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AVERAGE SECURITY VALUE

A.G.Z. KEMNA AND A.C.F. VORST

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THE VALUE OF AN OPTION BASED ON AN AVERAGE SECURITY VALUE

by

A.G.Z. Kemna and A.C.F. Vorst
Erasmus University Rotterdam

ABSTRACT

In this paper we shall discuss a financial option of which the payoff depends on the average value of the underlying security over some final time interval. After explaining what an option is about we will derive a partial differential equation for the option which is different from the partial differential equation of a simple European call option. From this we will get an expectation formula for the option value. We will give an economical as well as a mathematical argument for this expectation formula.

CONTENTS

1. INTRODUCTION

The rapid development of option pricing theory and the application of that theory is caused by the path-breaking papers of Black and Scholes (Black and Scholes (1973)) and Merton (Merton (1973)). They derived a first explicit option pricing formula by using the theory of stochastic processes. This theory has also played a fundamental rôle in the further developments of option pricing and will also do so in this paper. A call option gives the owner the right to buy a specific share of stock at a specific future date (maturity date) for a fixed price (excercise price). In fact this is an European call option. One also has put options where the owner has the right to sell instead of to buy a share of stock and one has American options where the owner has the right to buy or sell at any time before the maturity date instead of just at the maturity date.
Now the Black-Scholes formula gives the price of such an European call option in the financial markets and Black and Scholes suggested that their solution to the option pricing problem could also be used for more complex options. This resulted in numerous extensions of the basic Black-Scholes model. In this paper we will also give an extension for a specific kind of option. To explain this kind of option we first remark that in practice the owner of the option doesn't actually buy the option at the maturity date but he simply gets the difference between the share price and the exercise price if this is positive and nothing if the difference is negative from the issuer of the option. Hence the issuer doesn't have to possess a share of stock.
The option we want to discuss in this paper gives the owner the right of getting the difference between the average value of the underlying security (in our case a share of stock) over some final time interval preceeding the maturity date and the exercise price. This kind of option is often part of a commodity-linked bond contract. The value of a commodity-linked bond is determined by the value of a reference bundle of the commodity. A recent example is the guilder oil bond issued by the Dutch venture capital company Oranje Nassau of which the payoff is the maximum of the price of 10,5 barrels of North Sea oil and 1.000 guilders. By the price of 10,5 barrels of North Sea oil is meant the average price over the last year of the contract. A pricing model and more exact description can be found in Kemna (1986). The whole contract

can be split in a normal bond of 1.000 guilders and an option of the kind we will consider here, where we take a share of stock as reference bundle.

The value of this option can be determined in two stages. In section 2 we derive a partial differential equation for the option price during the final time interval. The option price during the final time interval differs from a simple option price if the share price is high during the first part of the time interval. Then one can be sure of a positive payoff before the maturity date, while this is never true for a normal option. It will also be shown that we only need to solve the partial differential equation for the case that there is no positive payoff with certainty. The derivation will follow the lines of the Black-Scholes equilibrium approach. In section 3 we derive an expectation formula for this option. An economic as well as a mathematical argument is used. This approach is applied in section 4 for the option before the final time interval. Before the final time interval we have a simple option whose boundary value equals the starting value of the option during the final time interval. Due to this boundary condition there is no explicit formula for this simple option. In section 5 we explore under what conditions it would be possible to find an explicit formula for our complex option. It turns out that none of the conditions can be economically justified. This means that we have to solve the problem numerically or we have to use Monte-Carlo-simulation. Finally, section 6 summarizes the results and offers some concluding remarks.

2. A PARTIAL DIFFERENTIAL EQUATION FOR THE OPTION PRICE.

In this section we shall give a PDE for the value of an option of which the payoff is not only based on the value of the underlying security at the excercise time T but also on the average value of the security over some final time interval. If t_1 is the first moment from where on we take the average and hence $T-t_1$ is the length of the time interval we assume that the payoff of the option is equal to

$$\max(A_T-K,0) \qquad (1)$$

with

$$A_T = \frac{1}{T-t_1} \int_{t_1}^{T} S_\tau \, d\tau \qquad (2)$$

where K is the fixed excercise price and S_τ is the value of the underlying security at time τ.
In this section we will only give the PDE for $t_1 \le t \le T$ while in the next section we will give a PDE for $t < t_1$. Hence we will assume $t_1 \le t \le T$. If we put

$$A_t = \frac{1}{T-t_1} \int_{t_1}^{t} S_\tau \, d\tau \qquad (3)$$

then it is clear that the value of the option at any time t will depend on t, S_t and A_t. As said in the preceeding section we will assume that our underlying security is a share of common stock and as usual we will assume that the stock price is governed by the following stochastic differential equation

$$dS_t = \alpha S_t dt + \sigma S_t dW_t \qquad (4)$$

where W_t is a Wiener process and α and σ are constants. Since S_t is a stochastic process we have to interpret formulas (2) and (3) as stochastic integrals. If we put $X_t = (\ S_t\ A_t)'$ and $\beta = 1/(T-t_1)$ we can combine (3) and (4) to the following system of stochastic differential equations:

$$dX_t = \begin{pmatrix} \alpha & 0 \\ \beta & 0 \end{pmatrix} X_t \, dt + \begin{pmatrix} \sigma & 0 \\ 0 & 0 \end{pmatrix} X_t \, dW_t \qquad (5)$$

If $C(S_t, A_t, t)$ is the value of an option at time t, where the underlying stock has a value S_t and the average up to t is given by A_t we have by Ito's formula that

$$dC_t = \left(\frac{\delta C}{\delta t} + \alpha S \frac{\delta C}{\delta S} + \beta S \frac{\delta C}{\delta A} + x\sigma^2 S^2 \frac{\delta^2 C}{(\delta S)^2}\right) dt + \sigma S \frac{\delta C}{\delta S} dW_t \qquad (6)$$

Furthermore let r be the interest rate on riskless default free bonds. Hence if we invest an amount B_t in such a bond our investment is governed by the following differential equation

$$dB_t = rB_t dt \tag{7}$$

For the following argument we must assume that the stock markets are frictionless, that there are no transaction costs for buying or selling options, stocks or bonds and that the interest rates on lending and borrowing are equal. These are also the underlying assumptions for the Black-Scholes option pricing formula and are often used for deriving theoretical results.
Instead of buying an option we could also buy $(\delta C/\delta S)$ shares of stock and borrow an amount of $((\delta C/\delta S)S - C)$. This last strategy has the same risk as holding the option. Or stated in another way : if we buy $(\delta C/\delta S)$ shares of stock, borrow $((\delta C/\delta S)S - C)$ and sell to someone the option we would bring ourselves in a riskless position since the disturbance term would be

$$\left(\sigma S \frac{\delta C}{\delta S} - \sigma S \frac{\delta C}{\delta S} \right) dW_t = 0 \tag{8}$$

Furthermore we see that this would cost or bring us no money at this moment since

$$- \frac{\delta C}{\delta S} S + \left(\frac{\delta C}{\delta S} S - C \right) + C = 0 \tag{9}$$

Such a strategy of buying or selling shares of stock and lending or borrowing against a riskfree interest rate is called a hedging strategy if it comes with the same risk and initial investment.
Bensoussan (Bensoussan (1984)) showed using martingales that a hedging strategy exists for more general claims as the option claim we discuss here. We also like to remark that in the hedging strategy one constantly has to adjust the amount of shares which one is holding and the amount one is borrowing. Hence the assumption that there are no transaction costs is essential. Since the hedging strategy has the same risk and the same investment costs as buying an option the expected profit of the option and the hedging strategy must be the same, otherwise one could make an arbitrage profit by buying the one with the highest expected profit and selling the other one.

Hence in financial markets investment opportunities which require no investment and bring no risk must have a zero expected profit. And the combination of the option and the hedging strategy is such an opportunity. From this it follows that

$$\frac{\delta C}{\delta t} + \alpha S \frac{\delta C}{\delta S} + \beta S \frac{\delta C}{\delta A} + \tfrac{1}{2}\sigma^2 S^2 \frac{\delta^2 C}{(\delta S)^2} = \alpha S \frac{\delta C}{\delta S} + r(C - S \frac{\delta C}{\delta S}) \tag{10}$$

and we get the following PDE for the value of the option

$$\frac{\delta C}{\delta t} + \beta S \frac{\delta C}{\delta A} + \tfrac{1}{2}\sigma^2 S^2 \frac{\delta^2 C}{(\delta S)^2} + r(S \frac{\delta C}{\delta S} - C) = 0 \tag{11}$$

on the region $R_1 = \{(S,A,t) \,^{u}\, S \geq 0,\ A \geq 0,\ t_1 \leq t \leq T\}$.
Of course we have some boundary conditions and these are

$$C(S,A,T) \quad = \mathrm{Max}(A-K,0) \tag{12}$$

$$C(0,A,t) \quad = \mathrm{Max}(e^{-r(T-t)}(A-K),0) \tag{13}$$

$$\frac{\delta C}{\delta S}(\infty,A,T) = \frac{T-t}{T-t_1} e^{-r(T-t)} \tag{14}$$

Before we can state the last boundary condition we first calculate $C(S,A,t)$ for $A \geq K$. If $A \geq K$ we know that we will get a positive payoff in the end. This payoff will be

$$(A-K) + \beta \int_t^T S_s d\tau \tag{15}$$

There is also another way to reach this payoff without using the option and that is the following : put $(A-K)e^{-r(T-t)}$ in riskless bonds and besides for every time interval $(t,t+\Delta t)$ we put the amount $\beta e^{-r(T-t)} S_t \Delta t$ in riskless bonds. If we do this for every time interval $(t,t+\Delta t)$ and let Δt go to zero we will also end up with (15) as final amount (the factor $e^{-r(T-t)}$ reflects that we earn interest on our bonds). To do this we need the following amount of money, which must therefore be the option price :

$$C(S,A,t) = (A-K)e^{-r(T-t)} + \beta \int_t^T e^{-r(T-\tau)} S_\tau d\tau =$$

$$(A-K)e^{-r(T-t)} + \frac{\beta}{r} S(1-e^{-r(T-t)}) \qquad (16)$$

Hence (16) gives the value of $C(S,A,t)$ if $A \geq K$. Of course (16) fulfils the PDE (11). So we only have to solve (11) on the region $R_2 = \{(S,A,t) \mid S \geq 0,\ 0 \leq A \leq K,\ t_1 \leq t \leq T\}$ with boundary conditions (12), (13), (14) and

$$C(S,K,t) = \frac{\beta}{r} S(1-e^{-r(T-t)}) \qquad (17)$$

3. THE OPTION PRICE AS AN EXPECTATION

There are two ways which lead to the same expectation formula for the option price $C(S,A,t)$. One uses an economic argument and the other is purely mathematically. We will start off with the economic argument which was first introduced by Cox and Ross (Cox and Ross (1976)). It is assumed that most investors are risk-averse, which means that if two financial objects have the same expected profit the investors prefer the object with the lowest risk i.e. with the smallest variance. Hence they will not buy the object with the higher risk and because of this the price of the object will decrease and hence its expected profit will increase. So in a risk-averse economy one assumes that objects with a higher risk must have a higher expected profit.

In a hypothetical risk-neutral economy investors only consider the expected profits and do not worry about the risk. Investors will prefer the objects with the highest expected profit. Because of this the object with lower expected profit will see their prices decrease and hence their expected profits increase until these are on the higher level. Hence in a risk-neutral economy all objects will have the same expected profit and the equilibrium rate of return must be the riskless interest rate. In the previous section we have seen that the share price and the bond price play a rBle in the forming of a hedging strategy and we have seen that such a riskless hedging strategy can be formed. But then the valuation of the objects should be independent of the investors attitude towards risk. Hence we might assume that all investors are risk neutral and we can find the option price in a risk-neutral economy. Hence we may assume that

$$\alpha = r \tag{18}$$

since in a risk neutral economy we must have

$$\exp(\alpha(t-t_0)) = E_{S_0}(S_t) = E_{B_0}(B_t) = \exp(r(t-t_0)) \tag{19}$$

and the price of the option must be the expected terminal payoff, discounted at the riskless interest rate r i.e. :

$$C(S,A,t) = e^{-r(T-t)} E_{S,A}\{\max(A_T - K, 0)\} \tag{20}$$

Now we will give a mathematical argument which leads to the same expectation formula. If we substitute

$$C(S,A,t) = e^{-r(T-t)}\, D(S,A,t) \tag{21}$$

then our PDE (11) becomes

$$\frac{\delta D}{\delta t} + \beta S \frac{\delta D}{\delta A} + \tfrac{1}{2}\sigma^2 S^2 \frac{\delta^2 D}{(\delta S)^2} + rS \frac{\delta D}{\delta S} = 0 \tag{22}$$

Now we see that this is the Kolmogorov backward equation of the following system of stochastic differential equations

$$dX_t = \begin{pmatrix} r & 0 \\ \beta & 0 \end{pmatrix} X_t\, dt + \begin{pmatrix} \sigma & 0 \\ 0 & 0 \end{pmatrix} X_t dW_t \tag{23}$$

with $X'_t = (S_t \; A_t)$.

But then it is well known that the solution of (20) is given by an expectation formula. Of course we have to use the boundary conditions for D instead of those for C but if we consider D on the region R_1 the first exit time will be T since we never hit the boundary before time T. Hence we find from (23) that

$$D(S,A,t) = E_{S,A}\,(D(S_T,A_T,T)) = E_{S,A}\,\{\max(A_T - K, 0)\} \tag{24}$$

and we see that the economical and mathematical argument give rise to the same formula. If $A \geq K$ (24) reduces of course to

$$D(S,A,t) = E_{S,A}\,(A_T) - K \tag{25}$$

and we will show in section 5 that in general

$$E_{S,A}(A_T) = A - \frac{\beta}{r} S(1 - e^{r(T-t)}) \tag{26}$$

And hence for $A \geq K$

$$D(S,A,t) = (A - K) + \frac{\beta}{r} S(e^{r(T-t)} - 1) \tag{27}$$

which is of course exactly the same as formula (14) for $A \geq K$. We can also give on expectation formula if we restrict ourselves to the region R_2 and this becomes

$$D(S,A,t) = E_{S,A}\,\{I_{\tau \leq T} \cdot \frac{\beta}{r} S(e^{(T-\tau)} - 1)\} \tag{28}$$

where τ is the first exit time from R_2 and I is the indicator function i.e.

$$I_{\tau \leq T} = \begin{cases} 1 \text{ if } \tau \leq T \\ 0 \text{ if } \tau > T \end{cases} \tag{29}$$

In practical applications one wants closed analytical formulas for the value of an option instead of (20), (24) or (28). If we had a simple option this would be possible and we would get the Black-Scholes formula. In the next section we will derive this formula. We do this for two reasons. The first is that for the Black-Scholes we need a PDE which also plays a rôle in our problem if $t < t_1$ and the second is of expositional nature.

4. THE BLACK-SCHOLES FORMULA

In the preceeding section we derived a PDE for the option price $C(S,A,t)$ if $t_1 \le t \le T$. In this section we are focussing on the option price if $t < t_1$ i.e. if we are not yet in the final time interval over which we take the average share price. It is clear that for $t < t_1$ A doesn't play a rôle and C only depends on S and t. Let us write $C^*(S,t)$ for the option price if $t < t_1$. By $C^*(S,t_1)$ we will of course mean $\lim_{t \to t_1} C^*(S,t)$ and we know that

$$C^*(S,t_1) = C(S,0,t_1) \tag{30}$$

We still assume that S_t follows the stochastic differential equation (4) and again we can apply Ito's lemma to $C^*(S,t)$. We can also form a riskless hedge and arguing as in section 2 we derive the following PDE for C^* if $t < t_1$:

$$\frac{\delta C^*}{\delta t} + \tfrac{1}{2}\sigma^2 S^2 \frac{\delta^2 C^*}{(\delta S)^2} + r(S\frac{\delta C^*}{\delta S} - C^*) = 0 \tag{31}$$

This is the PDE for a normal European call option which pays off $\max(S_T-K,0)$ instead of $\max(A_T-K,0)$ and has been derived by Black and Scholes. Hence for an European call option we have (31) with boundary condition

$$C^*(S,T) = \max(S_T-K,0) \tag{32}$$

The solution to (31) with our boundary condition (28) is by the same economical or mathematical argument as in the preceeding section given by

$$C^*(S,t) = e^{-r(t-t_1)}E_S\{C(S_{t_1},0,t_1)\} \tag{33}$$

with S_t given by the following stochastic differential equation

$$dS_t = rS_t dt + \sigma S_t dW_t \tag{34}$$

As we said before for a simple option we will also have (31) but now with boundary condition (32). Hence for a simple option the price $C^1(S,t)$ is given by

$$C^1(S,t) = e^{-r(T-t)} E\{\max(S_T-K,0)\} \tag{35}$$

but since S_t is again given by (34) we know that S_T is normally distributed and it easily follows that

$$C^1(S,t) = SN(x_1) - Ke^{-r(T-t)}N(x_2) \tag{36}$$

with $$x_1 = [\ln(S/K) + (r+\tfrac{1}{2}\sigma^2)(T-t)]/\sigma\sqrt{T-t} \tag{37}$$

$$x_2 = x_1 - \sigma\sqrt{T-t} \tag{38}$$

and where N is the normal distribution function. (36), (37) and (38) form the well-known Black-Scholes formula for the valuation of an European call option. For practical purposes it would be very convenient if we could find explicit formulas like (36), (37) and (38) for $C(S,A,t)$ and $C^*(S,t)$. We will comment on this in the next section.

5. PROBLEMS WITH AN EXPLICIT FORMULA

If we want to find explicit formula's for the option price if $t_1 \le t \le T$ then we have seen in section 3 formula (24) that we have to calculate expectations of a process given by the following system of stochastic differential equations

$$dX_t = (AX_t + a)dt + (BX_t + b)dW_t \tag{39}$$

with $a = b = 0$

$$A = \begin{pmatrix} r & 0 \\ \beta & 0 \end{pmatrix} \quad , \quad B = \begin{pmatrix} \sigma & 0 \\ 0 & 0 \end{pmatrix} \tag{40}$$

Now it seems impossible to us to give explicit formulas for

$$D(S,A,t) = E_{S,A}\{\max(A_T-K,0)\} \tag{24}$$

since X_t is not a Gaussian process. In the case of a simple European call option we indeed have a Gaussian process as we have seen in the preceeding section and we could give the explicit formulas (36)-(38). Although X_t is not Gaussian in our case we can calculate the expectation and variance of X_t but this of course alone is not enough to give us an explicit formula for (24). To calculate the expectation and variance of X_t we know that (see Arnold (1974)) $m_t = EX_t$ follows the following differential equation

$$dm_t = Am_t dt \tag{41}$$

and $P_t = EX_tX_t'$ is the unique nonnegative definite symmetric solution of the system of differential equation

$$dP_t = AP_t + P_tA' + BP_tB' \tag{42}$$

where we have already used that $a = b = 0$.
Now (41) can be solved easily and gives

$$ES_t = S_{t_0}e^{r(t-t_0)} \tag{43}$$

$$EA_t = A_{t_0} + \frac{\beta}{r} S_{t_0}(e^{r(t-t_0)} - 1) \tag{44}$$

Since A and B have very special forms one can also find successively explicit formulas for ES_t^2, ES_tA_t and EA_t^2. The results are as follows

$$ES_t^2 = S_0^2 \, e^{(2r+\sigma^2)(t-t_0)} \tag{45}$$

$$ES_tA_t = \frac{\beta S_0^{\,2}}{(r+\sigma^2)} e^{(2r+\sigma^2)(t-t_0)} + (A_0S_0 - \frac{\beta S_0^{\,2}}{(r+\sigma^2)}) \, e^{r(t-t_0)} \tag{46}$$

$$EA_t^2 = \frac{2\beta^2 S_0^{\,2}}{(r+\sigma^2)(2r+\sigma^2)} e^{(2r+\sigma^2)(t-t_0)} + \frac{2\beta}{r} (A_0S_0 - \frac{\beta S_0^{\,2}}{(r+\sigma^2)}) e^{r(t-t_0)} + C_0 \tag{47}$$

where C_0 is such that $EA_0^2 = A_0^2$.

As we said before this is not enough to find an explicit solution for (24) although (44) gives the solution of (24) if $A_{t_0} \geq K$ as we explained in section 3.

One might wonder whether it is possible to change the model such that we end up with a Gaussian process as for a simple European call option. This is indeed possible (see Arnold (1974) pg. 136) if we assume that S_t is governed by

$$dS_t = \alpha S_t dt + \sigma dW_t \tag{48}$$

instead of (4). In this case one easily sees that (37) gives rise to a Gaussian process and we could give explicit formulas for (24). However with a specification like (48) there is always a positive probability that S_t becomes negative and from an economic point of view this must be impossible since shares of common stock always have a positive value. An owner of a share of common stock is not responsible for the debt of the firm. Another way to get a Gaussian process would be to make a model with $a = b = 0$ and such that A and B commute (see Arnold (1974) pg. 144). This would happen for example if we assume

$$dA_t = \beta S_t dt + \sigma S_t dW_t \tag{49}$$

instead of (3). However this specification implies that there is a disturbance term in the measurement of the average share price. It will be clear that the owner of an option will never accept a downward disturbance, while the person who issued the option will never accept an upward disturbance. Hence from an economic point of view (49) doesn't make sense.

6. CONCLUSIONS

In this paper we have studied an option of which the payoff depends on the average value over some final time interval of the underlying security. We derived two PDE's for the option value, one for the case that we are already in the final time interval and one for the case where we are not yet in the interval. These PDE's can be seen as the Kolmogorov backward equations of two systems of stochastic differential equations. From this it follows that the option value can be written in an expectation formula. Unfortunately this formula is not as explicit as the Black-Scholes formula for a simple European call option. Hence if one wants to find the option value in a practical case one has to solve the PDE's numerically or one has to rely on simulation methods. Since the most important PDE has three variables it seems much cheaper to use simulation methods. All in all it seems interesting to find a reliable and fast method for computing the expectations and hence the option values.

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