be straightforward, indeed trivial, when that series is modelled as a Gaussian autoregressive process of finite order. That objective is met here through the application of an algorithm for Monte Carlo integration of the posterior density described elsewhere (Geweke, 1988a) that facilitates extension of the methods for univariate time series described in Geweke (1988b). The second objective was to investigate the sensitivity of the posterior distributions of the roots to alternative diffuse priors typically employed in Bayesian approaches to multiple time series. Ultimately it is important to employ substantive, informative priors in the economic interpretation of multiple time series, but the formulation of such priors is challenging, and diffuse reference priors will probably continue to be used in public reporting even when such informative priors become practical. Broadly, the results reported here indicate that prior robustness breaks down at about the point at which conventional rules of thumb would suggest the model has been overparameterized. The final objective of the project was to provide some indication of the sample sizes needed to discriminate among alternative hypotheses about the stationarity of linear combinations of a possibly nonstationary, multiple time series. The results suggest that the sample sizes required are near the upper limit of those typically available to macroeconomists.

The three main sections of this article address these three objectives, respectively.

1. Bayesian inference about the roots of vector autoregressions

A Gaussian vector autoregression for an m-variate multiple time series may be written

$$A(B)y_t = \varepsilon_t, \quad \varepsilon_t \sim IIDN(\alpha, \Sigma)$$

with

$$A(B) = \sum_{s=0}^{p} A_s B^s, \quad A_0 \equiv I,$$

where B is the conventional back-operator. The mp roots of the autoregression are those of the determinantal equation $\det[A(z^{-1})] = 0$; an algorithm for the determination of these roots is given by Robinson (1983, pp. 160-164). Equivalently the roots are the eigenvalues of the mp x mp matrix

$$Q \equiv \begin{bmatrix} -A_1 & -A_2 & -A_3 & \cdots & -A_{p-1} & -A_p \\ I & 0 & 0 & \cdots & 0 & 0 \\ 0 & I & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & I & 0 \end{bmatrix}$$

(Anderson, 1971, pp. 177-181). Note that in this formulation the explosive roots are those whose modulus exceeds one; thus $\{y_t\}$ is stationary if and only if all mp roots are *inside* the unit circle.

Given T + p successive observations on $\{y_t\}$ the log-likelihood function for the last T observations conditional on the first p is

$$-[(T+r)/2]\log|\Sigma| - (1/2)\sum_{t=p+1}^{T+p}[A(B)y_t - \alpha]'\Sigma^{-1}[A(B)y_t - \alpha] \qquad (1)$$

with $r = 0$, reflecting the Gaussian multivariate regression model for each y_t. If a standard conjugate prior is employed the log posterior density is of the same form with $r = m+1$ (up to an additive constant), and the marginal posterior densities of the parameters α, $A(B)$, and Σ may be obtained analytically (Zellner, 1971, pp. 224-233). If the prior is not standard conjugate or there are functions of interest nonlinear in the parameters, then with rare exception posterior moments cannot be determined analytically; that is certainly the case for the amplitude of the roots of the vector autoregression. However, in these cases Monte Carlo integration provides good numerical approximations so longs as the prior density remains diffuse.

To provide a concise summary of these methods, collect the parameters of the model in a vector θ, and denote (1) with $r = m+1$ by $p(\theta)$. As described in Geweke (1988a, Appendix B) it is straightforward to draw a pseudo-random sample $\{\theta_i\}_{i=1}^{n}$ from a density proportional to this function. Let $\pi(\theta)$ denote the ratio of the prior density to that of the standard conjugate prior and let $g(\theta)$ be the function of interest. If $\pi(\theta)$ is bounded above then by the law of large numbers,

$$g_n = \sum_{i=1}^{n} g(\theta_i)\pi(\theta_i) \Big/ \sum_{i=1}^{n}\pi(\theta_i)$$

converges almost surely to $E[g(\theta)]$, where the expectation is with respect to the posterior density. Moreover, $n^{1/2}(g_n - E[g(\theta)]) \Rightarrow N(0, \sigma^2)$; expressions for σ^2 and its consistent (in n) computable approximation are given in Kloek and van Dijk (1978) and Geweke (1989), so the accuracy of the numerical approximation can be appraised. As a byproduct of this procedure graphs of numerical approximations to the posterior densities of functions of interest may also be produced.

In this study, $n = 2000$, which provided two significant figures for most functions of interest. Each Monte Carlo replication entailed determination of the roots by means of the eigenvalues of Q, the eigenvectors of Q (for use in a subsequent project) and several hundred derived functions of interest associated with several projects and the construction of graphs. The time required ranged from 20 seconds (T=20, p=1, m=2) to 700 seconds (T=250, p=3, m=2) with a Vax 11/780.

This study concentrates on the largest root amplitudes, providing numerical approximations to their posterior densities. It also concentrates on the posterior probabilities that roots occur with amplitudes in ten specified regions. These regions are indicated in Table 1. Five (S5 - S1) correspond to stationary roots defined by ranges of amplitudes. Corresponding to a given amplitude, γ, there is an associated halving time $\log(.5)/\log(\gamma)$, the length of time for a shock subjected to the exponential decay γ^s to reach half its initial value. These are provided in Table 1, and were used as the basis for selecting the regions, in order to provide some economic interpretation base on one year as the unit time interval: e.g., roots in S3 are stationary but reflect persistence of shocks through a period as long as a business cycle. Similarly E1 - E5 correspond to explosive roots, classified by doubling times $\log(2)/\log(\gamma)$. Subsequently we report the posterior expectations of the number of roots with amplitudes in each of the ten regions.

2. Sensitivity to priors

Six alternative prior densities, detailed in Table 2, were considered. Prior A assigns a standard normal to each coefficient; prior B does the same except that the coefficient on the first lag of the left-side variable is -1, corresponding to a random walk. Prior C is centered at 0, with a standard error for the first lag that is smaller than that in prior A, and becomes smaller with increasing lag. Prior D again assigns a mean of -1 to the first lag of the left-side variable. Prior E provides a very diffuse, yet proper, prior on the coefficients. Prior L is the standard conjugate prior. Priors B and D are similar to those employed by Doan, Litterman, and Sims (1984), while A, E, and L provide a progression to the standard conjugate prior frequently employed in Bayesian treatments of autoregressions (e.g., Geweke, 1988b).

All of the results that follow employ bivariate autoregressions, with p = 1 (one lag, two roots, nine parameters) or with p = 3 (three lags, six roots, seventeen parameters). Table 3 indicates the prior expectation of the number of roots with amplitudes in each of the ten regions (by row in each panel) for each prior (by column) for the two parameterizations (the respective panels). Except for prior L, which is trivial, all prior expectations were computed by Monte Carlo integration, drawing directly from the priors. Four observations may be made about the results in Table 3. (1) Corresponding to the diffusion of the priors, amplitudes are dispersed across regions. (2) As prior variance increases (C to A to E to L, or D to B) the implied prior distribution of amplitudes shifts upward. (3) Shifting the prior mean of a_{ii1} from 0 to -1 has only a modest and mixed tendency to shift the prior distribution toward roots with greater amplitude. (4) Scaling the prior standard error of the coefficients by $.7^s$ consistently shifts the distribution toward smaller root amplitudes.

Posterior densities as a function of prior densities are examined directly using an artificially generated process. The process is a bivariate autoregression with $p = 1$, and

$$A_1 = \begin{bmatrix} -1 & 0 \\ -1 & 0 \end{bmatrix}, \qquad \Sigma = I$$

The process is cointegrated (in the sense of Engle and Granger, 1987), with the difference of the two constituent series stationary, while any other independent linear combinations, and the process itself, are nonstationary. The roots are 0 and 1. If the process is embedded in a higher-order autoregression with $A_2 = \ldots = A_p = 0$, then there are 2p-1 roots of 0 and one of 1. A sample of size 360 was generated from $y_0 = 0$, using the NAG normal random number generator G05DDF and an initial seed of 12345. From these samples four subsamples were created, from observations 111 through 130 (T = 20), 111 through 150 (T = 40), 111 through 210 (T = 100), and 111 through 360 (T = 250).

Posterior densities of the largest and second-largest root amplitudes, are provided in Figure 1. (In all panels, posteriors under priors E and F are hardly distinguishable, and so there often appear to be five rather than six lines.) The posterior densities from a first-order autoregression (p = 1) show little sensitivity to the choice of prior for T = 20 (first row of Figure 1): differences are more pronounced for the smaller than for the larger root amplitude, but still are not great. For the third-order autoregression the effects of the prior standard error are clearly manifest in the posterior densities of the first and second largest root amplitudes (second row of Figure 1): posteriors under priors C and D are more concentrated than those under A and B, which are in turn somewhat more concentrated than those under E and L. In view of the fact that this model has seventeen parameters and 40 initial degrees of freedom this sensitivity is not surprising. When the sample size is doubled (T = 40, last row of Figure 1) the sensitivity of the posterior distributions to the choice of prior is small, with the only notable effects being some remaining tendency for the posterior densities of the second largest root amplitude to be more concentrated under priors C and D.

Sensitivity of the posterior expectation of the number of roots in each of the ten regions to the choice of prior is documented systematically in Tables 4, 5, 6, and 7 for each of the four sample sizes respectively. For the smaller sample sizes, the evidence in Tables 4 and 5 in consistent with that in Figure 1. In the nine parameter model (Panels A) the only sensitivity to the prior appears in the way the expected number of roots is allocated among the small amplitude regions S5 and S4. Even this sensitivity rarely reaches 10%. In a seventeen-parameter model sensitivity persists at T = 40, amounting to as much as 40%. For $T \geq 100$, sensitivity to the prior has disappeared for this model, in that there are no differences in posterior expectations greater than 10%. In both panels A and B, there is evidence of a substantial range -- including T = 40 and T = 100 for p = 1, and T = 100 and T = 250 for p = 3 -- in which sensitivity to the prior is negligible but posterior densities have not yet attained asymptotic degeneracy.

In none of these examples, even when p = 3 and T = 20, is the discrepancy among posterior expectations anywhere near as great as among prior expectations: compare Table 3 with Table 4. The concern, sometimes expressed as a verbal assertion, that posterior densities of root amplitudes simply mirror the prior density, receives no support from these examples. Instead, the results are consistent with a working hypothesis that when the number of parameters is reasonable relative to initial degrees of freedom, sensitivity to reasonably diffuse priors is low. This hypothesis can, and should, be checked as part of responsible public reporting employing diffuse priors.

3. Discrimination among processes

Discrimination among stationary, nonstationary but cointegrated, and nonstationary and non-cointegrated processes is of concern to some macroeconometricians. How realistic is this objective given macroeconomic time series and no additional assumptions beyond those of a Gaussian vector autoregression? If the posterior distribution of the amplitudes of the roots are similar in the three cases, then prospects for discrimination are not good. Hence it is of some interest to compare these distributions in various sample sizes. Here we do so using a standard conjugate prior using three artificially generated Gaussian time series. Process WN is serially uncorrelated with variance $\Sigma = \mathbf{I}$, or white noise; process RW is a vector autoregression with $p = 1$, $A_1 = -\mathbf{I}$, and $\Sigma = \mathbf{I}$, a random walk; and process CI is the one employed in the investigation of prior robustness described in Section 2. In all three cases artificial data were generated as described previously.

As before, we compare the posterior densities of the ordered root amplitudes, and the expected number of roots with amplitudes in each of ten regions. The posterior densities are provided in Figures 2, 3, 4, and 5 for the four respective sample sizes considered, T = 20, T = 40, T = 100, and T = 250. The results indicate a marked contrast between the nine parameter model (p = 1) and the seventeen parameter model (p = 3).

When p = 1, the three cases are distinguished very well for T = 40, T = 100, and T = 250. The roots of the WN process are clearly smaller than those of the RW process, and each pair is closer to each other than to those of the other process. The roots of the CI process are clearly disentangled, the smaller distributed much like the roots of WN and the larger much like the roots of RW.

When p =3, it is not possible to distinguish among the three processes when T = 20. When T = 40 there is still little discrimination, with the posterior density of the three largest roots of WN shifted down only slightly from those of RW. When T = 100 and T = 250 there is discrimination among the three processes, although the amplitude of the second root of CI remains quite high for T = 100.

These conclusions are reinforced in Tables 8 and 9. For T = 20 all three models support a strongly serially correlated process in which a shock may persist through most of the sample. For T = 40 there is still little to distinguish even WN and RW, when p = 3.

The need for large amounts of data to discriminate among the three processes when $p = 3$ is sobering for two reasons. First, the cases studied here are extreme, in that all roots are zero or one: an extraneous stationary root with an amplitude of, say, .8 would presumably make it even more difficult to tell one model from another. Second, in applications there is often reason to believe that reasonable specification demands parameterizations on the order of $p = 3$: Geweke (1986) argues from prior theoretical considerations for exactly this specification in vector autoregressions for annual macroeconomic time series. The side-by-side comparison of the cases $p = 1$ and $p = 3$ indicates that difficulties arise with the more profligately parameterized model because the distribution of the largest of 4 or 5 identical amplitudes remains much higher than the common value even in samples whose sizes are in the range of 40 to 100. It is likely that incorporation of a prior that the small roots are of about the same size or are all small would mitigate the problem. But priors like these seem unfounded in macroeconomics -- indeed, it is the absence of such knowledge, prior or data-based, that motivates the use of equilibrium relationships without specifying disequilibrium dynamics in the first place.

The ability to distinguish among stationary, nonstationary, and cointegrated processes is most likely to be improved through the use of prior knowledge about those linear combinations that are apt to be stationary or nonstationary. Information of this sort is delivered by simple equilibrium models; it was not used here, nor is it used in approaches that estimate cointegrating vectors. This possibility will be taken up in subsequent research.

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Table 1: Classification of Amplitudes of Roots

Region	Range of amplitudes	Halving/doubling times
S5	.00000 - .50000	0.0 - 1.0
S4	.50000 - .75786	1.0 - 2.5
S3	.75786 - .91700	2.5 - 8.0
S2	.91700 - .97716	8.0 - 30.0
S1	.97716 - 1.00000	30.0 - ∞
E1	1.00000 - 1.02337	30.0 - ∞
E2	1.02337 - 1.09051	8.0 - 30.0
E3	1.09051 - 1.39151	2.5 - 8.0
E4	1.39151 - 2.00000	0.0 - 1.0

Table 2: Prior Densities

Prior A: $a_{ijs} \sim N(0,1)$

Prior B: $a_{ii1} \sim N(-1,1)$; $a_{ijs} \sim N(0,1)$ otherwise

Prior C: $a_{ijs} \sim N(0, .7^{2s})$

Prior D: $a_{ii1} \sim N(-1, .7^2)$; $a_{ijs} \sim N(0, .7^{2s})$ otherwise

Prior E: $a_{ijs} \sim N(0, 8^2)$

Prior L: $a_{ijs} \sim N(0, \sigma^2)$, $\sigma^2 \rightarrow \infty$

In each case, coefficient distributions are mutually independent. The prior density for Σ is indpendent of that for A(B) and is improper, proportional to $[\det(\Sigma)]^{-(m+1)/2}$.

Table 3: Prior Distribution of Roots

A: Bivariate autoregression, 1 lag

Region	Prior expectation of number of roots under prior A	B	C	D	E	L
S5	.48	.38	.75	.42	.05	.00
S4	.31	.23	.41	.25	.02	.00
S3	.20	.14	.24	.15	.02	.00
S2	.07	.04	.08	.05	.01	.00
S1	.02	.02	.03	.02	*	.00
E1	.03	.02	.03	.03	*	.00
E2	.07	.06	.07	.07	.01	.00
E3	.31	.26	.22	.33	.03	.00
E4	.35	.43	.15	.43	.07	.00
E5	.17	.42	.02	.23	1.80	2.00

B. Bivariate autoregression, 3 lags

Region	Prior expectation of number of roots under prior A	B	C	D	E	L
S5	.84	.92	1.54	1.87	.79	.00
S4	.87	.96	1.61	1.55	.65	.00
S3	.67	.69	1.01	.67	.41	.00
S2	.26	.27	.34	.22	.14	.00
S1	.10	.12	.11	.09	.05	.00
E1	.12	.09	.10	.07	.05	.00
E2	.33	.30	.27	.18	.13	.00
E3	1.31	1.03	.69	.52	.52	.00
E4	1.18	1.06	.29	.58	.59	.00
E5	.32	.56	.03	.25	2.66	6.00

*In the interval (0, .005).

Table 4: Posterior Distribution of Roots, T = 20
Simple co-integrated process

A. Bivariate autoregression, 1 lag

	Posterior expectation of number of roots under prior					
Region	A	B	C	D	E	L
S5	.88	.91	.89	.94	.86	.86
S4	.28	.25	.27	.22	.29	.29
S3	.67	.65	.68	.64	.66	.66
S2	.12	.13	.12	.14	.13	.13
S1	.02	.02	.02	.02	.02	.02
E1	.01	.01	.01	.01	.01	.01
E2	.02	.02	.01	.02	.02	.02
E3	*	*	*	*	*	*
E4	*	*	*	*	*	*
E5	*	*	*	*	*	*

B. Bivariate autoregression, 3 lags

	Posterior expectation of number of roots under prior					
Region	A	B	C	D	E	L
S5	1.27	1.34	1.85	1.99	1.13	1.13
S4	2.37	2.35	2.46	2.40	2.21	2.21
S3	1.85	1.79	1.38	1.31	2.01	2.01
S2	.30	.31	.21	.20	.34	.34
S1	.07	.07	.04	.04	.09	.09
E1	.04	.04	.02	.02	.05	.05
E2	.07	.07	.03	.03	.11	.11
E3	.04	.03	.01	.01	.07	.07
E4	*	*	*	*	*	*
E5	*	*	*	*	*	*

*In the interval (0, .005).

Table 5: Posterior Distribution of Roots, T = 40
Simple co-integrated process

A. Bivariate autoregression, 1 lag

Region	Posterior expectation of number of roots under prior A	B	C	D	E	L
S5	.93	.95	.94	.97	.91	.91
S4	.19	.17	.18	.14	.20	.20
S3	.76	.75	.76	.75	.75	.75
S2	.11	.11	.10	.12	.11	.11
S1	.02	.02	.01	.02	.02	.02
E1	*	.01	*	.01	.01	.01
E2	*	*	*	*	*	*
E3	*	*	*	*	*	*
E4	*	*	*	*	*	*
E5	*	*	*	*	*	*

B. Bivariate autoregression, 3 lags

Region	Posterior expectation of number of roots under prior A	B	C	D	E	L
S5	1.62	1.63	1.98	1.96	1.55	1.55
S4	2.55	2.56	2.72	2.75	2.44	2.44
S3	1.63	1.62	1.18	1.16	1.78	1.78
S2	.15	.15	.10	.11	.17	.17
S1	.02	.02	.01	.01	.03	.03
E1	.01	.01	.01	.01	.02	.02
E2	.01	.01	*	*	.01	.01
E3	*	*	*	*	.01	.01
E4	*	*	*	*	*	*
E5	*	*	*	*	*	*

*In the interval (0, .005).

Table 6: Posterior Deistribution of Roots, T = 100
Simple co-integrated process

A: Bivariate autoregression, 1 lag

Region	A	B	C	D	E	L
	Posterior expectation of number of roots under prior					
S5	1.00**	1.00**	1.00**	1.00**	1.00**	1.00**
S4	*	*	*	*	*	*
S3	*	*	*	*	*	*
S2	.21	.20	.21	.21	.20	.20
S1	.41	.41	.42	.41	.41	.41
E1	.30	.30	.30	.30	.30	.30
E2	.08	.08	.08	.08	.08	.08
E3	*	*	*	*	*	*
E4	*	*	*	*	*	*
E5	*	*	*	*	*	*

B. Bivariate autoregression, 3 lags

Region	A	B	C	D	E	L
	Posterior expectation of number of roots under prior					
S5	3.21	3.21	3.35	3.36	3.18	3.18
S4	1.78	1.78	1.64	1.64	1.81	1.81
S3	.01	.01	.01	.01	.01	.01
S2	.08	.08	.08	.08	.09	.09
S1	.36	.36	.37	.37	.36	.36
E1	.44	.44	.45	.45	.43	.43
E2	.11	.11	.10	.10	.11	.11
E3	*	*	*	*	*	*
E4	*	*	*	*	*	*
E5	*	*	*	*	*	*

*In the interval (0, .005).
**In the interval (.995, 1)

Table 7: Posterior Distribution of Roots, T = 250
Simple co-integrated process

A. Bivariate autoregression, 1 lag

	Posterior expectation of number of roots under prior					
Region	A	B	C	D	E	L
S5	1.00**	1.00**	1.00**	1.00**	1.00**	1.00**
S4	*	*	*	*	*	*
S3	*	*	*	*	*	*
S2	*	*	*	*	*	*
S1	.83	.82	.83	.82	.83	.83
E1	.17	.18	.17	.18	.17	.17
E2	*	*	*	*	*	*
E3	*	*	*	*	*	*
E4	*	*	*	*	*	*
E5	*	*	*	*	*	*

B. Bivariate autoregression, 3 lags

	Posterior expectation of number of roots under prior					
Region	A	B	C	D	E	L
S5	4.39	4.39	4.42	4.42	4.39	4.39
S4	.61	.61	.58	.58	.61	.61
S3	*	*	*	*	*	*
S2	*	*	*	*	*	*
S1	.83	.83	.83	.83	.83	.83
E1	.17	.17	.17	.17	.17	.17
E2	*	*	*	*	*	*
E3	*	*	*	*	*	*
E4	*	*	*	*	*	*
E5	*	*	*	*	*	*

*In the interval (0, .005).
**In the interval (.995, 1)

Table 8: Posterior Distribution of Roots, Smaller Samples
Standard conjugate prior

A. Bivariate autoregression, 1 lag

Sample size	T=20			T=40		
Process	WN	RW	CI	WN	RW	CI
Region						
S5	1.72	.25	.86	1.94	.05	.91
S4	.24	.79	.29	.06	.77	.20
S3	.03	.76	.66	*	1.04	.75
S2	*	.12	.13	*	.11	.11
S1	*	.03	.02	*	.14	.02
E1	*	.01	.01	*	.11	.01
E2	*	.03	.02	*	.11	*
E3	*	.02	*	*	*	*
E4	*	*	*	*	*	*
E5	*	*	*	*	*	*

B. Bivariate autoregression, 3 lags

Sample size	T=20			T=40		
Process	WN	RW	CI	WN	RW	CI
Region						
S5	.90	1.11	1.13	1.45	1.48	1.55
S4	2.59	2.04	2.21	3.27	2.18	2.44
S3	1.85	1.92	2.01	1.17	1.55	1.78
S2	.31	.48	.34	.09	.51	.17
S1	.08	.12	.09	.01	.12	.03
E1	.07	.08	.05	.01	.06	.02
E2	.10	.14	.11	*	.09	.01
E3	.10	.08	.07	*	.01	.01
E4	*	.01	*	*	*	*
E5	*	*	*	*	*	*

*In the interval (0, .005).

Table 9: Posterior Distribution of Roots, Larger Samples
Standard conjugate prior

A. Bivariate autoregression, 1 lag

Sample size	T=100			T=250		
Process	WN	RW	CI	WN	RW	CI
Region						
S5	2.00***	*	1.00**	2.00***	*	1.00**
S4	*	*	*	*	*	*
S3	*	.69	*	*	.02	*
S2	*	.57	.20	*	.85	*
S1	*	.37	.41	*	.92	.83
E1	*	.27	.30	*	.21	.17
E2	*	.09	.08	*	*	*
E3	*	*	*	*	*	*
E4	*	*	*	*	*	*
E5	*	*	*	*	*	*

B. Bivariate autoregression, 3 lags

Sample size	T=100			T=250		
Process	WN	RW	CI	WN	RW	CI
Region						
S5	1.96	3.40	3.18	4.21	3.95	4.39
S4	3.93	.79	1.81	1.79	.05	.61
S3	1.04	.62	.01	*	.03	*
S2	*	.33	.09	*	.78	*
S1	*	.39	.36	*	.99	.83
E1	*	.37	.44	*	.20	.17
E2	*	.10	.11	*	*	*
E3	*	*	*	*	*	*
E4	*	*	*	*	*	*
E5	*	*	*	*	*	*

*In the interval (0, .005).
**In the interval (.995, 1)
***In the interval (1.995, 2)

Figure 1: Posterior Densities of Root Amplitudes of Co-integrate Process under Alternative Priors

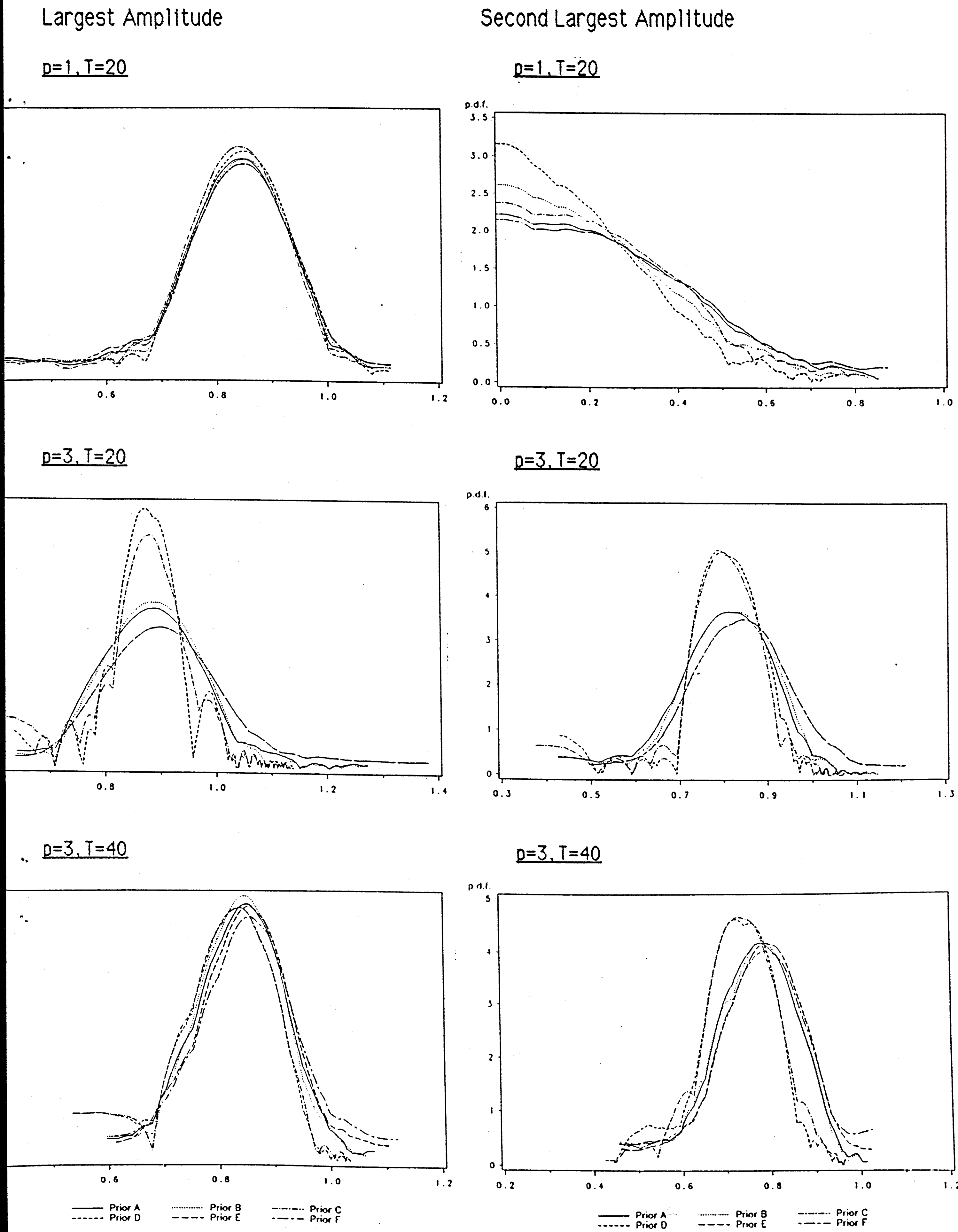


Figure 2: Posterior Distributions of Root Amplitudes of Three Processes
Standard Conjugate Prior, T=20

p=1 (9 parameters) p=3 (17 parameters)

Process WN Process WN

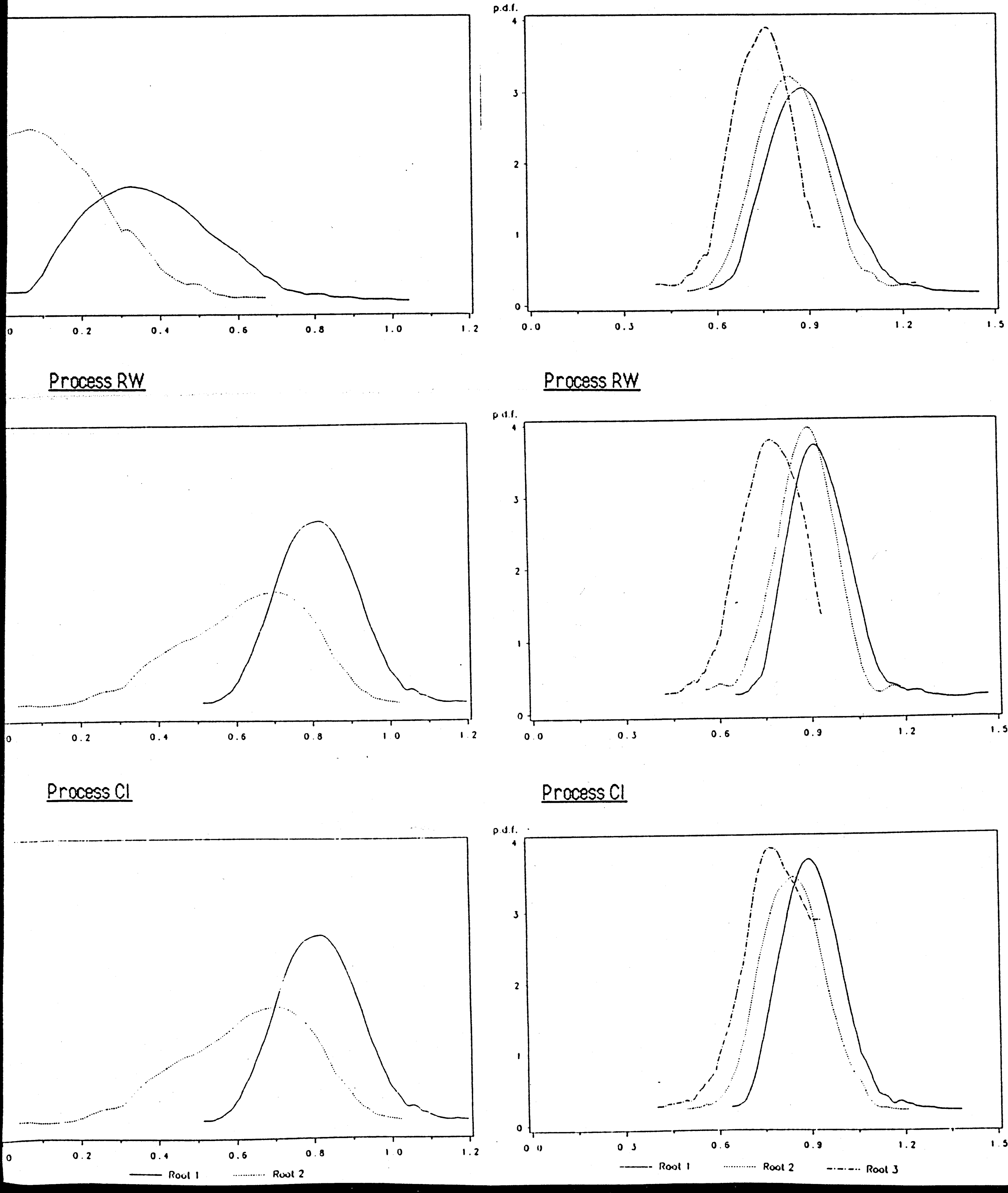


Figure 3: Posterior Distributions of Root Amplitudes of Three Processes
Standard Conjugate Prior, T=40

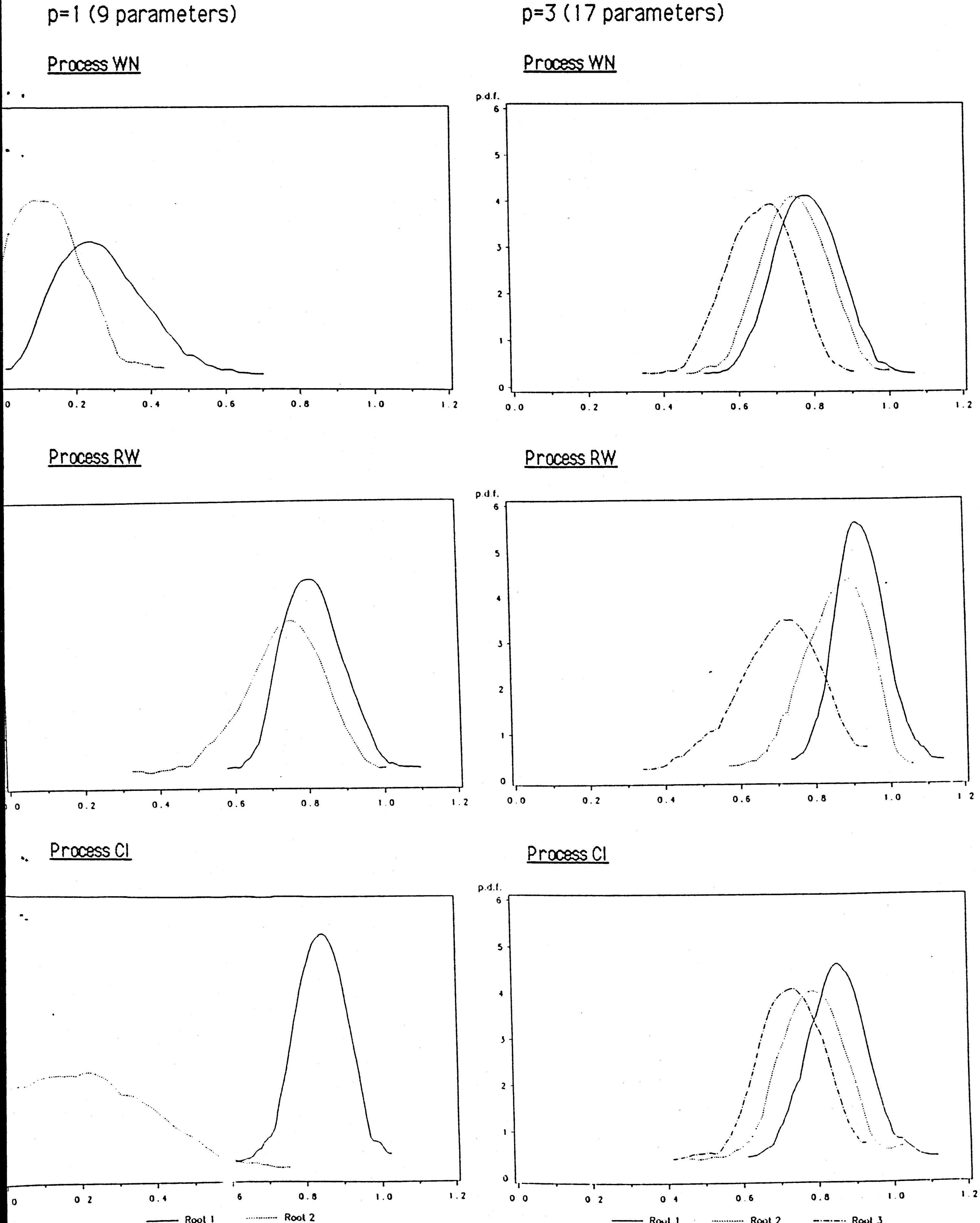


Figure 4: Posterior Distributions of Root Amplitudes of Three Processes
Standard Conjugate Prior, T=100

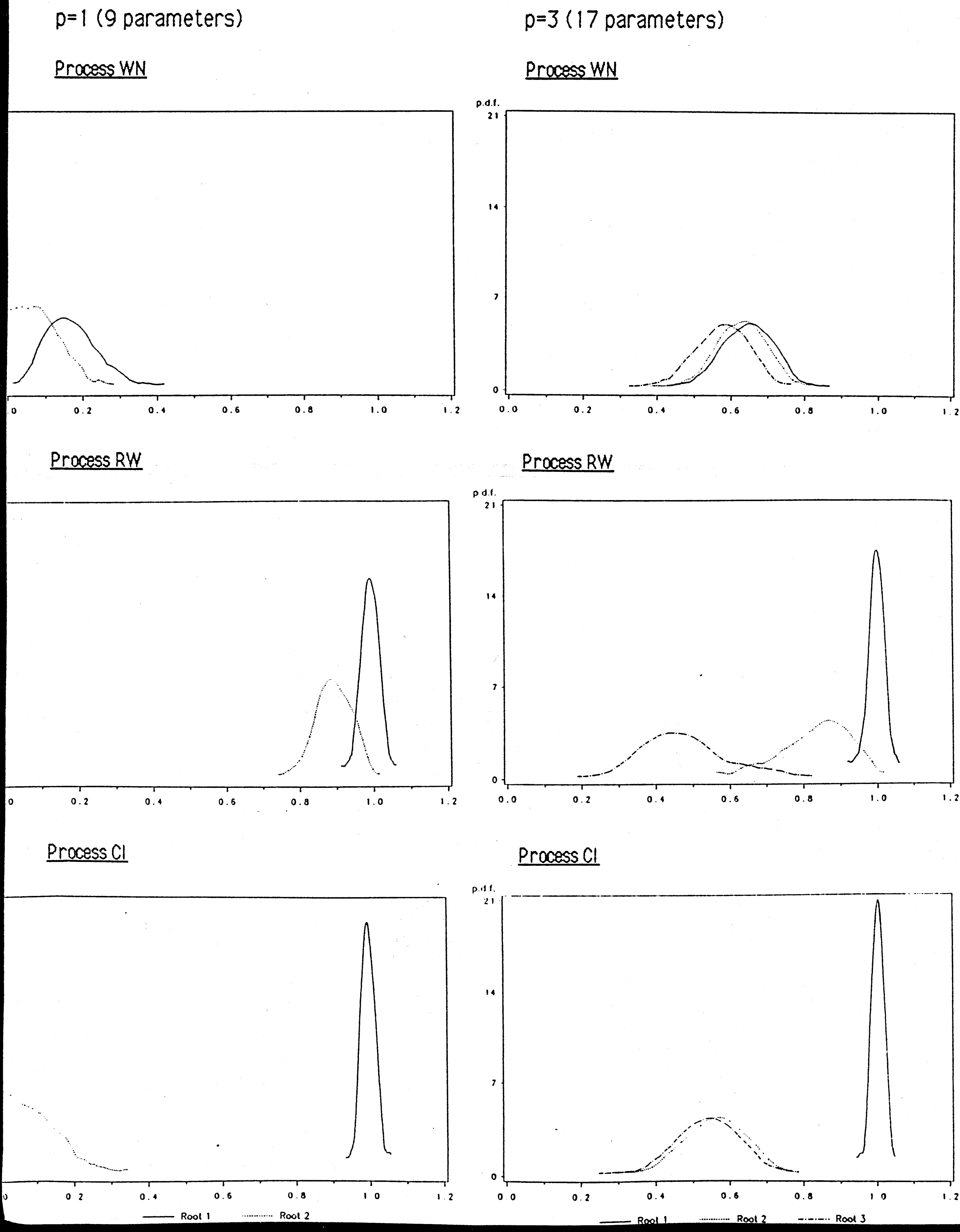
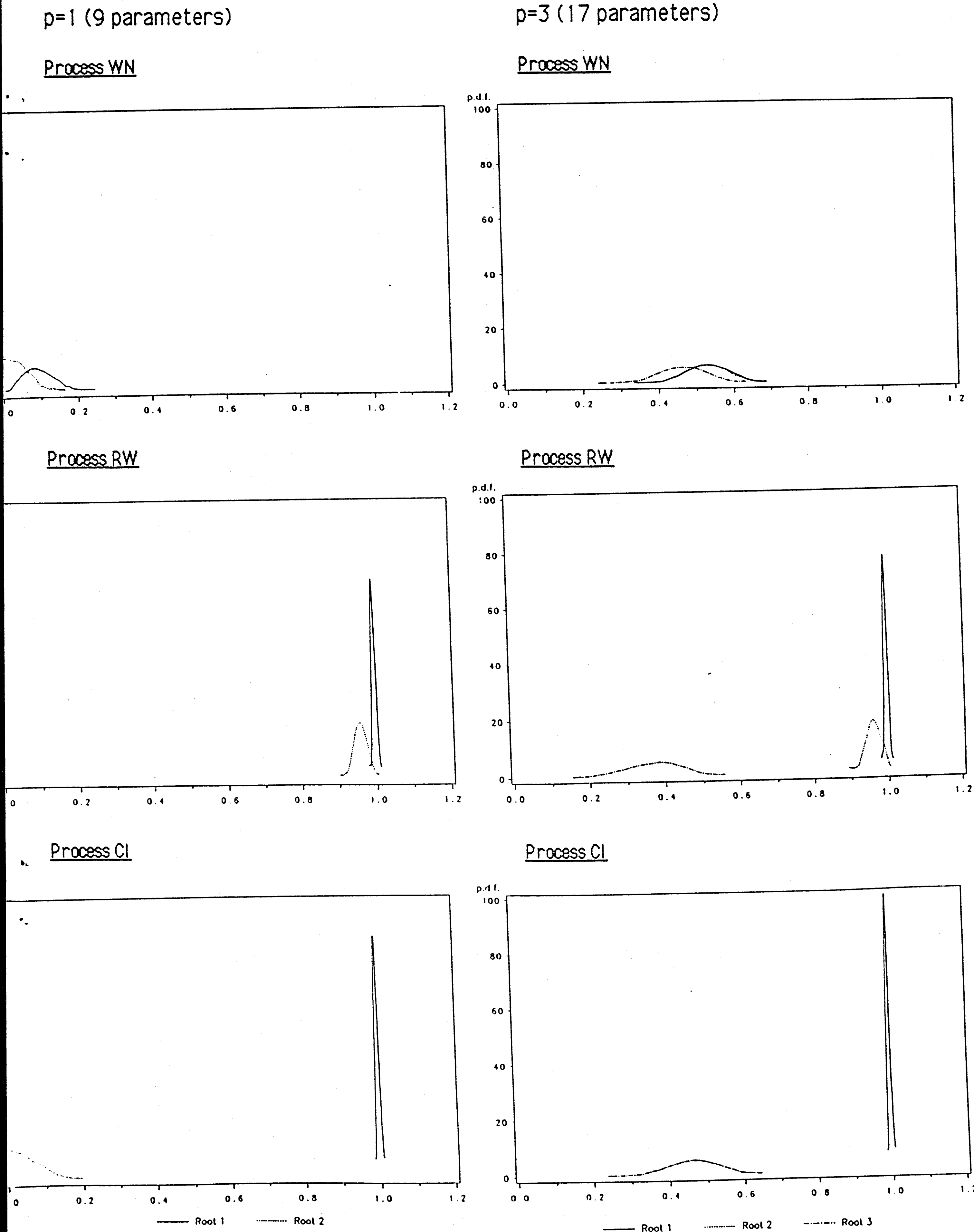


Figure 5: Posterior Distributions of Root Amplitudes of Three Processes
Standard Conjugate Prior, T=250



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