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AN ALGORITHM FOR THE COMPUTATION OF  
POSTERIOR MOMENTS AND DENSITIES USING  
SIMPLE IMPORTANCE SAMPLING

H.K. VAN DIJK, J.P. HOP AND A.S. LOUTER

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AN ALGORITHM FOR THE COMPUTATION OF POSTERIOR MOMENTS AND  
DENSITIES USING SIMPLE IMPORTANCE SAMPLING

by

Herman K. van Dijk<sup>1</sup>, J. Peter Hop<sup>1</sup> and Adri S. Louter<sup>2</sup>

Abstract

In earlier work (van Dijk (1984, Chapter 3)) one of the authors discussed the use of Monte Carlo integration methods for the computation of the multi-variate integrals that are defined in the posterior moments and the posterior densities of the parameters of interest of econometric models. In the present paper we describe the computational steps of one Monte Carlo method, mentioned in that work, which is known in the literature as importance sampling. Further, we have prepared a set of standard programs, which may be used for the implementation of a simple case of importance sampling. The computer programs have been written in Fortran.

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## 1. INTRODUCTION

In earlier work [see van Dijk (1984, Chapter 3), and the references cited there] one of the authors discussed the use of Monte Carlo integration methods for the computation of the multivariate integrals that are defined in the posterior moments and the posterior densities of the parameters of interest of econometric models. In the present paper we describe the computational steps of one Monte Carlo method, mentioned in that work. This method is a simple application of a technique that is known in the literature as importance sampling [see Hammersley and Handscomb (1964)]. Further, we have prepared a set of standard computer programs, which can be used for the application of importance sampling. The computer programs have been written in Fortran 77.

The multivariate integrals that we consider may be described briefly as follows. Let  $\theta$  be an  $l$ -vector of parameters of interest and let  $g(\theta)$  be an integrable function of  $\theta$ . The posterior mean of  $g(\theta)$  is defined as

$$Eg(\theta) = \frac{\int g(\theta)p(\theta)d\theta}{\int p(\theta)d\theta} \quad (1.1)$$

where  $p(\theta)$  is a kernel of a posterior density function. That is,  $p(\theta)$  is proportional and not equal to a density function and the denominator of (1.1) plays the role of integrating constant, similar to the role of  $\sqrt{2\pi}$  in the case of the normal distribution. We assume that  $g(\theta)p(\theta)$  is integrable on a certain region of integration. Simple examples of  $g(\theta)$  are  $g(\theta) = \theta$  and  $g(\theta) = \theta\theta'$ . Note that  $g$  may be a vector or a matrix. We emphasize that  $g(\theta)$  may also be a complicated nonlinear function of  $\theta$  such as the implied multipliers of the structural parameters of a simultaneous equation model [see, e.g., van Dijk and Kloek (1980) and van Dijk (1984), Chapter 4]. There exist several other examples of nontrivial nonlinear functions of  $\theta$ . For an example in the statistical literature we refer to Kass (1985), and for some examples in the econometric literature we refer to van Dijk (1985), Zellner (1985), and Geweke (1986).

Monte Carlo (MC) integration methods make use of the property that generating a large sample of random numbers is very easy using a computer procedure. The value of an integral is then estimated in the sampling theory tradition using this set of random numbers. So, MC methods change the integration problem into a statistical estimation problem. A clear and concise introduction to Monte Carlo has been given by Hammersley and Handscomb (1964).



The contents of this paper has been organized as follows. In Section 2 we describe a Monte Carlo algorithm that is based on the principle of importance sampling. Some suggestions for further work are given in Section 3. In the Appendix we present examples of computer output using the prepared computer programs for a simple case of importance sampling.

## 2. SIMPLE IMPORTANCE SAMPLING

In this section we discuss the application of a simple case of importance sampling to the computation of the integrals defined in posterior first-order and second-order moments and the application of importance sampling to the computation of univariate and bivariate marginal posterior densities. For more details on the principle of importance sampling we refer to Hammersley and Handscomb (1964, Chapter 5) and van Dijk (1984, Chapter 3).

The vector of posterior first-order moments is obtained from (1.1) by defining  $g(\theta) = \theta$ . This yields

$$E\theta = \frac{\int \theta p(\theta) d\theta}{\int p(\theta) d\theta} \quad (2.1)$$

Suppose that it is not known how one can generate a sample of random drawings from a distribution with density equal or proportional to  $p(\theta)$ , but it is known how one can generate a random sample from a distribution with a density equal (or proportional) to  $I(\theta)$ , which is different from  $p(\theta)$ . Suppose further, that  $I(\theta)$  is a reasonable approximation of  $p(\theta)$ . One can replace  $p(\theta)$  in (2.1) by  $w(\theta)I(\theta)$  where the weight function  $w(\theta)$  is defined as  $w(\theta) = p(\theta)/I(\theta)$ . This yields

$$E\theta = \frac{\int \theta w(\theta) I(\theta) d\theta}{\int w(\theta) I(\theta) d\theta} \quad (2.2)$$

where  $I(\theta)$  is restricted to be positive on the region of integration.  $I(\theta)$  is known in the literature as importance function. In this paper we make use of a simple choice with respect to the class of importance functions. That is, we opt for the multivariate Student-t class of density functions [see below]. We make use of the term Simple Importance Sampling (SIS) in this case. For more details on the choice of an importance function and for some alternatives to simple importance sampling we refer to van Dijk (1984, Chapter 3) and van Dijk and Kloek (1985).

Next, let  $\theta^{(1)}, \dots, \theta^{(N)}$  be a random sample from a distribution with a density function equal (or proportional) to  $I(\theta)$ . That is,  $\theta^{(1)}, \dots, \theta^{(N)}$  is a sequence of independently distributed random variables with a common distribution function. Let  $\theta^{(i)}$  be the typical  $i$ -th element of this sequence. Then the importance sampling estimator of the  $j$ -th element of the vector  $E\theta$  is given as

$$\begin{aligned} \hat{E}(\theta_j) &= \frac{\frac{1}{N} \sum_{i=1}^N \theta_j^{(i)} w(\theta^{(i)})}{\frac{1}{N} \sum_{i=1}^N w(\theta^{(i)})} \quad (j = 1, \dots, \ell) \quad (2.3) \\ &= \sum_{j=1}^N \theta_j^{(i)} w^*(\theta^{(i)}) \end{aligned}$$

where

$$w^*(\theta^{(i)}) = \frac{w(\theta^{(i)})}{\sum_{i=1}^N w(\theta^{(i)})}$$

This estimator may be interpreted as a weighted sample mean of the above mentioned random sample where  $w^*(\theta^{(1)}), \dots, w^*(\theta^{(N)})$  are the weights. The weighted sample mean is a good approximation of  $E\theta_j$  if the sample size  $N$  is sufficiently large and the variation in the weights is bounded. In order to evaluate the numerical accuracy of the estimator (2.3), we are interested in the variation of  $w(\theta)$  and of  $\theta_j w(\theta)$ ,  $j = 1, \dots, \ell$ . More details on the numerical accuracy of the approximation (2.3) are given below.

Second-order posterior moments can be computed in a similar way as first-order moments. The matrix of second-order moments around the mean is given as

$$E(\theta - E\theta)(\theta - E\theta)' = E(\theta\theta)' - (E\theta)(E\theta)' \quad (2.4)$$

The first term on the right hand side of (2.4) is defined as the matrix of second-order moments around zero. By making use of  $p(\theta) = w(\theta)I(\theta)$  one can write

$$E(\theta\theta)' = \frac{\int \theta\theta' w(\theta) I(\theta) d\theta}{\int w(\theta) I(\theta) d\theta} \quad (2.5)$$

An importance sampling estimator for the  $(j,k)$ -th element of (2.5) is given as

$$\hat{E}(\theta_j \theta_k) = \frac{\frac{1}{N} \sum_{i=1}^N \theta_j^{(i)} \theta_k^{(i)} w(\theta^{(i)})}{\frac{1}{N} \sum_{i=1}^N w(\theta^{(i)})} \quad (j, k = 1, \dots, \ell) \quad (2.6)$$

Practical details with respect to the computation of the estimators (2.3) and (2.6) are discussed below.

The approximation of the posterior first-order moments by means of the weighted sample mean, given in (2.3), involves a numerical error. Estimates of this numerical error can be obtained by making use of results from large sample theory [see, e.g., Cramér (1946) or Rao (1973)]. In particular, given certain regularity conditions from central limit theory, it follows that the estimator  $\hat{E}(\theta_j)$  is approximately normally distributed with mean  $E\theta_j$  and variance  $\sigma_j^2/N$ . [This normal approximation becomes more accurate as  $N$  becomes larger.] The estimator  $\hat{E}(\theta_j)$  is a ratio of correlated random variables [compare the first line of (2.3)]. The formula for  $\sigma_j^2$  is, therefore, more complex than the usual definition of a variance. The expression for  $\sigma_j^2$  can be written as follows. We start with (2.3) and define

$$\hat{t}_j = \frac{1}{N} \sum_{i=1}^N \theta_j^{(i)} w(\theta^{(i)}) \quad (j = 1, \dots, \ell) \quad (2.7)$$

$$\hat{t}_0 = \frac{1}{N} \sum_{i=1}^N w(\theta^{(i)}) \quad (2.8)$$

Then we can rewrite (2.3), using (2.7) and (2.8). Next, we make use of the approximation for  $\sigma_j^2$  given in Cramér (1946, Section 28.4). Then one can write

$$\sigma_j^2 \approx \left( \frac{\partial \hat{\theta}_j}{\partial \hat{t}_j} \right)^2 \text{var } \hat{t}_j + 2 \frac{\partial \hat{\theta}_j}{\partial \hat{t}_j} \frac{\partial \hat{\theta}_j}{\partial \hat{t}_0} \text{cov}(\hat{t}_j, \hat{t}_0) + \left( \frac{\partial \hat{\theta}_j}{\partial \hat{t}_0} \right)^2 \text{var } \hat{t}_0 \quad (2.9)$$

where, for convenience, we made use of the notation  $\hat{\theta}_j$ , instead of the more cumbersome notation  $\hat{E}(\theta_j)$ , in order to denote the right hand side of (2.3). An estimator for  $\sigma_j^2$  follows directly from (2.9) once estimators for  $\text{var } \hat{t}_j$ ,  $\text{var } \hat{t}_0$  and  $\text{cov}(\hat{t}_j, \hat{t}_0)$  are determined. By making use of (2.7) and (2.8) and standard theory on sample moments [compare, e.g., Mood, Graybill and Boes (1974, Chapters 2 and 6)] one can write the Monte Carlo estimators, using importance sampling, for the moments given at the right hand side of (2.9) as



$$\hat{\text{var}}(\hat{t}_j) = \frac{1}{N} \sum_{i=1}^N [\theta_j^{(i)} w(\theta^{(i)})]^2 - (\hat{t}_j)^2 \quad (2.10)$$

$$\hat{\text{var}}(\hat{t}_0) = \frac{1}{N} \sum_{i=1}^N [w(\theta^{(i)})]^2 - (\hat{t}_0)^2 \quad (2.11)$$

$$\hat{\text{cov}}(\hat{t}_j, \hat{t}_0) = \frac{1}{N} \sum_{i=1}^N \theta_j^{(i)} [w(\theta^{(i)})]^2 - \hat{t}_j \hat{t}_0 \quad (2.12)$$

Given an estimator  $\hat{\sigma}_j^2$  for  $\sigma_j^2$ , one can define in the usual way a 95 per cent confidence interval for  $E\theta_j$ . Let  $B_j = 1.96 \hat{\sigma}_j / \sqrt{N}$ , where 1.96 is taken from the table of the standard normal integral that are listed in most textbooks on statistics [see, e.g., Mood, Graybill, and Boes (1974, p. 522)]. Then the interval  $[E\theta_j - B_j, E\theta_j + B_j]$  contains the value of  $E\theta_j$  with a probability equal to 0.95. A value for  $\sigma_j / \sqrt{N}$  will be defined as an absolute numerical error and  $(\hat{\sigma}_j / \sqrt{N}) / \sigma_j$  will be defined as a relative numerical error.

Summarizing, for importance sampling estimates of the posterior first- and second-order moments we have to compute the following sums:

$$\sum_{i=1}^N w(\theta^{(i)}), \quad \sum_{i=1}^N \theta_j^{(i)} w(\theta^{(i)}), \quad \sum_{i=1}^N \theta_j^{(i)} \theta_k^{(i)} w(\theta^{(i)}) \quad (2.13)$$

(j, k = 1, ..., l)

[compare (2.3) and (2.6)]. For the evaluation of numerical errors of the posterior first-order moments we have to compute, in addition to (2.13), the sums:

$$\sum_{i=1}^N [w(\theta^{(i)})]^2, \quad \sum_{i=1}^N [\theta_j^{(i)} w(\theta^{(i)})]^2, \quad \sum_{i=1}^N \theta_j^{(i)} [w(\theta^{(i)})]^2 \quad (2.14)$$

(j = 1, ..., l)

[compare (2.7)-(2.12)]. These sums are listed in the computer program for simple importance sampling.

Univariate marginal posterior densities of  $\theta_j$ ,  $j = 1, \dots, l$ , can be approximated by so-called frequency histograms or frequency polygons using MC methods. We start by defining  $(a_{k-1}, a_k)$ ,  $k = 1, \dots, K$ , as a bounded interval for the parameter  $\theta_j$ ,  $j = 1, \dots, l$ . Further, let  $d(\theta)$  be a dummy variable defined as

$$\begin{aligned}
 d(\theta) &= 1 & \text{if } a_{k-1} < \theta_j < a_k \\
 &= 0 & \text{elsewhere}
 \end{aligned}
 \quad (2.15)$$

Then the posterior probability  $P_k$ , defined as  $P_k = P[a_{k-1} < \theta_j < a_k]$ , is given as

$$P_k = \frac{\int d(\theta) p(\theta) d\theta}{\int p(\theta) d\theta} \quad (k = 1, \dots, K) \quad (2.16)$$

The probabilities  $P_1, \dots, P_K$  can be used for the construction of a frequency histogram. Further, the posterior density of  $\theta_j$  evaluated at  $\frac{1}{2}(a_{k-1} + a_k)$  can be approximated by  $P_k/(a_k - a_{k-1})$  if the interval  $(a_{k-1}, a_k)$  is sufficiently small. This approximation of the posterior density at  $K$  points can be used for the construction of a frequency polygon.

Given that  $I(\theta)$  is an importance function for  $p(\theta)$  we can rewrite (2.16) as

$$P_k = \frac{\int d(\theta) w(\theta) I(\theta) d\theta}{\int w(\theta) I(\theta) d\theta} \quad (2.17)$$

An importance sampling estimator for (2.17) may be derived as follows. Let  $\theta^{(1)}, \dots, \theta^{(N)}$  be a random sample generated from a distribution with density  $I(\theta)$ . Further, let  $\bar{\theta}^{(h)} = \theta^{(h(i))}$ , where  $h = h(i)$  is generated by the following rule

$$\begin{aligned}
 h(0) &= 0 \\
 h(i) &= h(i-1) + d(i) \quad (i = 1, \dots, N)
 \end{aligned}
 \quad (2.18)$$

where

$$\begin{aligned}
 d(i) &= 1 & \text{if } a_{k-1} < \theta_j^{(i)} < a_k \\
 &= 0 & \text{elsewhere}
 \end{aligned}$$

Finally, let  $N_1$  be defined as  $N_1 = h(N)$ . Then an importance sampling estimator for (2.17) is given as

$$\hat{P}_k = \frac{\frac{1}{N} \sum_{h=1}^{N_1} w(\bar{\theta}^{(h)})}{\frac{1}{N} \sum_{i=1}^N w(\theta^{(i)})} \quad (2.19)$$

The definition of the estimator  $\hat{P}_k$  is a bit tedious, but the computation of  $\hat{P}_k$  is very simple. In fact, one has only to determine the particular interval to which a weight  $w(\theta^{(i)})$  belongs. This is especially simple when the interval width  $a_k - a_{k-1}$  is the same for all  $k$ . Let  $b$  be the common interval width for  $k = 1, \dots, K$ . Let  $r$  be a real number given as

$$r = (\theta_j^{(i)} - a_0)/b + 1 \quad (2.20)$$

That is,  $r$  is a real number in the interval  $[1, K+1]$ . Truncate  $r$  at its decimal point in order to make  $r$  an integer, defined as  $ir$ . Then it follows that  $ir$  is the interval to which a particular weight  $w(\theta^{(i)})$  belongs.<sup>1</sup> So, estimates for  $P_k$  are computed by adding the weights that belong in each interval and by dividing the sum of the weights in each interval by the total sum of the weights. Details are presented in the computer program. Minor modifications of the procedure described here are necessary when the intervals have unequal width. Further, the extreme values  $a_0$  and  $a_K$  may be equal to minus and plus infinity. Finally, we note that the computation of bivariate marginal posterior densities proceeds in a similar way as the computation of the univariate marginal posterior densities. Details are presented in the computer programs listed in the Appendix.

Next, we discuss the structure of a computer program for Simple Importance Sampling (SIS). The different computational steps are shown in the flow diagram, given in Figure 1. The computer program starts with statements that refer to the initial value of a random number generator and to initial zero-values. Note that we make use of an arrow sign (instead of an equality sign) in several statements. For instance, one interprets  $S^{(0)} \leftarrow 0$  as: 'the value zero is assigned to the variable (or the array of variables)  $S$  with superindex 0.' The major part of the program refers to two so-called do loops. In the inner loop, with the index  $i$ , one has as typical statement

$$S^{(i)} \leftarrow S^{(i-1)} + g(\theta^{(i)})w(\theta^{(i)}) \quad (i = 1, \dots, N) \quad (2.21)$$

The symbol  $S^{(i)}$  denotes the  $i$ -th partial sum of a sequence of random function values, defined as

1. The case where  $ir$  is exactly equal to  $K+1$  is not important, since it has probability measure zero.



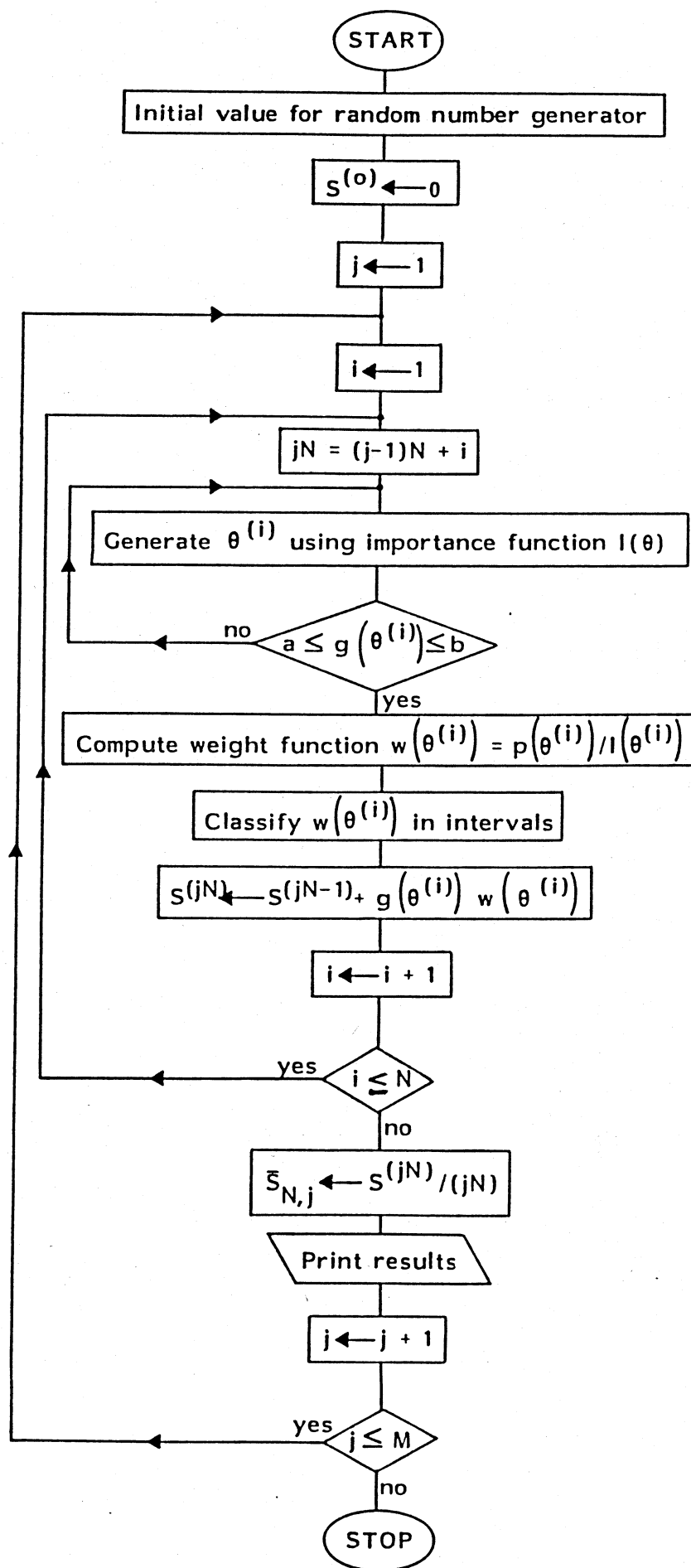


Figure 1. Flow diagram for simple importance sampling

$$S^{(i)} = g(\theta^{(1)})w(\theta^{(1)}) + g(\theta^{(2)})w(\theta^{(2)}) + \dots + g(\theta^{(i)})w(\theta^{(i)}) \quad (2.22)$$

The assignment statement (2.21) indicates that one does not store a large sample of random numbers from which sample moments are computed (as is suggested by equation (2.22)), but one makes use of a statement that updates the value of  $S^{(i)}$  after each accepted Monte Carlo drawing of  $\theta^{(i)}$ . After  $N$  random drawings one computes sample averages such as

$$\bar{S}_N = \frac{S^{(N)}}{N} \quad (2.23)$$

The accuracy of the sample means may be studied by increasing the size of the sample from  $N$  to  $2N$ ,  $3N$ , ...,  $MN$ . The outer loop of the program, with the index  $j$ , enables one to print results at each value of  $jN$ , with  $j = 1, \dots, M$ . This explains why we make use of the index  $jN$  at several places in the flow diagram. Given certain regularity conditions, the Monte Carlo estimator  $\hat{E}(\theta)$  converges with probability one to  $E(\theta)$ .

Apart from the two do loops that refer to updating procedures, there are two other major computational procedures in the computer program for simple importance sampling. First, the user has to supply a procedure that describes the computation of the posterior kernel studied in each particular case. Second, we have opted for a simple procedure that generates random vectors  $\theta^{(1)}, \dots, \theta^{(N)}$ , which may be described as follows.

The most obvious solution to the restriction of the light tails of the multivariate normal importance function is the choice of the multivariate Student  $t$  density as a functional form of the importance function, with the multivariate normal density as a limiting case. A multivariate Student  $t$  density may be written as

$$I(\theta) = c[\lambda + (\theta - \theta^0)'V^{-1}(\theta - \theta^0)]^{-\frac{1}{2}(\lambda + \ell)} \quad (2.24)$$

where  $\theta^0$  is the center of the distribution,  $V$  is a positive definite scaling matrix;  $\lambda > 0$  is the degrees of freedom parameter and  $\ell$  the dimension of  $\theta$  [compare, e.g., Zellner (1971, Appendix B2), where the numerical constant  $c$  is spelled out].

Random  $\ell$ -vectors  $\theta^{(1)}, \dots, \theta^{(N)}$ , distributed according to a multivariate Student  $t$  distribution, are generated as follows.

Step 1. Generate an  $(\ell + \lambda)$ -vector  $u^{(i)}$  of independent standard normally

distributed random variables. [Efficient techniques for this step can be found in, e.g. Newman and Odell (1971), Atkinson and Pearce (1976), Rubinstein (1981), Marsaglia (1984), and the references cited there. We make use of the normal random number generator from the NAG-Library, given as subroutine G05 DDF.] Note that  $\ell$  is usually considered as a positive integer instead of a positive real variable when it is used for the generation of Student  $t$  random variables.

Step 2. Partition the vector  $u^{(i)}$  in a subvector  $u_1^{(i)}$  that contains the first  $\ell$  elements of  $u^{(i)}$  and a subvector  $u_2^{(i)}$  that contains the remaining  $\ell$  elements of  $u^{(i)}$ . Then, obtain an  $\ell$ -vector of so-called standard Student  $t$  distributed random variables  $v^{(i)}$  from

$$v^{(i)} = \frac{u_1^{(i)}}{(u_2^{(i)'} u_2^{(i)} / \lambda)^{\frac{1}{2}}} \quad (2.25)$$

Step 3. Premultiply the  $\ell$ -vector  $v^{(i)}$  by a matrix  $A$  which satisfies  $V = AA'$ . One may obtain  $A$  from the eigenvalues and eigenvectors of  $V$  but it is also possible to construct a triangular matrix  $A$  by a Cholesky technique. Then, obtain the  $\ell$ -vector  $\theta^{(i)}$  from

$$\theta^{(i)} = \theta^0 + Av^{(i)} \quad (2.26)$$

In case the importance function is multivariate normal take  $\ell = 0$  in Step 1. Step 2 can be deleted and  $v^{(i)}$  is replaced by  $u^{(i)}$  in Step 3.

If we adopt the multivariate Student  $t$  density as a functional form for the importance function, we need a way to specify its parameters. It seems reasonable to take for  $\theta^0$  the posterior mode of  $\theta$  and for  $V$  minus the inverse of the Hessian of the log posterior density, evaluated at the posterior mode, possibly multiplied by a scalar. The posterior mode is determined by means of numerical minimization of minus the log of the posterior kernel. It is our experience that the use of an analytical gradient increases the numerical precision, in particular around the optimum where the gradient is almost zero. The reason is that the rounding errors of a numerically approximated gradient may cause convergence problems when the function mentioned above is relatively flat around its optimum value. Further, the evaluation of minus the Hessian of this function at the optimum values is simplified by making use of an analytical gradient. That is, instead of using second-order numerical differentiation formulae with respect to the objective function one makes use



of the analytical gradient and of first-order numerical differentiation formulae [compare Goldfeld and Quandt (1972, p. 19)].

We end the discussion of the structure of the computer program with two remarks. First, the computation of the weights  $w(\theta^{(i)})$  is a key step in any importance sampling procedure. The distribution of the weights contains relevant information on the accuracy of importance sampling. The weights are nearly equal if the approximation of  $I(\theta)$  to  $p(\theta)$  is very accurate. In the opposite case one finds great variation in the weights. An approximation of the distribution of the weights  $w(\theta^{(i)})$ ,  $i = 1, \dots, N$  is computed as a byproduct in the computer program for simple importance sampling. Second, we assume that the region of integration is bounded. Therefore, we have inserted a rejection step in the program since the Student  $t$  density is defined on the entire region  $\mathbb{R}^l$ , where  $l$  is the dimension of the vector  $\theta$ .

We emphasize that one can make use of the program SIS in an sequential way. That is, one starts with the posterior mode and minus the inverse of the Hessian matrix, evaluated at the mode, as location and scale parameters of the multivariate Student  $t$  importance function. After a round of Monte Carlo of, for instance,  $N = 2000$  random drawings one uses the posterior mean and the posterior covariance matrix as new starting values for the parameters of the Student  $t$  importance function.

We end this section with some examples. In van Dijk (1984, Chapter 4), the author discussed the computation of the prior and posterior moments of the structural parameters and the implied multipliers of a well-known simultaneous econometric model, which is known as Klein's Model I [see Klein (1950)]. Two parameters in this model are particularly interesting, that is, the government-expenditures-multiplier and the tax-multiplier in the reduced form equation for national income,  $Y$ . The government-expenditures-multiplier indicates the change in  $Y$  when government expenditures,  $G$ , change with, for instance, one billion dollars and the tax-multiplier indicates the change in  $Y$  when taxes,  $T$ , change with such an amount. If one takes a uniform prior on the structural parameters of interest on a bounded region [in this case the unit region, see van Dijk (op.cit.)] then one can compute the bivariate prior density of the government- and tax-multipliers by direct simulation and simple rejection. The result is shown in Figure 2. It is seen that the relevant interval for the tax multiplier is  $[-4.0, -.5]$  and the relevant interval for the government multiplier is  $[.5, 4.]$ . The results indicate that the prior distribution is skew, and that it has a long tail. The posterior results have

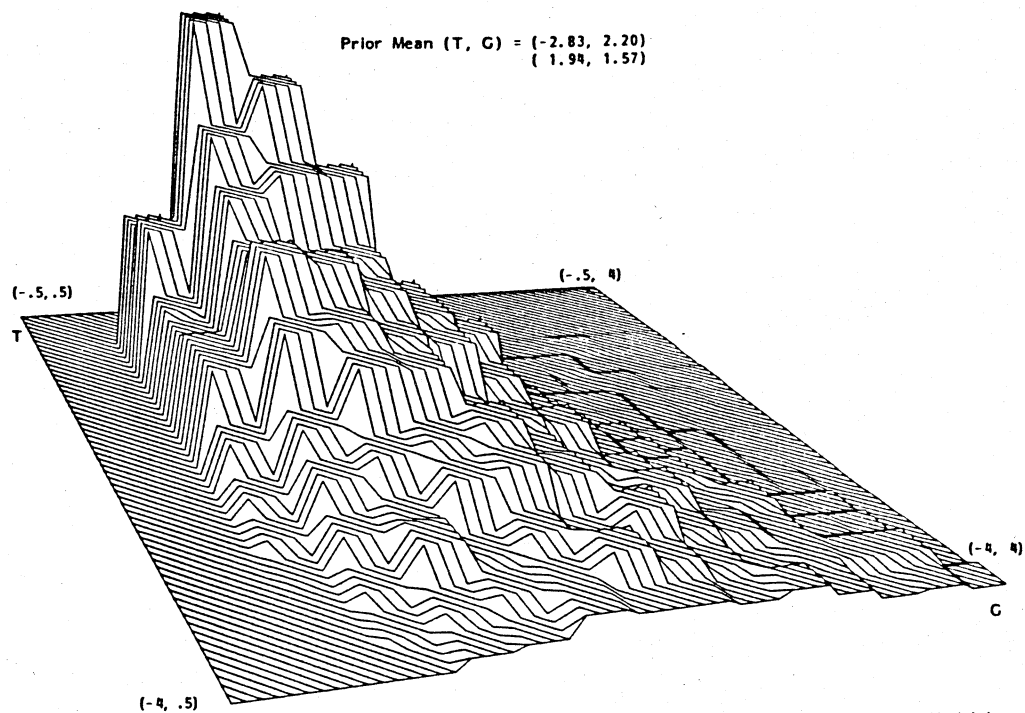


Figure 2A Bivariate marginal prior densities of short-run government and tax multipliers for Klein's Model I

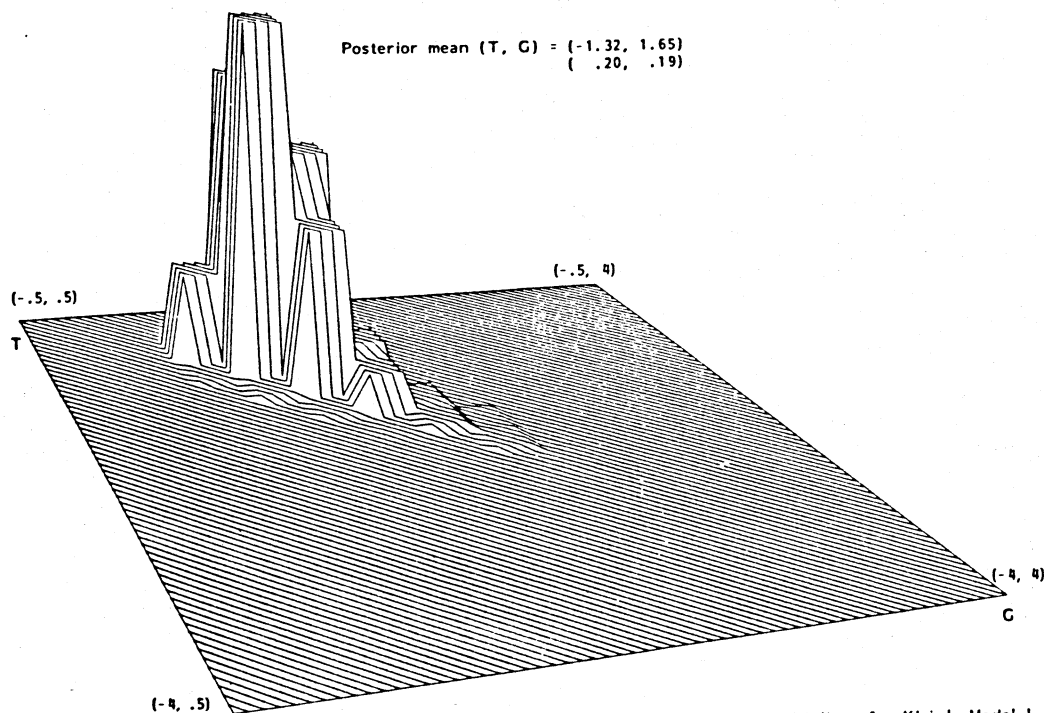


Figure 2B Bivariate marginal posterior densities of short-run government and tax multipliers for Klein's Model I

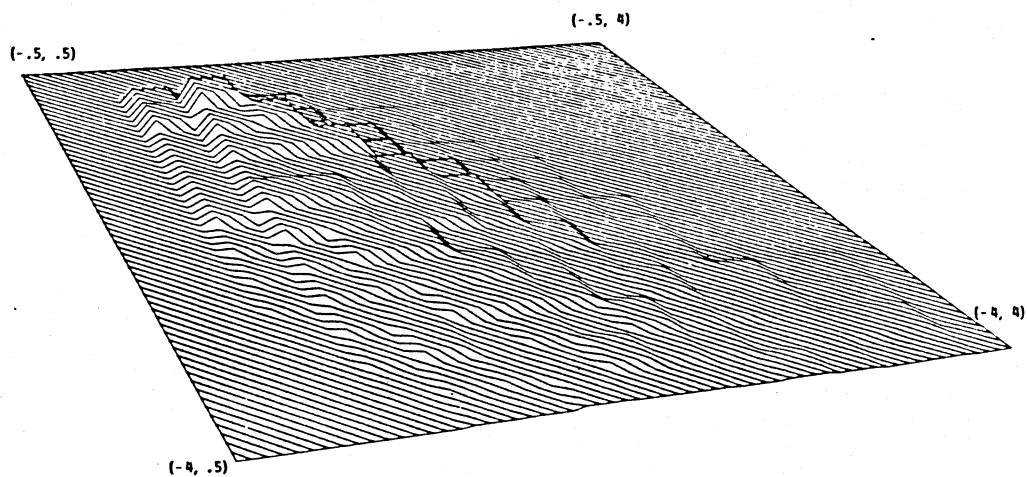


Figure 2C Scaled bivariate marginal prior densities

been computed by SIS. These posterior results indicate that the data are quite informative and dominate the prior information strongly. For a more detailed discussion of the results we refer to van Dijk (1984).

In the Appendix we present examples of computer output using simple importance sampling. The posterior densities refer to a well-known textbook model of simultaneous equations, i.e., Johnston's model, which involves three-dimensional numerical integration. For details on the specification of the model we refer to Johnston (1963, p.269) and van Dijk (1984, Chapter 3).

### 3. REMARKS

The computer program for simple importance sampling is a first step towards the development of standard software for Bayesian analysis of econometric and statistical models. Further developments in this area are needed. An immediate extension is to construct a family of importance functions that is more flexible than the symmetric multivariate Student  $t$  density. Some preliminary experiments with an importance function that consists of a finite mixture of conjugate densities appear promising. The results will be reported in a forthcoming paper.

Another field of research is to prepare a standard Fortran program for the method of mixed integration [van Dijk, Kloek, and Boender (1985)]. This is also a topic of current research.

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C -----
C -----
C -----
C -----
C
C MAIN - PROGRAM FOR "SIMPLE IMPORTANCE SAMPLING" (S.I.S)
C
C SEE: VAN DIJK, H.K., 1984, POSTERIOR ANALYSIS OF ECONOMETRIC
C       MODELS USING MONTE CARLO INTEGRATION, DOCTORAL
C       DISSERTATION (ERASMUS UNIVERSITY ROTTERDAM).
C -----
C
C INTEGER    MAXD
C
C THE NEXT PARAMETER-STATEMENT DETERMINES THE MAXIMUM DIMENSION OF
C A PROBLEM THE PROGRAM CAN HANDLE. IF TOO SMALL, THE CURRENT NUMBER
C HAS TO BE CHANGED HERE AS WELL AS IN THE SUBROUTINE "SISSUB".
C
C PARAMETER (MAXD=10)
C
C CHARACTER TITLE*60,XFILE*19,HFILE*19,SFILE*19
C LOGICAL    IUMPD,IBMPD,ISAVE
C INTEGER    I,J,IPRIN,NDIM,LAMBDA,ISTRT,NDRAW,MROUND,NDCL,IROTA,
C &          MROTA,IFCNT
C DOUBLE PRECISION FPOST,SCFAC
C DOUBLE PRECISION HINV(MAXD,MAXD),PMODE(MAXD),LBOUND(MAXD),
C &          UBOUND(MAXD),MEANPO(MAXD),COVPO(MAXD,MAXD),
C &          THETA(MAXD)
C
C DATA      IPRIN/3/,NDCL/MAXD/
C
C 3505 FORMAT (' TYPE TITLE OF PROBLEM (MAX. 60 CHARACTERS)')
C 3510 FORMAT (A)
C 3515 FORMAT (' TYPE THE DIMENSION OF THE VECTOR "THETA"')
C 3520 FORMAT (' TYPE NAME OF OUTPUT-FILE ')
C 3525 FORMAT (' TYPE NAME OF INPUT-FILE FOR "PMODE" AND "HINV" ')
C 3530 FORMAT (' TYPE LOWER BOUNDS FOR PARAMETERS (FREE FORMAT)')
C 3535 FORMAT (' TYPE UPPER BOUNDS FOR PARAMETERS (FREE FORMAT)')
C 3540 FORMAT (' TYPE BOOLEAN PARAMETER FOR COMPUTATION OF UNIVARIATE',
C &          ' POSTERIOR DENSITIES: 0 = NO, 1 = YES ')
C 3545 FORMAT (' TYPE BOOLEAN PARAMETER FOR COMPUTATION OF BIVARIATE',
C &          ' POSTERIOR DENSITIES: 0 = NO, 1 = YES ')
C 3550 FORMAT (' TYPE INITIAL VALUE OF RANDOM NUMBER GENERATOR ')
C 3555 FORMAT (' TYPE DEGREES OF FREEDOM FOR THE STUDENT -T- IMPORT',
C &          ' ANCE FUNCTION')
C 3560 FORMAT (' TYPE NUMBER OF ROUNDS ')
C 3565 FORMAT (' TYPE NUMBER OF RANDOM DRAWINGS FOR EACH ROUND ')
C 3570 FORMAT (' TYPE NUMBER OF ROTATIONS')
C 3585 FORMAT (' TYPE BOOLEAN PARAMETER FOR SAVING POSTERIOR MEAN AND',
C &          ' COVARIANCE-MATRIX: 0 = NO, 1 = YES')
C 3590 FORMAT (' TYPE NAME OF THE SAVE-FILE')
C
C
C TITLE OF THE PROBLEM FOR THIS RUN OF THE PROGRAM
C
C WRITE(6,3505)
C READ(5,3510) TITLE

```

```

C
C   THE DIMENSION OF THE PARAMETER VECTOR "THETA"
C
  WRITE(6,3515)
  READ(5,*) NDIM
C
C   THE NAME OF THE OUTPUT PRINT-FILE
C
  WRITE(6,3520)
  READ(5,3510) XFILE
C
  OPEN (UNIT=IPRIN, FILE=XFILE, STATUS='NEW')
C
C   THE NAME OF THE INPUT-FILE FOR "PMODE" AND "HINV"
C
  WRITE(6,3525)
  READ(5,3510) HFILE
C
C   READ POSTERIOR MODE AS "PMODE" AND MINUS-HESSIAN-INVERSE AS "HINV"
C
  OPEN (UNIT=23, FILE=HFILE, STATUS='OLD', FORM='FORMATTED')
  READ(23,*) (PMODE(I), I = 1,NDIM)
  DO 100 I = 1,NDIM
    READ(23,*) (HINV(I,J), J = 1,I)
100 CONTINUE
  CLOSE (UNIT=23)
C
C   INITIALIZE POSTERIOR FUNCTION ROUTINE
C
  CALL PSTROR(THETA,NDIM,FPOST,IFCNT,SCFAC,.TRUE.)
C
C   LOWER AND UPPER BOUNDS FOR PARAMETER VECTOR "THETA"
  WRITE(6,3530)
  READ(5,*) (LBOUND(I), I = 1,NDIM)
  WRITE(6,3535)
  READ(5,*) (UBOUND(I), I = 1,NDIM)
C
C   BOOLEAN PARAMETER THAT DETERMINES WHETHER UNIVARIATE MARGINAL
C   POSTERIOR DENSITIES ARE COMPUTED
C
  WRITE(6,3540)
  READ(5,*) I
  IUMPD = I .EQ. 1
C
C   BOOLEAN PARAMETER THAT DETERMINES WHETHER BIVARIATE MARGINAL
C   POSTERIOR DENSITIES ARE COMPUTED
C
  WRITE(6,3545)
  READ(5,*) I
  IBMPD = I .EQ. 1
C
C   INITIAL VALUE FOR THE RANDOM NUMBER GENERATOR
C
  WRITE(6,3550)
  READ(5,*) ISTRT
  CALL G05CBF(ISTRT)
C
C   DEGREES OF FREEDOM FOR THE STUDENT -T- IMPORTANCE FUNCTION
C
  WRITE(6,3555)
  READ(5,*) LAMBDA

```

```

C
C   NUMBER OF ROUNDS IS DEFINED AS "MROUND"
C
C   WRITE(6,3560)
C   READ(5,*) MROUND
C
C   NUMBER OF DRAWINGS IN EACH ROUND IS DEFINED AS "NDRAW"
C
C   WRITE(6,3565)
C   READ(5,*) NDRAW
C
C   NUMBER OF ROTATIONS
C
C   WRITE(6,3570)
C   READ(5,*) MROTA
C
C   DO 200 IROTA = 1,MROTA
C       CALL SISSUB(NDCL,NDIM,PMODE,HINV,LAMBDA,LBOUND,UBOUND,NDRAW,
C   &               MROUND,IROTA,ISTRN,IPRIN,IUMPD,IBMPD,THETA,FPOST,
C   &               IFCNT,MEANPO,COVPO,TITLE,*400)
C
C       DO 250 I = 1,NDIM
C           PMODE(I) = MEANPO(I)
C           DO 260 J = 1,NDIM
C               HINV(I,J) = COVPO(I,J)
C           CONTINUE
C       CONTINUE
C   250 CONTINUE
C   200 CONTINUE
C
C   SAVE "MEANPO" AND "COVPO"
C
C   WRITE(6,3585)
C   READ(5,*) I
C   ISAVE = I .EQ. 1
C   IF (ISAVE) THEN
C       WRITE(6,3590)
C       READ(5,3510) SFILE
C       OPEN (UNIT=25, FILE=SFILE, STATUS='NEW', FORM='FORMATTED')
C       WRITE(25,*) (MEANPO(I), I = 1,NDIM)
C       DO 300 I = 1,NDIM
C           WRITE(25,*) (COVPO(I,J), J = 1,I)
C       CONTINUE
C       CLOSE (UNIT=25)
C   END IF
C
C   400 CLOSE (UNIT=IPRIN)
C
C   STOP
C   END

```

```

C -----
C
C SUBROUTINE SISSUB(NDCL,LDIM,PMODE,HINV,LAMBDA,LBOUND,UBOUND,NDRAW,
& MROUND,IROTA,START,IPRIN,IUMPD,IBMPD,THETA,
& FPOST,IFCNT,MEANPO,COVPO,TITLE,*)
C CHARACTER TITLE*60
C LOGICAL IUMPD,IBMPD
C INTEGER NDCL,LDIM,LAMBDA,NDRAW,MROUND,IROTA,START,IPRIN,IFCNT
C DOUBLE PRECISION FPOST
C DOUBLE PRECISION PMODE(LDIM),HINV(NDCL,LDIM),LBOUND(LDIM),
& UBOUND(LDIM),MEANPO(LDIM),COVPO(NDCL,LDIM),
& THETA(LDIM)
C -----
C
C SIMPLE IMPORTANCE SAMPLING (S.I.S)
C
C VAR. NAME TYPE I/O DESCRIPTION
C
C NDCL I4 I FIRST DIMENSION OF THE TWO-DIMENSIONAL
C ADJUSTABLE ARRAYS AS DECLARED IN THE CALLING
C PROGRAM
C LDIM I4 I DIMENSION OF THE PARAMETER VECTOR "THETA",
C ALSO SECOND DIMENSION OF TWO-DIMENSIONAL
C ADJUSTABLE ARRAYS AND THE DIMENSION OF
C THE ONE-DIMENSIONAL ADJUSTABLE ARRAYS
C PMODE(LDIM) R8 I POSTERIOR MODE
C HINV(NDCL,LDIM) R8 I MINUS-HESSIAN-INVERSE OF THE LOG POSTERIOR
C LAMBDA I4 I DEGREES OF FREEDOM OF THE STUDENT'T
C IMPORTANCE FUNCTION
C LBOUND(LDIM) R8 I LOWER BOUND OF THE PARAMETER VECTOR "THETA"
C UBOUND(LDIM) R8 I UPPER BOUND OF THE PARAMETER VECTOR "THETA"
C NDRAW I4 I NUMBER OF DRAWINGS PER ROUND
C MROUND I4 I NUMBER OF ROUNDS WHERE OUTPUT IS PRINTED
C IROTA I4 I SEQUENCE NUMBER OF THE CURRENT ROTATION
C START I4 I INITIAL VALUE FOR THE RANDOM NUMBER GENERATOR
C IPRIN I4 I FILE NUMBER OF THE OUTPUT PRINT FILE
C IUMPD L4 I = TRUE: U.M.P.D. ARE COMPUTED,
C = FALSE: U.M.P.D. ARE NOT COMPUTED
C IBMPD L4 I = TRUE: B.M.P.D. ARE COMPUTED,
C = FALSE: B.M.P.D. ARE NOT COMPUTED
C THETA(LDIM) R8 O THE PARAMETER VECTOR
C FPOST R8 O VALUE OF THE LOG OF THE POSTERIOR KERNEL FUNCTION
C IFCNT I4 O NUMBER OF FUNCTION EVALUATIONS
C MEANPO(LDIM) R8 O THE POSTERIOR MEAN
C COVPO(NDCL,LDIM) R8 O THE POSTERIOR COVARIANCE MATRIX
C TITLE C60 I A TITLE FOR THE CURRENT PROBLEM
C * -- O RETURN LABEL FOR ERROR CONDITION
C -----
C
C POSTERIOR -- I A USER-SUPPLIED SUBROUTINE TO EVALUATE THE
C POSTERIOR FUNCTION:
C SUBROUTINE PSTOR(PAR,N,FP,NF,SC,IC)
C LOGICAL IC
C INTEGER N,NF
C DOUBLE PRECISION PAR(N),FP,SC
C PAR(N) : THE PARAMETER VECTOR
C N : THE DIMENSION OF THE POSTERIOR FUNCTION

```

## S.2 .

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C          FP      : THE COMPUTED VALUE OF THE LOG POSTERIOR
C          NF      : THE "NF"TH COMPUTATION OF THE
C                   POSTERIOR FUNCTION
C          SC      : SCALE FACTOR
C          IC      : IF (TRUE) SUBROUTINE CALL ONLY FOR
C                   INITIALISATION PURPOSES,
C                   IF (FALSE) SUBROUTINE CALL TO COMPUTE
C                   THE VALUE OF THE POSTERIOR FUNCTION
C RESTRICT      -- I A USER-SUPPLIED SUBROUTINE TO EVALUATE THE
C                   RESTRICTIONS ON THE PARAMETERS:
C                   SUBROUTINE RSTRCT(PAR,N,IC)
C                   LOGICAL IC
C                   INTEGER N
C                   DOUBLE PRECISION PAR(N)
C                   PAR(N) : THE PARAMETER VECTOR
C                   N      : THE DIMENSION OF THE PARAMETER VECTOR
C                   IC      : IF (TRUE) PARAMETERS DO NOT MEET THE
C                           RESTRICTION,
C                           IF (FALSE) PARAMETERS MEET THE
C                           RESTRICTION.
C
C -----
C
C          INTEGER  MAXD,MDX
C
C          SEE REMARK IN THE MAIN-PROGRAM FOR THE NEXT STATEMENT
C
C          PARAMETER (MAXD=10)
C
C          PARAMETER (MDX=MAXD*(MAXD-1)/2)
C          CHARACTER DAY*9
C          LOGICAL  FAIL,PAGE,PPOW
C          INTEGER  I,II,I1,I2,J,JJ1,JJ2,K,NCLASS,IFAIL,IRNACC,IRNREJ,
C          &        JROUND,IDRAW,IDD,NDD,ITERM,ITIME,ERRMSG,NDRAW2
C          INTEGER  IPOW(80),RELBMI(MDX,15,15),RELBMP(MDX,15,15),
C          &        JJX(MAXD),NALF(11)
C          DOUBLE PRECISION SUMW1,SUMW2,VALIMP,WEIGHT,WW,DUMMY,RNACC,
C          &        XX,YY,MEAND1,MEAND2,VARD,STDD,MEANN1,MEANN2,VARN,
C          &        COVAR,VARCOF,XNU,RN1,RN2,TOTUMP,TOTUMI,TERM,
C          &        MAXBMP,MAXBMI,SCFAC
C          DOUBLE PRECISION HISTD(MAXD),HICOR(MAXD,MAXD),YHELP(MAXD),
C          &        WIDTH(MAXD),EIGVEC(MAXD,MAXD),EIGVAL(MAXD),
C          &        BOUND(MAXD,16),SUMIM1(MAXD),MEANIM(MAXD),
C          &        STDIM(MAXD),SUMIM2(MAXD,MAXD),COVIM(MAXD,MAXD),
C          &        RHOIM(MAXD,MAXD),SUMPO1(MAXD),STDPO(MAXD),
C          &        SUMPO2(MAXD,MAXD),RHOPO(MAXD,MAXD),
C          &        SUMER1(MAXD),SUMER2(MAXD),
C          &        ERROR(MAXD).RELEFF(MAXD).CORREL(MAXD),
C          &        MAXW(10),MAXIME(10).HALFOS(10),MAXTHE(10,MAXD),
C          &        SUMUMP(MAXD,15),SUMUMI(MAXD,15),
C          &        RELUMP(MAXD,15),RELUMI(MAXD,15),
C          &        SUMBMP(MDX,15,15),SUMBMI(MDX,15,15)
C
C          EXTERNAL  X02AAF,X02ADF
C
C          DATA     NCLASS/15/,ERRMSG/0/,
C          &        NALF/' ','1','2','3','4','5','6','7','8','9','*'/
C

```

```

5000 FORMAT (/// BOUNDS OF THE PARAMETERS')
5001 FORMAT ('0',10F11.4/' ',10F11.4/' ',10F11.4)
5002 FORMAT (/// IMPORTANCE MEANS AND STANDARDDEVIATIONS')
5003 FORMAT (/// IMPORTANCE CORRELATION MATRIX')
5004 FORMAT ('0',10F11.5/' ',10F11.5/' ',10F11.5)
5005 FORMAT (/// IMPORTANCE COVARIANCE MATRIX')
5006 FORMAT (/// EIGENVALUES OF MIN INVERSE HESSIAN MATRIX'/
& '0',10F12.6/' ',10F12.6/' ',10F12.6)
5007 FORMAT (/// INITIAL VALUE OF RANDOM NUMBER GENERATOR :',
& ' CALL G05CBF(',I5,')')
5008 FORMAT (// IMPORTANCE MEANS AND STANDARDDEVIATIONS (TRUNCATED)')
5009 FORMAT (// IMPORTANCE CORRELATION MATRIX (TRUNCATED)')
5010 FORMAT (// IMPORTANCE COVARIANCE MATRIX (TRUNCATED)')
5011 FORMAT (// POSTERIOR MEAN AND STANDARDDEVIATION OF THE',
& ' PARAMETER VECTOR "THETA"')
5012 FORMAT (// POSTERIOR CORRELATION MATRIX OF THE PARAMETER',
& ' VECTOR "THETA"')
5013 FORMAT (// POSTERIOR COVARIANCE MATRIX OF THE PARAMETER',
& ' VECTOR "THETA"')
5014 FORMAT (/// NUMERICAL ERROR ESTIMATES OF POSTERIOR MEAN',
& '/// ERROR :',10F11.6/' ',17X,10F11.6/
& ' ',17X,10F11.6)
5015 FORMAT (// RELATIVE ERROR :',10F11.6/' ',17X,10F11.6/
& ' ',17X,10F11.6)
5016 FORMAT (// CORREL. COEFF. :',10F11.6/' ',17X,10F11.6/
& ' ',17X,10F11.6)
5017 FORMAT (/// FREQUENCIES OF IPOW. WEIGHT=0.*****E+IPOW//)
5018 FORMAT (' ',15I8)
5019 FORMAT (/// TEN DRAWINGS WITH LARGEST WEIGHT//9X,'W',12X,
& 'LN(IMP)',8X,'LN(POS)',10X,'(THETA(I), I = 1,NDIM)')
5020 FORMAT (' ',E15.7,2F15.7,3X,8F10.5/' ',48X,8F10.5/
& ' ',48X,8F10.5/' ',48X,6F10.5)
5021 FORMAT (/// MARGINAL POSTERIOR DENSITIES "P", AND',
& ' MARGINAL IMPORTANCE DENSITIES (TRUNCATED) "I"')
5022 FORMAT ('0',3(A1,10X,'PARAMETER',I3,13X))
5023 FORMAT (' ',3(A1,17X,'P',7X,'I',9X))
5024 FORMAT (' ',3(A1,'(',F5.2,',',F5.2,')',2F8.3,6X))
5025 FORMAT (// BIVARIATE MARGINAL POSTERIOR DENSITIES')
5026 FORMAT (// BIVARIATE MARGINAL DENSITIES OF IMPORTANCE FUNCTION',
& ' (TRUNCATED)')
5027 FORMAT (/// ',2(A1,15X,'VERT:',I2,3X,'HOR:',I2,29X))
5028 FORMAT (' ',2(F5.2,2X,34('-',),20X))
5029 FORMAT (' ',2(A1,6X,'| ',15(A1,1X), '| ',20X))
5030 FORMAT (' ',2(7X,F5.2,24X,F5.2,20X))
9101 FORMAT ('0','ERROR IN THE BOUNDS OF THE PARAMETERS')
9102 FORMAT ('0','ERROR IN THE NAG-ROUTINE "F02AMF", CALCULATING THE',
& ' EIGENVALUES AND EIGENVECTORS OF THE COVARIANCE-'
& ' ', 'MATRIX OF THE PARAMETERS; ERROR-CODE "IFAIL" = ',I2)
9103 FORMAT ('0','ONE OR MORE EIGENVALUES ARE LESS OR EQUAL TO ZERO')
9104 FORMAT ('0','THE NUMBER OF REJECTED DRAWINGS EXCEEDS 500 TIMES',
& ' THE INITIAL NUMBER OF DRAWINGS:'
& ' ', 'NUMBER OF ACCEPTED DRAWINGS: ',I10/
& ' ', 'NUMBER OF REJECTED DRAWINGS: ',I10)
9198 FORMAT ('0','PROGRAM TERMINATED BECAUSE OF UNKNOWN ERROR ',
& 'CONDITION')
9199 FORMAT ('0','PROGRAM TERMINATED BECAUSE OF ERROR CONDITION')

```

C

CALL DATE(DAY)

C

NDRAW2 = 500 \* NDRAW

NDD = LDIM \* (LDIM - 1) / 2



```

C
C   COMPUTE BOUNDS FOR INTERVALS THAT ARE USED FOR THE COMPUTATION
C   OF MARGINAL POSTERIOR DENSITIES
C
DUMMY = DFLOAT(NCLASS)
DO 100 I = 1,LDIM
    XX = LBOUND(I)
    YY = (UBOUND(I) - XX) / DUMMY
    IF (YY .LT. 1.0D-25) ERRMSG = 1
    WIDTH(I) = YY
    DO 110 J = 1,NCLASS+1
        BOUND(I,J) = XX + DFLOAT(J-1) * YY
110    CONTINUE
100 CONTINUE
C
    CALL PRTHD(IPRIN,DAY,TITLE,IROTA,0)
C
C   WRITE INPUT DATA ON BOUNDS THAT DEFINE REGION OF INTEGRATION
C
    WRITE(IPRIN,5000)
    WRITE(IPRIN,5001) (LBOUND(I), I = 1,LDIM)
    WRITE(IPRIN,5001) (UBOUND(I), I = 1,LDIM)
    IF (ERRMSG .GT. 0) GOTO 9000
C
C   WRITE INPUT DATA ON POSTERIOR MODE AND MINUS HESSIAN INVERSE
C
    WRITE(IPRIN,5002)
    WRITE(IPRIN,5001) (PMODE(I), I = 1,LDIM)
    DO 120 I = 1,LDIM
        HISTD(I) = DSQRT(DABS(HINV(I,I)))
120 CONTINUE
    WRITE(IPRIN,5001) (HISTD(I), I = 1,LDIM)
    WRITE(IPRIN,5003)
    DO 130 I = 1,LDIM
        XX = HISTD(I)
        DO 140 J = 1,I
            HICOR(I,J) = HINV(I,J) / XX / HISTD(J)
140    CONTINUE
        WRITE(IPRIN,5004) (HICOR(I,J), J = 1,I)
130 CONTINUE
    WRITE(IPRIN,5005)
    DO 150 I = 1,LDIM
        WRITE(IPRIN,5004) (HINV(I,J), J = 1,I)
150 CONTINUE
C
C   COMPUTE MATRIX P SUCH THAT  $PP' = -(H-INV)$  BY EIGENVALUE DECOMPOSITION
C   (NOTE THAT P IS DEFINED AS HINV IN THE PROGRAM) THE CALCULATION OF
C   EIGENVALUES AND EIGENVECTORS OF MINUS INVERSE-HESSIAN MATRIX OCCURS
C   BY MEANS OF HOUSEHOLDER REDUCTIE
C
    CALL F01AJF(LDIM,X02ADF(DUMMY),HINV,NDCL,EIGVAL,YHELP,EIGVEC,NDCL)
C
C   EIGENVALUES AND EIGENVECTORS
C
    IFAIL = 1
    CALL F02AMF(LDIM,X02AAF(DUMMY),EIGVAL,YHELP,EIGVEC,NDCL,IFAIL)
    IF (IFAIL .GT. 0) THEN
        ERRMSG = 2
        GOTO 9000
    END IF

```

```

C
WRITE(IPRIN,5006) (EIGVAL(J), J = 1,LDIM)
IF (EIGVAL(1) .LT. 1.0D-25) THEN
  ERRMSG = 3
  GOTO 9000
END IF
DO 160 J = 1,LDIM
  XX = DSQRT(EIGVAL(J))
  DO 170 I = 1,LDIM
    HINV(I,J) = XX * EIGVEC(I,J)
170   CONTINUE
160 CONTINUE
C
C   INITIAL VALUE OF RANDOM NUMBER GENERATOR
C
IF (IROTA .EQ. 1) WRITE(IPRIN,5007) START
C
C   INITIAL ZERO-VALUES FOR THE NUMBER OF ACCEPTED AND REJECTED
C   RANDOM DRAWINGS
C
IRNACC = 0
IRNREJ = 0
C
C   INITIAL ZERO-VALUES FOR PARTIAL SUMS FOR IMPORTANCE MOMENTS,
C   POSTERIOR MOMENTS AND NUMERICAL ERRORS
C
SUMW1 = 0.0D0
SUMW2 = 0.0D0
DO 200 I = 1,LDIM
  SUMIM1(I) = 0.0D0
  SUMPO1(I) = 0.0D0
  SUMER1(I) = 0.0D0
  SUMER2(I) = 0.0D0
  DO 210 J = 1,LDIM
    SUMIM2(I,J) = 0.0D0
    SUMPO2(I,J) = 0.0D0
210   CONTINUE
200 CONTINUE
C
C   INITIAL ZERO-VALUES FOR TEN DRAWINGS WITH MAX. WEIGHT
C
DO 220 I = 1,80
  IPOW(I) = 0
220 CONTINUE
DO 230 I = 1,10
  MAXW(I) = 0.0D0
  MAXIMP(I) = 0.0D0
  MAXPOS(I) = 0.0D0
  DO 240 J = 1,LDIM
    MAXTHE(I,J) = 0.0D0
240   CONTINUE
230 CONTINUE
C
C   INITIAL ZERO-VALUES FOR COMPUTATION OF UNIVARIATE M. P. D.
C
IF (IUMPD) THEN
  DO 250 J = 1,NCLASS
    DO 260 I = 1,LDIM
      SUMUMP(I,J) = 0.0D0

```

```

                SUMUMI(I,J) = 0.000
260          CONTINUE
250          CONTINUE
          END IF
C
C          INITIAL ZERO-VALUES FOR COMPUTATION OF BIVARIATE M. P. D.
C
          IF (IBMPD) THEN
            DO 270 I = 1,NDD
              DO 280 J = 1,NCLASS
                DO 290 K = 1,NCLASS
                  SUMBMI(I,J,K) = 0.000
                  SUMBMP(I,J,K) = 0.000
290          CONTINUE
280          CONTINUE
270          CONTINUE
          END IF
C
C          NUMBER OF ROUNDS IS DEFINED AS "MROUND"
C
          DO 800 JROUND = 1,MROUND
C
C          NUMBER OF DRAWINGS IN EACH ROUND IS DEFINED AS "NDRAW"
C
          DO 400 IDRAW = 1,NDRAW
            IF (IRNREJ .GT. NDRAW2) THEN
              ERRMSG = 4
              GOTO 9000
            END IF
C
C          GENERATE STUDENT'S -T- WITH "LAMBDA" DEGREES OF FREEDOM
C
300          VALIMP = 0.000
          XNU      = DFLOAT(LAMBDA)
          RN2      = 0.000
          DO 500 I = 1,LAMBDA
            XX      = G05DDF(0.000,1.000)
            RN2     = RN2 + XX * XX
500          CONTINUE
          RN2      = DSQRT(RN2 / XNU)
          DO 510 I = 1,LDIM
            RN1     = G05DDF(0.000,1.000)
            YHELP(I) = RN1 / RN2
            VALIMP  = VALIMP + YHELP(I) * YHELP(I)
510          CONTINUE
          VALIMP = -0.500 * (XNU + DFLOAT(LDIM)) *
&              DLOG(1.000 + VALIMP / XNU)
          DO 520 I = 1,LDIM
            XX      = 0.000
            DO 530 J = 1,LDIM
              XX = XX + HINV(I,J) * YHELP(J)
530          CONTINUE
            THETA(I) = PMODE(I) + XX
520          CONTINUE
C
C          TEST RESTRICTIONS ON THE PARAMETERS BY AN USER-SUPPLIED ROUTINE
C
          CALL RSTRCT(THETA,LDIM,FAIL).
          IF (FAIL) THEN
            IRNREJ = IRNREJ + 1
            GOTO 300
          END IF

```

```

C
C   TEST ON BOUNDS OF THE REGION OF INTEGRATION
C
      DO 540 J = 1,LDIM
        IF (THETA(J) .LT. LBOUND(J) .OR.
          &   THETA(J) .GT. UBOUND(J)) THEN
          IRNREJ = IRNREJ + 1
          GOTO 300
        END IF
540    CONTINUE

C
C   COMPUTE POSTERIOR KERNEL AND WEIGHT FUNCTION VALUE
C
      IRNACC = IRNACC + 1
      CALL PSTRROR(THETA,LDIM,FPOST,IFCNT,SCFAC,.FALSE.)
      WEIGHT = DEXP(FPOST - VALIMP)

C
C   CLASSIFY WEIGHTS AS POWERS OF TEN AND DETERMINE THE TEN DRAWINGS
C   WITH LARGEST WEIGHT
C
      IDD = -40
      DUMMY = WEIGHT
      IF (DUMMY .GT. 1.0D0) DUMMY = 1.0D1 * DUMMY
      IF (DUMMY .GT. 0.0D0) IDD = DLOG10(DUMMY)
      IPOW(IDD+41) = IPOW(IDD+41) + 1
      IF (WEIGHT .GT. MAXW(10)) THEN
        MAXW(10) = WEIGHT
        MAXIMP(10) = VALIMP
        MAXPOS(10) = FPOST
        DO 550 I = 1,LDIM
          MAXTHE(10,I) = THETA(I)
550    CONTINUE
        DO 560 II = 1,9
          DO 570 JJ = II+1,10
            IF (MAXW(II) .LT. MAXW(JJ)) THEN
              DUMMY = MAXW(II)
              MAXW(II) = MAXW(JJ)
              MAXW(JJ) = DUMMY
              DUMMY = MAXIMP(II)
              MAXIMP(II) = MAXIMP(JJ)
              MAXIMP(JJ) = DUMMY
              DUMMY = MAXPOS(II)
              MAXPOS(II) = MAXPOS(JJ)
              MAXPOS(JJ) = DUMMY
              DO 580 I = 1,LDIM
                DUMMY = MAXTHE(II,I)
                MAXTHE(II,I) = MAXTHE(JJ,I)
                MAXTHE(JJ,I) = DUMMY
580            CONTINUE
              END IF
            END IF
          CONTINUE
670    CONTINUE
560    CONTINUE
      END IF

C
C   COMPUTE PARTIAL SUMS FOR IMP. AND POST. MOMENTS AND ERROR ESTIMATES
C
      WW = WEIGHT * WEIGHT
      SUMW1 = SUMW1 + WEIGHT
      SUMW2 = SUMW2 + WW

```

```

DO 600 I = 1,LDIM
  XX = THETA(I)
  SUMIM1(I) = SUMIM1(I) + XX
  SUMP01(I) = SUMP01(I) + XX * WEIGHT
  SUMER1(I) = SUMER1(I) + XX * WW
  SUMER2(I) = SUMER2(I) + XX * XX * WW
  DO 610 J = 1,I
    YY = XX * THETA(J)
    SUMIM2(I,J) = SUMIM2(I,J) + YY
    SUMP02(I,J) = SUMP02(I,J) + YY * WEIGHT
610    CONTINUE
600    CONTINUE
C
C    COMPUTE PARTIAL SUMS FOR UNIVARIATE M. P. D.
C
DO 620 I = 1,LDIM
  JJX(I) = (THETA(I) - LBOUND(I)) / WIDTH(I) + 1.0D0
620  CONTINUE
  IF (IUMPD) THEN
    DO 630 I = 1,LDIM
      JJ1 = JJX(I)
      SUMUMP(I,JJ1) = SUMUMP(I,JJ1) + WEIGHT
      SUMUMI(I,JJ1) = SUMUMI(I,JJ1) + 1.0D0
630    CONTINUE
    END IF
C
C    COMPUTE PARTIAL SUMS FOR BIVARIATE M. P. D.
C
  IF (IBMPD) THEN
    II = 0
    DO 640 I1 = 1,LDIM-1
      JJ1 = JJX(I1)
      DO 650 I2 = I1+1,LDIM
        JJ2 = JJX(I2)
        II = II + 1
        SUMBMP(II,JJ1,JJ2) = SUMBMP(II,JJ1,JJ2) +
&                               WEIGHT
&                               SUMBMI(II,JJ1,JJ2) = SUMBMI(II,JJ1,JJ2) +
650                               1.0D0
640    CONTINUE
      CONTINUE
    END IF
C
400  CONTINUE
C
C    COMPUTE IMPORTANCE AND POSTERIOR MOMENTS
C
  RNACC = DFLOAT(IRNACC)
  CALL PMC(SUMIM1,SUMIM2,LDIM,NDCL,RNACC,MEANIM,COVIM,STDIM,
&          RHOIM)
  CALL PMC(SUMP01,SUMP02,LDIM,NDCL,SUMW1,MEANPO,COVPO,STDPO,
&          RHOPO)
C
C    COMPUTE NUMERICAL ERRORS OF POSTERIOR MEANS
C
  MEAND1 = SUMW1 / RNACC
  MEAND2 = MEAND1 * MEAND1
  VARD = SUMW2 / RNACC - MEAND2
  STDD = DSQRT(VARD)

```

```

DO 900 I = 1,LDIM
  MEANN1 = SUMPO1(I) / RNACC
  MEANN2 = MEANN1 * MEANN1
  VARN = SUMER2(I) / RNACC - MEANN2
  COVAR = SUMER1(I) / RNACC - MEANN1 * MEAND1
  VRCOF = VARN / MEANN2 + VARD / MEAND2 -
&      2.0DO * COVAR / (MEANN1 * MEAND1)
  ERROR(I) = DSQRT(VRCOF * MEANPO(I) * MEANPO(I) / RNACC)
  RELERR(I) = ERROR(I) / STDP0(I)
  CORREL(I) = COVAR / DSQRT(VARD * VARN)
900  CONTINUE
C
C  COMPUTE UNIVARIATE M. P. D.
C
  IF (IUMPD) THEN
    DO 910 I = 1,LDIM
      TOTUMP = 0.0DO
      TOTUMI = 0.0DO
      DO 920 J = 1,NCLASS
        TOTUMP = TOTUMP + SUMUMP(I,J)
        TOTUMI = TOTUMI + SUMUMI(I,J)
920  CONTINUE
      DO 930 J = 1,NCLASS
        RELUMP(I,J) = SUMUMP(I,J) / TOTUMP
        RELUMI(I,J) = SUMUMI(I,J) / TOTUMI
930  CONTINUE
910  CONTINUE
      END IF
C
C  COMPUTE BIVARIATE M. P. D.
C
  IF (IBMPD) THEN
    DO 940 I = 1,NDD
      MAXBMP = 0.0DO
      MAXBMI = 0.0DO
      DO 950 J = 1,NCLASS
        DO 960 K = 1,NCLASS
          TERM = SUMBMP(I,J,K)
          IF (TERM .GT. MAXBMP) MAXBMP = TERM
          TERM = SUMBMI(I,J,K)
          IF (TERM .GT. MAXBMI) MAXBMI = TERM
960  CONTINUE
950  CONTINUE
        DO 970 J = 1,NCLASS
          DO 980 K = 1,NCLASS
            TERM = SUMBMP(I,J,K) / MAXBMP + 0.05DO
            ITERM = 10.0DO * TERM
            RELBMP(I,J,K) = NALF(ITERM+1)
            TERM = SUMBMI(I,J,K) / MAXBMI + 0.05DO
            ITERM = 10.0DO * TERM
            RELBMI(I,J,K) = NALF(ITERM+1)
980  CONTINUE
970  CONTINUE
940  CONTINUE
      END IF
C
C  WRITE MOMENTS OF TRUNCATED IMPORTANCE DISTRIBUTION
C
  CALL PRTHD(IPRIN,DAY,TITLE,IROTA,JROUND)
  CALL PRTSBH(IPRIN,IRNACC,IRNREJ,IFCNT,MEAND1,STDD)

```

```

C
WRITE(IPRIN,5008)
WRITE(IPRIN,5001) (MEANIM(I), I = 1,LDIM)
WRITE(IPRIN,5001) (STDIM(I), I = 1,LDIM)
WRITE(IPRIN,5009)
DO 1000 I = 1,LDIM
    WRITE(IPRIN,5004) (RHOIM(I,J), J = 1,I)
1000 CONTINUE
WRITE(IPRIN,5010)
DO 1010 I = 1,LDIM
    WRITE(IPRIN,5004) (COVIM(I,J), J = 1,I)
1010 CONTINUE
C
C WRITE POSTERIOR MOMENTS AND ERROR ESTIMATES
C
CALL PRTHD(IPRIN,DAY,TITLE,IROTA,JROUND)
CALL PRTSBH(IPRIN,IRNACC,IRNREJ,IFCNT,MEAND1,STDD)
C
WRITE(IPRIN,5011)
WRITE(IPRIN,5001) (MEANPO(I), I = 1,LDIM)
WRITE(IPRIN,5001) (STDPO(I), I = 1,LDIM)
WRITE(IPRIN,5012)
DO 1020 I = 1,LDIM
    WRITE(IPRIN,5004) (RHOP0(I,J), J = 1,I)
1020 CONTINUE
WRITE(IPRIN,5013)
DO 1030 I = 1,LDIM
    WRITE(IPRIN,5004) (COVPO(I,J), J = 1,I)
1030 CONTINUE
WRITE(IPRIN,5014) (ERROR(I), I = 1,LDIM)
WRITE(IPRIN,5015) (RELERR(I), I = 1,LDIM)
WRITE(IPRIN,5016) (CORREL(I), I = 1,LDIM)
C
C WRITE DISTRIBUTION OF WEIGHT FUNCTION VALUES AND THE TEN DRAWINGS
C WITH LARGEST WEIGHT
C
CALL PRTHD(IPRIN,DAY,TITLE,IROTA,JROUND)
CALL PRTSBH(IPRIN,IRNACC,IRNREJ,IFCNT,MEAND1,STDD)
C
WRITE(IPRIN,5017)
DO 1040 JJ = 1,6
    J2 = JJ * 15
    J1 = J2 - 14
    IF (J2 .GT. 80) J2 = 80
    PPOW = .FALSE.
    DO 1050 J = J1,J2
        IF (IPOW(J) .NE. 0) PPOW = .TRUE.
1050 CONTINUE
    IF (PPOW) THEN
        WRITE(IPRIN,5018) (1, 1 = J1-41,J2-41)
        WRITE(IPRIN,5018) (IPOW(I), I = J1,J2)
        WRITE(IPRIN,5018)
    END IF
1040 CONTINUE
WRITE(IPRIN,5019)
DO 1060 I = 1,10
    WRITE(IPRIN,5020) MAXW(I),MAXIMP(I),MAXPOS(I),
        & (MAXTHE(I,J), J = 1,LDIM)
1060 CONTINUE

```



```

C
C   WRITE UNIVARIATE M. P. D.
C
      IF (IUMPD) THEN
C
          CALL PRTHD(IPRIN, DAY, TITLE, IROTA, JROUND)
          CALL PRTSBH(IPRIN, IRNACC, IRNREJ, IFCNT, MEAND1, STDD)
C
          WRITE(IPRIN, 5021)
          III = FLOAT(LDIM) / 3.0E0 + 0.7E0
          DO 1070 IK = 1, III
              I2 = IK * 3
              I1 = I2 - 2
              IF (I2 .GT. LDIM) I2 = LDIM
              WRITE(IPRIN, 5022) (NALF(1), I, I = I1, I2)
              WRITE(IPRIN, 5023) (NALF(1), I = I1, I2)
              DO 1080 J = 1, NCLASS
                  WRITE(IPRIN, 5024) (NALF(1), BOUND(I, J),
&                                     BOUND(I, J+1), RELUMP(I, J),
&                                     RELUMI(I, J), I = I1, I2)
1080          CONTINUE
1070      CONTINUE
          END IF
C
C   WRITE BIVARIATE M. P. D.
C
      IF (IBMPD) THEN
          DO 1100 ITIME = 1, 2
              IIE = 0
              PAGE = .TRUE.
              DO 1110 I1 = 1, LDIM-1
1200          I2E = I1
                  I2B = I2E + 1
                  I2E = I2B + 1
                  IF (I2E .GT. LDIM) I2E = LDIM
                  IIB = IIE + 1
                  IIE = IIB + (I2E - I2B)
                  IF (PAGE) THEN
C
                      CALL PRTHD(IPRIN, DAY, TITLE, IROTA, JROUND)
                      CALL PRTSBH(IPRIN, IRNACC, IRNREJ, IFCNT, MEAND1,
&                                  STDD)
C
                      IF (ITIME .EQ. 1) THEN
                          WRITE(IPRIN, 5025)
                      ELSE
                          WRITE(IPRIN, 5026)
                      END IF
                  END IF
                  PAGE = .NOT. PAGE
                  WRITE(IPRIN, 5027) (NALF(1), I1, I2, I2 = I2B, I2E)
                  WRITE(IPRIN, 5028) (UBOUND(I1), I2 = I2B, I2E)
                  IF (ITIME .EQ. 1) THEN
                      DO 1120 JJ = NCLASS, 1, -1
                          WRITE(IPRIN, 5029) (NALF(1),
&                                  (RELBMP(I1, JJ, K), K=1, NCLASS), I1=IIB, IIE)
1120          CONTINUE
                      ELSE

```

S.12

```
DO 1130 JJ = NCLASS,1,-1
      WRITE(IPRIN,5029) (NALF(1),
      (RELBMI(II,JJ,K),K=1,NCLASS),II=IIB,IIE)
      &
1130      CONTINUE
      END IF
      WRITE(IPRIN,5028) (LBOUND(I1), I2 = I2B,I2E)
      WRITE(IPRIN,5030) (LBOUND(I2),UBOUND(I2),
      &
      I2 = I2B,I2E)
      IF (I2E .LT. LDIM) GOTO 1200
1110      CONTINUE
1100      CONTINUE
      END IF
800 CONTINUE
C      RETURN
C
C
9000 IF (ERRMSG .EQ. 1) THEN
      WRITE(IPRIN,9101)
ELSE IF (ERRMSG .EQ. 2) THEN
      WRITE(IPRIN,9102) IFAIL
ELSE IF (ERRMSG .EQ. 3) THEN
      WRITE(IPRIN,9103)
ELSE IF (ERRMSG .EQ. 4) THEN
      WRITE(IPRIN,9104) IRNACC,IRNREJ
ELSE
      WRITE(IPRIN,9198)
END IF
      WRITE(IPRIN,9199)
C
      RETURN 1
      END
```

```

C -----
C -----
SUBROUTINE PMC(SUM1,SUM2,LDIM,NDCL,SUM,MEAN,COV,STD,RHO)
INTEGER LDIM,NDCL
DOUBLE PRECISION SUM
DOUBLE PRECISION SUM1(LDIM),MEAN(LDIM),STD(LDIM),
& SUM2(NDCL,LDIM),COV(NDCL,LDIM),RHO(NDCL,LDIM)
C
C CALCULATE MEANS, STANDARD-DEVIATIONS, COVARIANCES AND CORRELATIONS
C
INTEGER I,J
DOUBLE PRECISION XMEAN,YMEAN,XCOV,YCOV,XSTD,YSTD
C
DO 100 I = 1,LDIM
  XMEAN = SUM1(I) / SUM
  MEAN(I) = XMEAN
  XCOV = SUM2(I,I) / SUM - XMEAN * XMEAN
  COV(I,I) = XCOV
  XSTD = DSQRT(XCOV)
  STD(I) = XSTD
  RHO(I,I) = XCOV / XSTD / XSTD
  DO 200 J = 1,I-1
    YMEAN = MEAN(J)
    YCOV = SUM2(I,J) / SUM - YMEAN * XMEAN
    COV(I,J) = YCOV
    YSTD = STD(J)
    RHO(I,J) = YCOV / YSTD / XSTD
  200 CONTINUE
  100 CONTINUE
C
RETURN
END

```

```

C -----
C -----
SUBROUTINE PRTHD(IPRIN, DAY, TITLE, IROTA, JROUND)
CHARACTER*9 DAY
CHARACTER*60 TITLE
INTEGER IPRIN, IROTA, JROUND
C
C PRINT THE HEADING OF AN OUTPUT PAGE
C
4000 FORMAT ('1')
4001 FORMAT (' ', 130('*'))
4002 FORMAT (' ', '*', 128X, '*')
4003 FORMAT (' ', '* --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)',
& T101, 'DATE: ', A9, T131, '*')
4004 FORMAT (' ', '* ', A60, T101, 'ROTATION: ', I5, T131, '*')
4005 FORMAT (' ', '* ', T101, 'ROUND: ', I5, T131, '*')
C
WRITE(IPRIN, 4000)
WRITE(IPRIN, 4001)
WRITE(IPRIN, 4002)
WRITE(IPRIN, 4003) DAY
WRITE(IPRIN, 4004) TITLE, IROTA
IF (JROUND .GT. 0) WRITE(IPRIN, 4005) JROUND
WRITE(IPRIN, 4002)
WRITE(IPRIN, 4001)
C
RETURN
END

```

```

C -----
C -----
SUBROUTINE PRTSBH(IPRIN, IRNACC, IRNREJ, IFCNT, MEAN, STD)
INTEGER IPRIN, IRNACC, IRNREJ, IFCNT
DOUBLE PRECISION MEAN, STD
C
C PRINT THE SUB-HEADING OF AN OUTPUT PAGE
C
4050 FORMAT ('0', ' NUMBER OF ACCEPTED RANDOM DRAWINGS', I8)
4051 FORMAT (' ', ' NUMBER OF REJECTED RANDOM DRAWINGS', I8)
4052 FORMAT (' ', ' NUMBER OF FUNCTION EVALUATIONS ', I9, T61,
& 'DENOMINATOR: MEAN =', E13.6, ', STD. DEV. =', E13.6)
C
WRITE(IPRIN, 4050) IRNACC
WRITE(IPRIN, 4051) IRNREJ
WRITE(IPRIN, 4052) IFCNT, MEAN, STD
C
RETURN
END

```

```

C -----
C -----
SUBROUTINE PSTROR(PARAM,NDIM,FPOST,IFCNT,SCFAC,ICALL)
LOGICAL ICALL
INTEGER NDIM,IFCNT
DOUBLE PRECISION FPOST,SCFAC
DOUBLE PRECISION PARAM(NDIM)
C -----
C -----
C MINUS LOGPOSTERIOR, JOHNSTON MODEL [WITH BAYPAR PARAMETER = 5]
C -----
C -----
DOUBLE PRECISION BAYPAR,SCC,SY,SCY,SRIRI,SRIY,SRVRV,SRVY,
& SRVRI,SCRI,SCRV,A2,B2,B3,ECC,EII,ECI
DOUBLE PRECISION Y(10),C(10),RI(10),Z(10),RV(10)
C
C IF FIRST CALL: READ DATA AND CREATE DUMMY VARIABLES
C
IF (ICALL) THEN
  IFCNT = 0
  BAYPAR = 5.0D0
  SCFAC = -2.5D1
C
  OPEN (UNIT=20, FILE='JOHNSTON.INP', STATUS='OLD')
  READ(20,'(5F10.4)') (Y(I),C(I),RI(I),Z(I),RV(I), I = 1,10)
  CLOSE(UNIT=20)
C
  SCC = 0.0D0
  SY = 0.0D0
  SCY = 0.0D0
  SRIRI = 0.0D0
  SRIY = 0.0D0
  SRVRV = 0.0D0
  SRVY = 0.0D0
  SRVRI = 0.0D0
  SCRI = 0.0D0
  SCRIV = 0.0D0
  DO 100 I = 1,10
    SCC = SCC + C(I) * C(I)
    SY = SY + Y(I) * Y(I)
    SCY = SCY + C(I) * Y(I)
    SRIRI = SRIRI + RI(I) * RI(I)
    SRIY = SRIY + RI(I) * Y(I)
    SRVRV = SRVRV + RV(I) * RV(I)
    SRVY = SRVY + RV(I) * Y(I)
    SRVRI = SRVRI + RV(I) * RI(I)
    SCRI = SCRI + C(I) * RI(I)
    SCRIV = SCRIV + C(I) * RV(I)
100 CONTINUE
C
  RETURN
C
END IF
C

```

U.2

```

A2   = PARAM(1)
B2   = PARAM(2)
B3   = PARAM(3)
ECC  = SCC + A2 * (A2 * SYI - SCY - SCY)
EII  = SRIRI + B2 * (B2 * SYI - 2.0D0 * (SRIY - B3 * SRVY)) +
&      B3 * (B3 * SRVRV - SRVRI - SRVRI)
ECI  = SCRI - B2 * SCY - B3 * SCRIV -
&      A2 * (SRIY - B2 * SYI - B3 * SRVY)
FPOST = -1.0D0 * BAYPAR * DLOG(ECC * EII - ECI * ECI) +
&      1.0D1 * DLOG(DABS(1.0D0 - A2 - B2))
FPOST = FPOST + SCFAC
IFCNT = IFCNT + 1

```

C

```

RETURN
END

```

C

C

```

-----
SUBROUTINE RSTRCT(PARAM,NDIM,FAIL)
LOGICAL FAIL
INTEGER NDIM
DOUBLE PRECISION PARAM(NDIM)

```

C

C

C

C

C

C

C

C

C

C

```

-----
FAIL = .FALSE.
IF (DABS(1.0D0 - PARAM(1) - PARAM(2)) .LE. 1.0D-2) FAIL = .TRUE.

```

C

```

RETURN
END

```

```

$ SETD      [E.ECT.HOP.DIJK]
$ FORTRAN/OBJECT=JSISTEST/LIST=JSISTEST SISMAIN+SISSUB+POSJOHN+RESJOHN
$ LINK/NOMAP JSISTEST,CIW$SYS:NAGF/LIBRARY
$ REMOVE    JSISTEST.OBJ.*
$ RUN       JSISTEST
JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)          ! TITLE
3              ! DIMENSION OF THE VECTOR "THETA"
JSISTEST.DAT   ! NAME OF THE PRINT-OUTPUT-FILE
JOHNSTON.HHH   ! NAME OF THE INPUT-FILE WITH "PMODE", "HINV"
-2.  -1.7  -0.40 ! LOWER BOUNDS FOR "THETA"
.8   .25   1.00 ! UPPER BOUNDS FOR "THETA"
1              ! UNIVARIATE MARGINALS: 0=NO, 1=YES
1              ! BIVARIATE MARGINALS: 0=NO, 1=YES
79            ! INITIAL VALUE OF RANDOM NUMBER GENERATOR
1              ! DEGREES OF FREEDOM OF STUDENT -T- FUNCTION
2              ! NUMBER OF ROUNDS
20000         ! NUMBER OF DRAWINGS PER ROUND
2              ! NUMBER OF ROTATIONS
1              ! SAVE "MEANPO" AND "COVPO": 0=NO, 1=YES
JSISTEST.SAV   ! NAME OF THE SAVE-FILE FOR "MEAMPO" AND "COVPO"
$ REMOVE      JSISTEST.EXE.*

```

("JOHNSTON.HHH", "PMODE" AND "HINV")

0.457892800000000000E+00	8.929882000000000000E-02	0.362861500000000000E+00
1.02568634379867900E-02		
3.1452140481838555E-03	1.25411527436849796E-03	
1.97880731848330789E-03	-6.43554989639164606E-04	1.26391899220319765E-02

("JOHNSTON.INP", INPUT DATA)

-1.9019	-0.9288	-0.2249	-0.7482	-0.2104
-1.4359	-0.6188	-0.1799	-0.6372	-0.1564
-0.9719	-0.7798	-0.2509	0.0588	-0.1114
-0.9189	-0.8458	-0.3229	0.2498	-0.1824
-0.3279	-0.3948	-0.2299	0.2968	-0.2544
0.4011	0.1542	-0.0219	0.2688	-0.1614
0.9581	0.5742	0.1711	0.2132	0.0466
1.2681	0.6792	0.2881	0.3000	0.2396
1.5091	0.9332	0.3651	0.2108	0.3566
1.4201	1.2272	0.4061	-0.2132	0.4336



```

*****
*
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*****

```

```

DATE: 2-SEP-86
ROTATION: 1

```

# BOUNDS OF THE PARAMETERS

```

-2.0000  -1.7000  -0.4000
 0.8000   0.2500   1.0000

```

# IMPORTANCE MEANS AND STANDARD DEVIATIONS

```

0.4579  0.0893  0.3629
0.1013  0.0354  0.1124

```

# IMPORTANCE CORRELATION MATRIX

```

1.00000
0.87695  1.00000
0.17379 -0.16164  1.00000

```

# IMPORTANCE COVARIANCE MATRIX

```

0.01026
0.00315  0.00125
0.00198 -0.00064  0.01264

```

# EIGENVALUES OF MIN INVERSE HESSIAN MATRIX

```

0.000145  0.010156  0.013850

```

INITIAL VALUE OF RANDOM NUMBER GENERATOR : CALL G05CBF( 79)

```

*****
* --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
* JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*
*****

```

```

DATE: 2-SEP-86
ROTATION: 1
ROUND: 1

```

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 3789
NUMBER OF FUNCTION EVALUATIONS 20000

```

DENOMINATOR: MEAN = 0.543908E+08, STD. DEV. = 0.817992E+09

IMPORTANCE MEANS AND STANDARD DEVIATIONS (TRUNCATED)

```

0.4127 0.0751 0.3557
0.2286 0.0799 0.1938

```

IMPORTANCE CORRELATION MATRIX (TRUNCATED)

```

1.00000
0.88146 1.00000
0.17346 -0.07578 1.00000

```

IMPORTANCE COVARIANCE MATRIX (TRUNCATED)

```

0.05228
0.01611 0.00639
0.00769 -0.00117 0.03758

```

```

*****
*
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*
*****

```

```

DATE: 2-SEP-86
ROTATION: 1
ROUND: 1

```

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 3789
NUMBER OF FUNCTION EVALUATIONS 20000

```

DENOMINATOR: MEAN = 0.543908E+08, STD. DEV. = 0.817992E+09

POSTERIOR MEAN AND STANDARDDEVIATION OF THE PARAMETER VECTOR "THETA"

```

-0.6499 -0.2995 0.3188
0.8185 0.2923 0.1409

```

POSTERIOR CORRELATION MATRIX OF THE PARAMETER VECTOR "THETA"

```

1.00000
0.95902 1.00000
0.16079 0.26118 1.00000

```

POSTERIOR COVARIANCE MATRIX OF THE PARAMETER VECTOR "THETA"

```

0.66996
0.22947 0.08546
0.01855 0.01076 0.01986

```

NUMERICAL ERROR ESTIMATES OF POSTERIOR MEAN

```

ERROR      : 0.115859 0.041339 0.009103
RELATIVE ERROR : 0.141549 0.141408 0.064590
CORREL. COEFF. : -0.967603 -0.975136 0.964315

```

```

*****
* --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
* JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
* DATE: 2-SEP-86
* ROTATION: 1
* ROUND: 1
*
*****

```

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 3789
NUMBER OF FUNCTION EVALUATIONS 20000

```

DENOMINATOR: MEAN = 0.543908E+08, STD. DEV. = 0.817992E+09

FREQUENCIES OF IPOW. WEIGHT=0.\*\*\*\*\*E+IPOW

-25 0	-24 0	-23 0	-22 0	-21 0	-20 0	-19 0	-18 0	-17 0	-16 0	-15 0	-14 0	-13 0	-12 0	-11 2
-10 1	-9 7	-8 5	-7 9	-6 10	-5 21	-4 22	-3 33	-2 40	-1 66	0 65	1 112	2 137	3 225	4 275
5 504	6 1013	7 8001	8 8624	9 715	10 97	11 16	12 0	13 0	14 0	15 0	16 0	17 0	18 0	19 0

TEN DRAWINGS WITH LARGEST WEIGHT

W	LN(IMP)	LN(POS)	(THETA(I), I = 1,NDIM)		
0.6760783E+11	-12.8183133	12.1186763	-1.91397	-0.76367	0.29509
0.5245519E+11	-12.6695467	12.0136784	-1.87514	-0.72200	0.27256
0.3226033E+11	-12.6639374	11.5331668	-1.91723	-0.66222	0.25886
0.2810649E+11	-12.9272047	11.1320615	-1.86109	-0.78067	0.15572
0.2336449E+11	-12.6988862	11.1755969	-1.90996	-0.69381	0.45441
0.2226337E+11	-12.2585543	11.5676544	-0.95208	-0.56388	0.23280
0.1795778E+11	-12.5997756	11.0115135	-1.18861	-0.63431	0.10482
0.1654026E+11	-11.3891124	12.1399510	-0.89523	-0.47869	0.25977
0.1551928E+11	-11.3356672	12.1296818	-0.89816	-0.47139	0.22500
0.1532318E+11	-11.1230665	12.3295658	-1.10488	-0.46485	0.27607



```

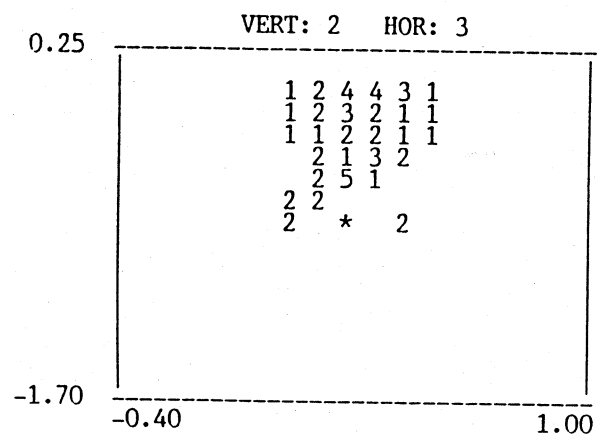
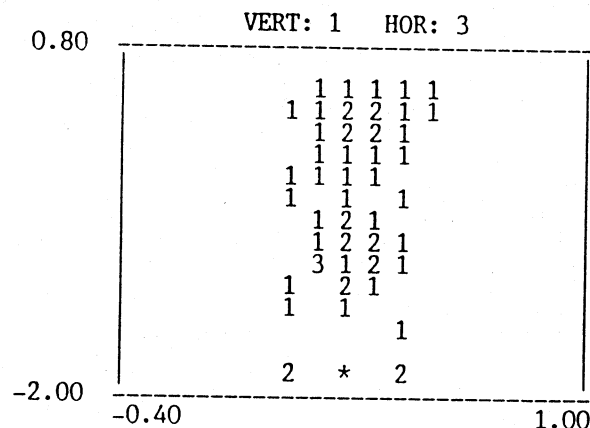
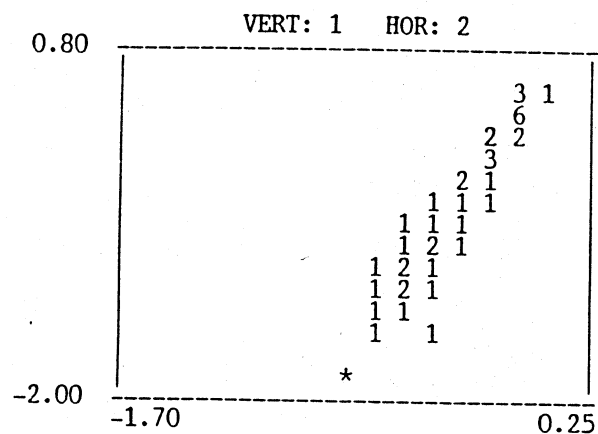
*****
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*****
DATE: 2-SEP-86
ROTATION: 1
ROUND: 1

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000  
 NUMBER OF REJECTED RANDOM DRAWINGS 3789  
 NUMBER OF FUNCTION EVALUATIONS 20000

DENOMINATOR: MEAN = 0.543908E+08, STD. DEV. = 0.817992E+09

BIVARIATE MARGINAL POSTERIOR DENSITIES



```

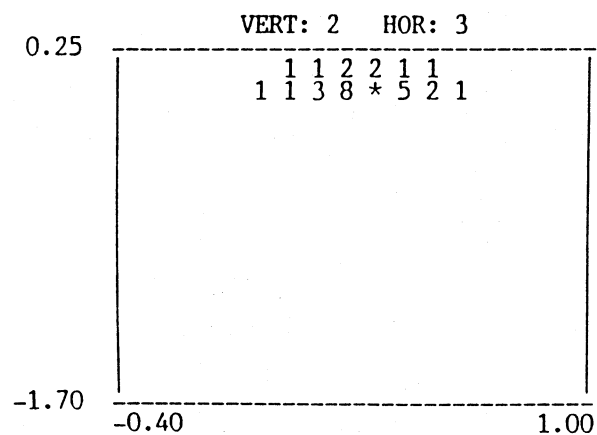
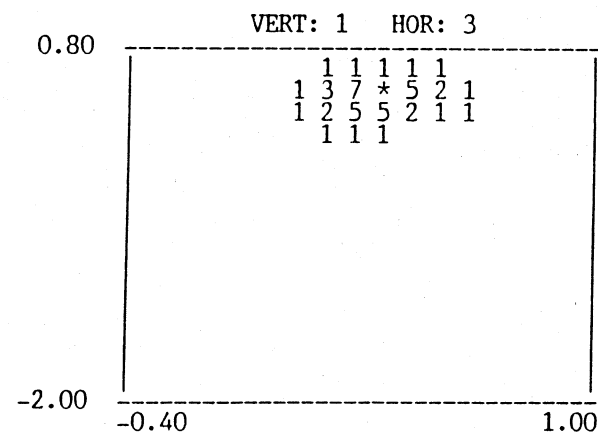
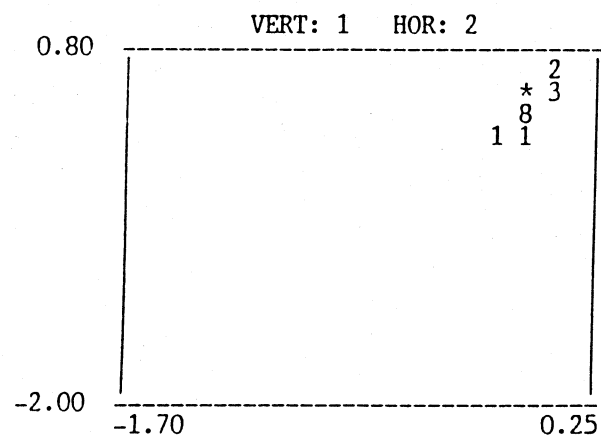
*****
* --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
* JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
* DATE: 2-SEP-86
* ROTATION: 1
* ROUND: 1
*
*****

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000  
 NUMBER OF REJECTED RANDOM DRAWINGS 3789  
 NUMBER OF FUNCTION EVALUATIONS 20000

DENOMINATOR: MEAN = 0.543908E+08, STD. DEV. = 0.817992E+09

BIVARIATE MARGINAL DENSITIES OF IMPORTANCE FUNCTION (TRUNCATED)





```

*****
* --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
* JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
* DATE: 2-SEP-86
* ROTATION: 1
* ROUND: 2
*
*****

```

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 7753
NUMBER OF FUNCTION EVALUATIONS 40000

```

DENOMINATOR: MEAN = 0.570712E+08, STD. DEV. = 0.890592E+09

IMPORTANCE MEANS AND STANDARDDEVIATIONS (TRUNCATED)

```

0.4132 0.0749 0.3559
0.2253 0.0802 0.1941

```

IMPORTANCE CORRELATION MATRIX (TRUNCATED)

```

1.00000
0.87748 1.00000
0.15840 -0.09902 1.00000

```

IMPORTANCE COVARIANCE MATRIX (TRUNCATED)

```

0.05077
0.01587 0.00644
0.00693 -0.00154 0.03767

```

```

*****
*
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*
*****

```

```

DATE: 2-SEP-86
ROTATION: 1
ROUND: 2

```

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 7753
NUMBER OF FUNCTION EVALUATIONS 40000

```

DENOMINATOR: MEAN = 0.570712E+08, STD. DEV. = 0.890592E+09

POSTERIOR MEAN AND STANDARD DEVIATION OF THE PARAMETER VECTOR "THETA"

```

-0.6573 -0.3193 0.3203
0.7948 0.3042 0.1343

```

POSTERIOR CORRELATION MATRIX OF THE PARAMETER VECTOR "THETA"

```

1.00000
0.95458 1.00000
0.14700 0.22498 1.00000

```

POSTERIOR COVARIANCE MATRIX OF THE PARAMETER VECTOR "THETA"

```

0.63179
0.23077 0.09251
0.01569 0.00919 0.01803

```

NUMERICAL ERROR ESTIMATES OF POSTERIOR MEAN

```

ERROR      : 0.078459 0.031534 0.005768
RELATIVE ERROR : 0.098710 0.103680 0.042961
CORREL. COEFF. : -0.968990 -0.973110 0.973590

```

```

*****
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*  DATE: 2-SEP-86
*  ROTATION: 1
*  ROUND: 2
*
*****

```

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 7753
NUMBER OF FUNCTION EVALUATIONS 40000

```

DENOMINATOR: MEAN = 0.570712E+08, STD. DEV. = 0.890592E+09

FREQUENCIES OF IPOW. WEIGHT=0.\*\*\*\*\*E+IPOW

-25 0	-24 0	-23 0	-22 0	-21 0	-20 0	-19 0	-18 0	-17 0	-16 0	-15 0	-14 0	-13 0	-12 0	-11 3
-10 2	-9 9	-8 14	-7 19	-6 26	-5 35	-4 52	-3 74	-2 93	-1 126	0 143	1 199	2 271	3 451	4 544
5 982	6 1965	7 16076	8 17224	9 1458	10 200	11 34	12 0	13 0	14 0	15 0	16 0	17 0	18 0	19 0

TEN DRAWINGS WITH LARGEST WEIGHT

W	LN(IMP)	LN(POS)	(THETA(I), I = 1,NDIM)		
0.7112271E+11	-13.7743725	11.2133000	-1.68069	-0.89169	0.28180
0.6760783E+11	-12.8183133	12.1186763	-1.91397	-0.76367	0.29509
0.6636652E+11	-12.8856034	12.0328551	-1.96643	-0.77804	0.34243
0.5245519E+11	-12.6695467	12.0136784	-1.87514	-0.72200	0.27256
0.4762237E+11	-12.7622241	11.8243443	-1.38556	-0.70372	0.25311
0.3292641E+11	-12.2291196	11.9884213	-1.24918	-0.62162	0.28113
0.3226033E+11	-12.6639374	11.5331668	-1.91723	-0.66222	0.25886
0.2893752E+11	-12.4016240	11.6867810	-1.20425	-0.63420	0.29757
0.2810649E+11	-12.9272047	11.1320615	-1.86109	-0.78067	0.15572
0.2336449E+11	-12.6988862	11.1755969	-1.90996	-0.69381	0.45441

```

*****
*
*   --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
*   JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*****
DATE: 2-SEP-86
ROTATION: 1
ROUND: 2

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000  
 NUMBER OF REJECTED RANDOM DRAWINGS 7753  
 NUMBER OF FUNCTION EVALUATIONS 40000

DENOMINATOR: MEAN = 0.570712E+08, STD. DEV. = 0.890592E+09

MARGINAL POSTERIOR DENSITIES "P", AND MARGINAL IMPORTANCE DENSITIES (TRUNCATED) "I"

PARAMETER 1			PARAMETER 2			PARAMETER 3		
	P	I		P	I		P	I
(-2.00, -1.81)	0.126	0.000	(-1.70, -1.57)	0.000	0.000	(-0.40, -0.31)	0.001	0.006
(-1.81, -1.63)	0.037	0.000	(-1.57, -1.44)	0.000	0.000	(-0.31, -0.21)	0.000	0.007
(-1.63, -1.44)	0.034	0.001	(-1.44, -1.31)	0.000	0.000	(-0.21, -0.12)	0.002	0.012
(-1.44, -1.25)	0.064	0.001	(-1.31, -1.18)	0.000	0.000	(-0.12, -0.03)	0.009	0.016
(-1.25, -1.07)	0.091	0.001	(-1.18, -1.05)	0.000	0.000	(-0.03, 0.07)	0.014	0.025
(-1.07, -0.88)	0.081	0.002	(-1.05, -0.92)	0.000	0.000	(0.07, 0.16)	0.069	0.049
(-0.88, -0.69)	0.057	0.002	(-0.92, -0.79)	0.037	0.000	(0.16, 0.25)	0.160	0.107
(-0.69, -0.51)	0.058	0.004	(-0.79, -0.66)	0.158	0.001	(0.25, 0.35)	0.366	0.232
(-0.51, -0.32)	0.057	0.006	(-0.66, -0.53)	0.092	0.001	(0.35, 0.44)	0.204	0.275
(-0.32, -0.13)	0.058	0.011	(-0.53, -0.40)	0.116	0.002	(0.44, 0.53)	0.115	0.139
(-0.13, 0.05)	0.067	0.023	(-0.40, -0.27)	0.124	0.005	(0.53, 0.63)	0.044	0.064
(0.05, 0.24)	0.082	0.065	(-0.27, -0.14)	0.108	0.012	(0.63, 0.72)	0.011	0.032
(0.24, 0.43)	0.115	0.310	(-0.14, -0.01)	0.138	0.058	(0.72, 0.81)	0.003	0.017
(0.43, 0.61)	0.072	0.477	(-0.01, 0.12)	0.215	0.720	(0.81, 0.91)	0.001	0.012
(0.61, 0.80)	0.001	0.099	(0.12, 0.25)	0.011	0.201	(0.91, 1.00)	0.000	0.007

```

*****
* --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
* JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*****

```

```

DATE: 2-SEP-86
ROTATION: 1
ROUND: 2

```

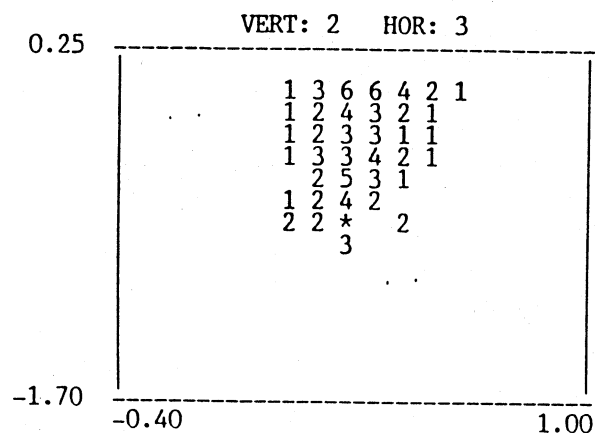
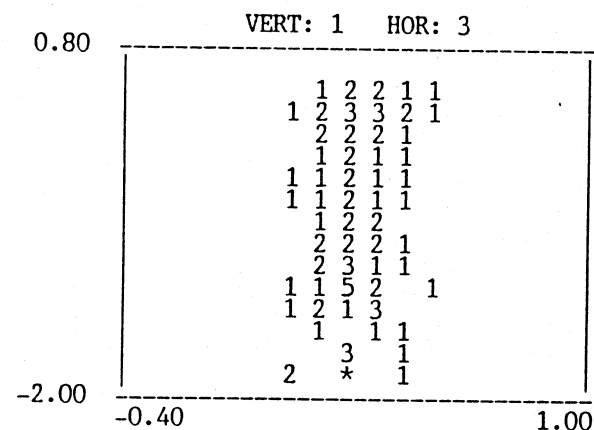
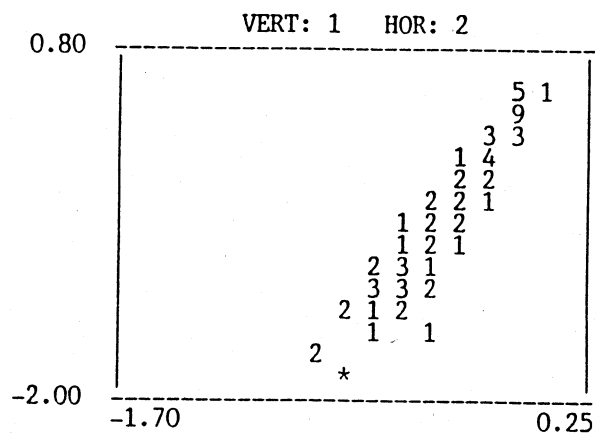
```

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 7753
NUMBER OF FUNCTION EVALUATIONS 40000

```

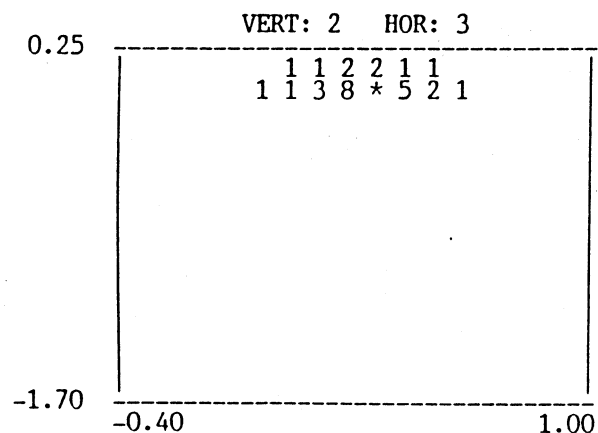
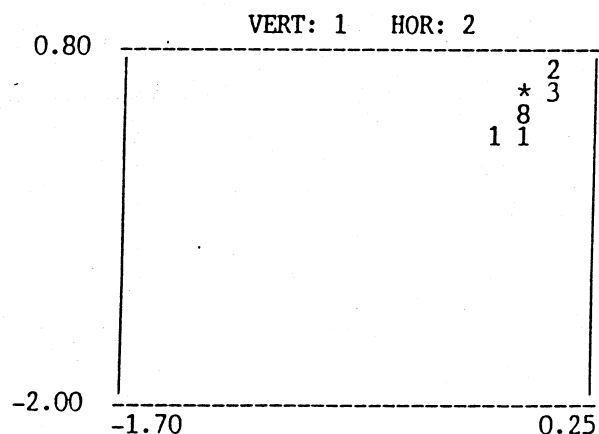
DENOMINATOR: MEAN = 0.570712E+08, STD. DEV. = 0.890592E+09

BIVARIATE MARGINAL POSTERIOR DENSITIES



[illegible]

DENOMINATOR: MEAN = 0.570712E+08, STD. DEV. = 0.890592E+09



```

*****
*
* --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
* JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*****

```

DATE: 2-SEP-86  
ROTATION: 2

#### BOUNDS OF THE PARAMETERS

-2.0000	-1.7000	-0.4000
0.8000	0.2500	1.0000

#### IMPORTANCE MEANS AND STANDARD DEVIATIONS

-0.6573	-0.3193	0.3203
0.7948	0.3042	0.1343

#### IMPORTANCE CORRELATION MATRIX

1.00000		
0.95458	1.00000	
0.14700	0.22498	1.00000

#### IMPORTANCE COVARIANCE MATRIX

0.63179		
0.23077	0.09251	
0.01569	0.00919	0.01803

#### EIGENVALUES OF MIN INVERSE HESSIAN MATRIX

0.006336	0.018467	0.717516
----------	----------	----------

```

*****
*
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*
*****

```

```

DATE: 2-SEP-86
ROTATION: 2
ROUND: 1

```

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 11489
NUMBER OF FUNCTION EVALUATIONS 60000

```

DENOMINATOR: MEAN = 0.109016E+07, STD. DEV. = 0.203716E+07

IMPORTANCE MEANS AND STANDARDDEVIATIONS (TRUNCATED)

```

-0.6653  -0.3325  0.3185
0.6301   0.2576  0.1772

```

IMPORTANCE CORRELATION MATRIX (TRUNCATED)

```

1.00000
0.84735  1.00000
0.05254  0.16261  1.00000

```

IMPORTANCE COVARIANCE MATRIX (TRUNCATED)

```

0.39706
0.13757  0.06638
0.00587  0.00743  0.03141

```



```

*****
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*****

```

```

DATE: 2-SEP-86
ROTATION: 2
ROUND: 1

```

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 11489
NUMBER OF FUNCTION EVALUATIONS 60000

```

DENOMINATOR: MEAN = 0.109016E+07, STD. DEV. = 0.203716E+07

POSTERIOR MEAN AND STANDARD DEVIATION OF THE PARAMETER VECTOR "THETA"

```

-0.5844 -0.3018 0.3126
0.7856 0.3235 0.1452

```

POSTERIOR CORRELATION MATRIX OF THE PARAMETER VECTOR "THETA"

```

1.00000
0.92121 1.00000
0.14811 0.28291 1.00000

```

POSTERIOR COVARIANCE MATRIX OF THE PARAMETER VECTOR "THETA"

```

0.61722
0.23411 0.10463
0.01689 0.01328 0.02107

```

NUMERICAL ERROR ESTIMATES OF POSTERIOR MEAN

```

ERROR      : 0.014525 0.005797 0.002065
RELATIVE ERROR : 0.018488 0.017920 0.014225
CORREL. COEFF. : 0.210403 -0.077054 0.917077

```

```

*****
*
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*****

```

```

DATE: 2-SEP-86
ROTATION: 2
ROUND: 1

```

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 11489
NUMBER OF FUNCTION EVALUATIONS 60000

```

DENOMINATOR: MEAN = 0.109016E+07, STD. DEV. = 0.203716E+07

FREQUENCIES OF IPOW. WEIGHT=0.\*\*\*\*\*E+IPOW

-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
1	2	4	4	7	6	6	13	24	35	63	92	142	244	382
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
943	12403	5450	179	0	0	0	0	0	0	0	0	0	0	0

TEN DRAWINGS WITH LARGEST WEIGHT

W	LN(IMP)	LN(POS)	(THETA(I), I = 1,NDIM)		
0.3932177E+08	-4.2693570	13.2179319	0.51246	0.07082	0.62155
0.3853749E+08	-2.9134306	14.5537116	0.51777	0.09146	0.47298
0.3825340E+08	-2.7768369	14.6829061	0.47654	0.08802	0.47074
0.3703270E+08	-2.3700851	15.0572268	0.45672	0.08229	0.41576
0.3590446E+08	-2.4400944	14.9562775	0.48720	0.09674	0.41912
0.3266503E+08	-2.2490453	15.0527703	0.44858	0.07809	0.39223
0.3257072E+08	-3.2006236	14.0983007	0.44394	0.07550	0.52568
0.3200778E+08	-2.2001544	15.0813351	0.44976	0.07998	0.37718
0.3147760E+08	-2.2515204	15.0132663	0.45691	0.09158	0.30632
0.3129323E+08	-2.3628494	14.8960629	0.49436	0.10901	0.30241

```
*****
*                                     *
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)                                *
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)                               *
*                                     *
*                                     DATE: 2-SEP-86                          *
*                                     ROTATION: 2                            *
*                                     ROUND: 1                               *
*                                     *
*****
```

MARGINAL POSTERIOR DENSITIES "P", AND MARGINAL IMPORTANCE DENSITIES (TRUNCATED) "I"

```

*****
*
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*****

```

```

DATE: 2-SEP-86
ROTATION: 2
ROUND: 1

```

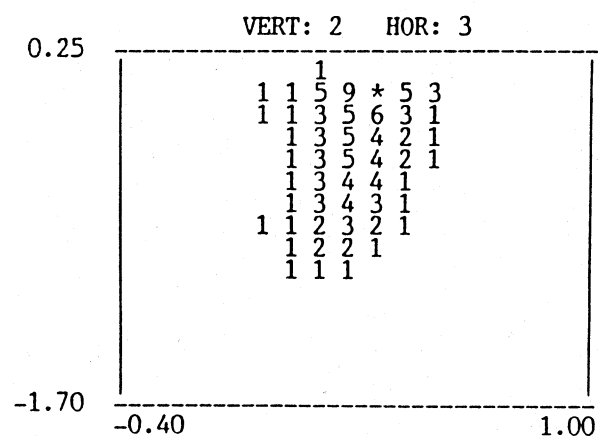
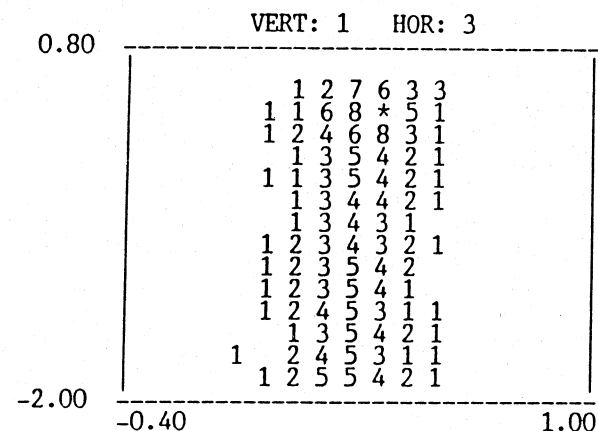
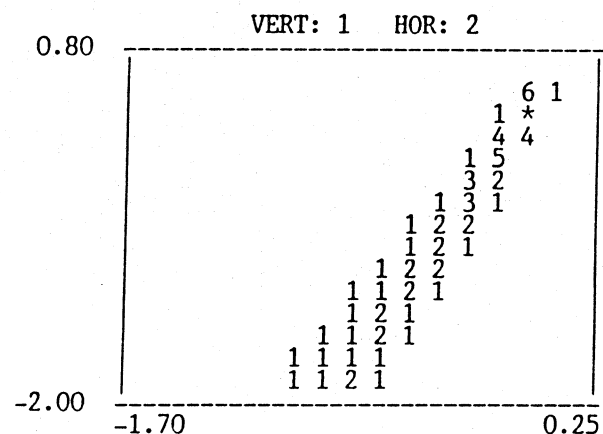
```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 11489
NUMBER OF FUNCTION EVALUATIONS 60000

```

DENOMINATOR: MEAN = 0.109016E+07, STD. DEV. = 0.203716E+07

BIVARIATE MARGINAL POSTERIOR DENSITIES



```

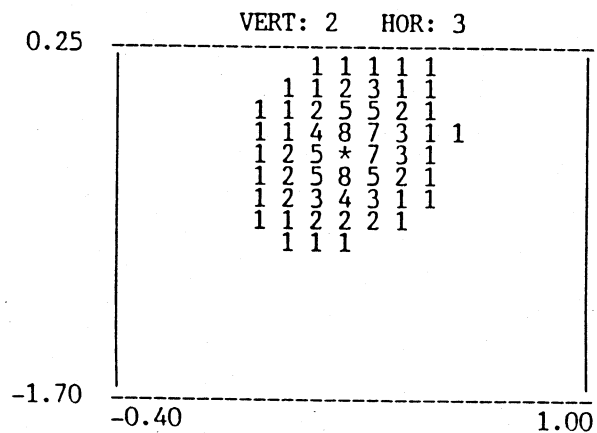
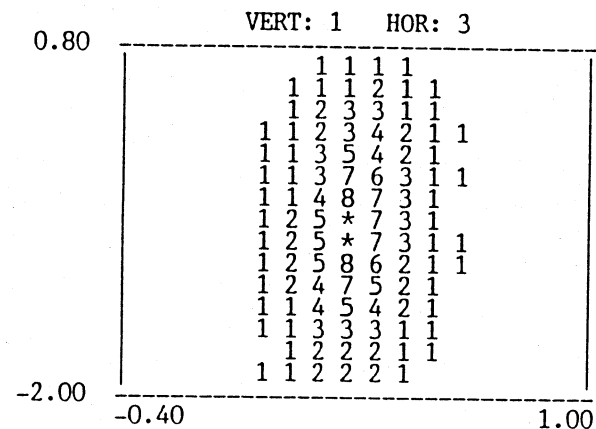
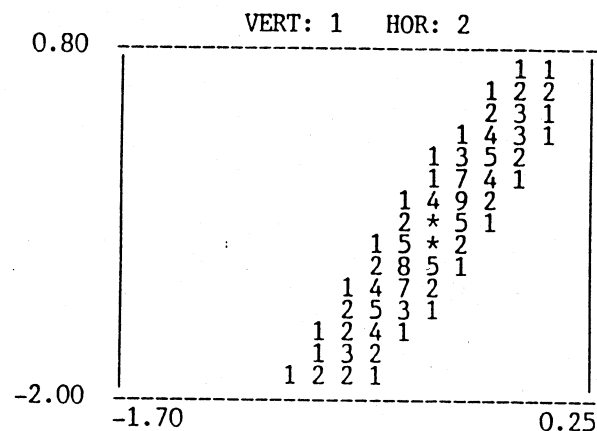
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* --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
* JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
* DATE: 2-SEP-86
* ROTATION: 2
* ROUND: 1
*
*****

```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000  
 NUMBER OF REJECTED RANDOM DRAWINGS 11489  
 NUMBER OF FUNCTION EVALUATIONS 60000

DENOMINATOR: MEAN = 0.109016E+07, STD. DEV. = 0.203716E+07

BIVARIATE MARGINAL DENSITIES OF IMPORTANCE FUNCTION (TRUNCATED)



```

*****
* --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
* JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
* DATE: 2-SEP-86
* ROTATION: 2
* ROUND: 2
*
*****

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NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 22792
NUMBER OF FUNCTION EVALUATIONS 80000

```

DENOMINATOR: MEAN = 0.108995E+07, STD. DEV. = 0.203021E+07

IMPORTANCE MEANS AND STANDARDDEVIATIONS (TRUNCATED)

```

-0.6653 -0.3314 0.3178
0.6307 0.2590 0.1780

```

IMPORTANCE CORRELATION MATRIX (TRUNCATED)

```

1.00000
0.84847 1.00000
0.06168 0.17852 1.00000

```

IMPORTANCE COVARIANCE MATRIX (TRUNCATED)

```

0.39774
0.13857 0.06706
0.00692 0.00823 0.03168

```

```

*****
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*****

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DATE: 2-SEP-86
ROTATION: 2
ROUND: 2

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NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 22792
NUMBER OF FUNCTION EVALUATIONS 80000

```

DENOMINATOR: MEAN = 0.108995E+07, STD. DEV. = 0.203021E+07

POSTERIOR MEAN AND STANDARD DEVIATION OF THE PARAMETER VECTOR "THETA"

```

-0.5993 -0.3092 0.3125
0.7862 0.3289 0.1465

```

POSTERIOR CORRELATION MATRIX OF THE PARAMETER VECTOR "THETA"

```

1.00000
0.91745 1.00000
0.16798 0.30885 1.00000

```

POSTERIOR COVARIANCE MATRIX OF THE PARAMETER VECTOR "THETA"

```

0.61807
0.23726 0.10820
0.01935 0.01488 0.02146

```

NUMERICAL ERROR ESTIMATES OF POSTERIOR MEAN

```

ERROR      : 0.010348 0.004266 0.001478
RELATIVE ERROR : 0.013163 0.012970 0.010088
CORREL. COEFF. : 0.158431 -0.127086 0.914430

```

```

*****
*
* --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
* JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*****

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```

DATE: 2-SEP-86
ROTATION: 2
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NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 22792
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```

DENOMINATOR: MEAN = 0.108995E+07, STD. DEV. = 0.203021E+07

FREQUENCIES OF IPOW. WEIGHT=0.\*\*\*\*\*E+IPOW

-25 0	-24 0	-23 0	-22 0	-21 0	-20 0	-19 0	-18 0	-17 0	-16 0	-15 0	-14 0	-13 0	-12 0	-11 1
-10 1	-9 3	-8 7	-7 9	-6 13	-5 16	-4 14	-3 23	-2 53	-1 76	0 121	1 186	2 298	3 457	4 771
5 1882	6 24755	7 10955	8 359	9 0	10 0	11 0	12 0	13 0	14 0	15 0	16 0	17 0	18 0	19 0

TEN DRAWINGS WITH LARGEST WEIGHT

W	LN(IMP)	LN(POS)	(THETA(I), I = 1,NDIM)		
0.3979926E+08	-3.3737670	14.1255919	0.46103	0.06702	0.53429
0.3932177E+08	-4.2693570	13.2179319	0.51246	0.07082	0.62155
0.3878236E+08	-2.6889068	14.7845692	0.51115	0.09543	0.44738
0.3864311E+08	-2.8858178	14.5840613	0.44728	0.07270	0.48715
0.3853749E+08	-2.9134306	14.5537116	0.51777	0.09146	0.47298
0.3825340E+08	-2.7768369	14.6829061	0.47654	0.08802	0.47074
0.3703270E+08	-2.3700851	15.0572268	0.45672	0.08229	0.41576
0.3590446E+08	-2.4400944	14.9562775	0.48720	0.09674	0.41912
0.3589604E+08	-2.2482294	15.1479082	0.46191	0.08886	0.38700
0.3501558E+08	-2.9264061	14.4448974	0.42811	0.06546	0.49489



```

*****
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)                                *
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)                                *
*                                                                                   *
*                                                                                   *
*****

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```

DATE: 2-SEP-86
ROTATION: 2
ROUND: 2

```

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NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 22792
NUMBER OF FUNCTION EVALUATIONS 80000

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DENOMINATOR: MEAN = 0.108995E+07, STD. DEV. = 0.203021E+07

MARGINAL POSTERIOR DENSITIES "P", AND MARGINAL IMPORTANCE DENSITIES (TRUNCATED) "I"

PARAMETER 1			PARAMETER 2			PARAMETER 3		
	P	I		P	I		P	I
(-2.00,-1.81)	0.072	0.034	(-1.70,-1.57)	0.001	0.000	(-0.40,-0.31)	0.001	0.003
(-1.81,-1.63)	0.070	0.041	(-1.57,-1.44)	0.001	0.000	(-0.31,-0.21)	0.001	0.006
(-1.63,-1.44)	0.063	0.052	(-1.44,-1.31)	0.003	0.001	(-0.21,-0.12)	0.004	0.009
(-1.44,-1.25)	0.064	0.066	(-1.31,-1.18)	0.005	0.002	(-0.12,-0.03)	0.011	0.017
(-1.25,-1.07)	0.061	0.082	(-1.18,-1.05)	0.010	0.003	(-0.03, 0.07)	0.032	0.033
(-1.07,-0.88)	0.059	0.096	(-1.05,-0.92)	0.022	0.008	( 0.07, 0.16)	0.081	0.072
(-0.88,-0.69)	0.060	0.113	(-0.92,-0.79)	0.047	0.026	( 0.16, 0.25)	0.187	0.165
(-0.69,-0.51)	0.059	0.113	(-0.79,-0.66)	0.076	0.063	( 0.25, 0.35)	0.272	0.279
(-0.51,-0.32)	0.061	0.104	(-0.66,-0.53)	0.100	0.112	( 0.35, 0.44)	0.239	0.225
(-0.32,-0.13)	0.065	0.089	(-0.53,-0.40)	0.108	0.165	( 0.44, 0.53)	0.115	0.102
(-0.13, 0.05)	0.070	0.069	(-0.40,-0.27)	0.113	0.205	( 0.53, 0.63)	0.041	0.045
( 0.05, 0.24)	0.095	0.055	(-0.27,-0.14)	0.120	0.180	( 0.63, 0.72)	0.010	0.021
( 0.24, 0.43)	0.123	0.041	(-0.14,-0.01)	0.151	0.126	( 0.72, 0.81)	0.004	0.012
( 0.43, 0.61)	0.077	0.028	(-0.01, 0.12)	0.233	0.073	( 0.81, 0.91)	0.002	0.007
( 0.61, 0.80)	0.001	0.017	( 0.12, 0.25)	0.010	0.036	( 0.91, 1.00)	0.000	0.004

```

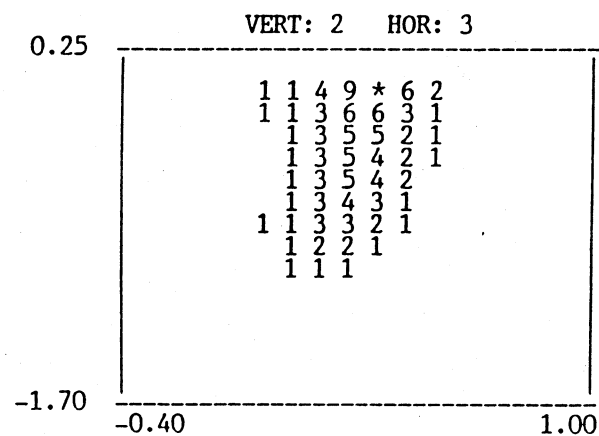
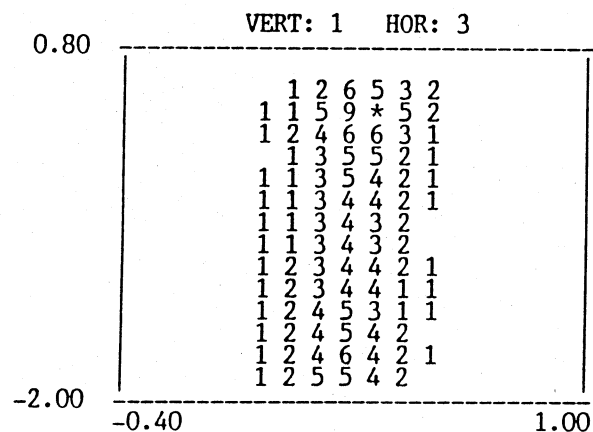
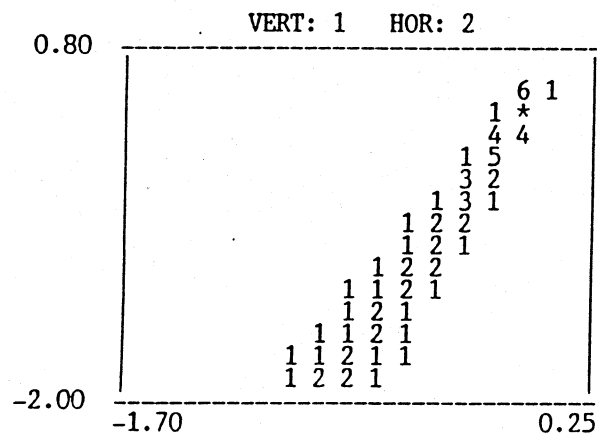
*****
*  ——— SIMPLE IMPORTANCE SAMPLING ——— (S.I.S)  DATE: 2-SEP-86
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)  ROTATION: 2
*  ROUND: 2
*
*****

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 NUMBER OF REJECTED RANDOM DRAWINGS 22792  
 NUMBER OF FUNCTION EVALUATIONS 80000

DENOMINATOR: MEAN = 0.108995E+07, STD. DEV. = 0.203021E+07

BIVARIATE MARGINAL POSTERIOR DENSITIES



```

*****
*  --- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
*  JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)
*
*
*****

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DATE: 2-SEP-86
ROTATION: 2
ROUND: 2

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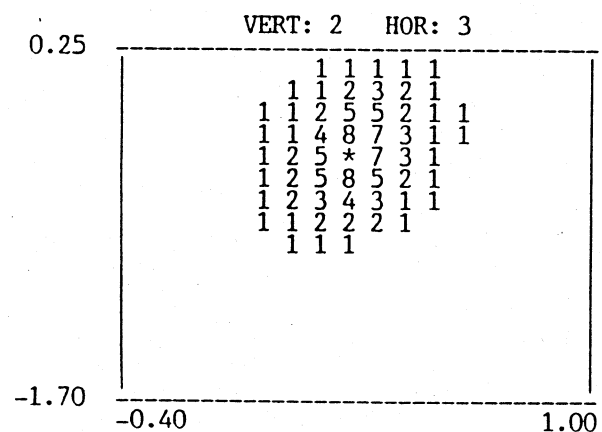
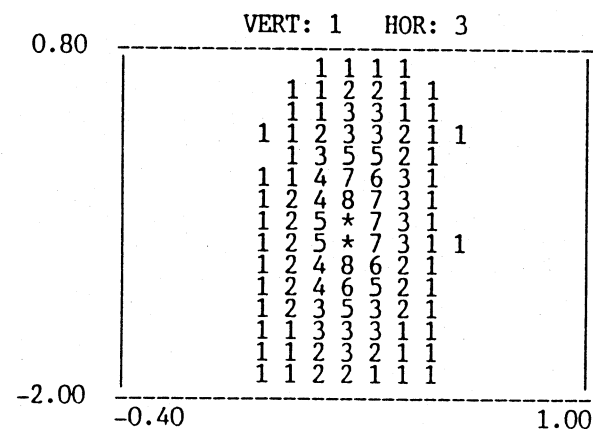
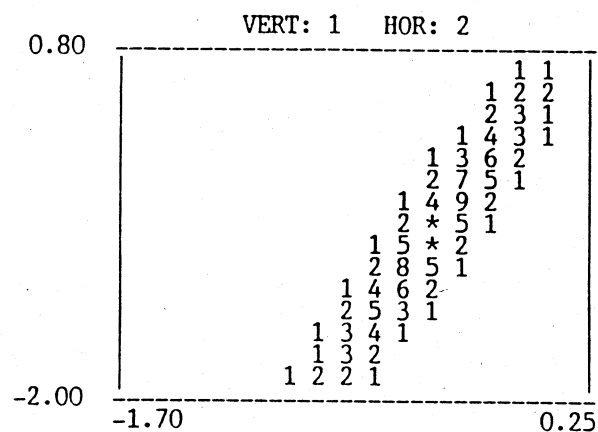
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DENOMINATOR: MEAN = 0.108995E+07, STD. DEV. = 0.203021E+07

BIVARIATE MARGINAL DENSITIES OF IMPORTANCE FUNCTION (TRUNCATED)





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