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THE CONSUMPTION OF DURABLE GOODS IN A COMPLETE DEMAND SYSTEM

S.R. WUNDERINK-VAN VEEN AND J. VAN DAAL

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S.R. Wunderink-van Veen
(University of Utrecht)
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Preliminary

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Summary

In this paper we introduce a model for investigating effects of income, family composition, wealth and stock of durables on the allocation of household budgets to the various budget items, among which are durable goods. The model, an extension of the indirect addilog system, is estimated on data obtained from a Danish Budget Survey of 1971. Two series of results (parameter estimates, elasticities and predictions) are presented; first, on the basis of the (observed) expenditure amounts and second, on the basis of a data set obtained from the original set by transforming all amounts spent on durables into consumption amounts.

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1. Introduction

1.1 In this paper we estimate, on the basis of cross-section data, a model for the allocation of total family budgets to the various consumption categories, among which the consumption of durable goods, as depending on these budgets as well as on other explanatory variables. The latter variables are the households' numbers of children, numbers of adults, wealth, stocks of durables and "normative budgets" (to be explained below). In section 2 we describe our model, based on the addi-log budget allocation system. The data is discussed in section 3. In that section we also set out the methods we adopted for estimation and computation. The estimation results and a number of elasticities are presented and discussed in section 4. In section 5 we present some forecasting results. We end with some concluding remarks. The remainder of this section is devoted to a justification of the choice of our model.

1.2 Since we are interested in individual household behaviour or, to be more precise, in individual Engel curves, we have chosen as a startingpoint a model that is very flexible with respect to the shape of these Engel curves and that, at the same time, meets the integrability conditions under fairly wide conditions on the parameter values.

As far as we know, the only model that shows Engel curves that are non-linear in income, nor in some transformation of income, while meeting the integrability conditions, is the "Indirect Addi-log" demand system; see Somermeyer and Langhout (1972). Other models may have more attractive properties with respect to aggregation over individuals; in this paper, however, we are not interested in market demand systems. Moreover, the indirect addi-log system is furthermore attractive, because of its easy interpretativity and because the pertinent computations (of elasticities, for instance) are not too complicated.

As a startingpoint for the addi-log demand system, we consider the following general expenditure allocation model. Total expenditure y of a household will be allocated to various goods and services, numbered $1,\ldots,K$. The amount y_k that is allocated to category k is determined by household characteristics that are either typical for category k (like prices and stocks of k), or general (like family composition and total expenditure). Let $x_1,\ldots,x_S \in \mathbb{R}^K$ be vectors of S kinds of category dependent explanatory variables and $z_1,\ldots,z_G$ the general explanatory variables; then y_k depends on these variables as follows

(A) $$y_k = \phi_k^*(x_{11} \ldots x_{S1} \ldots x_{1K} \ldots x_{SK}, z_1 \ldots z_G).$$

Since $y = \sum_{k=1}^{K} y_k$ the budgetshares w_k can be written as

(B) $$w_k = \frac{\phi_k(x_{11} \ldots x_{SK}, z_1 \ldots z_G)}{\sum_{\ell=1}^{K} \phi_\ell(x_{11} \ldots x_{SK}, z_1 \ldots z_G)}$$

for $k = 1,\ldots,K$, where the ϕ_k results from the ϕ_k^* by omitting all common factors.

Somermeyer (1956) was the first to study this model extensively. In the first instance he considered only one type of k-specific variables and one general variable, viz. prices $p_1,\ldots,p_K$ and total expenditure y, respectively. Moreover, he specified the functions ϕ_k. This resulted in:

(C) $$w_k = \frac{c_k \left(\frac{p_k}{y}\right)^{\alpha_k}}{\sum_{k=1}^{K} c_\ell \left(\frac{p_\ell}{y}\right)^{\alpha_\ell}},$$

where the α_k and c_k are parameters. The model (C) meets the integrability conditions if all c_k are positive and all $\alpha_k \leq 1$ with at most one $\alpha_k = 1$, irrespective of the values of the variables; see also Van Daal (1984). These conditions can easily be checked after parameter estimation.

Since $w_k = \frac{p_k q_k}{y}$ we find that the consumed quantity q_k can be written as

(D) $$q_k = \frac{c_k \left(\frac{p_k}{y}\right)^{\alpha_k - 1}}{\sum_{k=1}^{K} c_\ell \left(\frac{p_\ell}{y}\right)^{\alpha_\ell}}.$$

The shapes of the Engel curves are extensively investigated by Somermeyer and Langhout (1972). These curves all start in the origin because

$$\lim_{y \to 0} y_k = p_k \lim_{y \to 0} q_k = 0;$$

when $y \to \infty$ we have three possibilities:

$$\begin{aligned} \lim_{y \to \infty} y_k = p_k \lim_{y \to \infty} q_k &= \infty \quad \text{for all } k \text{ with } \alpha_k < 1 + \alpha_{\underline{\ell}}, \\ &= \text{finite} \quad \text{for } k \text{ such that } \alpha_k = 1 + \alpha_{\underline{\ell}}, \\ &= 0 \quad \text{for all } k \text{ with } \alpha_k > 1 + \alpha_{\underline{\ell}} \end{aligned}$$

in which $\underline{\ell}$ is the category with the lowest α (the most luxurous good).

The slopes of the Engel curves in the origin are either zero or one:

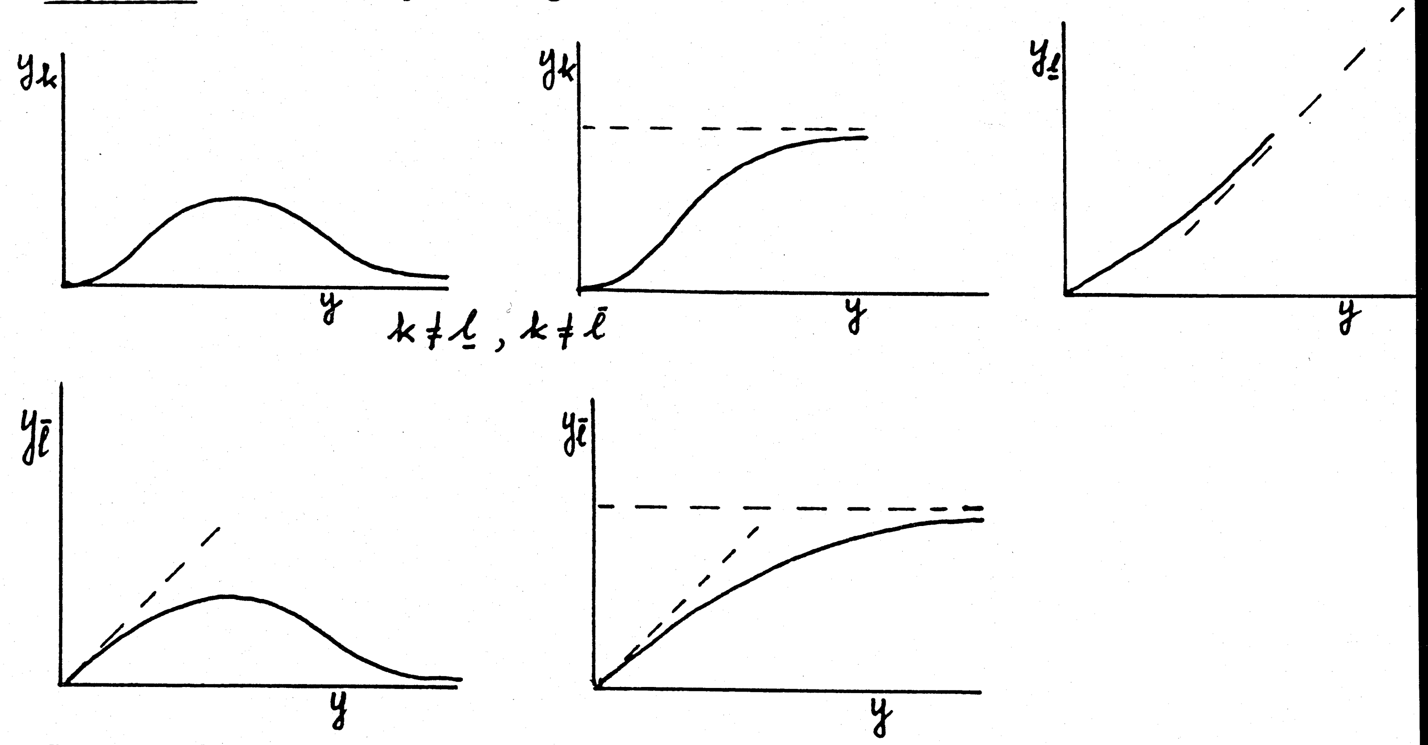
$$\begin{aligned} \lim_{y \to 0} \left(\frac{\partial y_k}{\partial y}\right) &= 0 \text{ for all } k \text{ with } \alpha_k < \alpha_{\bar{\ell}} \\ &= 1 \text{ for } k \text{ such that } \alpha_k = \alpha_{\bar{\ell}} \end{aligned}$$

in which $\bar{\ell}$ is the category with the highest α (the most necessary good). When $y \to \infty$ we find:

$$\begin{aligned} \lim_{y \to \infty} \left(\frac{\partial y_k}{\partial y}\right) &= 0 \text{ for } \alpha_k > \alpha_{\underline{\ell}} \\ &= 1 \text{ for } \alpha_k = \alpha_{\underline{\ell}} \end{aligned}$$

The slopes further appear to be such that there are five possible shapes; see Figure 1.

Figure 1. Possible shapes of Engel curves



Because of this great variety in shapes we chose for the addi-log model.

2. The model

2.1 Our starting point is, therefore, the following extension of (C):

$$(1) \qquad w_{kh} = \frac{c^{*}_{kh}\left(\frac{p_k}{y_h}\right)^{\alpha_k}}{\sum_{\ell=1}^{K} c^{*}_{\ell h}\left(\frac{p_\ell}{y_h}\right)^{\alpha_\ell}},$$

where w_{kh} = budget share of consumption category k (k = 1,...,K) by household h (h = 1,...,H),

p_k = price of item k, supposedly the same for all households,

y_h = budget of household h.

The α_k (k = 1,...,K) are "reaction parameters" that are supposed to be the same for all households and the c^{*}_{kh} are "urgency parameters" that depend on household characteristics in a way to be discussed below.

In our model (1) the variable y_h and the parameters c^{*}_{kh} are household-dependent. These dependencies are typical of this paper; they are, among others, inspired by the fact that our data is detailed enough to implement them empirically. We discuss the y_h and c^{*}_{kh} consecutively below.

2.2 First, we discuss the budget variable y_h. As we mentioned above, the system (1) can be seen as the result of maximizing a utility function under the condition of a budget restriction, provided some conditions on the α_k and c^{*}_{kh} are met with. This implies that the quantities consumed have to be non-negative and infinitely divisible; to be precise, they have to be non-negative real numbers. Moreover, in many models, such as ours, these quantities have to be positive. Not all households meet this condition with respect to the purchase of durable goods. All households, however, use durable goods and if we are able to measure the "utilization" per year, we get the consumption of durables per year which is an amount that fulfills the conditions of divisibility and positivity. For that reason we adopt consumption instead of purchase of durables in our model. The plausibility of this choice can be argued as follows.

Often, in particular in time-series demand analysis, the households are assumed to budget in two stages as follows:

(2) disposable income
- purchases (y_h^*)
  1. food (, beverages, tobacco)
  2. clothing (& footwear)
  3. housing
  4. transport (& communication)
  5. durables (purchases!)
  6. remaining goods and services
- savings.

The budget items 1 through 6 are those that will be considered throughout the remainder of this paper. If a ("primary") durable good (e.g., the washing machine) breaks down, one has to spend a big amount of money at once which usually goes at the cost of savings, sometimes even resulting in dissavings. This means that the budget depends, in this respect, on the purchases and not the other way round, which makes scheme (2) above less adequate.

If, however, <u>consumption</u> of durables is taken into account, then the two-stage budgeting process looks like this:

(3) diposable income + initial value of stock of durables

- consumption budget (y_h)
  1. food
  2. clothing
  3. housing
  4. transport
  5. durables (cons.!)
  6. remaining
- savings + final value of stock of durables

In this scheme, consumption of durables arises from stock and/or purchases; note that the amounts y_h^* in (2) and y_h in (3) will be different in general. In case of purchase of a durable good a (small) part of the purchase-money is considered as consumption in the concerning year, whereas the remainder is reserved for future consumption; the latter part will be called the investment part of these purchases. It is a moot question what price has to be attached to a unit of consumption of durables in the above setup. Fortunately, we do not have to solve this problem now because we are going to estimate the model on a cross-section of consumption data where all households are supposed to have paid the same price for the same item.

The variable y_h in (3), i.e. the total consumption budget of household h, can be written as follows:

(4) y_h = total purchases of non-durables + consumption of durables

= total purchases of non-durables + (consumption part of the initial stock of durables + purchases of durables - investment part of purchases of durables - consumption part of sold durables)

= (a) total purchases + (b) the consumption part of the initial stock of durables - (c) the investment part of the purchases of durables - (d) the consumption part of sold durables.

The difference between disposable income and y_h calculated as above is the sum of savings or dissavings and the change in the value of the stock of durable goods. The amounts obtained from durables sold during the periods of observation is thus a part of disposable income. These sales, as well as the purchases of durables, are all considered as having happened at the beginning of the period.

We cannot deny that the way, in which we are going to calculate the elements (b), (c) and (d) in (4), as described below, is somewhat arbitrary. This is partly because of the nature of the given data and partly because making arbitrary decisions is inherent in doing research.

For each kind of durable m bought τ periods before the period of observation by a household h and which would have a purchase value v_{mh} in the period of observation we calculate the consumption amount z_{mh} for the period of observation as follows:

(5) $$z_{mh} = v_{mh}(1 - d_m)^{\tau}(d_m + r),$$

where d_m = depreciation rate of m
r = real interest rate during the period of observation.

The data that are available to us, tell us the year in which a certain durable, which is still part of the stock of some household, was bought (new or second-hand). Therefore, we can find for each durable good the age-composition of the stock for the whole sample of households. If households would have bought durables only as a replacement of worn-out or defective pieces, we might have been able to use some reliability model to estimate normal lifetime of these durables. However, there are also households that buy these durables for the first time (in a growing market or because they are "young" households). The age composition of the stock of a certain durable for our sample of households shows the result of the combination of these aspects. For that reason we could not just rely on our sample data to estimate depreciation rates of the various durables; we also had to gather information from outside the sample (like from the Dutch Automobile Club, A.N.W.B., for cars).

Where possible, that means where replacements were an important part of the purchases, we estimated "normal" lifetime L_m from our data, such that only a fraction ε of the stock is older than L_m (ε some small number). The depreciation rate is chosen such, that d_m is the smallest number such that

$$(6) \qquad (1 - d_m)^{L_m} \leq \varepsilon.$$

For ε we took the value 0.02; we also tried the values 0.03 and 0.04, but this did not make much difference. Newly introduced durables were given depreciation rates corresponding to comparable, longer existing durables (dishwasher equal to washing machine; color TV equal to black en white TV et cetera). The parameter estimates have not shown great sensitivity for varying depreciation rates. We shall comment on this in subsection 4.4.

If a durable good m has been bought in the period of observation ($\tau = 0$) then we take for v_{mh} simply the purchase amount. For the other present values v_{mh} we took, for all but six kinds of durables, the means of the purchase amounts paid in the period of observation; for these goods the values v_{mh} are, in fact, household-independent ($\tau \geq 1$). In six cases, however, (new and second-hand cars and new mopeds, washing machines, freezers and car radios) there appeared to exist a significant relation between the amount v_{mh} paid by household h for a unit of such a kind of durable good bought during the observation period on the one hand, and, on the other, observed disposable income Y_h^d:

$$(7) \qquad v_{mh} = \phi_m(Y_h^d).$$

We suppose that the same relation held for these goods when bought in the past by a household. Hence we use (7) for the estimation of the present value v_{mh} of such a durable good m bought τ periods ago by household h where we take an estimate $\hat{Y}_{h\tau}^d$ of the household's disposable income τ periods ago. The estimate $\hat{Y}_{h\tau}^d$ is calculated by means of an age-income profile derived from the sample of bread-earners in the cross-section. Lack of data prevented us to use a more appropriate profile. Now we can calculate total consumption D_h of durables by household h as follows:

(8) $$D_h = \sum_{m=1}^{M} \sum_{\tau=0}^{T} (d_m + r)(1 - d_m)^{\tau} v_{mh} \delta_{\tau m},$$

where M = number of categories of durables,

T = number of periods in the past,

$\delta_{\tau m}$ = 1 if a purchase of kind m took place τ periods before the period of observation

= 0 if otherwise.

We admit that it is fairly rough to throw all kinds of durables on one big pile as has been done in (8). On the other hand, we are now allowed to consider the consumption of durables as a real decision variable because every household could choose to spend an amount on durables during the observation period and was, therefore, not completely rationed in the consumption of durables, which would be the case when all durables were bought in the past; see, e.g., Deaton (1981). In fact, the decision to take housing as unrationed, as we did, is likewise questionable.

2.3 Second, we discuss the "parameters" c^*_{kh}. One of our maintained hypotheses is that households that are identical with respect to a number of characteristics behave approximately the same with respect to consumption. So we assume the c^*_{kh} to depend on a number of characteristics in a way that is independent of the households. We assume that each household consists of a certain number of "standard consumers." This number is a linear function of the number of children and adults in the household such that the bread-earner can be considered as precisely one standard consumer; the parameters of this function have to be estimated. The c^*_{kh} are specified such that each household's budget is divided by its number of standard consumers. The other variables in the c^*_{kh} are introduced straightforwardly in an exponential form. So we get

(9) $$c^*_{kh} = c^*_k \cdot \exp(\beta_k W_h + \gamma_k B_h + \delta_k S_{kh}) \cdot [1 + fm_h + g(a_h - 1)]^{\alpha_k}$$

where W_h = net financial wealth

B_h = the rate of deviation from normative budget

S_{kh} = the estimated value of the stock of k-specific durables

m_h = number of children

a_h = number of adults of household h;

c_k^*, α_k, β_k, γ_k, δ_k, f and g are parameters.

The variables W_h, B_h and S_{kh} are introduced with parameters that may differ per budget item; the exponential form is chosen because these variables are not necessarily positive, while c_{kh}^* must be positive. The parameters f and g are taken the same for k = 1,...,K. This is for empirical reasons as will be set out below. The power factor of (9) can now be combined with the powers in (1). The complete model then can be written as follows

$$(10) \qquad w_{kh} = \frac{c_k \cdot \exp(\beta_k W_h + \gamma_k B_h + \delta_k S_{kh}) \cdot [y_h/(1 + fm_h + g(a_h - 1))]^{-\alpha_k}}{\sum_{\ell=1}^{K} c_\ell \cdot \exp(\beta_\ell W_h + \gamma_\ell B_h + \delta_\ell S_{\ell h}) \cdot [y_h/(1 + fm_h + g(a_h - 1))]^{-\alpha_\ell}},$$

where $c_k = c_k^* p_k^{\alpha_k}$; because the prices are unknown but the same for all households we could do no better than combining them with the c_k^* into one parameter c_k for each k = 1,...,K.

The choice of the variables is fairly straightforward with two exceptions, viz. B_h and S_{kh}. The variable B_h is a more or less ad hoc variable deviced to further make plausible our choice of y_h (see (4) above), i.e. to show that the households adapt their purchase budget in case of purchase of a costly durable good (the so-called "fur coat" effect; see 6.1). This adaptation is necessary, as we assume, in order to prevent that the consumption of other goods decreases too much. Our purpose was to find a measure for these indicidental adaptations that can be positive (if purchases take place) or negative (if a purchase is foreseen for, say, the next period). For all households in the sample we found that there is a fairly stable relation of the type

$$(11) \qquad y_h^* = \mu_0 + \mu_1 y_h^d + \mu_2 (y_h^d)^2 + \mu_3 \log(a_h + m_h)$$

between y_h^* = total budget

y_h^d = disposable income, and

$a_h + m_h$ = family size of household h.

We have chosen for this specific form, simply because it showed a high R^2, compared to other forms that we tried. We took its unelegant construction for granted since we use it only to create the variable B_h. The estimation result is:

$$\hat{y}_h^* = \underset{(1223.91)}{1943.56} + \underset{(0.0456)}{0.8889}\, y_h^d - \underset{(0.0033)}{0.0075}\left(\frac{y_h^d}{100}\right)^2 + \underset{(612.43)}{2685.43}\, \log(a_h + m_h) \tag{12}$$

$[R^2 = 0.78,\ H = 917]$.

The variable B_h, that we introduced in our model is defined on the difference between the actual consumption rate and the estimated consumption rate

$$B_h = \frac{y_h^* - \hat{y}_h^*}{y_h^d}\,. \tag{13}$$

Note that the choice between saving and expenditure is supposed to be made independent of the decision, how to spread the budget over the consumption categories, so that we don't introduce any circularity in our model. See section 2, scheme (2).

Concerning the variables S_{kh} we remark that the total value of the stock of durables of each household h has been split up into the values S_{3h}, S_{4h} and S_{6h} of durables belonging to housing, transport and remaining goods, respectively. There are no stock values known for the categories 1 (food) and 2 (clothing) so that there are no parameters δ_1 and δ_2. For S_{5h} we have taken the total stock of durables.

3. Data and data handling

3.1 The data are taken from the budget survey 1971 performed by the Danish Statistical Office (Danmarks Statistik). About 1000 Danish households of wage earners were sampled and inquired. For a discussion of the questionnaire and the answers given by the households we refer to Danmarks Statistik (1977) and to Sloth Jensen (1980). All the figures that we need for the construction of our variables are at our disposal[1]. There is, in particular, very detailed information on 37 kinds of major durables (clothing and furniture are not considered as durables in this survey, however).

In the remainder of this paper we shall distinguish two data sets, further to be called the "original data" and the "revised data"; all computations will be performed on both sets. The first data set is the set as we found it on the tapes: the amounts with respect to durables were purchases and sales done during the period of observation; all these purchases and sales were combined into our category 5 in (3) above. In the revised data set the category 5 consists of the total consumption of durables as calculated according to (8) above. There are no further differences between both sets. In tables 1 and 2 average values of budget shares for seven income classes are presented. In table 3 one can find some more averages.

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1) We thank the Economic Institute of the Copenhagen School of Economics (in particular Professor B. Sloth Jensen) and Danmarks Statistik (in particular Mr. A. Nielsen) for this.

Table 1. Income ranges and average observed budget shares for 7 income classes (original data, standard deviations between brackets).

Income class and range	food	clothing	housing	transp.	durables	rem.	number of h.h.
1 D.Kr. 0- 30.000	0.295 (0.133)	0.078 (0.050)	0.266 (0.102)	0.108 (0.077)	0.057 (0.116)	0.196 (0.092)	112
2 30.000- 40.000	0.307 (0.100)	0.078 (0.045)	0.268 (0.098)	0.117 (0.066)	0.052 (0.088)	0.178 (0.075)	115
3 40.000- 50.000	0.255 (0.105)	0.082 (0.036)	0.278 (0.099)	0.122 (0.080)	0.082 (0.117)	0.181 (0.073)	97
4 50.000- 65.000	0.272 (0.108)	0.074 (0.037)	0.281 (0.112)	0.124 (0.061)	0.052 (0.089)	0.197 (0.074)	88
5 65.000- 80.000	0.241 (0.098)	0.095 (0.042)	0.322 (0.105)	0.119 (0.049)	0.027 (0.044)	0.196 (0.066)	29
6 80.000-110.000	0.215 (0.089)	0.078 (0.044)	0.341 (0.101)	0.129 (0.092)	0.082 (0.074)	0.155 (0.058)	10
7 110.000- ∞	0.201 (0.091)	0.057 (0.029)	0.359 (0.173)	0.116 (0.027)	0.074 (0.125)	0.193 (0.062)	7

Table 2. Average budgetshares for 7 income classes (revised data, standard deviations between brackets).

class	food	clothing	housing	transp.	durables	rem.	number
1	0.291 (0.127)	0.078 (0.050)	0.262 (0.098)	0.107 (0.076)	0.067 (0.071)	0.195 (0.090)	112
2	0.301 (0.098)	0.077 (0.044)	0.260 (0.094)	0.114 (0.062)	0.075 (0.062)	0.174 (0.073)	115
3	0.254 (0.101)	0.083 (0.037)	0.277 (0.098)	0.119 (0.073)	0.089 (0.066)	0.179 (0.069)	97
4	0.266 (0.105)	0.073 (0.037)	0.272 (0.106)	0.121 (0.057)	0.077 (0.052)	0.192 (0.069)	88
5	0.227 (0.092)	0.090 (0.041)	0.302 (0.095)	0.111 (0.045)	0.084 (0.048)	0.184 (0.062)	29
6	0.213 (0.087)	0.076 (0.041)	0.331 (0.083)	0.126 (0.091)	0.099 (0.052)	0.154 (0.064)	10
7	0.199 (0.098)	0.055 (0.026)	0.345 (0.160)	0.114 (0.031)	0.101 (0.074)	0.186 (0.056)	7

Table 3. Average wealth, deviation and stock per income class (standard deviation between brackets).

cl.	net wealth[1)]	"deviation"[2)]	housing[3)] stocks	transp.[3)] stocks	rem.1[3)] stocks	total[3)] stocks
1	0.371 (0.519)	-0.000 (0.250)	0.665 (0.595)	2.440 (3.564)	0.819 (1.280)	3.995 (3.690)
2	0.546 (0.645)	-0.011 (0.169)	0.958 (0.743)	4.695 (5.086)	1.341 (1.931)	6.994 (5.406)
3	0.705 (0.973)	(0.013) (0.212)	1.271 (1.008)	5.570 (5.832)	1.389 (1.189)	8.230 (6.306)
4	1.085 (1.083)	-0.007 (0.162)	1.259 (0.850)	7.942 (6.841)	1.810 (2.046)	11.012 (7.462)
5	1.460 (1.092)	-0.077 (0.139)	1.979 (1.276)	11.864 (7.322)	2.000 (2.402)	15.843 (8.758)
6	2.350 (1.710)	0.068 (0.197)	1.747 (1.290)	12.235 (13.032)	3.139 (3.682)	17.121 (14.584)
7	5.555 (2.201)	-0.039 (0.151)	2.534 (1.537)	24.539 (11.325)	2.603 (1.596)	29.676 (12.140)

1) One hundred thousands of Danish Crowns.

2) See (13).

3) In thousands of Danish Crowns.

3.2 For estimation purposes we introduce additive disturbances into the model (10); this can then be written as

$$(14) \qquad w_{kh} = f_k(X_h, \theta) + u_{kh},$$

where $f_k(X_h,\theta)$ means, for short, the right-hand member of (10) with X_h the vector of the explanatory variables for household h, and θ the parameter vector of the deterministic part of the model. The u_{kh}, combined into the vector u_h, are the disturbances that are supposed to be normal variates with zero expectation and a covariance matrix Ω^* that is common to all households; because of the budget restriction we have $\iota^T\Omega^* = 0$. The whole parameter set consists now of the collection of all components of θ and Ω^*. To estimate them, in particular θ, we adopt a full information maximum likelihood method with as log-likelihood function

$$(15) \quad \ell(\theta,\Omega^*) = -\tfrac{1}{2}H(K-1)\log 2\pi - \tfrac{1}{2}H\log\det\Omega - \tfrac{1}{2}\sum_{h=1}^{H} e_h^T\Omega^{-1}e_h,$$

where Ω is the matrix that results if we delete the last row and the last column from Ω^*; H is the number of households of the sample and e_h is an (K - 1)-dimensional vector of residuals depending on θ, given the data, such that for all h = 1,...,H and k = 1,...,K-1

$$(16) \quad e_{kh} = w_{kh} - f_k(X_h,\theta).$$

In fact, we let out of consideration for each h all relations in (14) with k = K. It is well-known that $\ell(\theta,\Omega^*)$ remains the same if we delete for each h another equation of (14) instead of the Kth one; see Barten (1969). The log-likelihood function (15) can be concentrated by differentiating the right-hand member with respect to Ω, equating the result to zero, solving Ω from this in terms of θ, given X, and substituting the latter result in the right-hand member of (15); see, e.g., Bard (1974, p. 66). We then get the concentrated log-likelihood function

$$(17) \quad \ell^*(\theta) = \tfrac{1}{2}H(K-1)(\log(H/2\pi) - 1) - \tfrac{1}{2}H\log\det M(\theta),$$

where $M(\theta)$ is a matrix of moments defined as

$$(18) \quad M(\theta) = \sum_{h=1}^{H} e_h \cdot e_h^T.$$

Hence a vector $\hat{\theta}$ which minimizes $\det M(\theta)$ is a FIML estimate of θ.

This minimization we performed by means of a Gauss-Marquardt numerical procedure (see Bard (1974, pp. 94-99)) where the Hessian of the objective function $\det M(\theta)$ has been approximated by using only first order derivatives of $\det M(\theta)$ with respect to the elements of θ (see Berndt et al. (1974)). The derivatives have been computed analytically.

In fact, the assumption of normality of the disturbances is not correct because this implies the possibility of budget shares that are negative or that exceed 1. A possible way-out is to assume that the budget shares themselves are random variables with a Dirichlet distribution as suggested by

Woodland (1979). Woodland's calculations, though on a fairly simple model, give evidence that using a normal distribution instead of a Dirichlet distribution does not result in considerable differences in parameter estimates. We have the same experience on the basis of the estimations we performed on more complicated models with both assumptions alternatively. Where the assumption of a Dirichlet distribution entailed complicated calculations, in particular with respect to the asymptotic standard errors, we retained the assumption of normality.

Finally, we remark that all estimations are performed on one half of the data (to be called part 1); the other half (part 2) is used in section 5 for forecasting purposes using the estimation results of section 4.

4. Estimation results

4.1 In this section we present estimation results of the model

$$(19) \quad w_{kh} = \frac{c_k \cdot \exp(\beta_k W_h + \gamma_k B_h + \delta_k S_{kh}) \cdot [y_h/(1 + fm_h + g(a_h - 1))]^{-\alpha_k}}{\sum_{\ell=1}^{K} c_\ell \cdot \exp(\beta_\ell W_h + \gamma_\ell B_h + \delta_\ell S_{\ell h}) \cdot [y_h/(1 + fm_h + g(a_h - 1))]^{-\alpha_\ell}}$$

developed in section 2 on the basis of the data described in section 3.

For reasons of identification we were compelled to impose the following restrictions:

(i) Because there are no price variations known between the units of observation we cannot estimate the levels of the reaction parameters α_k. Only the mutual differences between these parameters can be identified; this can easily be inferred from the fact that augmenting all exponents α_k with the same number does not change the w_{kh} of (19). Fortunately, this is sufficient for calculating income elasticities; see below. It is regrettable, however, that now it is impossible to test the condition of negative-semi-definiteness of the Slutsky matrices of the households. Attempts to identify the levels of the α_k parameters by replacing the parameters f and g in (19) by k-dependent parameters (in the numerator we than would have f_k and g_k, respectively, and in the denominator f_ℓ and g_ℓ) were not successful. The same negative experience is gained by Van Driel (1985) and Janssen (1984); see also Blokland (1976, pp. 74 ff.) and Cramer (1969, pp. 161 ff.). Consequently, we fix α_1 = 0.5; this is a plausible value because the α_k are more close to 1 the more k is a necessary budget item; see Somermeyer and Langhout (1972).

(ii) The right-hand member of (19) is zero-homogeneous in the parameters c_k. Therefore, we fix $c_6 = 1$.

(iii) Just as the parameters α_k, the β_k and γ_k are only identifiable up to an additive constant. Therefore, we fix $\beta_1 = \gamma_1 = 0$.

4.2 In table 4 we present our estimation results.

Table 4. Estimation results

	Original data: parameter estimates	Original data: asymptotic standard errors	Original data: asymptotic t-values	Revised data: parameter estimates	Revised data: asymptotic standard errors	Revised data: asymptotic t-values
α_1	0.5	--	--	0.5	--	--
α_2	0.184	0.113	2.80*)	0.253	0.083	2.98*)
α_3	-0.009	0.100	5.09	0.081	0.085	4.93
α_4	-0.049	0.125	4.39	0.108	0.097	4.04
α_5	-0.947	0.215	6.73	0.227	0.104	2.63
α_6	-0.290	0.100	7.90	-0.050	0.096	5.73
f	0.297	0.076	3.91	0.547	0.221	2.48
g	1.112	0.179	6,26	2.585	0.730	3.54
β_1	0	--	--	0	--	--
β_2	0.05	0.03	1.53	0.06	0.03	1.95
β_3	0.08	0.03	2.83	0.10	0.03	3.66
β_4	-0.07	0.04	-1.83	-0.04	0.04	-1.03
β_5	-0.32	0.09	-3.42	-0.10	0.04	-2.50
β_6	-0.05	0.03	-1.58	-0.01	0.03	-4.26
γ_1	0	--	--	0	--	--
γ_2	-0.350	0.212	-1.65	-0.365	0.201	-1.81
γ_3	-0.639	0.186	-3.44	-0.411	0.169	-2.44
γ_4	-0.728	0.245	-2.96	-0.413	0.219	-1.88
γ_5	0.953	0.212	4.50	-0.415	0.244	-1.70
γ_6	-0.817	0.181	-4.52	-0.437	0.159	-2.75
δ_1	--	--	--	--	--	--
δ_2	--	--	--	--	--	--
δ_3	0.119	0.023	5.24	0.129	0.023	5.68
δ_4	0.029	0.004	7.24	0.032	0.004	7.86
δ_5	-0.000	0.010	-0.00	0.054	0.004	14.07
δ_6	0.019	0.011	1.80	0.023	0.011	2.04
c_1	63.425	31.453	2.02	16.321	7.861	2.08
c_2	3.850	1.962	1.96	1.536	0.580	2.65
c_3	4.579	1.983	2.31	2.118	0.600	3.53
c_4	1.725	0.911	1.89	1.078	0.370	2.91
c_5	0.015	0.016	0.95	0.953	0.451	2.11
c_6	1	--	--	1	--	--
minimum function value:	852.34**			564.76		
H	458			458		

*) The t-values for the α-parameters refer to the estimated values of the differences $\alpha_1 - \alpha_k$.

**) $\frac{1}{2}H \log \det(M(\hat{\theta}))$.

4.3 From (19) we can compute the income elasticities $E(q_{kh},y_h)$ of the quantities q_{kh} of good k consumed by household h with respect to y_h:

$$(20) \qquad E(q_{kh},y_h) = 1 - \alpha_k + \sum_{\ell=1}^{L} w_{\ell h}\alpha_\ell = 1 - (\alpha_k - \alpha_1) + \sum_{\ell=1}^{L} w_{\ell h}(\alpha_\ell - \alpha_1)$$

because $\Sigma_\ell w_{\ell h} = 1$. Hence these elasticities can be computed since we know all differences $\alpha_k - \alpha_1$. The elasticities all differ per household; in tables 5 and 6 we tabulate their averages over all the households of seven classes of disposable income (see table 1). Because (20) is linear in the budget shares and the α_k do not depend on h, we get the averages of the income elasticities if we substitute the averages of the budget shares from tables 1 and 2 into (20). The elasticities in tables 5 and 6 are computed in this way. The standard errors are computed by means of the covariance matrix of the estimates, given the average budget shares.

Table 5. Estimated average income elasticities and there standard errors (original data).

cl.	food		clothing		housing		transp.		durables		rem.	
	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error
1	0.544	0.06	0.860	0.27	1.053	0.23	1.093	0.29	1.991	0.52	1.333	0.23
2	0.559	*	0.875		1.068		1.108		2.006		1.348	
3	0.504		0.820		1.013		1.053		1.951		1.293	
4	0.535		0.851		1.044		1.084		1.982		1.324	
5	0.547		0.863		1.056		1.096		1.994		1.336	
6	0.489	0.07	0.805		0.998		1.039		1.936		1.279	
7	0.477		0.793		0.985		1.026		1.924		1.266	

* The same as above if not mentioned

Table 6. Estimated income elasticities (revised data).

cl.	food elast.	food st. error	clothing elast.	clothing st. error	housing elast.	housing st. error	transp. elast.	transp. st. error	durables elast.	durables st. error	rem. elast.	rem. st. error
1	0.704	0.05	0.951	0.06	1.122	0.07	1.096	0.08	0.977	0.06	1.253	0.08
2	0.712		0.959		1.130		1.104		0.985		1.261	
3	0.694	0.06	0.941		1.113	0.08	1.086		0.968	0.07	1.244	
4	0.694		0.941		1.113		1.086		0.968		1.244	
5	0.683		0.930		1.102		1.075		0.956		1.233	
6	0.681		0.928		1.100		1.073		0.954		1.231	
7	0.667		0.915		1.086		1.059		0.941		1.217	

4.4 The elasticities of the budget shares with respect to net wealth, durables and deviation from normative budget are computed by means of the following formulae

(21) $$E(w_{kh},W_h) = (\beta_k - \sum_{\ell=1}^{K} w_{\ell h}\beta_\ell)\cdot W_h,$$

(22) $$E(w_{kh},B_h) = (\gamma_k - \sum_{\ell=1}^{K} w_{\ell h}\gamma_\ell)\cdot B_h,$$

(23) $$E(w_{kh},S_{k\ell}) = (1 - w_{kh})\delta_k\cdot S_{kh},$$

and, for $k \neq \ell$,

(24) $$E(w_{\ell h},S_{kh}) = -w_{kh}\delta_k\cdot S_{kh}.$$

The appearance of the last factors W_h, B_h and S_{kh} in the 4 above formulae can be explained by the fact that multiplication, by a certain number μ of the unit of measurement of such a factor implies estimates of the matching parameters that are divided by the same number μ. This phenomenon does not appear in (20) because a change in the unit of measurement of income only effects the estimates of the parameters c_k but not the α_k. In tables 7 through 12 we present estimates of the first factors $(\beta_k - \sum_\ell w_{\ell h}\beta_\ell)$, $(\gamma_k - \sum_\ell w_{\ell h}\gamma_\ell)$ and $(1 - w_{kh})\delta_k$ of the elasticities of the budget shares with respect to net wealth, deviation from normative budget and own stock, respectively; these figures have been computed in the same way as adopted for tables 5 and 6. The averages of the remaining factors can be found in table 3.

Table 7. Wealth elasticities (original data).

cl.	food		clothing		housing		transp.		durables		rem.	
	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error
1	0.01	0.02	0.06	0.08	0.09	0.07	-0.06	0.09	-0.31	0.22	-0.04	0.07
2	0.01		0.06		0.09		-0.06		-0.31		-0.04	
3	0.02		0.07		0.10		-0.05		-0.30		-0.03	
4	0.01		0.06		0.09		-0.06		-0.31		-0.04	
5	-0.01		0.05		0.08		-0.08		-0.32		-0.05	
6	0.01		0.06		0.09		-0.06		-0.31		-0.04	
7	0.01		0.06		0.09		-0.06		-0.31		-0.04	

Table 8. Wealth elasticities (revised data).

cl	food		clothing		housing		transp.		durables		rem.	
	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error
1	-0.02	0.02	0.05	0.08	0.08	0.06	-0.06	0.09	-0.12	0.10	-0.03	0.07
2	-0.02		0.05		0.09		-0.06		-0.12		-0.03	
3	-0.02		0.05		0.09		-0.06		-0.12		-0.03	
4	-0.02		0.05		0.09		-0.06		-0.12		-0.03	
5	-0.02		0.04		0.08		-0.06		-0.12		-0.03	
6	-0.02		0.04		0.08		-0.06		-0.12		-0.03	
7	-0.02		0.04		0.08		-0.06		-0.12		-0.03	

Table 9. "Deviation" elasticities (original data).

cl.	food		clothing		housing		transp.		durables		rem.	
	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error
1	0.382	0.10	0.033	0.50	-0.257	0.43	-0.346	0.58	1.336	0.51	-0.435	0.42
2	0.379	0.11	0.030		-0.260		-0.348		1.333		-0.438	
3	0.364		0.015		-0.275		-0.363		1.318		-0.453	
4	0.408		0.058		-0.232		-0.320		1.361		-0.410	
5	0.460	0.12	0.110		-0.180		-0.268		1.413		-0.358	
6	0.387		0.038		-0.252		-0.340		1.341		-0.430	
7	0.421		0.072		-0.218		-0.307		1.375		-0.396	

Table 10. "Deviation" elasticities (revised data).

cl	food		clothing		housing		transp.		durables		rem.	
	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error
1	0.293	0.10	-0.072	0.48	-0.117	0.39	-0.119	0.52	-0.122	0.58	-0.144	0.37
2	0.289		-0.076		-0.122		-0.124		-0.126		-0.149	
3	0.308		-0.057		-0.103		-0.105		-0.107		-0.129	
4	0.304		-0.061		-0.107		-0.109		-0.111		-0.134	
5	0.319		-0.046		-0.092		-0.094		-0.096		-0.119	
6	0.325	0.11	-0.040		-0.086		-0.088		-0.090		-0.113	
7	0.332		-0.033		-0.079		-0.081		-0.083		-0.106	

Table 11. Own stock elasticities (original data).

cl.	food	clothing	housing		transp.		durables		rem.	
			elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error
1			0.088	0.007	0.026	0.012	0.000	0.009	0.016	0.005
2			0.087		0.026		0.000		0.016	
3			0.086		0.026		0.000		0.016	
4			0.086		0.026		0.000		0.016	
5			0.081	0.006	0.026		0.000		0.016	
6			0.079		0.026	0.011	0.000		0.016	
7			0.076		0.026		0.000		0.016	

Table 12. Own stock elasticities (revised data).

cl.	food	clothing	housing		transp.		duralbes		rem.	
			elast.	st. error	elast.	st. error	elast.	st. error	elast.	st. error
1			0.095	0.006	0.029	0.007	0.050	0.004	0.019	0.005
2			0.096		0.029		0.050		0.019	
3			0.093		0.028		0.049		0.019	
4			0.094		0.028		0.050		0.019	
5			0.090	0.005	0.029		0.049		0.019	
6			0.086		0.028		0.048		0.020	
7			0.085		0.029		0.048		0.019	

5. Forecasts

5.1 The model (19) can be used for making predictions of the budget shares of individual households given their budgets. This will be done in this subsection; in the next subsection we present some predictions of the purchase of durable goods. The word prediction has to be taken with a pinch of salt in this case because we take the (consumption) budgets of the households as given. In fact, the only thing we do, as is usual in this field of research, is calculating how the households of part 2 of our data (see subsection 3.1) would have allocated their budgets (the purchase budget as well as the (revised) consumption budget) according to the model (19) with parameters from table 4, i.e. obtained on the basis of part 1 of our data. These predictions will be compared with the observed shares. Again, we report averages over our seven income classes. This has been done in table 13.

5.2 Finally, we used the predictions above for calculating predictions of the purchases of durable goods. For each household we computed consumption from stock. If the predicted total consumption is less than or equal to consumption from stocks then the household is considered not to have purchased any durable good. If there is a postive difference between predicted total consumption of durables and consumption from stock then we convert this difference into a prediction of the amount spent on purchasing durables. The results are presented in table 14 together with the predictions obtained by using the original data.

Table 13. Predicted budget shares and observed budgetshares

		Original data (part 2)			Revised data (part 2)		
cat.	cl	pred.	st. error	observed	pred.	st. error	observed
food	1	0.392	0.110	0.325	0.383	0.105	0.329
	2	0.401		0.301	0.392		0.297
	3	0.368		0.294	0.368		0.291
	4	0.365		0.267	0.368		0.261
	5	0.350		0.258	0.358		0.253
	6	0.330		0.254	0.333		0.256
	7	0.296		0.280	0.296		0.286
clothing	1	0.072	0.045	0.079	0.074	0.044	0.079
	2	0.074		0.081	0.073		0.080
	3	0.072		0.073	0.072		0.072
	4	0.073		0.082	0.072		0.080
	5	0.072		0.080	0.072		0.078
	6	0.075		0.065	0.073		0.067
	7	0.074		0.073	0.071		0.074
housing	1	0.170	0.100	0.238	0.224	0.096	0.237
	2	0.176		0.289	0.223		0.280
	3	0.185		0.266	0.234		0.262
	4	0.191		0.291	0.234		0.284
	5	0.201		0.312	0.244		0.303
	6	0.233		0.313	0.263		0.314
	7	0.234		0.309	0.272		0.308
transp.	1	0.066	0.069	0.095	0.095	0.067	0.095
	2	0.068		0.105	0.095		0.103
	3	0.072		0.118	0.098		0.116
	4	0.076		0.104	0.102		0.102
	5	0.077		0.120	0.102		0.112
	6	0.080		0.109	0.103		0.107
	7	0.092		0.116	0.115		0.118

Table 13. (continued)

durables	1	0.195	0.098	0.063	0.056	0.057	0.059
	2	0.177		0.051	0.058		0.069
	3	0.193		0.073	0.063		0.083
	4	0.188		0.061	0.070		0.081
	5	0.189		0.045	0.071		0.070
	6	0.172		0.082	0.071		0.080
	7	0.180		0.121	0.087		0.112
rem.	1	0.104	0.075	0.200	0.168	0.071	0.200
	2	0.104		0.173	0.159		0.170
	3	0.110		0.176	0.165		0.176
	4	0.108		0.195	0.154		0.192
	5	0.112		0.186	0.154		0.182
	6	0.119		0.177	0.158		0.177
	7	0.125		0.099	0.159		0.102

Table 14. Predicted total amount of purchase of durables for 7 income classes

Income class	Observed purchases	Predictions on revised data	Predictions on original data	Number of households
1	1420	1536	5010	32
2	1688	1949	5169	69
3	1856	1941	6810	83
4	4044	2554	8306	112
5	3670	2901	9695	73
6	2977	3071	10277	62
7	5559	6312	13297	25

6. Conclusion

6.1 The results of the two foregoing sections give rise to the following remarks.

(i) For both data sets food turns out to be the most necessary good as could be expected. Durable goods are the most luxurious goods on the bases of the original data; the income elasticities are in the order of magnitude of 2. The use of the revised data changes this picture drastically; the income elasticities of durables are now in the order of magnitude of somewhat less than 1 and the category remaining goods becomes the most luxurious one. The overestimation of the income elasticities for durables for the original data set is possibly an example of the so-called "fur coat effect." This may be illustrated by the following example. Suppose that we measure the total expenditure of the households in some sample during a week together with the expenditure on clothing during that week. If clothing is a necessary good, we expect its budget share to decrease when total expenditure increases. But if one household would have bought a fur coat during that particular week both its total expenditure and its budget share for clothing increase considerably. This may result in an overestimation of the slope of the "budget share line" and the corresponding elasticity.

(ii) The wealth elasticities are all fairly small; they can be computed by multiplying the estimates reported in tables 7 and 8 by a factor W_h, which is in most cases far less than 2 because the observed amounts of wealth are seldom more than 200,000 Danish Crowns. Again, revision of the data makes the category durables less prominent.

(iii) The "deviation elasticities" are computed in order to check whether the households use a part of (expected) savings to adapt their budgets in case of purchase of costly durable goods. We see that, for the original data, these elasticities are considerably positive for durables, while the elasticities for the categories 3, 4 and 6 are negative albeit non-significantly. The positive elasticities for food and clothing (the latter non-significant) can possibly be explained as follows. The adaptation of the budget intended to finance the purchase of a durable good may sometimes be not enough to cover completely this extra expenditure. This necessarily goes at the cost of the amount for the consumption of non-durables. Apparently, the consumption of food is not harmed by this (buying a car, e.g. does not imply eating less).
For the revised data we see, moreover, that the category durables lost its

status of an exceptional case. Remarkably, all food elasticities are still, positive but less, and the other ones are negative; most of the latter are non-significant as could be expected from a point of theory.

(iv) The parameters δ_k can be identified in level, given the units of measurement of the stocks (here in thousands of Crowns). Our results on the basis of the original data turn out that the value of the total stock has no significant influence on the purchase of durables. This also changes for the revised data because the consumption of durables is partly consumption out of stock, as we assume.

(v) The estimates of the parameters f and g, resulting from the original data set, indicate that the cost of a child is about 30 per cent of the cost of a standard consumer. The cost of a married couple (of two adults) is more than twice the cost of a standard consumer and, therefore, the cost of a child is about 14 per cent of the cost of two adults ("the parents"). The estimates that result, when using the revised data set, give rise to a different view. There we find that the cost of a child is about 55 per cent of the standard consumer and 15 per cent of a (married) couple which means that the cost of children in a "one-parent" household is felt as a much more heavy financial burden than in a "two-parents" household. Although this result is quite acceptable, we are not content with the fact that for the revised data the costs of the two adults is three and a half times that of the standard consumer. The differences between both data sets cannot explain this drastic change in results. We think that this may be caused by a dependence between the c_1-parameter and the f,g-combination. The reason for this dependence could be that the f and g parameters are some averages of f_1 through f_6 and respectively, g_1 through g_6; see (i) of subsection 4.1. This average f is far less than the values that we can expect for f_1; consequently the preference parameter c_1 takes a high value. Fixing the c_1 parameter at the level that was estimated from the original data (63.425), we found the values 0.346 and 1.589 for the estimated parameters f and g, respectively. Moreover, we found that the parameter estimates for the αs, βs and δs correspond much better to the values estimated from the original data set. A simple likelihood ratio test, however, obliged us to reject the hypothesis of a fixed parameter c_1.

(vi) The standard errors of the forecasts in table 13 are all substantial and, except for durables, do not differ much between our two data sets. The predicted shares for food and clothing show about the same picture for both sets. For the other categories the predictions based on the revised data set are much better than those based on the original one; for durables this difference is most striking. Knowing this, the predictions reported in table 14 will be no surprise.

6.2 The size of our data set is rather small for a cross-section investigation of the complexity of our study. Our results should, therefore, be considered as illustrations rather than as "hard facts." Nevertheless, in our opinion, the above results support the use of the revised data set as preferred to the original one. This is in accordance with our theoretical expositions in section 2. Further research is needed on the sensitivity of the estimates and the predictions with respect to the way the data are manipulated.

Altogether, it appears reasonable to propagate working, if possible, with a kind of data like our revised data set. Anyway, the category (or categories) of durable goods has (or have) to be treated in a special way. Using data like our original set precludes, strictly spoken, the assumption of normality (or some other continuous distribution) of the disturbances of the equations relating to durable goods. The density function for durables should then display one or more discontinuities; fitting in this complication into a simultaneous system of equations is rather complicated (see, e.g., Amemiya (1974)).

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