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AN ALGORITHM FOR THE COMPUTATION OF POSTERIOR MOMENTS AND DENSITIES USING SIMPLE IMPORTANCE SAMPLING

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AN ALGORITHM FOR THE COMPUTATION OF POSTERIOR MOMENTS AND DENSITIES USING SIMPLE IMPORTANCE SAMPLING

by

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Abstract

In earlier work (van Dijk (1984, Chapter 3)) one of the authors discussed the use of Monte Carlo integration methods for the computation of the multivariate integrals that are defined in the posterior moments and the posterior densities of the parameters of interest of econometric models. In the present paper we describe the computational steps of one Monte Carlo method, mentioned in that work, which is known in the literature as importance sampling. Further, we have prepared a set of standard programs, which may be used for the implementation of a simple case of importance sampling. The computer programs have been written in Fortran.

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1. INTRODUCTION

In earlier work [see van Dijk (1984, Chapter 3), and the references cited there] one of the authors discussed the use of Monte Carlo integration methods for the computation of the multivariate integrals that are defined in the posterior moments and the posterior densities of the parameters of interest of econometric models. In the present paper we describe the computational steps of one Monte Carlo method, mentioned in that work. This method is a simple application of a technique that is known in the literature as importance sampling [see Hammersley and Handscomb (1964)]. Further, we have prepared a set of standard computer programs, which can be used for the application of importance sampling. The computer programs have been written in Fortran 77.

The multivariate integrals that we consider may be described briefly as follows. Let θ be an ℓ-vector of parameters of interest and let $g(\theta)$ be an integrable function of θ. The posterior mean of $g(\theta)$ is defined as

$$Eg(\theta) = \frac{\int g(\theta)p(\theta)d\theta}{\int p(\theta)d\theta} \tag{1.1}$$

where $p(\theta)$ is a kernel of a posterior density function. That is, $p(\theta)$ is proportional and not equal to a density function and the denominator of (1.1) plays the role of integrating constant, similar to the role of $\sqrt{2\pi}$ in the case of the normal distribution. We assume that $g(\theta)p(\theta)$ is integrable on a certain region of integration. Simple examples of $g(\theta)$ are $g(\theta) = \theta$ and $g(\theta) = \theta\theta'$. Note that g may be a vector or a matrix. We emphasize that $g(\theta)$ may also be a complicated nonlinear function of θ such as the implied multipliers of the structural parameters of a simultaneous equation model [see, e.g., van Dijk and Kloek (1980) and van Dijk (1984), Chapter 4]. There exist several other examples of nontrivial nonlinear functions of θ. For an example in the statistical literature we refer to Kass (1985), and for some examples in the econometric literature we refer to van Dijk (1985), Zellner (1985), and Geweke (1986).

Monte Carlo (MC) integration methods make use of the property that generating a large sample of random numbers is very easy using a computer procedure. The value of an integral is then estimated in the sampling theory tradition using this set of random numbers. So, MC methods change the integration problem into a statistical estimation problem. A clear and concise introduction to Monte Carlo has been given by Hammersley and Handscomb (1964).

The contents of this paper has been organized as follows. In Section 2 we describe a Monte Carlo algorithm that is based on the principle of importance sampling. Some suggestions for further work are given in Section 3. In the Appendix we present examples of computer output using the prepared computer programs for a simple case of importance sampling.

2. SIMPLE IMPORTANCE SAMPLING

In this section we discuss the application of a simple case of importance sampling to the computation of the integrals defined in posterior first-order and second-order moments and the application of importance sampling to the computation of univariate and bivariate marginal posterior densities. For more details on the principle of importance sampling we refer to Hammersley and Handscomb (1964, Chapter 5) and van Dijk (1984, Chapter 3).

The vector of posterior first-order moments is obtained from (1.1) by defining $g(\theta) = \theta$. This yields

$$E\theta = \frac{\int \theta p(\theta)d\theta}{\int p(\theta)d\theta} \tag{2.1}$$

Suppose that it is not known how one can generate a sample of random drawings from a distribution with density equal or proportional to $p(\theta)$, but it is known how one can generate a random sample from a distribution with a density equal (or proportional) to $I(\theta)$, which is different from $p(\theta)$. Suppose further, that $I(\theta)$ is a reasonable approximation of $p(\theta)$. One can replace $p(\theta)$ in (2.1) by $w(\theta)I(\theta)$ where the weight function $w(\theta)$ is defined as $w(\theta) = p(\theta)/I(\theta)$. This yields

$$E\theta = \frac{\int \theta w(\theta)I(\theta)d\theta}{\int w(\theta)I(\theta)d\theta} \tag{2.2}$$

where $I(\theta)$ is restricted to be positive on the region of integration. $I(\theta)$ is known in the literature as importance function. In this paper we make use of a simple choice with respect to the class of importance functions. That is, we opt for the multivariate Student-t class of density functions [see below]. We make use of the term Simple Importance Sampling (SIS) in this case. For more details on the choice of an importance function and for some alternatives to simple importance sampling we refer to van Dijk (1984, Chapter 3) and van Dijk and Kloek (1985).

Next, let $\theta^{(1)}, \ldots, \theta^{(N)}$ be a random sample from a distribution with a density function equal (or proportional) to $I(\theta)$. That is, $\theta^{(1)}, \ldots, \theta^{(N)}$ is a sequence of independently distributed random variables with a common distribution function. Let $\theta^{(i)}$ be the typical i-th element of this sequence. Then the importance sampling estimator of the j-th element of the vector $E\theta$ is given as

$$\hat{E}(\theta_j) = \frac{\frac{1}{N}\sum_{i=1}^{N} \theta_j^{(i)} w(\theta^{(i)})}{\frac{1}{N}\sum_{i=1}^{N} w(\theta^{(i)})} \qquad (j = 1, \ldots, \ell) \qquad (2.3)$$

$$= \sum_{j=1}^{N} \theta_j^{(i)} w^*(\theta^{(i)})$$

where $$w^*(\theta^{(i)}) = \frac{w(\theta^{(i)})}{\sum_{i=1}^{N} w(\theta^{(i)})}$$

This estimator may be interpreted as a weighted sample mean of the above mentioned random sample where $w^*(\theta^{(1)}), \ldots, w^*(\theta^{(N)})$ are the weights. The weighted sample mean is a good approximation of $E\theta_j$ if the sampe size N is sufficiently large and the variation in the weights is bounded. In order to evaluate the numerical accuracy of the estimator (2.3), we are interested in the variation of $w(\theta)$ and of $\theta_j w(\theta)$, $j = 1, \ldots, \ell$. More details on the numerical accuracy of the approximation (2.3) are given below.

Second-order posterior moments can be computed in a similar way as first-order moments. The matrix of second-order moments around the mean is given as

$$E(\theta - E\theta)(\theta - E\theta)' = E(\theta\theta)' - (E\theta)(E\theta)' \qquad (2.4)$$

The first term on the right hand side of (2.4) is defined as the matrix of second-order moments around zero. By making use of $p(\theta) = w(\theta)I(\theta)$ one can write

$$E(\theta\theta)' = \frac{\int \theta\theta' w(\theta)I(\theta)d\theta}{\int w(\theta)I(\theta)d\theta} \qquad (2.5)$$

An importance sampling estimator for the (j,k)-th element of (2.5) is given as

$$\hat{E}(\theta_j\theta_k) = \frac{\frac{1}{N}\sum_{i=1}^{N} \theta_j^{(i)}\theta_k^{(i)}w(\theta^{(i)})}{\frac{1}{N}\sum_{i=1}^{N} w(\theta^{(i)})} \qquad (j,k = 1, \ldots, \ell) \qquad (2.6)$$

Practical details with respect to the computation of the estimators (2.3) and (2.6) are discussed below.

The approximation of the posterior first-order moments by means of the weighted sample mean, given in (2.3), involves a numerical error. Estimates of this numerical error can be obtained by making use of results from large sample theory [see, e.g., Cramér (1946) or Rao (1973)]. In particular, given certain regularity conditions from central limit theory, it follows that the estimator $\hat{E}(\theta_j)$ is approximately normally distributed with mean $E\theta_j$ and variance σ_j^2/N. [This normal approximation becomes more accurate as N becomes larger.] The estimator $\hat{E}(\theta_j)$ is a ratio of correlated random variables [compare the first line of (2.3)]. The formula for σ_j^2 is, therefore, more complex than the usual definition of a variance. The expression for σ_j^2 can be written as follows. We start with (2.3) and define

$$\hat{t}_j = \frac{1}{N}\sum_{i=1}^{N} \theta_j^{(i)}w(\theta^{(i)}) \qquad (j = 1, \ldots, \ell) \qquad (2.7)$$

$$\hat{t}_0 = \frac{1}{N}\sum_{i=1}^{N} w(\theta^{(i)}) \qquad (2.8)$$

Then we can rewrite (2.3), using (2.7) and (2.8). Next, we make use of the approximation for σ_j^2 given in Cramér (1946, Section 28.4). Then one can write

$$\sigma_j^2 \approx \left(\frac{\partial\hat{\theta}_j}{\partial\hat{t}_j}\right)^2 \text{var } \hat{t}_j + 2\frac{\partial\hat{\theta}_j}{\partial\hat{t}_j}\frac{\partial\hat{\theta}_j}{\partial\hat{t}_0}\text{cov}(\hat{t}_j, \hat{t}_0) + \left(\frac{\partial\hat{\theta}_j}{\partial\hat{t}_0}\right)^2 \text{var } \hat{t}_0 \qquad (2.9)$$

where, for convenience, we made use of the notation $\hat{\theta}_j$, instead of the more cumbersome notation $\hat{E}(\theta_j)$, in order to denote the right hand side of (2.3). An estimator for σ_j^2 follows directly from (2.9) once estimators for var $\hat{t}_j$, var $\hat{t}_0$ and cov($\hat{t}_j$, $\hat{t}_0$) are determined. By making use of (2.7) and (2.8) and standard theory on sample moments [compare, e.g., Mood, Graybill and Boes (1974, Chapters 2 and 6)] one can write the Monte Carlo estimators, using importance sampling, for the moments given at the right hand side of (2.9) as

$$\hat{var}(\hat{t}_j) = \frac{1}{N} \sum_{i=1}^{N} [\theta_j^{(i)} w(\theta^{(i)})]^2 - (\hat{t}_j)^2 \tag{2.10}$$

$$\hat{var}(\hat{t}_0) = \frac{1}{N} \sum_{i=1}^{N} [w(\theta^{(i)})]^2 - (\hat{t}_0)^2 \tag{2.11}$$

$$\hat{cov}(\hat{t}_j, \hat{t}_0) = \frac{1}{N} \sum_{i=1}^{N} \theta_j^{(i)} [w(\theta^{(i)})]^2 - \hat{t}_j \hat{t}_0 \tag{2.12}$$

Given an estimator $\hat{\sigma}_j^2$ for σ_j^2, one can define in the usual way a 95 per cent confidence interval for $E\theta_j$. Let $B_j = 1.96\ \hat{\sigma}_j/\sqrt{N}$, where 1.96 is taken from the table of the standard normal integral that are listed in most textbooks on statistics [see, e.g., Mood, Graybill, and Boes (1974, p. 522)]. Then the interval $[\hat{E\theta}_j - B_j, \hat{E\theta}_j + B_j]$ contains the value of $E\theta_j$ with a probability equal to 0.95. A value for $\sigma_j/\sqrt{N}$ will be defined as an <u>absolute numerical error</u> and $(\hat{\sigma}_j/\sqrt{N})/\sigma_j$ will be defined as a <u>relative numerical error</u>.

Summarizing, for importance sampling estimates of the posterior first- and second-order moments we have to compute the following sums:

$$\sum_{i=1}^{N} w(\theta^{(i)}),\quad \sum_{i=1}^{N} \theta_j^{(i)} w(\theta^{(i)}),\quad \sum_{i=1}^{N} \theta_j^{(i)} \theta_k^{(i)} w(\theta^{(i)}) \tag{2.13}$$

$$(j,k = 1, \ldots, \ell)$$

[compare (2.3) and (2.6)]. For the evaluation of numerical errors of the posterior first-order moments we have to compute, in addition to (2.13), the sums:

$$\sum_{i=1}^{N} [w(\theta^{(i)})]^2,\quad \sum_{i=1}^{N} [\theta_j^{(i)} w(\theta^{(i)})]^2,\quad \sum_{i=1}^{N} \theta_j^{(i)} [w(\theta^{(i)})]^2 \tag{2.14}$$

$$(j = 1, \ldots, \ell)$$

[compare (2.7)-(2.12)]. These sums are listed in the computer program for simple importance sampling.

Univariate marginal posterior densities of θ_j, $j = 1, \ldots, \ell$, can be approximated by so-called frequency <u>histograms</u> or frequency <u>polygones</u> using MC methods. We start by defining (a_{k-1}, a_k), $k = 1, \ldots, K$, as a bounded interval for the parameter θ_j, $j = 1, \ldots, \ell$. Further, let $d(\theta)$ be a dummy variable defined as

$$d(\theta) = 1 \quad \text{if } a_{k-1} < \theta_j < a_k \qquad (2.15)$$
$$= 0 \quad \text{elsewhere}$$

Then the posterior probability P_k, defined as $P_k = P[a_{k-1} < \theta_j < a_k]$, is given as

$$P_k = \frac{\int d(\theta)p(\theta)d\theta}{\int p(\theta)d\theta} \quad (k = 1, \ldots, K) \qquad (2.16)$$

The probabilities $P_1, \ldots, P_K$ can be used for the construction of a frequency histogram. Further, the posterior density of θ_j evaluated at $\frac{1}{2}(a_{k-1} + a_k)$ can be approximated by $P_k/(a_k - a_{k-1})$ if the interval (a_{k-1}, a_k) is sufficiently small. This approximation of the posterior density at K points can be used for the construction of a frequency polygon.

Given that $I(\theta)$ is an importance function for $p(\theta)$ we can rewrite (2.16) as

$$P_k = \frac{\int d(\theta)w(\theta)I(\theta)d\theta}{\int w(\theta)I(\theta)d\theta} \qquad (2.17)$$

An importance sampling estimator for (2.17) may be derived as follows. Let $\theta^{(1)}, \ldots, \theta^{(N)}$ be a random sample generated from a distribution with density $I(\theta)$. Further, let $\bar{\theta}^{(h)} = \theta^{(h(i))}$, where $h = h(i)$ is generated by the following rule

$$h(0) = 0 \qquad (2.18)$$
$$h(i) = h(i-1) + d(i) \qquad (i = 1, \ldots, N)$$

where

$$d(i) = 1 \qquad \text{if } a_{k-1} < \theta_j^{(i)} < a_k$$
$$= 0 \qquad \text{elsewhere}$$

Finally, let N_1 be defined as $N_1 = h(N)$. Then an importance sampling estimator for (2.17) is given as

$$\hat{P}_k = \frac{\frac{1}{N}\sum_{h=1}^{N_1} w\left(\bar{\theta}^{(h)}\right)}{\frac{1}{N}\sum_{i=1}^{N} w\left(\theta^{(i)}\right)} \qquad (2.19)$$

The definition of the estimator $\hat{P}_k$ is a bit tedious, but the computation of $\hat{P}_k$ is very simple. In fact, one has only to determine the particular interval to which a weight $w(\theta^{(i)})$ belongs. This is especially simple when the interval width $a_k - a_{k-1}$ is the same for all k. Let b be the common interval width for k = 1, ..., K. Let r be a real number given as

$$r = (\theta_j^{(i)} - a_0)/b + 1 \tag{2.20}$$

That is, r is a real number in the interval [1, K+1]. Truncate r at its decimal point in order to make r an integer, defined as ir. Then it follows that ir is the interval to which a particular weight $w(\theta^{(i)})$ belongs.[1] So, estimates for P_k are computed by adding the weights that belong in each interval and by dividing the sum of the weights in each interval by the total sum of the weights. Details are presented in the computer program. Minor modifications of the procedure described here are necessary when the intervals have unequal width. Further, the extreme values a_0 and a_K may be equal to minus and plus infinity. Finally, we note that the computation of bivariate marginal posterior densities proceeds in a similar way as the computation of the univariate marginal posterior densities. Details are presented in the computer programs listed in the Appendix.

Next, we discuss the structure of a computer program for Simple Importance Sampling (SIS). The different computational steps are shown in the flow diagram, given in Figure 1. The computer program starts with statements that refer to the initial value of a random number generator and to initial zero-values. Note that we make use of an arrow sign (instead of an equality sign) in several statements. For instance, one interprets $S^{(0)} \leftarrow 0$ as: 'the value zero is assigned to the variable (or the array of variables) S with superindex 0.' The major part of the program refers to two so-called do loops. In the inner loop, with the index i, one has as typical statement

$$S^{(i)} \leftarrow S^{(i-1)} + g(\theta^{(i)})w(\theta^{(i)}) \qquad (i = 1,\dots,N) \tag{2.21}$$

The symbol $S^{(i)}$ denotes the i-th partial sum of a sequence of random function values, defined as

1. The case where ir is exactly equal to K+1 is not important, since it has probability measure zero.

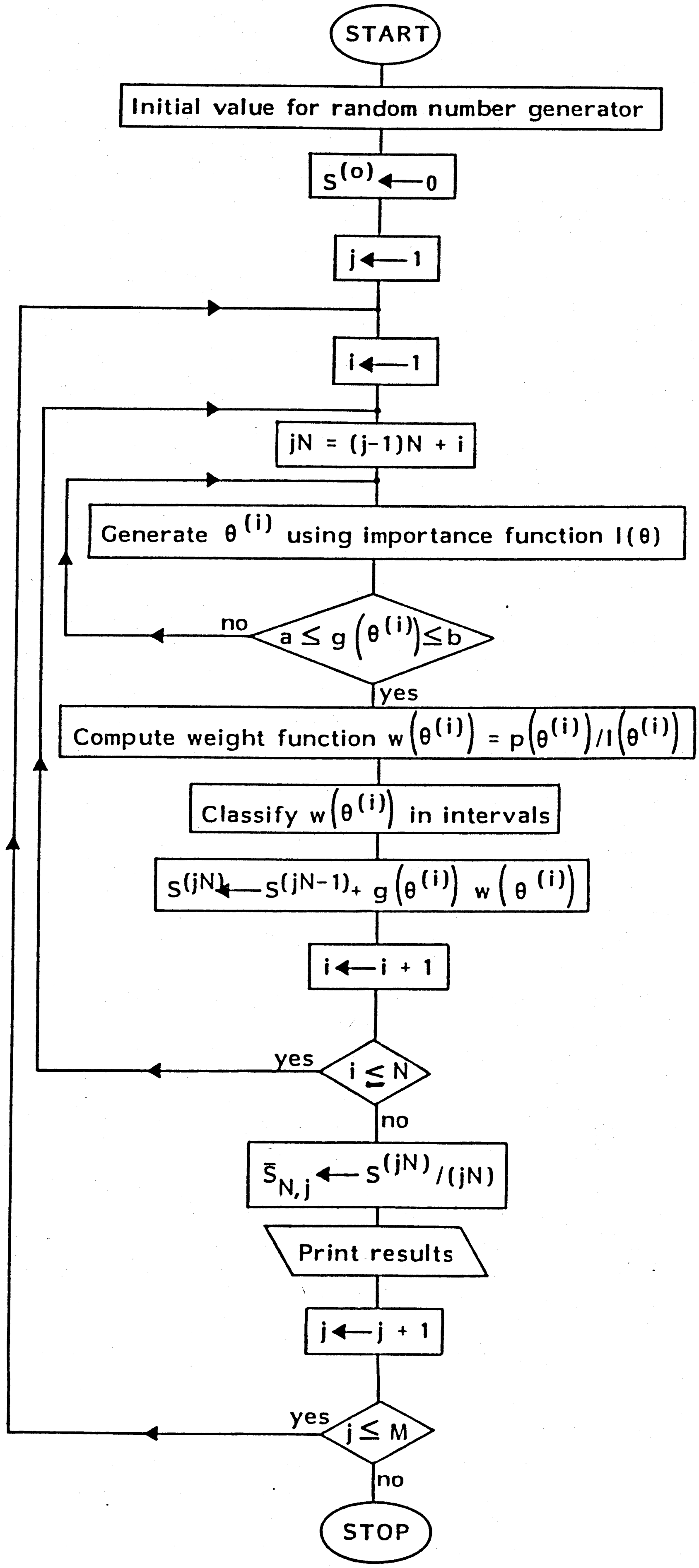

Figure 1. Flow diagram for simple importance sampling

$$S^{(i)} = g(\theta^{(1)})w(\theta^{(1)}) + g(\theta^{(2)})w(\theta^{(2)}) + \ldots + g(\theta^{(i)})w(\theta^{(i)}) \qquad (2.22)$$

The assignment statement (2.21) indicates that one does not store a large sample of random numbers from which sample moments are computed (as is suggested by equation (2.22)), but one makes use of a statement that updates the value of $S^{(i)}$ after each accepted Monte Carlo drawing of $\theta^{(i)}$. After N random drawings one computes sample averages such as

$$\bar{S}_N \leftarrow \frac{S^{(N)}}{N} \qquad (2.23)$$

The accuracy of the sample means may be studied by increasing the size of the sample from N to 2N, 3N, ..., MN. The outer loop of the program, with the index j, enables one to print results at each value of jN, with j = 1,...,M. This explains why we make use of the index jN at several places in the flow diagram. Given certain regularity conditions, the Monte Carlo estimator $\hat{E}(\theta)$ converges with probability one to $E(\theta)$.

Apart from the two do loops that refer to updating procedures, there are two other major computational procedures in the computer program for simple importance sampling. First, the user has to supply a procedure that describes the computation of the posterior kernel studied in each particular case. Second, we have opted for a simple procedure that generates random vectors $\theta^{(1)}, \ldots, \theta^{(N)}$, which may be described as follows.

The most obvious solution to the restriction of the light tails of the multivariate normal importance function is the choice of the multivariate Student t density as a functional form of the importance function, with the multivariate normal density as a limiting case. A multivariate Student t density may be written as

$$I(\theta) = c[\lambda + (\theta-\theta^0)'V^{-1}(\theta-\theta^0)]^{-\frac{1}{2}(\lambda+\ell)} \qquad (2.24)$$

where θ^0 is the center of the distribution, V is a positive definite scaling matrix; $\lambda > 0$ is the degrees of freedom parameter and ℓ the dimension of θ [compare, e.g., Zellner (1971, Appendix B2), where the numerical constant c is spelled out].

Random ℓ-vectors $\theta^{(1)}, \ldots, \theta^{(N)}$, distributed according to a multivariate Student t distribution, are generated as follows.

Step 1. Generate an $(\ell+\lambda)$-vector $u^{(i)}$ of independent standard normally distributed random variables. [Efficient techniques for this step can be found in, e.g. Newman and Odell (1971), Atkinson and Pearce (1976), Rubinstein (1981), Marsaglia (1984), and the references cited there. We make use of the normal random number generator from the NAG-Library, given as subroutine G05 DDF.] Note that ℓ is usually considered as a positive integer instead of a positive real variable when it is used for the generation of Student t random variables.

Step 2. Partition the vector $u^{(i)}$ in a subvector $u_1^{(i)}$ that contains the first ℓ elements of $u^{(i)}$ and a subvector $u_2^{(i)}$ that contains the remaining ℓ elements of $u^{(i)}$. Then, obtain an ℓ-vector of so-called standard Student t distributed random variables $v^{(i)}$ from

$$v^{(i)} = \frac{u_1^{(i)}}{(u_2^{(i)\prime} u_2^{(i)}/\lambda)^{\frac{1}{2}}} \tag{2.25}$$

Step 3. Premultiply the ℓ-vector $v^{(i)}$ by a matrix A which satisfies V = AA'. One may obtain A from the eigenvalues and eigenvectors of V but it is also possible to construct a triangular matrix A by a Cholesky technique. Then, obtain the ℓ-vector $\theta^{(i)}$ from

$$\theta^{(i)} = \theta^0 + Av^{(i)} \tag{2.26}$$

In case the importance function is multivariate normal take $\ell = 0$ in Step 1. Step 2 can be deleted and $v^{(i)}$ is replaced by $u^{(i)}$ in Step 3.

If we adopt the multivariate Student t density as a functional form for the importance function, we need a way to specify its parameters. It seems reasonable to take for θ^0 the posterior mode of θ and for V minus the inverse of the Hessian of the log posterior density, evaluated at the posterior mode, possibly multiplied by a scalar. The posterior mode is determined by means of numerical minimization of minus the log of the posterior kernel. It is our experience that the use of an analytical gradient increases the numerical precision, in particular around the optimum where the gradient is almost zero. The reason is that the rounding errors of a numerically approximated gradient may cause convergence problems when the function mentioned above is relatively flat around it's optimum value. Further, the evaluation of minus the Hessian of this function at the optimum values is simplified by making use of an analytical gradient. That is, instead of using second-order numerical differentiation formulae with respect to the objective function one makes use of the analytical gradient and of first-order numerical differentiation formulae [compare Goldfeld and Quandt (1972, p. 19)].

We end the discussion of the structure of the computer program with two remarks. First, the computation of the weights $w(\theta^{(i)})$ is a key step in any importance sampling procedure. The distribution of the weights contains relevant information on the accuracy of importance sampling. The weights are nearly equal if the approximation of $I(\theta)$ to $p(\theta)$ is very accurate. In the opposite case one finds great variation in the weights. An approximation of the distribution of the weights $w(\theta^{(i)})$, $i = 1, \ldots, N$ is computed as a byproduct in the computer program for simple importance sampling. Second, we assume that the region of integration is bounded. Therefore, we have inserted a rejection step in the program since the Student t density is defined on the entire region $\mathbb{R}^{\ell}$, where ℓ is the dimension of the vector θ.

We emphasize that one can make use of the program SIS in an <u>sequential</u> way. That is, one starts with the posterior mode and minus the inverse of the Hessian matrix, evaluated at the mode, as location and scale parameters of the multivariate Student t importance function. After a round of Monte Carlo of, for instance, N = 2000 random drawings one uses the posterior mean and the posterior covariance matrix as new starting values for the parameters of the Student t importance function.

We end this section with some examples. In van Dijk (1984, Chapter 4), the author discussed the computation of the prior and posterior moments of the structural parameters and the implied multipliers of a well-known simultaneous econometric model, which is known as Klein's Model I [see Klein (1950)]. Two parameters in this model are particularly interesting, that is, the government-expenditures-multiplier and the tax-multiplier in the reduced form equation for national income, Y. The government-expenditures-multiplier indicates the change in Y when government expenditures, G, change with, for instance, one billion dollars and the tax-multiplier indicates the change in Y when taxes, T, change with such an amount. If one takes a uniform prior on the structural parameters of interest on a bounded region [in this case the unit region, see van Dijk (op.cit.)] then one can compute the bivariate prior density of the government- and tax-multipliers by direct simulation and simple rejection. The result is shown in Figure 2. It is seen that the relevant interval for the tax multiplier is [-4.0, -.5] and the relevant interval for the government multiplier is [.5, 4.]. The results indicate that the prior distribution is skew, and that it has a long tail. The posterior results have

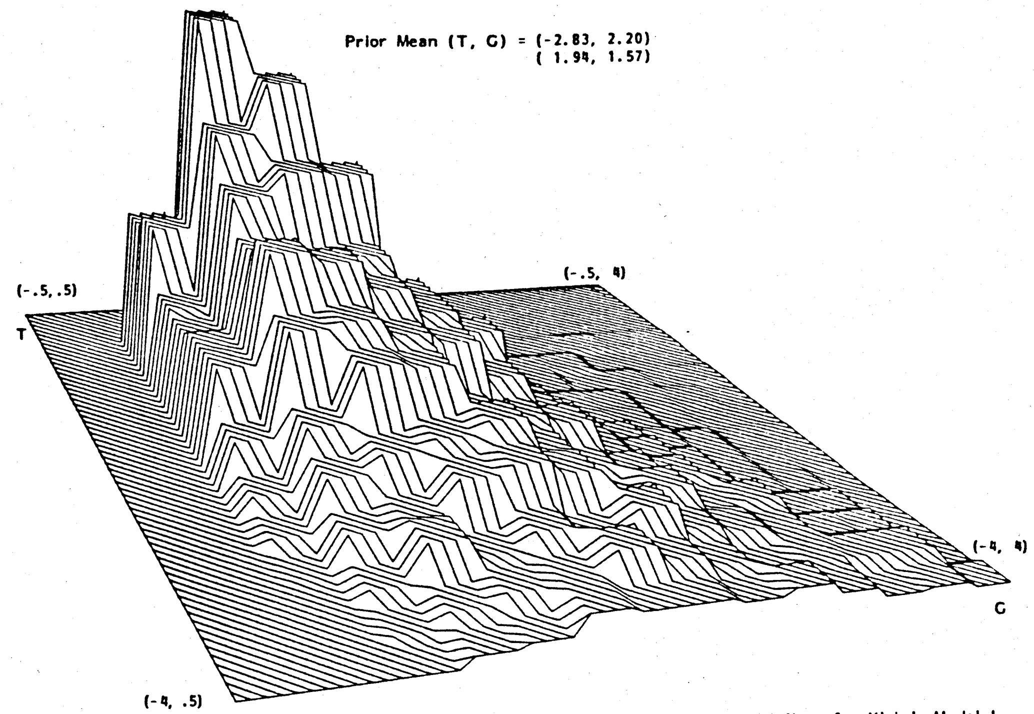

Figure 2A Bivariate marginal prior densities of short-run government and tax multipliers for Klein's Model I

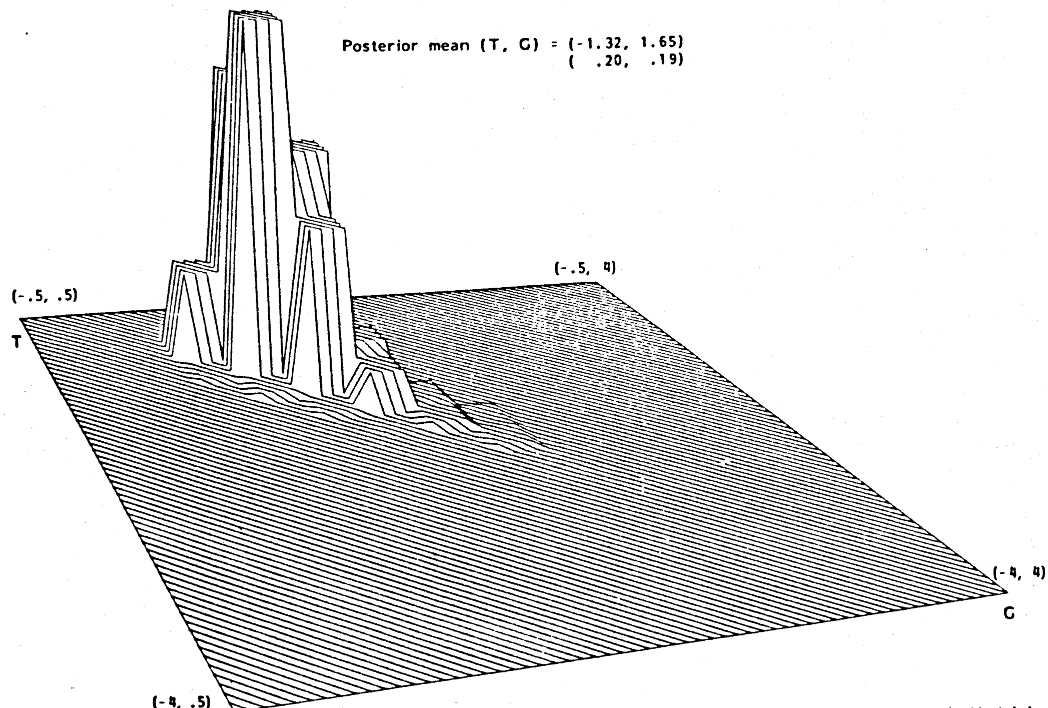

Figure 2B Bivariate marginal posterior densities of short-run government and tax multipliers for Klein's Model I

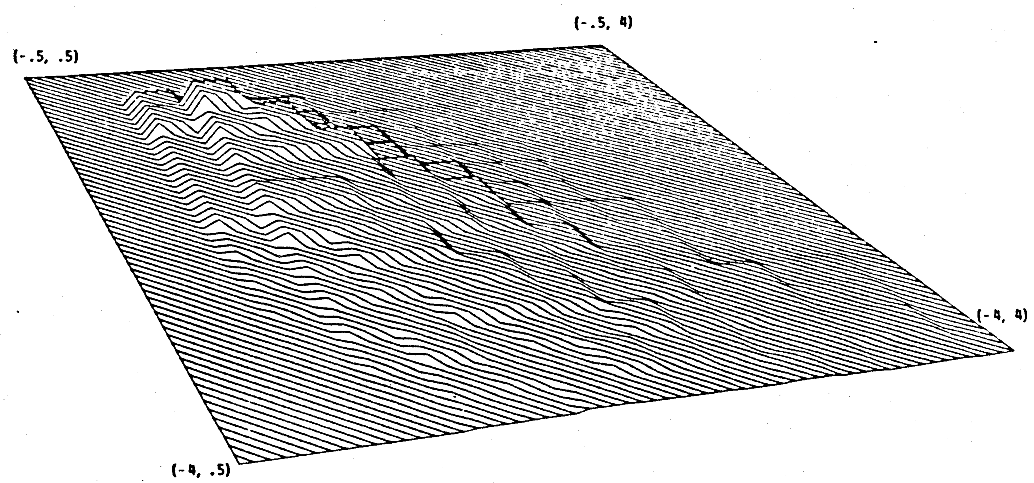

Figure 2C Scaled bivariate marginal prior densities been computed by SIS. These posterior results indicate that the data are quite informative and dominate the prior information strongly. For a more detailed discussion of the results we refer to van Dijk (1984).

In the Appendix we present examples of computer output using simple importance sampling. The posterior densities refer to a well-known textbook model of simultaneous equations, i.e., Johnston's model, which involves three-dimensional numerical integration. For details on the specification of the model we refer to Johnston (1963, p.269) and van Dijk (1984, Chapter 3).

3. REMARKS

The computer program for simple importance sampling is a first step towards the development of standard software for Bayesian analysis of econometric and statistical models. Further developments in this area are needed. An immediate extension is to construct a family of importance functions that is more flexible than the symmetric multivariate Student t density. Some preliminary experiments with an importance function that consists of a finite mixture of conjugate densities appear promising. The results will be reported in a forthcoming paper.

Another field of research is to prepare a standard Fortran program for the method of mixed integration [van Dijk, Kloek, and Boender (1985)]. This is also a topic of current research.

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C
C     MAIN - PROGRAM FOR "SIMPLE IMPORTANCE SAMPLING" (S.I.S)
C
C     SEE: VAN DIJK, H.K., 1984, POSTERIOR ANALYSIS OF ECONOMETRIC
C              MODELS USING MONTE CARLO INTEGRATION, DOCTORAL
C              DISSERTATION (ERASMUS UNIVERSITY ROTTERDAM).
C
C     - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
      INTEGER   MAXD
C
C     THE NEXT PARAMETER-STATEMENT DETERMINES THE MAXIMUM DIMENSION OF
C     A PROBLEM THE PROGRAM CAN HANDLE. IF TOO SMALL, THE CURRENT NUMBER
C     HAS TO BE CHANGED HERE AS WELL AS IN THE SUBROUTINE "SISSUB".
C
      PARAMETER (MAXD=10)
C
      CHARACTER TITLE*60,XFILE*19,HFILE*19,SFILE*19
      LOGICAL   IUMPD,IBMPD,ISAVE
      INTEGER   I,J,IPRIN,NDIM,LAMBDA,ISTRT,NDRAW,MROUND,NDCL,IROTA,
     &          MROTA,IFCNT
      DOUBLE PRECISION FPOST,SCFAC
      DOUBLE PRECISION HINV(MAXD,MAXD),PMODE(MAXD),LBOUND(MAXD),
     &                 UBOUND(MAXD),MEANPO(MAXD),COVPO(MAXD,MAXD),
     &                 THETA(MAXD)
C
      DATA      IPRIN/3/,NDCL/MAXD/
C
 3505 FORMAT (' TYPE TITLE OF PROBLEM (MAX. 60 CHARACTERS)')
 3510 FORMAT (A)
 3515 FORMAT (' TYPE THE DIMENSION OF THE VECTOR "THETA"')
 3520 FORMAT (' TYPE NAME OF OUTPUT-FILE ')
 3525 FORMAT (' TYPE NAME OF INPUT-FILE FOR "PMODE" AND "HINV" ')
 3530 FORMAT (' TYPE LOWER BOUNDS FOR PARAMETERS (FREE FORMAT)')
 3535 FORMAT (' TYPE UPPER BOUNDS FOR PARAMETERS (FREE FORMAT)')
 3540 FORMAT (' TYPE BOOLEAN PARAMETER FOR COMPUTATION OF UNIVARIATE',
     &        ' POSTERIOR DENSITIES: 0 = NO, 1 = YES ')
 3545 FORMAT (' TYPE BOOLEAN PARAMETER FOR COMPUTATION OF BIVARIATE',
     &        ' POSTERIOR DENSITIES: 0 = NO, 1 = YES ')
 3550 FORMAT (' TYPE INITIAL VALUE OF RANDOM NUMBER GENERATOR ')
 3555 FORMAT (' TYPE DEGREES OF FREEDOM FOR THE STUDENT -T- IMPORT',
     &        'ANCE FUNCTION')
 3560 FORMAT (' TYPE NUMBER OF ROUNDS ')
 3565 FORMAT (' TYPE NUMBER OF RANDOM DRAWINGS FOR EACH ROUND ')
 3570 FORMAT (' TYPE NUMBER OF ROTATIONS')
 3585 FORMAT (' TYPE BOOLEAN PARAMETER FOR SAVING POSTERIOR MEAN AND',
     &        ' COVARIANCE-MATRIX: 0 = NO, 1 = YES')
 3590 FORMAT (' TYPE NAME OF THE SAVE-FILE')
C
C
C     TITLE OF THE PROBLEM FOR THIS RUN OF THE PROGRAM
C
      WRITE(6,3505)
      READ(5,3510) TITLE
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C
C     THE DIMENSION OF THE PARAMETER VECTOR "THETA"
C
      WRITE(6,3515)
      READ(5,*) NDIM
C
C     THE NAME OF THE OUTPUT PRINT-FILE
C
      WRITE(6,3520)
      READ(5,3510) XFILE
C
      OPEN (UNIT=IPRIN, FILE=XFILE, STATUS='NEW')
C
C     THE NAME OF THE INPUT-FILE FOR "PMODE" AND "HINV"
C
      WRITE(6,3525)
      READ(5,3510) HFILE
C
C     READ POSTERIOR MODE AS "PMODE" AND MINUS-HESSIAN-INVERSE AS "HINV"
C
      OPEN (UNIT=23, FILE=HFILE, STATUS='OLD', FORM='FORMATTED')
      READ(23,*) (PMODE(I), I = 1,NDIM)
      DO 100 I = 1,NDIM
          READ(23,*) (HINV(I,J), J = 1,I)
  100 CONTINUE
      CLOSE (UNIT=23)
C
C     INITIALIZE POSTERIOR FUNCTION ROUTINE
C
      CALL PSTROR(THETA,NDIM,FPOST,IFCNT,SCFAC,.TRUE.)
C
C     LOWER AND UPPER BOUNDS FOR PARAMETER VECTOR "THETA"
      WRITE(6,3530)
      READ(5,*) (LBOUND(I), I = 1,NDIM)
      WRITE(6,3535)
      READ(5,*) (UBOUND(I), I = 1,NDIM)
C
C     BOOLEAN PARAMETER THAT DETERMINES WHETHER UNIVARIATE MARGINAL
C     POSTERIOR DENSITIES ARE COMPUTED
C
      WRITE(6,3540)
      READ(5,*) I
      IUMPD = I .EQ. 1
C
C     BOOLEAN PARAMETER THAT DETERMINES WHETHER BIVARIATE MARGINAL
C     POSTERIOR DENSITIES ARE COMPUTED
C
      WRITE(6,3545)
      READ(5,*) I
      IBMPD = I .EQ. 1
C
C     INITIAL VALUE FOR THE RANDOM NUMBER GENERATOR
C
      WRITE(6,3550)
      READ(5,*) ISTRT
      CALL G05CBF(ISTRT)
C
C     DEGREES OF FREEDOM FOR THE STUDENT -T- IMPORTANCE FUNCTION
C
      WRITE(6,3555)
      READ(5,*) LAMBDA
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```
C
C     NUMBER OF ROUNDS IS DEFINED AS "MROUND"
C
      WRITE(6,3560)
      READ(5,*) MROUND
C
C     NUMBER OF DRAWINGS IN EACH ROUND IS DEFINED AS "NDRAW"
C
      WRITE(6,3565)
      READ(5,*) NDRAW
C
C     NUMBER OF ROTATIONS
C
      WRITE(6,3570)
      READ(5,*) MROTA
C
      DO 200 IROTA = 1,MROTA
          CALL SISSUB(NDCL,NDIM,PMODE,HINV,LAMBDA,LBOUND,UBOUND,NDRAW,
     &                MROUND,IROTA,ISTRT,IPRIN,IUMPD,IBMPD,THETA,FPOST,
     &                IFCNT,MEANPO,COVPO,TITLE,*400)
C
          DO 250 I = 1,NDIM
              PMODE(I) = MEANPO(I)
              DO 260 J = 1,NDIM
                  HINV(I,J) = COVPO(I,J)
  260         CONTINUE
  250     CONTINUE
  200 CONTINUE
C
C     SAVE "MEANPO" AND "COVPO"
C
      WRITE(6,3585)
      READ(5,*) I
      ISAVE = I .EQ. 1
      IF (ISAVE) THEN
          WRITE(6,3590)
          READ(5,3510) SFILE
          OPEN (UNIT=25, FILE=SFILE, STATUS='NEW', FORM='FORMATTED')
          WRITE(25,*) (MEANPO(I), I = 1,NDIM)
          DO 300 I = 1,NDIM
              WRITE(25,*) (COVPO(I,J), J = 1,I)
  300     CONTINUE
          CLOSE (UNIT=25)
      END IF
C
  400 CLOSE (UNIT=IPRIN)
C
      STOP
      END
```

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C     ------------------------------------------------------------
C     ------------------------------------------------------------
      SUBROUTINE SISSUB(NDCL,LDIM,PMODE,HINV,LAMBDA,LBOUND,UBOUND,NDRAW,
     &                  MROUND,IROTA,START,IPRIN,IUMPD,IBMPD,THETA,
     &                  FPOST,IFCNT,MEANPO,COVPO,TITLE,*)
      CHARACTER TITLE*60
      LOGICAL   IUMPD,IBMPD
      INTEGER   NDCL,LDIM,LAMBDA,NDRAW,MROUND,IROTA,START,IPRIN,IFCNT
      DOUBLE PRECISION FPOST
      DOUBLE PRECISION PMODE(LDIM),HINV(NDCL,LDIM),LBOUND(LDIM),
     &                 UBOUND(LDIM),MEANPO(LDIM),COVPO(NDCL,LDIM),
     &                 THETA(LDIM)
C
C     - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C     SIMPLE IMPORTANCE SAMPLING (S.I.S)
C
C
C VAR. NAME        TYPE I/O DESCRIPTION
C
C NDCL              I4   I  FIRST DIMENSION OF THE TWO-DIMENSIONAL
C                           ADJUSTABLE ARRAYS AS DECLARED IN THE CALLING
C                           PROGRAM
C LDIM              I4   I  DIMENSION OF THE PARAMETER VECTOR "THETA",
C                           ALSO SECOND DIMENSION OF TWO-DIMENSIONAL
C                           ADJUSTABLE ARRAYS AND THE DIMENSION OF
C                           THE ONE-DIMENSIONAL ADJUSTABLE ARRAYS
C PMODE(LDIM)       R8   I  POSTERIOR MODE
C HINV(NDCL,LDIM)   R8   I  MINUS-HESSIAN-INVERSE OF THE LOG POSTERIOR
C LAMBDA            I4   I  DEGREES OF FREEDOM OF THE STUDENT'T
C                           IMPORTANCE FUNCTION
C LBOUND(LDIM)      R8   I  LOWER BOUND OF THE PARAMETER VECTOR "THETA"
C UBOUND(LDIM)      R8   I  UPPER BOUND OF THE PARAMETER VECTOR "THETA"
C NDRAW             I4   I  NUMBER OF DRAWINGS PER ROUND
C MROUND            I4   I  NUMBER OF ROUNDS WHERE OUTPUT IS PRINTED
C IROTA             I4   I  SEQUENCE NUMBER OF THE CURRENT ROTATION
C START             I4   I  INITIAL VALUE FOR THE RANDOM NUMBER GENERATOR
C IPRIN             I4   I  FILE NUMBER OF THE OUTPUT PRINT FILE
C IUMPD             L4   I  = TRUE: U.M.P.D. ARE COMPUTED,
C                           = FALSE: U.M.P.D. ARE NOT COMPUTED
C IBMPD             L4   I  = TRUE: B.M.P.D. ARE COMPUTED,
C                           = FALSE: B.M.P.D. ARE NOT COMPUTED
C THETA(LDIM)       R8   O  THE PARAMETER VECTOR
C FPOST             R8   O  VALUE OF THE LOG OF THE POSTERIOR KERNEL FUNCTION
C IFCNT             I4   O  NUMBER OF FUNCTION EVALUATIONS
C MEANPO(LDIM)      R8   O  THE POSTERIOR MEAN
C COVPO(NDCL,LDIM)R8     O  THE POSTERIOR COVARIANCE MATRIX
C TITLE             C60  I  A TITLE FOR THE CURRENT PROBLEM
C *                 --   O  RETURN LABEL FOR ERROR CONDITION
C
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C POSTERIOR         --   I  A USER-SUPPLIED SUBROUTINE TO EVALUATE THE
C                           POSTERIOR FUNCTION:
C                            SUBROUTINE PSTROR(PAR,N,FP,NF,SC,IC)
C                            LOGICAL IC
C                            INTEGER N,NF
C                            DOUBLE PRECISION PAR(N),FP,SC
C                            PAR(N) : THE PARAMETER VECTOR
C                            N      : THE DIMENSION OF THE POSTERIOR FUNCTION
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C                            FP      : THE COMPUTED VALUE OF THE LOG POSTERIOR
C                                      FUNCTION
C                            NF      : THE "NF"TH COMPUTATION OF THE
C                                      POSTERIOR FUNCTION
C                            SC      : SCALE FACTOR
C                            IC      : IF (TRUE) SUBROUTINE CALL ONLY FOR
C                                      INITIALISATION PURPOSES,
C                                      IF (FALSE) SUBROUTINE CALL TO COMPUTE
C                                      THE VALUE OF THE POSTERIOR FUNCTION
C RESTRICT          --    I  A USER-SUPPLIED SUBROUTINE TO EVALUATE THE
C                            RESTRICTIONS ON THE PARAMETERS:
C                             SUBROUTINE RSTRCT(PAR,N,IC)
C                             LOGICAL IC
C                             INTEGER N
C                             DOUBLE PRECISION PAR(N)
C                             PAR(N) : THE PARAMETER VECTOR
C                             N      : THE DIMENSION OF THE PARAMETER VECTOR
C                             IC     : IF (TRUE) PARAMETERS DO NOT MEET THE
C                                      RESTRICTION,
C                                      IF (FALSE) PARAMETERS MEET THE
C                                      RESTRICTION.
C
C     - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
      INTEGER   MAXD,MDX
C
C     SEE REMARK IN THE MAIN-PROGRAM FOR THE NEXT STATEMENT
C
      PARAMETER (MAXD=10)
C
      PARAMETER (MDX=MAXD*(MAXD-1)/2)
      CHARACTER DAY*9
      LOGICAL   FAIL,PAGE,PPOW
      INTEGER   I,II,I1,I2,J,JJ1,JJ2,K,NCLASS,IFAIL,IRNACC,IRNREJ,
     &          JROUND,IDRAW,IDD,NDD,ITERM,ITIME,ERRMSG,NDRAW2
      INTEGER   IPOW(80),RELBMI(MDX,15,15),RELBMP(MDX,15,15),
     &          JJX(MAXD),NALF(11)
      DOUBLE PRECISION SUMW1,SUMW2,VALIMP,WEIGHT,WW,DUMMY,RNACC,
     &                 XX,YY,MEAND1,MEAND2,VARD,STDD,MEANN1,MEANN2,VARN,
     &                 COVAR,VARCOF,XNU,RN1,RN2,TOTUMP,TOTUMI,TERM,
     &                 MAXBMP,MAXBMI,SCFAC
      DOUBLE PRECISION HISTD(MAXD),HICOR(MAXD,MAXD),YHELP(MAXD),
     &                 WIDTH(MAXD),EIGVEC(MAXD,MAXD),EIGVAL(MAXD),
     &                 BOUND(MAXD,16),SUMIM1(MAXD),MEANIM(MAXD),
     &                 STDIM(MAXD),SUMIM2(MAXD,MAXD),COVIM(MAXD,MAXD),
     &                 RHOIM(MAXD,MAXD),SUMPO1(MAXD),STDPO(MAXD),
     &                 SUMPO2(MAXD,MAXD),RHOPO(MAXD,MAXD),
     &                 SUMER1(MAXD),SUMER2(MAXD),
     &                 ERROR(MAXD),RELERR(MAXD),CORREL(MAXD),
     &                 MAXW(10),MAXIHI(10),MAXPOS(10),MAXTHE(10,MAXD),
     &                 SUMUMP(MAXD,15),SUMUMI(MAXD,15),
     &                 RELUMP(MAXD,15),RELUMI(MAXD,15),
     &                 SUMBMP(MDX,15,15),SUMBMI(MDX,15,15)
C
      EXTERNAL      X02AAF,X02ADF
C
      DATA          NCLASS/15/,ERRMSG/0/,
     &              NALF/' ','1','2','3','4','5','6','7','8','9','*'/
C
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5000 FORMAT (//'  BOUNDS OF THE PARAMETERS')
5001 FORMAT ('0',10F11.4/' ',10F11.4/' ',10F11.4)
5002 FORMAT (//'  IMPORTANCE MEANS AND STANDARDDEVIATIONS')
5003 FORMAT (//'  IMPORTANCE CORRELATION MATRIX')
5004 FORMAT ('0',10F11.5/' ',10F11.5/' ',10F11.5)
5005 FORMAT (//'  IMPORTANCE COVARIANCE MATRIX')
5006 FORMAT (//'  EIGENVALUES OF MIN INVERSE HESSIAN MATRIX'/
    &        '0',10F12.6/' ',10F12.6/' ',10F12.6)
5007 FORMAT (//'  INITIAL VALUE OF RANDOM NUMBER GENERATOR :',
    &         ' CALL G05CBF(',I5,')')
5008 FORMAT (/'  IMPORTANCE MEANS AND STANDARDDEVIATIONS (TRUNCATED)')
5009 FORMAT (/'  IMPORTANCE CORRELATION MATRIX (TRUNCATED)')
5010 FORMAT (/'  IMPORTANCE COVARIANCE MATRIX (TRUNCATED)')
5011 FORMAT (/'  POSTERIOR MEAN AND STANDARDDEVIATION OF THE',
    &        ' PARAMETER VECTOR "THETA"')
5012 FORMAT (/'  POSTERIOR CORRELATION MATRIX OF THE PARAMETER',
    &        ' VECTOR "THETA"')
5013 FORMAT (/'  POSTERIOR COVARIANCE MATRIX OF THE PARAMETER',
    &        ' VECTOR "THETA"')
5014 FORMAT (//'  NUMERICAL ERROR ESTIMATES OF POSTERIOR MEAN',
    &        //'  ERROR          :',10F11.6/' ',17X,10F11.6/
    &                                   ' ',17X,10F11.6)
5015 FORMAT (/'  RELATIVE ERROR :',10F11.6/' ',17X,10F11.6/
    &                                   ' ',17X,10F11.6)
5016 FORMAT (/'  CORREL. COEFF. :',10F11.6/' ',17X,10F11.6/
    &                                   ' ',17X,10F11.6)
5017 FORMAT (//'  FREQUENCIES OF IPOW.    WEIGHT=0.*****E+IPOW'/)
5018 FORMAT (' ',15I8)
5019 FORMAT (//'  TEN DRAWINGS WITH LARGEST WEIGHT'//9X,'W',12X,
    &       'LN(IMP)',8X,'LN(POS)',10X,'(THETA(I), I = 1,NDIM)'/)
5020 FORMAT (' ',E15.7,2F15.7,3X,8F10.5/' ',48X,8F10.5/
    &        ' ',48X,8F10.5/' ',48X,6F10.5)
5021 FORMAT (//'  MARGINAL POSTERIOR DENSITIES "P", AND',
    &         ' MARGINAL IMPORTANCE DENSITIES (TRUNCATED) "I"')
5022 FORMAT ('0',3(A1,10X,'PARAMETER',I3,13X))
5023 FORMAT (' ',3(A1,17X,'P',7X,'I',9X))
5024 FORMAT (' ',3(A1,'(',F5.2,',',F5.2,')',2F8.3,6X))
5025 FORMAT (/'  BIVARIATE MARGINAL POSTERIOR DENSITIES')
5026 FORMAT (/'  BIVARIATE MARGINAL DENSITIES OF IMPORTANCE FUNCTION',
    &        ' (TRUNCATED)')
5027 FORMAT (//' ',2(A1,15X,'VERT:',I2,3X,'HOR:',I2,29X))
5028 FORMAT (' ',2(F5.2,2X,34('-'),20X))
5029 FORMAT (' ',2(A1,6X,'| ',15(A1,1X),' |',20X))
5030 FORMAT (' ',2(7X,F5.2,24X,F5.2,20X))
9101 FORMAT ('0','ERROR IN THE BOUNDS OF THE PARAMETERS')
9102 FORMAT ('0','ERROR IN THE NAG-ROUTINE "F02AMF", CALCULATING THE',
    &          ' EIGENVALUES AND EIGENVECTORS OF THE COVARIANCE-'/
    &        ' ','MATRIX OF THE PARAMETERS; ERROR-CODE "IFAIL" = ',I2)
9103 FORMAT ('0','ONE OR MORE EIGENVALUES ARE LESS OR EQUAL TO ZERO')
9104 FORMAT ('0','THE NUMBER OF REJECTED DRAWINGS EXCEEDS 500 TIMES',
    &          ' THE INITIAL NUMBER OF DRAWINGS:'/
    &        ' ','NUMBER OF ACCEPTED DRAWINGS: ',I10/
    &        ' ','NUMBER OF REJECTED DRAWINGS: ',I10)
9198 FORMAT ('0','PROGRAM TERMINATED BECAUSE OF UNKNOWN ERROR ',
    &           'CONDITION')
9199 FORMAT ('0','PROGRAM TERMINATED BECAUSE OF ERROR CONDITION')
C
      CALL DATE(DAY)
C
      NDRAW2 = 500 * NDRAW
      NDD    = LDIM * (LDIM - 1) / 2
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```
C
C     COMPUTE BOUNDS FOR INTERVALS THAT ARE USED FOR THE COMPUTATION
C     OF MARGINAL POSTERIOR DENSITIES
C
      DUMMY = DFLOAT(NCLASS)
      DO 100 I = 1,LDIM
          XX = LBOUND(I)
          YY = (UBOUND(I) - XX) / DUMMY
          IF (YY .LT. 1.0D-25) ERRMSG = 1
          WIDTH(I)  = YY
          DO 110 J = 1,NCLASS+1
              BOUND(I,J) = XX + DFLOAT(J-1) * YY
  110     CONTINUE
  100 CONTINUE
C
      CALL PRTHD(IPRIN,DAY,TITLE,IROTA,0)
C
C     WRITE INPUT DATA ON BOUNDS THAT DEFINE REGION OF INTEGRATION
C
      WRITE(IPRIN,5000)
      WRITE(IPRIN,5001) (LBOUND(I), I = 1,LDIM)
      WRITE(IPRIN,5001) (UBOUND(I), I = 1,LDIM)
      IF (ERRMSG .GT. 0) GOTO 9000
C
C     WRITE INPUT DATA ON POSTERIOR MODE AND MINUS HESSIAN INVERSE
C
      WRITE(IPRIN,5002)
      WRITE(IPRIN,5001) (PMODE(I), I = 1,LDIM)
      DO 120 I = 1,LDIM
          HISTD(I) = DSQRT(DABS(HINV(I,I)))
  120 CONTINUE
      WRITE(IPRIN,5001) (HISTD(I), I = 1,LDIM)
      WRITE(IPRIN,5003)
      DO 130 I = 1,LDIM
          XX = HISTD(I)
          DO 140 J = 1,I
              HICOR(I,J) = HINV(I,J) / XX / HISTD(J)
  140     CONTINUE
          WRITE(IPRIN,5004) (HICOR(I,J), J = 1,I)
  130 CONTINUE
      WRITE(IPRIN,5005)
      DO 150 I = 1,LDIM
          WRITE(IPRIN,5004) (HINV(I,J), J = 1,I)
  150 CONTINUE
C
C     COMPUTE MATRIX P SUCH THAT PP'=-(H-INVERSE) BY EIGENVALUE DECOMPOSITION
C     (NOTE THAT P IS DEFINED AS HINV IN THE PROGRAM) THE CALCULATION OF
C     EIGENVALUES AND EIGENVECTORS OF MINUS INVERSE-HESSIAN MATRIX OCCURS
C     BY MEANS OF HOUSEHOLDER REDUCTIE
C
      CALL F01AJF(LDIM,X02ADF(DUMMY),HINV,NDCL,EIGVAL,YHELP,EIGVEC,NDCL)
C
C     EIGENVALUES AND EIGENVECTORS
C
      IFAIL = 1
      CALL F02AMF(LDIM,X02AAF(DUMMY),EIGVAL,YHELP,EIGVEC,NDCL,IFAIL)
      IF (IFAIL .GT. 0) THEN
          ERRMSG = 2
          GOTO 9000
      END IF
```

```
C
      WRITE(IPRIN,5006) (EIGVAL(J), J = 1,LDIM)
      IF (EIGVAL(1) .LT. 1.0D-25) THEN
          ERRMSG = 3
          GOTO 9000
      END IF
      DO 160 J = 1,LDIM
          XX = DSQRT(EIGVAL(J))
          DO 170 I = 1,LDIM
              HINV(I,J) = XX * EIGVEC(I,J)
  170     CONTINUE
  160 CONTINUE
C
C     INITIAL VALUE OF RANDOM NUMBER GENERATOR
C
      IF (IROTA .EQ. 1) WRITE(IPRIN,5007) START
C
C     INITIAL ZERO-VALUES FOR THE NUMBER OF ACCEPTED AND REJECTED
C     RANDOM DRAWINGS
C
      IRNACC = 0
      IRNREJ = 0
C
C     INITIAL ZERO-VALUES FOR PARTIAL SUMS FOR IMPORTANCE MOMENTS,
C     POSTERIOR MOMENTS AND NUMERICAL ERRORS
C
      SUMW1 = 0.0D0
      SUMW2 = 0.0D0
      DO 200 I = 1,LDIM
          SUMIM1(I) = 0.0D0
          SUMPO1(I) = 0.0D0
          SUMER1(I) = 0.0D0
          SUMER2(I) = 0.0D0
          DO 210 J = 1,LDIM
              SUMIM2(I,J) = 0.0D0
              SUMPO2(I,J) = 0.0D0
  210     CONTINUE
  200 CONTINUE
C
C     INITIAL ZERO-VALUES FOR TEN DRAWINGS WITH MAX. WEIGHT
C
      DO 220 I = 1,80
          IPOW(I) = 0
  220 CONTINUE
      DO 230 I = 1,10
          MAXW(I)   = 0.0D0
          MAXIMP(I) = 0.0D0
          MAXPOS(I) = 0.0D0
          DO 240 J = 1,LDIM
              MAXTHE(I,J) = 0.0D0
  240     CONTINUE
  230 CONTINUE
C
C     INITIAL ZERO-VALUES FOR COMPUTATION OF UNIVARIATE M. P. D.
C
      IF (IUMPD) THEN
          DO 250 J = 1,NCLASS
              DO 260 I = 1,LDIM
                  SUMUMP(I,J) = 0.0D0
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                    SUMUMI(I,J) = 0.0D0
  260           CONTINUE
  250       CONTINUE
        END IF
C
C       INITIAL ZERO-VALUES FOR COMPUTATION OF BIVARIATE M. P. D.
C
        IF (IBMPD) THEN
            DO 270 I = 1,NDD
                DO 280 J = 1,NCLASS
                    DO 290 K = 1,NCLASS
                        SUMBMI(I,J,K) = 0.0D0
                        SUMBMP(I,J,K) = 0.0D0
  290               CONTINUE
  280           CONTINUE
  270       CONTINUE
        END IF
C
C       NUMBER OF ROUNDS IS DEFINED AS "MROUND"
C
        DO 800 JROUND = 1,MROUND
C
C       NUMBER OF DRAWINGS IN EACH ROUND IS DEFINED AS "NDRAW"
C
            DO 400 IDRAW = 1,NDRAW
                IF (IRNREJ .GT. NDRAW2) THEN
                    ERRMSG = 4
                    GOTO 9000
                END IF
C
C       GENERATE STUDENT'S -T- WITH "LAMBDA" DEGREES OF FREEDOM
C
  300           VALIMP = 0.0D0
                XNU    = DFLOAT(LAMBDA)
                RN2    = 0.0D0
                DO 500 I = 1,LAMBDA
                    XX  = G05DDF(0.0D0,1.0D0)
                    RN2 = RN2 + XX * XX
  500           CONTINUE
                RN2    = DSQRT(RN2 / XNU)
                DO 510 I = 1,LDIM
                    RN1      = G05DDF(0.0D0,1.0D0)
                    YHELP(I) = RN1 / RN2
                    VALIMP   = VALIMP + YHELP(I) * YHELP(I)
  510           CONTINUE
                VALIMP = -0.5D0 * (XNU + DFLOAT(LDIM)) *
     &                            DLOG(1.0D0 + VALIMP / XNU)
                DO 520 I = 1,LDIM
                    XX = 0.0D0
                    DO 530 J = 1,LDIM
                        XX = XX + HINV(I,J) * YHELP(J)
  530               CONTINUE
                    THETA(I) = PMODE(I) + XX
  520           CONTINUE
C
C       TEST RESTRICTIONS ON THE PARAMETERS BY AN USER-SUPPLIED ROUTINE
C
                CALL RSTRCT(THETA,LDIM,FAIL)
                IF (FAIL) THEN
                    IRNREJ = IRNREJ + 1
                    GOTO 300
                END IF
```

```
C
C     TEST ON BOUNDS OF THE REGION OF INTEGRATION
C
            DO 540 J = 1,LDIM
                IF (THETA(J) .LT. LBOUND(J) .OR.
     &              THETA(J) .GT. UBOUND(J)) THEN
                    IRNREJ = IRNREJ + 1
                    GOTO 300
                END IF
  540       CONTINUE
C
C     COMPUTE POSTERIOR KERNEL AND WEIGHT FUNCTION VALUE
C
            IRNACC = IRNACC + 1
            CALL PSTROR(THETA,LDIM,FPOST,IFCNT,SCFAC,.FALSE.)
            WEIGHT = DEXP(FPOST - VALIMP)
C
C     CLASSIFY WEIGHTS AS POWERS OF TEN AND DETERMINE THE TEN DRAWINGS
C     WITH LARGEST WEIGHT
C
            IDD   = -40
            DUMMY = WEIGHT
            IF (DUMMY .GT. 1.0D0)  DUMMY = 1.0D1 * DUMMY
            IF (DUMMY .GT. 0.0D0)  IDD   = DLOG10(DUMMY)
            IPOW(IDD+41) = IPOW(IDD+41) + 1
            IF (WEIGHT .GT. MAXW(10)) THEN
                MAXW(10)    = WEIGHT
                MAXIMP(10) = VALIMP
                MAXPOS(10) = FPOST
                DO 550 I = 1,LDIM
                    MAXTHE(10,I) = THETA(I)
  550           CONTINUE
                DO 560 II = 1,9
                    DO 570 JJ = II+1,10
                        IF (MAXW(II) .LT. MAXW(JJ)) THEN
                            DUMMY      = MAXW(II)
                            MAXW(II)   = MAXW(JJ)
                            MAXW(JJ)   = DUMMY
                            DUMMY      = MAXIMP(II)
                            MAXIMP(II) = MAXIMP(JJ)
                            MAXIMP(JJ) = DUMMY
                            DUMMY      = MAXPOS(II)
                            MAXPOS(II) = MAXPOS(JJ)
                            MAXPOS(JJ) = DUMMY
                            DO 580 I = 1,LDIM
                                DUMMY        = MAXTHE(II,I)
                                MAXTHE(II,I) = MAXTHE(JJ,I)
                                MAXTHE(JJ,I) = DUMMY
  580                       CONTINUE
                        END IF
  570               CONTINUE
  560           CONTINUE
            END IF
C
C     COMPUTE PARTIAL SUMS FOR IMP. AND POST. MOMENTS AND ERROR ESTIMATES
C
            WW    = WEIGHT * WEIGHT
            SUMW1 = SUMW1  + WEIGHT
            SUMW2 = SUMW2  + WW
```

```
            DO 600 I = 1,LDIM
                XX = THETA(I)
                SUMIM1(I) = SUMIM1(I) + XX
                SUMPO1(I) = SUMPO1(I) + XX * WEIGHT
                SUMER1(I) = SUMER1(I) + XX * WW
                SUMER2(I) = SUMER2(I) + XX * XX * WW
                DO 610 J = 1,I
                    YY = XX * THETA(J)
                    SUMIM2(I,J) = SUMIM2(I,J) + YY
                    SUMPO2(I,J) = SUMPO2(I,J) + YY * WEIGHT
  610           CONTINUE
  600       CONTINUE
C
C     COMPUTE PARTIAL SUMS FOR UNIVARIATE M. P. D.
C
            DO 620 I = 1,LDIM
                JJX(I) = (THETA(I) - LBOUND(I)) / WIDTH(I) + 1.0D0
  620       CONTINUE
            IF (IUMPD) THEN
                DO 630 I = 1,LDIM
                    JJ1 = JJX(I)
                    SUMUMP(I,JJ1) = SUMUMP(I,JJ1) + WEIGHT
                    SUMUMI(I,JJ1) = SUMUMI(I,JJ1) + 1.0D0
  630           CONTINUE
            END IF
C
C     COMPUTE PARTIAL SUMS FOR BIVARIATE M. P. D.
C
            IF (IBMPD) THEN
                II = 0
                DO 640 I1 = 1,LDIM-1
                    JJ1 = JJX(I1)
                    DO 650 I2 = I1+1,LDIM
                        JJ2 = JJX(I2)
                        II = II + 1
                        SUMBMP(II,JJ1,JJ2) = SUMBMP(II,JJ1,JJ2) +
     &                                       WEIGHT
                        SUMBMI(II,JJ1,JJ2) = SUMBMI(II,JJ1,JJ2) +
     &                                       1.0D0
  650               CONTINUE
  640           CONTINUE
            END IF
C
  400     CONTINUE
C
C
C     COMPUTE IMPORTANCE AND POSTERIOR MOMENTS
C
          RNACC = DFLOAT(IRNACC)
          CALL PMC(SUMIM1,SUMIM2,LDIM,NDCL,RNACC,MEANIM,COVIM,STDIM,
     &             RHOIM)
          CALL PMC(SUMPO1,SUMPO2,LDIM,NDCL,SUMW1,MEANPO,COVPO,STDPO,
     &             RHOPO)
C
C     COMPUTE NUMERICAL ERRORS OF POSTERIOR MEANS
C
          MEAND1 = SUMW1 / RNACC
          MEAND2 = MEAND1 * MEAND1
          VARD   = SUMW2 / RNACC - MEAND2
          STDD   = DSQRT(VARD)
```

```
          DO 900 I = 1,LDIM
              MEANN1    = SUMPO1(I) / RNACC
              MEANN2    = MEANN1 * MEANN1
              VARN      = SUMER2(I) / RNACC - MEANN2
              COVAR     = SUMER1(I) / RNACC - MEANN1 * MEAND1
              VARCOF    = VARN / MEANN2 + VARD / MEAND2 -
     &                    2.0D0 * COVAR / (MEANN1 * MEAND1)
              ERROR(I)  = DSQRT(VARCOF * MEANPO(I) * MEANPO(I) / RNACC)
              RELERR(I) = ERROR(I) / STDPO(I)
              CORREL(I) = COVAR / DSQRT(VARD * VARN)
  900     CONTINUE
C
C     COMPUTE UNIVARIATE M. P. D.
C
          IF (IUMPD) THEN
              DO 910 I = 1,LDIM
                  TOTUMP = 0.0D0
                  TOTUMI = 0.0D0
                  DO 920 J = 1,NCLASS
                      TOTUMP = TOTUMP + SUMUMP(I,J)
                      TOTUMI = TOTUMI + SUMUMI(I,J)
  920             CONTINUE
                  DO 930 J = 1,NCLASS
                      RELUMP(I,J) = SUMUMP(I,J) / TOTUMP
                      RELUMI(I,J) = SUMUMI(I,J) / TOTUMI
  930             CONTINUE
  910         CONTINUE
          END IF
C
C     COMPUTE BIVARIATE M. P. D.
C
          IF (IBMPD) THEN
              DO 940 I = 1,NDD
                  MAXBMP = 0.0D0
                  MAXBMI = 0.0D0
                  DO 950 J = 1,NCLASS
                      DO 960 K = 1,NCLASS
                          TERM = SUMBMP(I,J,K)
                          IF (TERM .GT. MAXBMP) MAXBMP = TERM
                          TERM = SUMBMI(I,J,K)
                          SUMBMI(I,J,K) = TERM
                          IF (TERM .GT. MAXBMI) MAXBMI = TERM
  960                 CONTINUE
  950             CONTINUE
                  DO 970 J = 1,NCLASS
                      DO 980 K = 1,NCLASS
                          TERM = SUMBMP(I,J,K) / MAXBMP + 0.05D0
                          ITERM = 10.0D0 * TERM
                          RELBMP(I,J,K) = NALF(ITERM+1)
                          TERM = SUMBMI(I,J,K) / MAXBMI + 0.05D0
                          ITERM = 10.0D0 * TERM
                          RELBMI(I,J,K) = NALF(ITERM+1)
  980                 CONTINUE
  970             CONTINUE
  940         CONTINUE
          END IF
C
C     WRITE MOMENTS OF TRUNCATED IMPORTANCE DISTRIBUTION
C
          CALL PRTHD(IPRIN,DAY,TITLE,IROTA,JROUND)
          CALL PRTSBH(IPRIN,IRNACC,IRNREJ,IFCNT,MEAND1,STDD)
```

```
C
      WRITE(IPRIN,5008)
      WRITE(IPRIN,5001) (MEANIM(I), I = 1,LDIM)
      WRITE(IPRIN,5001) (STDIM(I),  I = 1,LDIM)
      WRITE(IPRIN,5009)
      DO 1000 I = 1,LDIM
          WRITE(IPRIN,5004) (RHOIM(I,J), J = 1,I)
 1000 CONTINUE
      WRITE(IPRIN,5010)
      DO 1010 I = 1,LDIM
          WRITE(IPRIN,5004) (COVIM(I,J), J = 1,I)
 1010 CONTINUE
C
C     WRITE POSTERIOR MOMENTS AND ERROR ESTIMATES
C
      CALL PRTHD(IPRIN,DAY,TITLE,IROTA,JROUND)
      CALL PRTSBH(IPRIN,IRNACC,IRNREJ,IFCNT,MEAND1,STDD)
C
      WRITE(IPRIN,5011)
      WRITE(IPRIN,5001) (MEANPO(I), I = 1,LDIM)
      WRITE(IPRIN,5001) (STDPO(I),  I = 1,LDIM)
      WRITE(IPRIN,5012)
      DO 1020 I = 1,LDIM
          WRITE(IPRIN,5004) (RHOPO(I,J), J = 1,I)
 1020 CONTINUE
      WRITE(IPRIN,5013)
      DO 1030 I = 1,LDIM
          WRITE(IPRIN,5004) (COVPO(I,J), J = 1,I)
 1030 CONTINUE
      WRITE(IPRIN,5014) (ERROR(I),  I = 1,LDIM)
      WRITE(IPRIN,5015) (RELERR(I), I = 1,LDIM)
      WRITE(IPRIN,5016) (CORREL(I), I = 1,LDIM)
C
C     WRITE DISTRIBUTION OF WEIGHT FUNCTION VALUES AND THE TEN DRAWINGS
C     WITH LARGEST WEIGHT
C
      CALL PRTHD(IPRIN,DAY,TITLE,IROTA,JROUND)
      CALL PRTSBH(IPRIN,IRNACC,IRNREJ,IFCNT,MEAND1,STDD)
C
      WRITE(IPRIN,5017)
      DO 1040 JJ = 1,6
          J2 = JJ * 15
          J1 = J2 - 14
          IF (J2 .GT. 80) J2 = 80
          PPOW = .FALSE.
          DO 1050 J = J1,J2
              IF (IPOW(J) .NE. 0) PPOW = .TRUE.
 1050     CONTINUE
          IF (PPOW) THEN
              WRITE(IPRIN,5018) (I, I = J1-41,J2-41)
              WRITE(IPRIN,5018) (IPOW(I), I = J1,J2)
              WRITE(IPRIN,5018)
          END IF
 1040 CONTINUE
      WRITE(IPRIN,5019)
      DO 1060 I = 1,10
          WRITE(IPRIN,5020) MAXW(I),MAXIMP(I),MAXPOS(I),
     &                      (MAXTHE(I,J), J = 1,LDIM)
 1060 CONTINUE
```

```
C
C     WRITE UNIVARIATE M. P. D.
C
          IF (IUMPD) THEN
C
              CALL PRTHD(IPRIN,DAY,TITLE,IROTA,JROUND)
              CALL PRTSBH(IPRIN,IRNACC,IRNREJ,IFCNT,MEAND1,STDD)
C
              WRITE(IPRIN,5021)
              III = FLOAT(LDIM) / 3.0E0 + 0.7E0
              DO 1070 IK = 1,III
                  I2 = IK * 3
                  I1 = I2 - 2
                  IF (I2 .GT. LDIM) I2 = LDIM
                  WRITE(IPRIN,5022) (NALF(1),I, I = I1,I2)
                  WRITE(IPRIN,5023) (NALF(1), I = I1,I2)
                  DO 1080 J = 1,NCLASS
                      WRITE(IPRIN,5024) (NALF(1),BOUND(I,J),
     &                                   BOUND(I,J+1),RELUMP(I,J),
     &                                   RELUMI(I,J), I = I1,I2)
 1080             CONTINUE
 1070         CONTINUE
          END IF
C
C     WRITE BIVARIATE M. P. D.
C
          IF (IBMPD) THEN
              DO 1100 ITIME = 1,2
                  IIE   = 0
                  PAGE = .TRUE.
                  DO 1110 I1 = 1,LDIM-1
                      I2E = I1
1200                  I2B = I2E + 1
                      I2E = I2B + 1
                      IF (I2E .GT. LDIM) I2E = LDIM
                      IIB = IIE + 1
                      IIE = IIB + (I2E - I2B)
                      IF (PAGE) THEN
C
                          CALL PRTHD(IPRIN,DAY,TITLE,IROTA,JROUND)
                          CALL PRTSBH(IPRIN,IRNACC,IRNREJ,IFCNT,MEAND1,
     &                                STDD)
C
                          IF (ITIME .EQ. 1) THEN
                              WRITE(IPRIN,5025)
                          ELSE
                              WRITE(IPRIN,5026)
                          END IF
                      END IF
                      PAGE = .NOT. PAGE
                      WRITE(IPRIN,5027) (NALF(1),I1,I2, I2 = I2B,I2E)
                      WRITE(IPRIN,5028) (UBOUND(I1), I2 = I2B,I2E)
                      IF (ITIME .EQ. 1) THEN
                          DO 1120 JJ = NCLASS,1,-1
                              WRITE(IPRIN,5029) (NALF(1),
     &                        (RELBMP(II,JJ,K),K=1,NCLASS),II=IIB,IIE)
 1120                     CONTINUE
                      ELSE
```

```
                                  DO 1130 JJ = NCLASS,1,-1
                                      WRITE(IPRIN,5029) (NALF(1),
     &                                (RELBMI(II,JJ,K),K=1,NCLASS),II=IIB,IIE)
 1130                             CONTINUE
                              END IF
                              WRITE(IPRIN,5028) (LBOUND(I1), I2 = I2B,I2E)
                              WRITE(IPRIN,5030) (LBOUND(I2),UBOUND(I2),
     &                                           I2 = I2B,I2E)
                              IF (I2E .LT. LDIM) GOTO 1200
 1110                     CONTINUE
 1100                 CONTINUE
                  END IF
  800 CONTINUE
C
      RETURN
C
C
 9000 IF (ERRMSG .EQ. 1) THEN
          WRITE(IPRIN,9101)
      ELSE IF (ERRMSG .EQ. 2) THEN
          WRITE(IPRIN,9102) IFAIL
      ELSE IF (ERRMSG .EQ. 3) THEN
          WRITE(IPRIN,9103)
      ELSE IF (ERRMSG .EQ. 4) THEN
          WRITE(IPRIN,9104) IRNACC,IRNREJ
      ELSE
          WRITE(IPRIN,9198)
      END IF
      WRITE(IPRIN,9199)
C
      RETURN 1
      END
```

```
C     ---------------------------------------------------------------
C     ---------------------------------------------------------------
      SUBROUTINE PMC(SUM1,SUM2,LDIM,NDCL,SUM,MEAN,COV,STD,RHO)
      INTEGER LDIM,NDCL
      DOUBLE PRECISION SUM
      DOUBLE PRECISION SUM1(LDIM),MEAN(LDIM),STD(LDIM),
     &                 SUM2(NDCL,LDIM),COV(NDCL,LDIM),RHO(NDCL,LDIM)
C
C     CALCULATE MEANS, STANDARD-DEVIATIONS, COVARIANCES AND CORRELATIONS
C
      INTEGER I,J
      DOUBLE PRECISION XMEAN,YMEAN,XCOV,YCOV,XSTD,YSTD
C
      DO 100 I = 1,LDIM
          XMEAN    = SUM1(I) / SUM
          MEAN(I)  = XMEAN
          XCOV     = SUM2(I,I) / SUM - XMEAN * XMEAN
          COV(I,I) = XCOV
          XSTD     = DSQRT(XCOV)
          STD(I)   = XSTD
          RHO(I,I) = XCOV / XSTD / XSTD
          DO 200 J = 1,I-1
              YMEAN    = MEAN(J)
              YCOV     = SUM2(I,J) / SUM - YMEAN * XMEAN
              COV(I,J) = YCOV
              YSTD     = STD(J)
              RHO(I,J) = YCOV / YSTD / XSTD
  200     CONTINUE
  100 CONTINUE
C
      RETURN
      END
```

```
C     ------------------------------------------------------------------
C     ------------------------------------------------------------------
      SUBROUTINE PRTHD(IPRIN,DAY,TITLE,IROTA,JROUND)
      CHARACTER*9  DAY
      CHARACTER*60 TITLE
      INTEGER      IPRIN,IROTA,JROUND
C
C     PRINT THE HEADING OF AN OUTPUT PAGE
C
 4000 FORMAT ('1')
 4001 FORMAT (' ',130('*'))
 4002 FORMAT (' ','*',128X,'*')
 4003 FORMAT (' ','*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)',
     &           T101,'DATE: ',A9,T131,'*')
 4004 FORMAT (' ','*  ',A60,T101,'ROTATION: ',I5,T131,'*')
 4005 FORMAT (' ','*',T101,'ROUND:    ',I5,T131,'*')
C
      WRITE(IPRIN,4000)
      WRITE(IPRIN,4001)
      WRITE(IPRIN,4002)
      WRITE(IPRIN,4003) DAY
      WRITE(IPRIN,4004) TITLE,IROTA
      IF (JROUND .GT. 0) WRITE(IPRIN,4005) JROUND
      WRITE(IPRIN,4002)
      WRITE(IPRIN,4001)
C
      RETURN
      END
```

```
C     ------------------------------------------------------------------
C     ------------------------------------------------------------------
      SUBROUTINE PRTSBH(IPRIN,IRNACC,IRNREJ,IFCNT,MEAN,STD)
      INTEGER IPRIN,IRNACC,IRNREJ,IFCNT
      DOUBLE PRECISION MEAN,STD
C
C     PRINT THE SUB-HEADING OF AN OUTPUT PAGE
C
 4050 FORMAT ('0',' NUMBER OF ACCEPTED RANDOM DRAWINGS',I8)
 4051 FORMAT (' ',' NUMBER OF REJECTED RANDOM DRAWINGS',I8)
 4052 FORMAT (' ',' NUMBER OF FUNCTION EVALUATIONS   ',I9,T61,
     &           'DENOMINATOR: MEAN =',E13.6,', STD. DEV. =',E13.6)
C
      WRITE(IPRIN,4050) IRNACC
      WRITE(IPRIN,4051) IRNREJ
      WRITE(IPRIN,4052) IFCNT,MEAN,STD
C
      RETURN
      END
```

```
C     ------------------------------------------------------------------
C     ------------------------------------------------------------------
      SUBROUTINE PSTROR(PARAM,NDIM,FPOST,IFCNT,SCFAC,ICALL)
      LOGICAL ICALL
      INTEGER NDIM,IFCNT
      DOUBLE PRECISION FPOST,SCFAC
      DOUBLE PRECISION PARAM(NDIM)
C
C     - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C     MINUS LOGPOSTERIOR, JOHNSTON MODEL [WITH BAYPAR PARAMETER = 5]
C
C     - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
      DOUBLE PRECISION BAYPAR,SCC,SYY,SCY,SRIRI,SRIY,SRVRV,SRVY,
     &                 SRVRI,SCRI,SCRV,A2,B2,B3,ECC,EII,ECI
      DOUBLE PRECISION Y(10),C(10),RI(10),Z(10),RV(10)
C
C     IF FIRST CALL: READ DATA AND CREATE DUMMY VARIABLES
C
      IF (ICALL) THEN
         IFCNT  = 0
         BAYPAR = 5.0D0
         SCFAC  =-2.5D1
C
         OPEN (UNIT=20, FILE='JOHNSTON.INP', STATUS='OLD')
         READ(20,'(5F10.4)') (Y(I),C(I),RI(I),Z(I),RV(I), I = 1,10)
         CLOSE(UNIT=20)
C
         SCC   = 0.0D0
         SYY   = 0.0D0
         SCY   = 0.0D0
         SRIRI = 0.0D0
         SRIY  = 0.0D0
         SRVRV = 0.0D0
         SRVY  = 0.0D0
         SRVRI = 0.0D0
         SCRI  = 0.0D0
         SCRV  = 0.0D0
         DO 100 I = 1,10
            SCC   = SCC   + C(I)  * C(I)
            SYY   = SYY   + Y(I)  * Y(I)
            SCY   = SCY   + C(I)  * Y(I)
            SRIRI = SRIRI + RI(I) * RI(I)
            SRIY  = SRIY  + RI(I) * Y(I)
            SRVRV = SRVRV + RV(I) * RV(I)
            SRVY  = SRVY  + RV(I) * Y(I)
            SRVRI = SRVRI + RV(I) * RI(I)
            SCRI  = SCRI  + C(I)  * RI(I)
            SCRV  = SCRV  + C(I)  * RV(I)
  100    CONTINUE
C
         RETURN
C
      END IF
C
```

```
      A2    = PARAM(1)
      B2    = PARAM(2)
      B3    = PARAM(3)
      ECC   = SCC + A2 * (A2 * SYY - SCY - SCY)
      EII   = SRIRI + B2 * (B2 * SYY - 2.0D0 * (SRIY - B3 * SRVY)) +
     &              B3 * (B3 * SRVRV - SRVRI - SRVRI)
      ECI   = SCRI -  B2 * SCY - B3 * SCRV -
     &              A2 * (SRIY - B2 * SYY - B3 * SRVY)
      FPOST = -1.0D0 * BAYPAR * DLOG(ECC * EII - ECI * ECI) +
     &                1.0D1 * DLOG(DABS(1.0D0 - A2 - B2))
      FPOST = FPOST + SCFAC
      IFCNT = IFCNT + 1
C
      RETURN
      END




C     ---------------------------------------------------------------
C     ---------------------------------------------------------------
      SUBROUTINE RSTRCT(PARAM,NDIM,FAIL)
      LOGICAL FAIL
      INTEGER NDIM
      DOUBLE PRECISION PARAM(NDIM)
C
C     - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C     JOHNSTON MODEL - RESTRICTION ON PARAMETERS
C
C     TEST ON JACOBIAN
C
C     - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
      FAIL = .FALSE.
      IF (DABS(1.0D0 - PARAM(1) - PARAM(2)) .LE. 1.0D-2) FAIL = .TRUE.
C
      RETURN
      END
```

```
$ SETD        [E.ECT.HOP.DIJK]
$ FORTRAN/OBJECT=JSISTEST/LIST=JSISTEST SISMAIN+SISSUB+POSJOHN+RESJOHN
$ LINK/NOMAP JSISTEST,CIW$SYS:NAGF/LIBRARY
$ REMOVE      JSISTEST.OBJ.*
$ RUN         JSISTEST
JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)              ! TITLE
3                             ! DIMENSION OF THE VECTOR "THETA"
JSISTEST.DAT                  ! NAME OF THE PRINT-OUTPUT-FILE
JOHNSTON.HHH                  ! NAME OF THE INPUT-FILE WITH "PMODE", "HINV"
-2.  -1.7   -0.40             ! LOWER BOUNDS FOR "THETA"
  .8   .25   1.00             ! UPPER BOUNDS FOR "THETA"
1                             ! UNIVARIATE MARGINALS: 0=NO, 1=YES
1                             ! BIVARIATE MARGINALS: 0=NO, 1=YES
79                            ! INITIAL VALUE OF RANDOM NUMBER GENERATOR
1                             ! DEGREES OF FREEDOM OF STUDENT -T- FUNCTION
2                             ! NUMBER OF ROUNDS
20000                         ! NUMBER OF DRAWINGS PER ROUND
2                             ! NUMBER OF ROTATIONS
1                             ! SAVE "MEANPO" AND "COVPO": 0=NO, 1=YES
JSISTEST.SAV                  ! NAME OF THE SAVE-FILE FOR "MEAMPO" AND "COVPO"
$ REMOVE     JSISTEST.EXE.*
```

("JOHNSTON.HHH", "PMODE" AND "HINV")

```
0.45789280000000000E+00   8.92988200000000000E-02   0.36286150000000000E+00
1.02568634379867900E-02
3.14521404818385555E-03   1.25411527436849796E-03
1.97880731848330789E-03  -6.43554989639164606E-04   1.26391899220319765E-02
```

("JOHNSTON.INP", INPUT DATA)

-1.9019	-0.9288	-0.2249	-0.7482	-0.2104
-1.4359	-0.6188	-0.1799	-0.6372	-0.1564
-0.9719	-0.7798	-0.2509	0.0588	-0.1114
-0.9189	-0.8458	-0.3229	0.2498	-0.1824
-0.3279	-0.3948	-0.2299	0.2968	-0.2544
0.4011	0.1542	-0.0219	0.2688	-0.1614
0.9581	0.5742	0.1711	0.2128	0.0466
1.2681	0.6792	0.2881	0.3008	0.2396
1.5091	0.9332	0.3651	0.2108	0.3566
1.4201	1.2272	0.4061	-0.2132	0.4336

```
*************************************************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---   (S.I.S)                                                DATE:  2-SEP-86         *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                                                 ROTATION:     1         *
*                                                                                                                       *
*************************************************************************************************************************
```

BOUNDS OF THE PARAMETERS

-2.0000	-1.7000	-0.4000
0.8000	0.2500	1.0000

IMPORTANCE MEANS AND STANDARDDEVIATIONS

0.4579	0.0893	0.3629
0.1013	0.0354	0.1124

IMPORTANCE CORRELATION MATRIX

1.00000		
0.87695	1.00000	
0.17379	-0.16164	1.00000

IMPORTANCE COVARIANCE MATRIX

0.01026		
0.00315	0.00125	
0.00198	-0.00064	0.01264

EIGENVALUES OF MIN INVERSE HESSIAN MATRIX

0.000145	0.010156	0.013850

INITIAL VALUE OF RANDOM NUMBER GENERATOR : CALL G05CBF(79)

```
*****************************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)                                                     *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                               DATE:  2-SEP-86       *
*                                                                             ROTATION:     1       *
*                                                                             ROUND:        1       *
*                                                                                                   *
*****************************************************************************************************
```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 3789
NUMBER OF FUNCTION EVALUATIONS 20000

DENOMINATOR: MEAN = 0.543908E+08, STD. DEV. = 0.817992E+09

IMPORTANCE MEANS AND STANDARDDEVIATIONS (TRUNCATED)

0.4127	0.0751	0.3557
0.2286	0.0799	0.1938

IMPORTANCE CORRELATION MATRIX (TRUNCATED)

1.00000		
0.88146	1.00000	
0.17346	-0.07578	1.00000

IMPORTANCE COVARIANCE MATRIX (TRUNCATED)

0.05228		
0.01611	0.00639	
0.00769	-0.00117	0.03758

--- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)

DATE: 2-SEP-86
ROTATION: 1
ROUND: 1

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 3789
NUMBER OF FUNCTION EVALUATIONS 20000

DENOMINATOR: MEAN = 0.543908E+08, STD. DEV. = 0.817992E+09

POSTERIOR MEAN AND STANDARDDEVIATION OF THE PARAMETER VECTOR "THETA"

-0.6499	-0.2995	0.3188
0.8185	0.2923	0.1409

POSTERIOR CORRELATION MATRIX OF THE PARAMETER VECTOR "THETA"

1.00000		
0.95902	1.00000	
0.16079	0.26118	1.00000

POSTERIOR COVARIANCE MATRIX OF THE PARAMETER VECTOR "THETA"

0.66996		
0.22947	0.08546	
0.01855	0.01076	0.01986

NUMERICAL ERROR ESTIMATES OF POSTERIOR MEAN

ERROR :	0.115859	0.041339	0.009103
RELATIVE ERROR :	0.141549	0.141408	0.064590
CORREL. COEFF. :	-0.967603	-0.975136	0.964315

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*************************************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---   (S.I.S)                                                             *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                                 DATE:   2-SEP-86             *
*                                                                               ROTATION:      1             *
*                                                                               ROUND:         1             *
*************************************************************************************************************
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NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 3789
NUMBER OF FUNCTION EVALUATIONS 20000

DENOMINATOR: MEAN = 0.543908E+08, STD. DEV. = 0.817992E+09

FREQUENCIES OF IPOW.   WEIGHT=0.*****E+IPOW

-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11
0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
1	7	5	9	10	21	22	33	40	66	65	112	137	225	275
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
504	1013	8001	8624	715	97	16	0	0	0	0	0	0	0	0

TEN DRAWINGS WITH LARGEST WEIGHT

W	LN(IMP)	LN(POS)	(THETA(I), I = 1,NDIM)		
0.6760783E+11	-12.8183133	12.1186763	-1.91397	-0.76367	0.29509
0.5245519E+11	-12.6695467	12.0136784	-1.87514	-0.72200	0.27256
0.3226033E+11	-12.6639374	11.5331668	-1.91723	-0.66222	0.25886
0.2810649E+11	-12.9272047	11.1320615	-1.86109	-0.78067	0.15572
0.2336449E+11	-12.6988862	11.1755969	-1.90996	-0.69381	0.45441
0.2226337E+11	-12.2585543	11.5676544	-0.95208	-0.56388	0.23280
0.1795778E+11	-12.5997756	11.0115135	-1.18861	-0.63431	0.10482
0.1654026E+11	-11.3891124	12.1399510	-0.89523	-0.47869	0.25977
0.1551928E+11	-11.3356672	12.1296818	-0.89816	-0.47139	0.22500
0.1532318E+11	-11.1230665	12.3295658	-1.10488	-0.46485	0.27607

--- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)

DATE: 2-SEP-86
ROTATION: 1
ROUND: 1

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 3789
NUMBER OF FUNCTION EVALUATIONS 20000

DENOMINATOR: MEAN = 0.543908E+08, STD. DEV. = 0.817992E+09

MARGINAL POSTERIOR DENSITIES "P", AND MARGINAL IMPORTANCE DENSITIES (TRUNCATED) "I"

PARAMETER 1

	P	I
(-2.00,-1.81)	0.188	0.001
(-1.81,-1.63)	0.000	0.000
(-1.63,-1.44)	0.027	0.001
(-1.44,-1.25)	0.034	0.001
(-1.25,-1.07)	0.072	0.001
(-1.07,-0.88)	0.090	0.002
(-0.88,-0.69)	0.082	0.003
(-0.69,-0.51)	0.054	0.003
(-0.51,-0.32)	0.045	0.005
(-0.32,-0.13)	0.059	0.011
(-0.13, 0.05)	0.062	0.022
(0.05, 0.24)	0.088	0.066
(0.24, 0.43)	0.123	0.311
(0.43, 0.61)	0.075	0.475
(0.61, 0.80)	0.001	0.098

PARAMETER 2

	P	I
(-1.70,-1.57)	0.000	0.000
(-1.57,-1.44)	0.000	0.000
(-1.44,-1.31)	0.000	0.000
(-1.31,-1.18)	0.000	0.000
(-1.18,-1.05)	0.000	0.000
(-1.05,-0.92)	0.000	0.000
(-0.92,-0.79)	0.000	0.000
(-0.79,-0.66)	0.193	0.001
(-0.66,-0.53)	0.069	0.001
(-0.53,-0.40)	0.120	0.002
(-0.40,-0.27)	0.127	0.004
(-0.27,-0.14)	0.111	0.012
(-0.14,-0.01)	0.140	0.057
(-0.01, 0.12)	0.227	0.721
(0.12, 0.25)	0.012	0.201

PARAMETER 3

	P	I
(-0.40,-0.31)	0.001	0.006
(-0.31,-0.21)	0.000	0.007
(-0.21,-0.12)	0.002	0.012
(-0.12,-0.03)	0.012	0.016
(-0.03, 0.07)	0.011	0.025
(0.07, 0.16)	0.097	0.049
(0.16, 0.25)	0.144	0.105
(0.25, 0.35)	0.359	0.231
(0.35, 0.44)	0.174	0.278
(0.44, 0.53)	0.133	0.138
(0.53, 0.63)	0.049	0.065
(0.63, 0.72)	0.012	0.032
(0.72, 0.81)	0.004	0.017
(0.81, 0.91)	0.001	0.012
(0.91, 1.00)	0.000	0.007

```
*******************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---   (S.I.S)
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                      DATE:   2-SEP-86      *
*                                                                    ROTATION:     1       *
*                                                                    ROUND:        1       *
*******************************************************************************************
```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 3789
NUMBER OF FUNCTION EVALUATIONS 20000

DENOMINATOR: MEAN = 0.543908E+08, STD. DEV. = 0.817992E+09

BIVARIATE MARGINAL POSTERIOR DENSITIES

```
                    VERT: 1   HOR: 2
  0.80   ----------------------------------
        |                                  |
        |                           3 1    |
        |                           6      |
        |                         2 2      |
        |                         3        |
        |                       2 1        |
        |                     1 1 1        |
        |                   1 1 1          |
        |                   1 2 1          |
        |                 1 2 1            |
        |                 1 2 1            |
        |                 1 1              |
        |                 1   1            |
        |                                  |
        |               *                  |
 -2.00   ----------------------------------
        -1.70                          0.25
```

```
                    VERT: 1   HOR: 3
  0.80   ----------------------------------
        |                                  |
        |             1 1 1 1 1            |
        |           1 1 2 2 1 1            |
        |             1 2 2 1              |
        |             1 1 1 1              |
        |           1 1 1 1                |
        |           1   1   1              |
        |             1 2 1                |
        |             1 2 2 1              |
        |             3 1 2 1              |
        |           1   2 1                |
        |           1   1                  |
        |                   1              |
        |                                  |
        |           2   *   2              |
 -2.00   ----------------------------------
        -0.40                          1.00
```

```
                    VERT: 2   HOR: 3
  0.25   ----------------------------------
        |                                  |
        |           1 2 4 4 3 1            |
        |           1 2 3 2 1 1            |
        |           1 1 2 2 1 1            |
        |             2 1 3 2              |
        |             2 5 1                |
        |           2 2                    |
        |           2   *   2              |
        |                                  |
        |                                  |
        |                                  |
        |                                  |
        |                                  |
        |                                  |
        |                                  |
 -1.70   ----------------------------------
        -0.40                          1.00
```

```
*****************************************************************************************************
*                                                                                                   *
*   --- SIMPLE IMPORTANCE SAMPLING ---   (S.I.S)                              DATE:  2-SEP-86       *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                               ROTATION:     1       *
*                                                                             ROUND:        1       *
*                                                                                                   *
*****************************************************************************************************
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```
NUMBER OF ACCEPTED RANDOM DRAWINGS   20000
NUMBER OF REJECTED RANDOM DRAWINGS    3789
NUMBER OF FUNCTION EVALUATIONS       20000          DENOMINATOR: MEAN = 0.543908E+08, STD. DEV. = 0.817992E+09
```

BIVARIATE MARGINAL DENSITIES OF IMPORTANCE FUNCTION (TRUNCATED)

```
               VERT: 1   HOR: 2                                  VERT: 1   HOR: 3
 0.80  ---------------------------------          0.80  ---------------------------------
      |                               2 |              |       1 1 1 1 1                 |
      |                             * 3 |              |     1 3 7 * 5 2 1               |
      |                             8   |              |     1 2 5 5 2 1 1               |
      |                           1 1   |              |       1 1 1                     |
      |                                 |              |                                 |
      |                                 |              |                                 |
      |                                 |              |                                 |
-2.00  ---------------------------------         -2.00  ---------------------------------
      -1.70                         0.25               -0.40                         1.00

               VERT: 2   HOR: 3
 0.25  ---------------------------------
      |          1 1 2 2 1 1            |
      |        1 1 3 8 * 5 2 1          |
      |                                 |
      |                                 |
      |                                 |
      |                                 |
-1.70  ---------------------------------
      -0.40                         1.00
```

```
*****************************************************************************************************
*                                                                                                   *
*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)                          DATE:  2-SEP-86            *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                          ROTATION:     1            *
*                                                                        ROUND:        2            *
*                                                                                                   *
*****************************************************************************************************
```

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 7753
NUMBER OF FUNCTION EVALUATIONS 40000

DENOMINATOR: MEAN = 0.570712E+08, STD. DEV. = 0.890592E+09

IMPORTANCE MEANS AND STANDARDDEVIATIONS (TRUNCATED)

0.4132	0.0749	0.3559
0.2253	0.0802	0.1941

IMPORTANCE CORRELATION MATRIX (TRUNCATED)

1.00000		
0.87748	1.00000	
0.15840	-0.09902	1.00000

IMPORTANCE COVARIANCE MATRIX (TRUNCATED)

0.05077		
0.01587	0.00644	
0.00693	-0.00154	0.03767

```
*****************************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)                                                      *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                               DATE:  2-SEP-86         *
*                                                                             ROTATION:      1        *
*                                                                             ROUND:         2        *
*                                                                                                     *
*****************************************************************************************************
```

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 7753
NUMBER OF FUNCTION EVALUATIONS 40000

DENOMINATOR: MEAN = 0.570712E+08, STD. DEV. = 0.890592E+09

POSTERIOR MEAN AND STANDARDDEVIATION OF THE PARAMETER VECTOR "THETA"

-0.6573	-0.3193	0.3203
0.7948	0.3042	0.1343

POSTERIOR CORRELATION MATRIX OF THE PARAMETER VECTOR "THETA"

1.00000		
0.95458	1.00000	
0.14700	0.22498	1.00000

POSTERIOR COVARIANCE MATRIX OF THE PARAMETER VECTOR "THETA"

0.63179		
0.23077	0.09251	
0.01569	0.00919	0.01803

NUMERICAL ERROR ESTIMATES OF POSTERIOR MEAN

ERROR :	0.078459	0.031534	0.005768
RELATIVE ERROR :	0.098710	0.103680	0.042961
CORREL. COEFF. :	-0.968990	-0.973110	0.973590

--- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)

DATE: 2-SEP-86
ROTATION: 1
ROUND: 2

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 7753
NUMBER OF FUNCTION EVALUATIONS 40000

DENOMINATOR: MEAN = 0.570712E+08, STD. DEV. = 0.890592E+09

FREQUENCIES OF IPOW. WEIGHT=0.*****E+IPOW

-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11
0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
2	9	14	19	26	35	52	74	93	126	143	199	271	451	544
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
982	1965	16076	17224	1458	200	34	0	0	0	0	0	0	0	0

TEN DRAWINGS WITH LARGEST WEIGHT

W	LN(IMP)	LN(POS)	(THETA(I), I = 1,NDIM)		
0.7112271E+11	-13.7743725	11.2133000	-1.68069	-0.89169	0.28180
0.6760783E+11	-12.8183133	12.1186763	-1.91397	-0.76367	0.29509
0.6636652E+11	-12.8856034	12.0328551	-1.96643	-0.77804	0.34243
0.5245519E+11	-12.6695467	12.0136784	-1.87514	-0.72200	0.27256
0.4762237E+11	-12.7622241	11.8243443	-1.38556	-0.70372	0.25311
0.3292641E+11	-12.2291196	11.9884213	-1.24918	-0.62162	0.28113
0.3226033E+11	-12.6639374	11.5331668	-1.91723	-0.66222	0.25886
0.2893752E+11	-12.4016240	11.6867810	-1.20425	-0.63420	0.29757
0.2810649E+11	-12.9272047	11.1320615	-1.86109	-0.78067	0.15572
0.2336449E+11	-12.6988862	11.1755969	-1.90996	-0.69381	0.45441

```
***************************************************************************************************
*                                                                                                 *
*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)                          DATE:  2-SEP-86          *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                          ROTATION:     1          *
*                                                                        ROUND:        2          *
*                                                                                                 *
***************************************************************************************************
```

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 7753
NUMBER OF FUNCTION EVALUATIONS 40000

DENOMINATOR: MEAN = 0.570712E+08, STD. DEV. = 0.890592E+09

MARGINAL POSTERIOR DENSITIES "P", AND MARGINAL IMPORTANCE DENSITIES (TRUNCATED) "I"

PARAMETER 1	P	I	PARAMETER 2	P	I	PARAMETER 3	P	I
(-2.00,-1.81)	0.126	0.000	(-1.70,-1.57)	0.000	0.000	(-0.40,-0.31)	0.001	0.006
(-1.81,-1.63)	0.037	0.000	(-1.57,-1.44)	0.000	0.000	(-0.31,-0.21)	0.000	0.007
(-1.63,-1.44)	0.034	0.001	(-1.44,-1.31)	0.000	0.000	(-0.21,-0.12)	0.002	0.012
(-1.44,-1.25)	0.064	0.001	(-1.31,-1.18)	0.000	0.000	(-0.12,-0.03)	0.009	0.016
(-1.25,-1.07)	0.091	0.001	(-1.18,-1.05)	0.000	0.000	(-0.03, 0.07)	0.014	0.025
(-1.07,-0.88)	0.081	0.002	(-1.05,-0.92)	0.000	0.000	(0.07, 0.16)	0.069	0.049
(-0.88,-0.69)	0.057	0.002	(-0.92,-0.79)	0.037	0.000	(0.16, 0.25)	0.160	0.107
(-0.69,-0.51)	0.058	0.004	(-0.79,-0.66)	0.158	0.001	(0.25, 0.35)	0.366	0.232
(-0.51,-0.32)	0.057	0.006	(-0.66,-0.53)	0.092	0.001	(0.35, 0.44)	0.204	0.275
(-0.32,-0.13)	0.058	0.011	(-0.53,-0.40)	0.116	0.002	(0.44, 0.53)	0.115	0.139
(-0.13, 0.05)	0.067	0.023	(-0.40,-0.27)	0.124	0.005	(0.53, 0.63)	0.044	0.064
(0.05, 0.24)	0.082	0.065	(-0.27,-0.14)	0.108	0.012	(0.63, 0.72)	0.011	0.032
(0.24, 0.43)	0.115	0.310	(-0.14,-0.01)	0.138	0.058	(0.72, 0.81)	0.003	0.017
(0.43, 0.61)	0.072	0.477	(-0.01, 0.12)	0.215	0.720	(0.81, 0.91)	0.001	0.012
(0.61, 0.80)	0.001	0.099	(0.12, 0.25)	0.011	0.201	(0.91, 1.00)	0.000	0.007

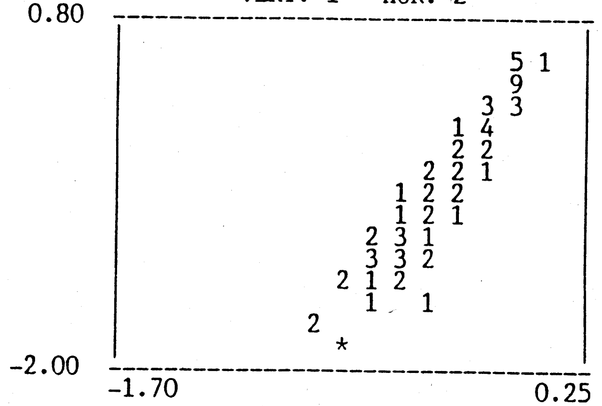
```
*****************************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)                                          *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                    DATE:  2-SEP-86       *
*                                                                  ROTATION:      1      *
*                                                                  ROUND:         2      *
*****************************************************************************************************
```

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 7753
NUMBER OF FUNCTION EVALUATIONS 40000

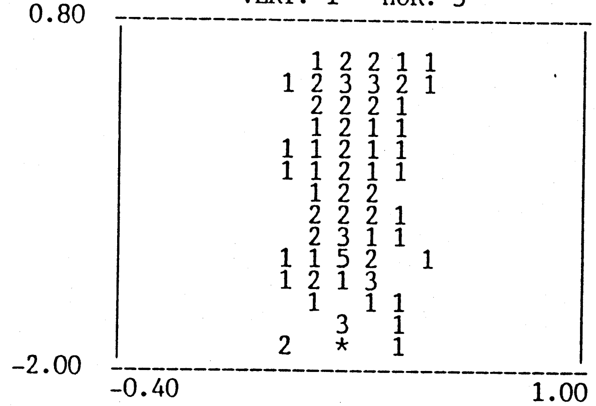
DENOMINATOR: MEAN = 0.570712E+08, STD. DEV. = 0.890592E+09

BIVARIATE MARGINAL POSTERIOR DENSITIES

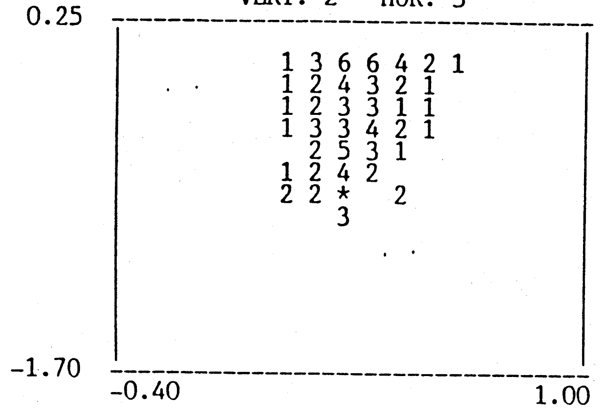
VERT: 1 HOR: 2



VERT: 1 HOR: 3



VERT: 2 HOR: 3



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****************************************************************************************************
*                                                                                                  *
*   --- SIMPLE IMPORTANCE SAMPLING ---   (S.I.S)                        DATE:   2-SEP-86           *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                         ROTATION:      1           *
*                                                                       ROUND:         2           *
*                                                                                                  *
****************************************************************************************************

 NUMBER OF ACCEPTED RANDOM DRAWINGS    40000
 NUMBER OF REJECTED RANDOM DRAWINGS     7753
 NUMBER OF FUNCTION EVALUATIONS        40000          DENOMINATOR: MEAN = 0.570712E+08, STD. DEV. = 0.890592E+09

 BIVARIATE MARGINAL DENSITIES OF IMPORTANCE FUNCTION (TRUNCATED)

                 VERT: 1   HOR: 2                                      VERT: 1   HOR: 3
  0.80  ----------------------------------                0.80  ----------------------------------
        |                              2 |                      |        1 1 1 1 1               |
        |                            * 3 |                      |      1 3 7 * 5 2 1             |
        |                            8   |                      |      1 2 5 5 2 1 1             |
        |                          1 1   |                      |        1 1 1                   |
        |                                |                      |                                |
        |                                |                      |                                |
        |                                |                      |                                |
        |                                |                      |                                |
 -2.00  ----------------------------------               -2.00  ----------------------------------
        -1.70                       0.25                        -0.40                       1.00

                 VERT: 2   HOR: 3
  0.25  ----------------------------------
        |          1 1 2 2 1 1           |
        |        1 1 3 8 * 5 2 1         |
        |                                |
        |                                |
        |                                |
        |                                |
        |                                |
 -1.70  ----------------------------------
        -0.40                       1.00
```

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*******************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)                                              *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                    DATE:  2-SEP-86         *
*                                                                   ROTATION:     2         *
*                                                                                           *
*******************************************************************************************
```

BOUNDS OF THE PARAMETERS

-2.0000	-1.7000	-0.4000
0.8000	0.2500	1.0000

IMPORTANCE MEANS AND STANDARDDEVIATIONS

-0.6573	-0.3193	0.3203
0.7948	0.3042	0.1343

IMPORTANCE CORRELATION MATRIX

1.00000		
0.95458	1.00000	
0.14700	0.22498	1.00000

IMPORTANCE COVARIANCE MATRIX

0.63179		
0.23077	0.09251	
0.01569	0.00919	0.01803

EIGENVALUES OF MIN INVERSE HESSIAN MATRIX

0.006336	0.018467	0.717516

014

```
***************************************************************************************************
*                                                                                                 *
*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)                              DATE:  2-SEP-86      *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                              ROTATION:      2     *
*                                                                            ROUND:         1     *
*                                                                                                 *
***************************************************************************************************
```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 11489
NUMBER OF FUNCTION EVALUATIONS 60000

DENOMINATOR: MEAN = 0.109016E+07, STD. DEV. = 0.203716E+07

IMPORTANCE MEANS AND STANDARDDEVIATIONS (TRUNCATED)

-0.6653	-0.3325	0.3185
0.6301	0.2576	0.1772

IMPORTANCE CORRELATION MATRIX (TRUNCATED)

1.00000		
0.84735	1.00000	
0.05254	0.16261	1.00000

IMPORTANCE COVARIANCE MATRIX (TRUNCATED)

0.39706		
0.13757	0.06638	
0.00587	0.00743	0.03141

015

```
****************************************************************************************************
*                                                                                                  *
*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)                                                    *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                        DATE:  2-SEP-86             *
*                                                                      ROTATION:     2             *
*                                                                      ROUND:        1             *
****************************************************************************************************
```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 11489
NUMBER OF FUNCTION EVALUATIONS 60000

DENOMINATOR: MEAN = 0.109016E+07, STD. DEV. = 0.203716E+07

POSTERIOR MEAN AND STANDARDDEVIATION OF THE PARAMETER VECTOR "THETA"

-0.5844	-0.3018	0.3126
0.7856	0.3235	0.1452

POSTERIOR CORRELATION MATRIX OF THE PARAMETER VECTOR "THETA"

1.00000		
0.92121	1.00000	
0.14811	0.28291	1.00000

POSTERIOR COVARIANCE MATRIX OF THE PARAMETER VECTOR "THETA"

0.61722		
0.23411	0.10463	
0.01689	0.01328	0.02107

NUMERICAL ERROR ESTIMATES OF POSTERIOR MEAN

ERROR :	0.014525	0.005797	0.002065
RELATIVE ERROR :	0.018488	0.017920	0.014225
CORREL. COEFF. :	0.210403	-0.077054	0.917077

016

--- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)

DATE: 2-SEP-86
ROTATION: 2
ROUND: 1

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 11489
NUMBER OF FUNCTION EVALUATIONS 60000

DENOMINATOR: MEAN = 0.109016E+07, STD. DEV. = 0.203716E+07

FREQUENCIES OF IPOW. WEIGHT=0.*****E+IPOW

-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
1	2	4	4	7	6	6	13	24	35	63	92	142	244	382

5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
943	12403	5450	179	0	0	0	0	0	0	0	0	0	0	0

TEN DRAWINGS WITH LARGEST WEIGHT

W	LN(IMP)	LN(POS)	(THETA(I), I = 1,NDIM)		
0.3932177E+08	-4.2693570	13.2179319	0.51246	0.07082	0.62155
0.3853749E+08	-2.9134306	14.5537116	0.51777	0.09146	0.47298
0.3825340E+08	-2.7768369	14.6829061	0.47654	0.08802	0.47074
0.3703270E+08	-2.3700851	15.0572268	0.45672	0.08229	0.41576
0.3590446E+08	-2.4400944	14.9562775	0.48720	0.09674	0.41912
0.3266503E+08	-2.2490453	15.0527703	0.44858	0.07809	0.39223
0.3257072E+08	-3.2006236	14.0983007	0.44394	0.07550	0.52568
0.3200778E+08	-2.2001544	15.0813351	0.44976	0.07998	0.37718
0.3147760E+08	-2.2515204	15.0132663	0.45691	0.09158	0.30632
0.3129323E+08	-2.3628494	14.8960629	0.49436	0.10901	0.30241

```
**************************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)                                                  *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                          DATE:  2-SEP-86         *
*                                                                        ROTATION:     2         *
*                                                                        ROUND:        1         *
**************************************************************************************************
```

NUMBER OF ACCEPTED RANDOM DRAWINGS 20000
NUMBER OF REJECTED RANDOM DRAWINGS 11489
NUMBER OF FUNCTION EVALUATIONS 60000

DENOMINATOR: MEAN = 0.109016E+07, STD. DEV. = 0.203716E+07

MARGINAL POSTERIOR DENSITIES "P", AND MARGINAL IMPORTANCE DENSITIES (TRUNCATED) "I"

PARAMETER 1

	P	I
(-2.00,-1.81)	0.072	0.035
(-1.81,-1.63)	0.063	0.038
(-1.63,-1.44)	0.061	0.052
(-1.44,-1.25)	0.063	0.066
(-1.25,-1.07)	0.061	0.082
(-1.07,-0.88)	0.062	0.100
(-0.88,-0.69)	0.061	0.113
(-0.69,-0.51)	0.057	0.110
(-0.51,-0.32)	0.061	0.104
(-0.32,-0.13)	0.064	0.088
(-0.13, 0.05)	0.069	0.067
(0.05, 0.24)	0.097	0.057
(0.24, 0.43)	0.125	0.042
(0.43, 0.61)	0.082	0.028
(0.61, 0.80)	0.001	0.017

PARAMETER 2

	P	I
(-1.70,-1.57)	0.001	0.000
(-1.57,-1.44)	0.000	0.000
(-1.44,-1.31)	0.002	0.001
(-1.31,-1.18)	0.003	0.001
(-1.18,-1.05)	0.009	0.003
(-1.05,-0.92)	0.024	0.008
(-0.92,-0.79)	0.045	0.026
(-0.79,-0.66)	0.076	0.063
(-0.66,-0.53)	0.101	0.112
(-0.53,-0.40)	0.107	0.168
(-0.40,-0.27)	0.112	0.204
(-0.27,-0.14)	0.120	0.180
(-0.14,-0.01)	0.151	0.124
(-0.01, 0.12)	0.237	0.071
(0.12, 0.25)	0.013	0.036

PARAMETER 3

	P	I
(-0.40,-0.31)	0.000	0.003
(-0.31,-0.21)	0.002	0.006
(-0.21,-0.12)	0.004	0.009
(-0.12,-0.03)	0.011	0.017
(-0.03, 0.07)	0.032	0.033
(0.07, 0.16)	0.079	0.071
(0.16, 0.25)	0.188	0.165
(0.25, 0.35)	0.271	0.279
(0.35, 0.44)	0.245	0.226
(0.44, 0.53)	0.110	0.101
(0.53, 0.63)	0.044	0.046
(0.63, 0.72)	0.009	0.022
(0.72, 0.81)	0.003	0.012
(0.81, 0.91)	0.001	0.007
(0.91, 1.00)	0.000	0.003

```
*****************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)                            DATE:  2-SEP-86   *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                            ROTATION:     2   *
*                                                                          ROUND:        1   *
*                                                                                            *
*****************************************************************************************
```

```
NUMBER OF ACCEPTED RANDOM DRAWINGS   20000
NUMBER OF REJECTED RANDOM DRAWINGS   11489
NUMBER OF FUNCTION EVALUATIONS       60000        DENOMINATOR: MEAN = 0.109016E+07, STD. DEV. = 0.203716E+07
```

BIVARIATE MARGINAL POSTERIOR DENSITIES

```
                VERT: 1   HOR: 2
 0.80  ---------------------------------
       |                               |
       |                     6 1       |
       |                   1 *         |
       |                   4 4         |
       |                 1 5           |
       |                 3 2           |
       |               1 3 1           |
       |             1 2 2             |
       |             1 2 1             |
       |           1 2 2               |
       |         1 1 2 1               |
       |         1 2 1                 |
       |       1 1 2 1                 |
       |     1 1 1 1                   |
       |     1 1 2 1                   |
-2.00  ---------------------------------
       -1.70                        0.25
```

```
                VERT: 1   HOR: 3
 0.80  ---------------------------------
       |                               |
       |           1 2 7 6 3 3         |
       |         1 1 6 8 * 5 1         |
       |         1 2 4 6 8 3 1         |
       |           1 3 5 4 2 1         |
       |         1 1 3 5 4 2 1         |
       |           1 3 4 4 2 1         |
       |           1 3 4 3 1           |
       |         1 2 3 4 3 2 1         |
       |         1 2 3 5 4 2           |
       |         1 2 3 5 4 1           |
       |         1 2 4 5 3 1 1         |
       |           1 3 5 4 2 1         |
       |       1   2 4 5 3 1 1         |
       |         1 2 5 5 4 2 1         |
-2.00  ---------------------------------
       -0.40                        1.00
```

```
                VERT: 2   HOR: 3
 0.25  ---------------------------------
       |             1                 |
       |         1 1 5 9 * 5 3         |
       |         1 1 3 5 6 3 1         |
       |           1 3 5 4 2 1         |
       |           1 3 5 4 2 1         |
       |           1 3 4 4 1           |
       |           1 3 4 3 1           |
       |         1 1 2 3 2 1           |
       |           1 2 2 1             |
       |           1 1 1               |
       |                               |
       |                               |
       |                               |
       |                               |
-1.70  ---------------------------------
       -0.40                        1.00
```

```
*************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)                                     *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)              DATE:  2-SEP-86        *
*                                                            ROTATION:     2        *
*                                                            ROUND:        1        *
*                                                                                   *
*************************************************************************************
```

```
NUMBER OF ACCEPTED RANDOM DRAWINGS    20000
NUMBER OF REJECTED RANDOM DRAWINGS    11489
NUMBER OF FUNCTION EVALUATIONS        60000        DENOMINATOR: MEAN = 0.109016E+07, STD. DEV. = 0.203716E+07
```

BIVARIATE MARGINAL DENSITIES OF IMPORTANCE FUNCTION (TRUNCATED)

```
              VERT: 1   HOR: 2
 0.80  ----------------------------------
      |                                  |
      |                       1 1        |
      |                     1 2 2        |
      |                     2 3 1        |
      |                   1 4 3 1        |
      |                 1 3 5 2          |
      |                 1 7 4 1          |
      |               1 4 9 2            |
      |               2 * 5 1            |
      |             1 5 * 2              |
      |             2 8 5 1              |
      |           1 4 7 2                |
      |           2 5 3 1                |
      |         1 2 4 1                  |
      |         1 3 2                    |
      |       1 2 2 1                    |
-2.00  ----------------------------------
     -1.70                          0.25
```

```
              VERT: 1   HOR: 3
 0.80  ----------------------------------
      |                                  |
      |         1 1 1 1                  |
      |       1 1 1 2 1 1                |
      |       1 2 3 3 1 1                |
      |     1 1 2 3 4 2 1 1              |
      |     1 1 3 5 4 2 1                |
      |     1 1 3 7 6 3 1 1              |
      |     1 1 4 8 7 3 1                |
      |     1 2 5 * 7 3 1                |
      |     1 2 5 * 7 3 1 1              |
      |     1 2 5 8 6 2 1 1              |
      |     1 2 4 7 5 2 1                |
      |     1 1 4 5 4 2 1                |
      |     1 1 3 3 3 1 1                |
      |       1 2 2 2 1 1                |
      |     1 1 2 2 2 1                  |
-2.00  ----------------------------------
     -0.40                          1.00
```

```
              VERT: 2   HOR: 3
 0.25  ----------------------------------
      |                                  |
      |           1 1 1 1 1              |
      |         1 1 2 3 1 1              |
      |       1 1 2 5 5 2 1              |
      |       1 1 4 8 7 3 1 1            |
      |       1 2 5 * 7 3 1              |
      |       1 2 5 8 5 2 1              |
      |       1 2 3 4 3 1 1              |
      |       1 1 2 2 2 1                |
      |         1 1 1                    |
      |                                  |
      |                                  |
      |                                  |
      |                                  |
-1.70  ----------------------------------
     -0.40                          1.00
```

020

```
*****************************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---   (S.I.S)                                                    *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                            DATE:  2-SEP-86          *
*                                                                          ROTATION:      2         *
*                                                                          ROUND:         2         *
*                                                                                                   *
*****************************************************************************************************

NUMBER OF ACCEPTED RANDOM DRAWINGS    40000
NUMBER OF REJECTED RANDOM DRAWINGS    22792
NUMBER OF FUNCTION EVALUATIONS        80000          DENOMINATOR: MEAN = 0.108995E+07, STD. DEV. = 0.203021E+07

IMPORTANCE MEANS AND STANDARDDEVIATIONS (TRUNCATED)

  -0.6653    -0.3314     0.3178

   0.6307     0.2590     0.1780

IMPORTANCE CORRELATION MATRIX (TRUNCATED)

  1.00000

  0.84847    1.00000

  0.06168    0.17852    1.00000

IMPORTANCE COVARIANCE MATRIX (TRUNCATED)

  0.39774

  0.13857    0.06706

  0.00692    0.00823    0.03168
```

```
*****************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---  (S.I.S)                                        *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                  DATE:  2-SEP-86       *
*                                                                ROTATION:      2      *
*                                                                ROUND:         2      *
*****************************************************************************************
```

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 22792
NUMBER OF FUNCTION EVALUATIONS 80000

DENOMINATOR: MEAN = 0.108995E+07, STD. DEV. = 0.203021E+07

POSTERIOR MEAN AND STANDARDDEVIATION OF THE PARAMETER VECTOR "THETA"

-0.5993	-0.3092	0.3125
0.7862	0.3289	0.1465

POSTERIOR CORRELATION MATRIX OF THE PARAMETER VECTOR "THETA"

1.00000		
0.91745	1.00000	
0.16798	0.30885	1.00000

POSTERIOR COVARIANCE MATRIX OF THE PARAMETER VECTOR "THETA"

0.61807		
0.23726	0.10820	
0.01935	0.01488	0.02146

NUMERICAL ERROR ESTIMATES OF POSTERIOR MEAN

ERROR :	0.010348	0.004266	0.001478
RELATIVE ERROR :	0.013163	0.012970	0.010088
CORREL. COEFF. :	0.158431	-0.127086	0.914430

022

--- SIMPLE IMPORTANCE SAMPLING --- (S.I.S)
JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)

DATE: 2-SEP-86
ROTATION: 2
ROUND: 2

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 22792
NUMBER OF FUNCTION EVALUATIONS 80000

DENOMINATOR: MEAN = 0.108995E+07, STD. DEV. = 0.203021E+07

FREQUENCIES OF IPOW. WEIGHT=0.*****E+IPOW

-25	-24	-23	-22	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
1	3	7	9	13	16	14	23	53	76	121	186	298	457	771
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1882	24755	10955	359	0	0	0	0	0	0	0	0	0	0	0

TEN DRAWINGS WITH LARGEST WEIGHT

W	LN(IMP)	LN(POS)	(THETA(I), I = 1,NDIM)		
0.3979926E+08	-3.3737670	14.1255919	0.46103	0.06702	0.53429
0.3932177E+08	-4.2693570	13.2179319	0.51246	0.07082	0.62155
0.3878236E+08	-2.6889068	14.7845692	0.51115	0.09543	0.44738
0.3864311E+08	-2.8858178	14.5840613	0.44728	0.07270	0.48715
0.3853749E+08	-2.9134306	14.5537116	0.51777	0.09146	0.47298
0.3825340E+08	-2.7768369	14.6829061	0.47654	0.08802	0.47074
0.3703270E+08	-2.3700851	15.0572268	0.45672	0.08229	0.41576
0.3590446E+08	-2.4400944	14.9562775	0.48720	0.09674	0.41912
0.3589604E+08	-2.2482294	15.1479082	0.46191	0.08886	0.38700
0.3501558E+08	-2.9264061	14.4448974	0.42811	0.06546	0.49489

---- SIMPLE IMPORTANCE SAMPLING ---- (S.I.S)
JOHNSTON MODEL (TEST FOR PROGRAM EXECUTION)

DATE: 2-SEP-86
ROTATION: 2
ROUND: 2

NUMBER OF ACCEPTED RANDOM DRAWINGS 40000
NUMBER OF REJECTED RANDOM DRAWINGS 22792
NUMBER OF FUNCTION EVALUATIONS 80000

DENOMINATOR: MEAN = 0.108995E+07, STD. DEV. = 0.203021E+07

MARGINAL POSTERIOR DENSITIES "P", AND MARGINAL IMPORTANCE DENSITIES (TRUNCATED) "I"

PARAMETER 1	P	I	PARAMETER 2	P	I	PARAMETER 3	P	I
(-2.00,-1.81)	0.072	0.034	(-1.70,-1.57)	0.001	0.000	(-0.40,-0.31)	0.001	0.003
(-1.81,-1.63)	0.070	0.041	(-1.57,-1.44)	0.001	0.000	(-0.31,-0.21)	0.001	0.006
(-1.63,-1.44)	0.063	0.052	(-1.44,-1.31)	0.003	0.001	(-0.21,-0.12)	0.004	0.009
(-1.44,-1.25)	0.064	0.066	(-1.31,-1.18)	0.005	0.002	(-0.12,-0.03)	0.011	0.017
(-1.25,-1.07)	0.061	0.082	(-1.18,-1.05)	0.010	0.003	(-0.03, 0.07)	0.032	0.033
(-1.07,-0.88)	0.059	0.096	(-1.05,-0.92)	0.022	0.008	(0.07, 0.16)	0.081	0.072
(-0.88,-0.69)	0.060	0.113	(-0.92,-0.79)	0.047	0.026	(0.16, 0.25)	0.187	0.165
(-0.69,-0.51)	0.059	0.113	(-0.79,-0.66)	0.076	0.063	(0.25, 0.35)	0.272	0.279
(-0.51,-0.32)	0.061	0.104	(-0.66,-0.53)	0.100	0.112	(0.35, 0.44)	0.239	0.225
(-0.32,-0.13)	0.065	0.089	(-0.53,-0.40)	0.108	0.165	(0.44, 0.53)	0.115	0.102
(-0.13, 0.05)	0.070	0.069	(-0.40,-0.27)	0.113	0.205	(0.53, 0.63)	0.041	0.045
(0.05, 0.24)	0.095	0.055	(-0.27,-0.14)	0.120	0.180	(0.63, 0.72)	0.010	0.021
(0.24, 0.43)	0.123	0.041	(-0.14,-0.01)	0.151	0.126	(0.72, 0.81)	0.004	0.012
(0.43, 0.61)	0.077	0.028	(-0.01, 0.12)	0.233	0.073	(0.81, 0.91)	0.002	0.007
(0.61, 0.80)	0.001	0.017	(0.12, 0.25)	0.010	0.036	(0.91, 1.00)	0.000	0.004

```
*****************************************************************************************************
*                                                                                                   *
*   --- SIMPLE IMPORTANCE SAMPLING ---   (S.I.S)                            DATE:  2-SEP-86         *
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                             ROTATION:    2          *
*                                                                           ROUND:       2          *
*                                                                                                   *
*****************************************************************************************************
```

```
NUMBER OF ACCEPTED RANDOM DRAWINGS    40000
NUMBER OF REJECTED RANDOM DRAWINGS    22792
NUMBER OF FUNCTION EVALUATIONS        80000          DENOMINATOR: MEAN = 0.108995E+07, STD. DEV. = 0.203021E+07
```

BIVARIATE MARGINAL POSTERIOR DENSITIES

```
                 VERT: 1   HOR: 2
 0.80   ----------------------------------
        |                                |
        |                   6 1          |
        |                 1 *            |
        |                 4 4            |
        |               1 5              |
        |               3 2              |
        |             1 3 1              |
        |           1 2 2                |
        |           1 2 1                |
        |         1 2 2                  |
        |       1 1 2 1                  |
        |       1 2 1                    |
        |     1 1 2 1                    |
        |   1 1 2 1 1                    |
        |   1 2 2 1                      |
-2.00   ----------------------------------
       -1.70                         0.25
```

```
                 VERT: 1   HOR: 3
 0.80   ----------------------------------
        |                                |
        |         1 2 6 5 3 2            |
        |       1 1 5 9 * 5 2            |
        |       1 2 4 6 6 3 1            |
        |         1 3 5 5 2 1            |
        |       1 1 3 5 4 2 1            |
        |       1 1 3 4 4 2 1            |
        |       1 1 3 4 3 2              |
        |       1 1 3 4 3 2              |
        |       1 2 3 4 4 2 1            |
        |       1 2 3 4 4 1 1            |
        |       1 2 4 5 3 1 1            |
        |       1 2 4 5 4 2              |
        |       1 2 4 6 4 2 1            |
        |       1 2 5 5 4 2              |
-2.00   ----------------------------------
       -0.40                         1.00
```

```
                 VERT: 2   HOR: 3
 0.25   ----------------------------------
        |                                |
        |       1 1 4 9 * 6 2            |
        |       1 1 3 6 6 3 1            |
        |         1 3 5 5 2 1            |
        |         1 3 5 4 2 1            |
        |         1 3 5 4 2              |
        |         1 3 4 3 1              |
        |       1 1 3 3 2 1              |
        |         1 2 2 1                |
        |         1 1 1                  |
        |                                |
        |                                |
        |                                |
-1.70   ----------------------------------
       -0.40                         1.00
```

025

```
****************************************************************************************
*
*   --- SIMPLE IMPORTANCE SAMPLING ---   (S.I.S)
*  JOHNSTON MODEL  (TEST FOR PROGRAM EXECUTION)                          DATE:  2-SEP-86        *
*                                                                        ROTATION:     2        *
*                                                                        ROUND:        2        *
*                                                                                               *
****************************************************************************************
```

NUMBER OF ACCEPTED RANDOM DRAWINGS   40000
NUMBER OF REJECTED RANDOM DRAWINGS   22792
NUMBER OF FUNCTION EVALUATIONS   80000

DENOMINATOR: MEAN = 0.108995E+07, STD. DEV. = 0.203021E+07

BIVARIATE MARGINAL DENSITIES OF IMPORTANCE FUNCTION (TRUNCATED)

```
                 VERT: 1   HOR: 2
  0.80  ----------------------------------
        |                           1 1  |
        |                         1 2 2  |
        |                         2 3 1  |
        |                       1 4 3 1  |
        |                     1 3 6 2    |
        |                     2 7 5 1    |
        |                   1 4 9 2      |
        |                   2 * 5 1      |
        |                 1 5 * 2        |
        |                 2 8 5 1        |
        |               1 4 6 2          |
        |               2 5 3 1          |
        |             1 3 4 1            |
        |             1 3 2              |
        |           1 2 2 1              |
 -2.00  ----------------------------------
       -1.70                          0.25
```

```
                 VERT: 1   HOR: 3
  0.80  ----------------------------------
        |             1 1 1 1            |
        |           1 1 2 2 1 1          |
        |           1 1 3 3 1 1          |
        |         1 1 2 3 3 2 1 1        |
        |           1 3 5 5 2 1          |
        |         1 1 4 7 6 3 1          |
        |         1 2 4 8 7 3 1          |
        |         1 2 5 * 7 3 1          |
        |         1 2 5 * 7 3 1 1        |
        |         1 2 4 8 6 2 1          |
        |         1 2 4 6 5 2 1          |
        |         1 2 3 5 3 2 1          |
        |         1 1 3 3 3 1 1          |
        |         1 1 2 3 2 1 1          |
        |         1 1 2 2 1 1 1          |
 -2.00  ----------------------------------
       -0.40                          1.00
```

```
                 VERT: 2   HOR: 3
  0.25  ----------------------------------
        |             1 1 1 1 1          |
        |           1 1 2 3 2 1          |
        |         1 1 2 5 5 2 1 1        |
        |         1 1 4 8 7 3 1 1        |
        |         1 2 5 * 7 3 1          |
        |         1 2 5 8 5 2 1          |
        |         1 2 3 4 3 1 1          |
        |         1 1 2 2 2 1            |
        |           1 1 1                |
        |                                |
        |                                |
        |                                |
        |                                |
        |                                |
        |                                |
 -1.70  ----------------------------------
       -0.40                          1.00
```

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