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ECONOMETRIC INSTITUTE

THE PROBLEM OF AGGREGATION OF
INDIVIDUAL ECONOMIC RELATIONS; CONSISTENCY AND
REPRESENTATIVITY IN A HISTORICAL PERSPECTIVE

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AND REPRESENTATIVITY IN A HISTORICAL PERSPECTIVE

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SUMMARY

It was already fairly late in the development of economic theory that aggregation of individual economic relations into macro relations was recognized as a serious problem. This is illustrated in this paper by pointing to the more or less incidental way in which aggregation problems are mentioned and solved in the older literature till the famous Econometrica debate in the late forties. In particular, we try to disentangle the notions of consistency and representativity.

Contents	page
1. Introduction	2
2. The period till 1945	3
3. The Econometrica debate	12
4. Consistency: Nataf's theorem	20
5. Representativity	25
6. Concluding remarks	31
References	33

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Preliminary

1. INTRODUCTION

The aggregation problem to be dealt with in this paper can be described as follows. Assume that for each member of a certain group of J individuals the magnitude of some variable (e.g. consumption, output, investment) depends, according to some theory, on the magnitudes of certain explanatory variables (such as prices, interest rates, incomes, inputs of labour and capital, et cetera). For each $j = 1, \dots, J$ this can be written as

$$(1) \quad y_j = f_j(x_{j1}, x_{j2}, \dots, x_{jM}),$$

where y_j is the magnitude of the variable to be explained for j and where the x_{jm} denote the magnitudes for j of the explanatory variable of kind m ($= 1, \dots, M$). The problem of consistent aggregation of relationship (1) over all J individuals (or, rather, individual decision units) is now whether there are aggregation formulae yielding aggregates y, x_1, x_2, \dots, x_M of the respective kinds of variables such that, for all values of these variables within a certain domain, there exists a macro relation F such that

$$(2) \quad y = F(x_1, x_2, \dots, x_M). \quad 1)$$

In cases where one is interested in macro relations such as (2) but where the real (logical, theoretical) relations are at the micro level, one has to pay attention to this aggregation problem. Although Cournot (1838) already touched on it, it was only some forty years ago that it was explicitly recognised as a serious problem. This will be illustrated in the next section from work of some famous economists as Cournot, Jevons, Walras, Marshall, Edgeworth, Cobb and Douglas and Tinbergen; the section has to be considered as a more or less personally flavoured historical treatment. In section 3 we deal with the famous Econometrica debate (1946-1948) on aggregation. Two notions played an important rôle in this debate. The first notion is that of consistency,

1) In (1) and (2) the functions all have the same number M of arguments; this, however, is not essential from a mathematical point of view as, e.g. all x_{1j} and x_{2j} can be combined into one single aggregate. From an economic point of view, however, it does make much difference because, then, aggregation over different goods is also in order. Furthermore, for the sake of convenience, we take the y_j and y as scalars.

dealing with the mathematical problem of the existence of aggregates and macro relations such as described above; the second one is that of representativity (whether or not together with consistency), dealing with the question whether the macro relation can be considered as the description of the behaviour of some (fictitious) representative agent acting more or less in the same way as individuals do. These two notions are the subject of sections 4 and 5, respectively. The paper ends with some concluding remarks.

2. THE PERIOD TILL 1945

2.1 Consistency of aggregation in our sense means that knowledge of the macro function and the aggregate explanatory variables gives us the same information on the aggregate dependent variable(s) as does knowledge of all micro relations together with all micro explanatory variables. Something like this could have been in Cournot's mind when he wrote in section²⁾ 21 of his Recherches sur les Principes Mathématiques de la theorie des Richesses (1838): 'Admettons donc que le débit ou la demande annuelle est, pour chaque denrée, une fonction particulière $F(p)$ du prix p de cette denrée. Connaître la forme de cette fonction, ce serait connaître ce que nous appelons la loi de la demande ou du débit. Elle dépend évidemment du mode d'utilité de la chose, de la nature des services qu'elle peut rendre ou des jouissances qu'elle procure, des habitudes et des mœurs de chaque peuple, de la richesse moyenne et de l'échelle suivant laquelle la richesse est répartie.' The reader may recognise some "cetera" of the ceteris paribus condition. Alternatively, however, we may assume that the aggregates as the habits and morals of the society and aggregate wealth (and income) as well as the income distribution are hidden in the symbol F and that they will become apparent as soon as F is specified. Both views seem to apply.

The latter idea might be to the point in the subsequent section 22 where Cournot tries to argue that the function F is continuous³⁾ in p . He does so

2) If possible we indicate the places in the older literature by means of sections, lessons or something like that, in order to facilitate checking in translations or reprints.

3) And, therefore, in his opinion, differentiable in p . Cournot needed this for discussing profit maximization. Note that at that time it was not yet generally known that continuity is not sufficient for differentiability; Weierstrasz' and Riemann's examples of everywhere continuous but nowhere differentiable functions still had to be devised. See, e.g. Cajori (1980, p. 425) who mentions 1861 as the year in which it was Weierstrasz who found for the first time such a function.

through a hardly convincing appeal on something like a law of large numbers: 'Mais plus le marché s'étendra [i.e., the number of demanders increases], plus les combinaisons des besoins, des fortunes ou même des caprices, seront variées parmi les consommateurs, plus la fonction $F(p)$ approchera de varier avec p d'une manière continue.' (emphasis added). This reasoning is only acceptable, however, if there are among the individual demand functions, which are not necessarily continuous in Cournot's view, decreasing as well as increasing functions of p . It may be that the word "caprices" in the last quotation is meant in this sense, but if all (or almost all) individual demand functions are decreasing then, in general, the discontinuities may just reinforce each other. Hence, after all, Cournot could not escape making the assumption that the aggregate demand function is continuous, as he actually did in the opening sentence of section 22. The decreasingness of F as a function of p is so evident for Cournot that he does not argue it; this too, however, is not without problems as we know now (see Hildenbrand (1983)).

The *ceteris paribus* idea appears to prevail in most parts of the Recherches. In the chapters IV to X, on market behaviour, it is tacitly present. In chapter XI, entitled "Du revenu social", it is even explicitly stated; now the prices of other goods are also mentioned. In section 74 Cournot remarks, with regret, that the *ceteris paribus* condition, in his opinion, is necessary because the complete solution of an economic system, with all its interdependencies, 'surpasserait les forces de l'analyse mathématique et de nos méthodes pratiques de calcul, quand même toutes les valeurs des constantes pourraient être numériquement assignées.' See on this point also Theocharis (1983, pp.138-140).

Cournot's Recherches remained for more than thirty years nearly completely unnoticed possibly due to the fact that "political economy" was considered as a "general" science the results of which had to be understood also by non-economists such as lawyers, merchants, bankers and politicians, who usually did not have much mathematical training. It was Jevons who brought Cournot's Recherches to the attention of the English economists. In the preface to the second edition of his book The Theory of Political Economy (1st ed. 1871, 2nd ed. 1879) Jevons writes: 'It is strange that it should have remained for me among Englishmen to discover its value.'

2.2 Jevons seems to be guilty of the same kind of ambiguity as Cournot with respect to the discrepancy between the behaviour of individuals on the one hand, and the "regularity" of aggregates on the other, as we shall see below. Jevons does not derive or use demand functions. His "pièce de résistance" is his treatment of the subject of exchange. In chapter IV of his Theory he derives his "Theory of exchange": the exchange ratio of two goods (i.e., the reciprocal of their price ratio) is the reciprocal of the ratio of the marginal utilities after the exchange is completed. The exchangers in this process are so-called "trading bodies"; see the section entitled "Definition of Trading Body" in chapter IV. A trading body can be any group of individuals from only one person to 'the whole inhabitants of a continent'. Where Cournot aggregates the results of utility maximization (i.e., the individual demand functions), Jevons aggregates, before maximizing the utility, the individuals into a (big) group and assumes that this group, the trading body, as a whole, behaves itself again as an individual⁴⁾. Both Jevons and Cournot tried to show that differences in powers, wants, habits and possessions cancel against each other⁵⁾. In the introductory chapter I, section "Measurement of Feelings and Motives", Jevons states his general starting-point: 'I must here point out that, though the theory presumes to investigate the condition of a mind, and bases upon this investigation the whole of economics⁶⁾, practically it is an aggregate of individuals which will be treated. The general forms of the laws of economics are the same in the case of individuals and the nations; and, in reality, it is a law operating in the case of multitudes of individuals which gives rise to the aggregate represented in the transactions of a nation.' Of course, these somewhat obscure ideas met with a lot of discussion and criticism.

A serious criticism is the seeming incompatibility of perfect competition with

4) This contradistinction is just one of the points in the Econometrica debate to be discussed in the next section.

5) Both use the term "caprices" in discussing individual behaviour.

6) Jevons was one of the first who spoke of "economics" instead of "political economy" (preface second edition).

the presence of only two agents in a market, viz. the demanding trading body and the supplying one; in other (modern) words: the core consists of more than one point and, therefore, the contract between the two trading bodies is indeterminate. It appears that Jevons is not unaware of this (see, e.g. the section entitled "Problems in the Theory of Exchange" in his chapter IV) but it is Edgeworth who is Jevons' advocate in this respect. In Appendix V of his Mathematical Psychics (1881) Edgeworth writes: 'His [i.e. Jevons'] couple of dealers are, I take it, a sort of typical couple, clothed with the property of "Indifference" (...). Each (...) is a "representative particular"; an individual only is presented, but there is presupposed a class of competitors in the background.' And he continues: 'exchange is indeterminate, if either (1) one of the trading bodies (quâ individual or quâ union) or (2) the commodity supplied by one of the dealers, be indivisible or not perfectly divisible.' Nevertheless Edgeworth, too, admits that Jevons should have been more explicit on this point. Walras (1889 and further, section 163 of (1926)) criticized the fact that Jevons apparently considered 'le probleme (...) comme résolu avec le cas de deux échangeurs.' According to Walras, Jevons left reality by considering 'moyennes fictives', i.e. trading bodies. Apparently, however, he (Walras) was not aware as Edgeworth was of the necessity of imposing some extra condition (here perfect competition) in order to prevent indeterminateness; Edgeworth touched the heart of the problem of indeterminateness while Walras intuitively sensed it. Further, we mention Marshall (1920, III.III.6) who rejects the possibility of a price as measuring final (= marginal) utility to a trading body.

Marshall has also to be mentioned because he introduced the notion of the representative firm; in Book IV, Chapter XIII, section 2 of his Principles of Economics (8th ed., 1920 (1st ed. 1890)). The representative firm is not strictly an aggregate in the sense of the preceding section, i.e. it is not a firm with as input some kind of average of the inputs of all the firms of the industry in question and as output an average of all outputs. It is neither a marginal firm, nor an extremely successful firm; it is just a firm between these extreme ones. Marshall's analysis of industries (an important part of his work) is carried out in terms of an analysis of the representative firm. In particular, the "normal supply price" of that firm (that is, the expenses of production (including gross earnings of management) per unit of output) is such that as the market supply price equals this price (1) the representative

firm's output remains constant, (ii) some (marginal) firms are "falling" and decrease their outputs (iii) other (successful) firms are "rising" and increase their outputs, and (iv) aggregate output remains constant. All this is in the more or less short run (V.III.4). It is conceivable that the notion of representativity has, among others, its roots in Marshall's representative firm; Pigou (1938, II.XI,3 and App. III.I.2), e.g., borrows the conception under the name "equilibrium firm". Anyway, the notion has already a long life in spite of its obscurity. In this connection the following quotation of Schumpeter (1954, p.997, n.14) is interesting: 'Marshall's authority as a teacher secured mechanical acceptance of the concept. But it received neither the criticism nor the development it deserves.'

2.3 The position of Walras and Pareto with respect to the problem of aggregation is manifestly enigmatic. Walras aggregates without any comment in his general equilibrium analysis, whereas Pareto, without comments either, painfully abstains from aggregating in discussing general economic equilibrium; nevertheless the latter admits that he continues, in this respect, the work of the former. For each consumer Walras carefully derives, from utility maximization⁷⁾, demand and supply functions of consumption goods (les produits) and production factors (les services), respectively; these are functions of the prices of the respective goods and factors; see, e.g. the 20th lesson, section 201 of his Eléments d'Economie Politique Pure (Edition définitive, 1926). For instance, in his notation, an individual's supply of factor t is a function of the prices p_t, p_p, p_k, \dots of factors t, p, k, \dots and of the prices p_b, p_c, p_d, \dots of goods b, c, d, \dots (good a being the numéraire), written as

$$(3) \quad o_t = f_t(p_t, p_p, p_k, \dots, p_b, p_c, p_d, \dots)$$

and that individual's demand function for b is likewise

$$(4) \quad d_b = f_b(p_t, p_p, p_k, \dots, p_b, p_c, p_d, \dots);$$

7) Both Walras and Jevons considered marginal utility of a good to depend only on the quantity of that good. Edgeworth, Fisher and Pareto assumed more general utility functions.

the demand for good a is not written in the form (4), but in a form that is derived from the budget restriction of the individual in question.

In the subsequent section 202 Walras writes 'Et maintenant, en désignant par O_t, O_p, O_k, \dots , les offres totales des services, soit les excédents des o_t, o_p, o_k, \dots positifs sur les o_t, o_p, o_k, \dots négatifs, par $D_a, D_b, D_c, D_d, \dots$ les demandes totales des produits, par $F_t, F_p, F_k, \dots, F_b, F_c, F_d, \dots$ les sommes des fonctions $f_t, f_p, f_k, \dots, f_b, f_c, f_d, \dots$ on aurait déjà, (...), le système suivant de n équations d'offre totale des services:

$$O_t = F_t(p_t, p_p, p_k, \dots, p_b, p_c, p_d, \dots),$$

$$(\cdot \cdot \cdot) \quad [1]$$

et le système suivant de m équations de demande totale des produits:

$$D_b = F_b(p_t, p_p, p_k, \dots, p_b, p_c, p_d, \dots)$$

$$(\cdot \cdot \cdot); \quad [2]$$

soit en tout $n + m$ équations.' (emphasis added). These $n + m$ equations are combined with technical and market behaviour equations of the firms into Walras' well-known model of general economic equilibrium (section 206); see also Van Daal, Henderiks and Vorst (1985). The whole sentence just cited has 21 lines in Walras' book, which is in no sense an exception in his works. The reader would expect some comments on Walras' bold example of aggregation, but throughout the whole book any such comment is lacking. Apparently, Walras saw no problem in arriving at the equations of the systems [1] and [2] (Walras' numbering) of the quotation above; this impression is further enhanced by the fact that nor the earlier editions of his Eléments, neither his correspondence (Jaffé (1965)) give further clues. Just as in the case of Cournot, aggregates such as those of the initial endowments q_t, q_p, q_k, \dots are hidden somewhere in the functions F . Nevertheless, here we have the first explicit and comprehensive (to say the least) description in mathematical form of what happens if consumers and firms "meet" each other in the various markets of an economy under circumstances of perfect competition. The only contemporaneous author (as far as we know) who explicitly points to Walras' specific way of

arriving at collective demand and supply functions is Edgeworth; he does this with the appreciation Walras deserves (Edgeworth (1881, p.42)).

As mentioned above, Pareto, in analyzing general economic equilibrium, does not aggregate at all over the consumers, neither in his Cours d'Economie Politique (1896, note 100), nor in his Manuel d'Economie Politique (1909, Appendix, section 85), nor in his article "Economie Mathématique" (1911, section 31); these publications are the most important sources of (the development of) Pareto's ideas on general economic equilibrium and, in particular, of its mathematical formulation. All individuals remain identifiable in Pareto's general equilibrium models which have, therefore, a large number of equations, viz. the number of individuals times the number of first-order conditions (the budget restrictions and the equalities of the ratios of prices and corresponding marginal utilities) in addition to the technical equations (cost minimization) and those stemming from market behaviour (price taking and/or price setting) of the firms. In this sense Pareto's models are forerunners of the models of Arrow and Debreu and their successors⁸). It is a challenge to find out why Pareto developed his models as he did. One reason may have been that for his purpose, i.e., demonstrating consistency in general economic equilibrium models, he did not need aggregation. It remains a moot question, however, why he did not explain why he abstained from following Walras on this point.

2.4 An aggregate that did appear in Pareto's mathematical economics is total welfare of a whole community. Starting from an apparently cardinal utility concept, Pareto discussed shortly (and obscurely) the problem of maximization of this concept in a given economy; see Pareto (1909, Appendix, section 89 and 1911, section 28). In fact, Pareto considers in this kind of analysis, the economy as if it consists of only one consumption unit, apart from the firms. This is more or less in line with the ideas that lie behind Jevons' trading body.

With respect to total welfare also Edgeworth (1881) is noteworthy. He is more

8) In fact, Pareto was not the first precursor of Arrow and Debreu in this respect. Fisher, in his doctor's thesis (1925 (1892), Part I, Chapter IV, V, VI), developed similar models. Pareto (1911, n.44) refers to Fisher's equations.

explicit on the subject. In a note in the beginning of his section entitled "Utilitarian Calculus" he considers total utility as a triple integral over 'Number of enjoyers \times duration of enjoyment \times degree thereof' (p.57). Two pages further he states in an axiom the measurability of utility as well as commensurability between different individuals.

2.5 During the period between both world wars aggregation over individuals was still not generally recognized as a problem⁹⁾. It was passed over in silence. A notorious example of this is the study by Cobb and Douglas (1928) in which the famous Cobb-Douglas production function is introduced and estimated on time series (1899-1922) of indices for output (P), labour (L) and capital (C) for the American economy as a whole. The latter is considered as one big profit maximizing firm whose production function fits remarkably well the equation¹⁰⁾ $P = 1.015 L^{3/4} C^{1/4}$.

The same is the case with the macro models built by Tinbergen (1936 and 1939). Professor Tinbergen explained to us that his models were so extremely simple from the point of view of economic theory ('merely linear common sense macro relations') that aggregation over individuals was hardly relevant; the mathematical and statistical problems asked all attention. Moreover, the final aim of his work was (and has always been) to contribute to the solution of real economic problems. So his main interest was in models that "work" anyhow, i.e. that explain numerically the economic phenomena in their interdependence.

Other macroeconomic work in that period (non-econometric classical models as well as models devised by Keynes(ians)) started usually by formulating a macro theory simply stating the relationship between a few (mostly monetary) aggregates with at most some lip service to the microeconomic theories of

9) The problem of aggregation over goods was already widely investigated at that time. We mention the work on price and quantity indices by Fisher (1912, 1922), Frisch (1936), Konüs (1924), Leontief (1936), Bowley (1928), Divisia (1925/26, 1928); as professor Tinbergen pointed out to us this problem was highly topical at the meetings of the Econometric Society in the thirties, in contrast to aggregation over individuals.

10) We estimated the equation $\ln(P/C) = a + g \ln(L/C)$ on Cobb and Douglas' data and found OLS estimates $\hat{g} = .749$ and $\hat{a} = .0145 (= \ln 1.0147)$; for \hat{g} we found a t-value of 18.1. It is evident that Cobb (who was responsible for the concerning part of the article) did the same, but he does not say anything about it.

individual behaviour of households and firms. It cannot be denied that this method seems to be the most sensible if one wishes to get at macro results without the risk of running ashore somewhere between the micro starting-point and the macro end. Moreover, the models were not precise in an econometric sense. Their main aim was to describe, qualitatively, general tendencies. Therefore, it is clear that most of the relations showed at most some kind of "representativity" as far as micro behaviour was considered.

2.6 The idea of macro analysis in the sense above was not new. Quesnay (1759) already gave utterance to it in his Tableau Economique. We permit ourselves a little digression in describing it. In the tableau streams of total money expenditures and receipts between the classes of farmers ("classe productive"), landlords, and other workers ("classe sterile") are depicted for one time period of a stationary economy. Quesnay assumes that at the beginning of the period the landlords spend one half of total, cumulated rent of the preceding period (their only income) on agricultural products and the other half on products of the classe sterile; during the period they are assumed not to spend anymore. The classe sterile spends also one half of its receipts to agricultural products; the other half is spent in the class itself. Further, Quesnay assumes the farmers to be "productive" in the sense that they succeed (by farming) in doubling their receipts from the other classes. Half of each doubled amount is paid as rent to the landlords, whereas the other half is halfly spent for 'consommation de production fournies par cette même classe, & l'autre moitié en entretien de vêtements, utensiles, instruments, & c. qu'il paye à la classe des dépenses sterils.' (Quesnay (1759, p.ii). This process of producing and spending peters out just at the end of the period, leaving the landlords with the same total amount of rent as that with which they started, after which a new period begins. The tableau is perhaps the first economic application of a geometric series (with ratio 2). More important for our subject, however, is that Quesnay's exposition is rather obscure in the sense that he mixes micro and macro considerations. Clearly, he has in mind some prototype of a macroeconomic flow model; this can be inferred from the fact that in his numerical example the landlords' total income is about national income of France at that time. In describing the model, however, he sometimes gives the impression that he discusses the behaviour of only one landlord and one single tenant; in other places he describes the individuals as identical per class which gives him the possibility of simply adding individual amounts.

Ricardo (1821) paid likewise attention to individual (entrepreneurial) behaviour. He also let the social totals of individual quantities live their own life eventually, after having started his analysis from the individual firm. See, for instance, his chapter 6 in The Principles of Political Economy and Taxation where he explains the decrease of the profit rate as population increases. He did not refer back too much to the individuals' acts in making clear the argument of that difficult chapter. It can be argued that there is a whole chain of authors from the latter two to Keynes and further, who worked in the same line; see, e.g., Schumpeter (1954, pp.987-8).

3. THE ECONOMETRICA DEBATE

3.1 The beginning of the after-war period is characterised by three facts that are important for the subject of this paper: (i) more statistical data were gathered and became available, (ii) more and better econometric techniques were devised and (iii) research facilities (organizational as well as in manpower) increased considerably, in particular in the United States; see Roll (1973, pp.523ff.). This made possible a quantitative and qualitative progress of econometric work, microeconomic as well as macroeconomic. As a consequence, a first explicit statement of the problem of aggregation over individuals could not fail to appear. It was L.R. Klein who in 1946 in an article in Econometrica, entitled "Macroeconomics and the Theory of Rational Behavior," opened the discussion. He started by remarking that 'Many of the newly constructed mathematical models of economic systems, especially the business-cycle theories, are very loosely related to the behaviour of individual households or firms which must form the basis of all the theories of economic behaviour.' (p.93; emphasis added). As an example he mentions that 'the demand equations for factors of production in the economy as a whole are derived from the assumption that entrepreneurs collectively attempt to maximize some aggregate profit; whereas the usually accepted assumption is that the individual firm attempts to maximize its own profit.' (p.93).

In his aim to dot the i's and cross the t's in (mathematical) macroeconomics he suggested the aggregates to have to satisfy two criteria. The first one looks like what we called consistency in the introduction of this paper. He expressed it as follows (p.94): 'For example we have for the firm in

microeconomics,

$$(5)^{11)} \quad F_{\alpha}(x_{1\alpha}, \dots, x_{m\alpha}; n_{1\alpha}, \dots, n_{r\alpha}; z_{1\alpha}, \dots, z_{s\alpha}) = 0,$$

$$\alpha = 1, 2, \dots, A.$$

This relation states that the α th firm produces the m commodities $\{x_{i\alpha}\}$ through the input of the services of r kinds of labor $\{n_{i\alpha}\}$ and of s kinds of capital $\{z_{i\alpha}\}$. We now demand that there exist a function, in macroeconomics,

$$(6)^{11)} \quad F(X, N, Z) = 0$$

which states that the entire community of firms produces the aggregate X through the input of the services of [aggregate] labor N and [aggregate] capital Z .¹¹⁾ As far as we know, this was the first time that a requirement of consistency in aggregation was stated in an explicit and mathematically precise form. Walras cannot be given that honour because he only considered sums as aggregates and took the existence of his functions F for granted, irrespective of the form of his functions f ; see the quotation above.

Klein's problem in macroeconomics was how to find mathematical expressions for the aggregates X , N and Z such that not only the technological macro relation (6) is fulfilled but that, in addition, a second criterion is satisfied, viz. that of representativity, or better, analogy. This means here that the marginal productivity equations for perfect competition hold in the aggregate:

$$(7) \quad \frac{\partial X}{\partial N} = \frac{W}{P}$$

and

$$(8) \quad \frac{\partial X}{\partial Z} = \frac{Q}{P},$$

where W , the wage aggregate, depends on the wages w_j ($j = 1, \dots, r$) of the different types j of labour and where P and Q are output-price and capital-service-price aggregates, respectively, depending on the prices p_i ($i = 1, \dots, m$) of commodities i and prices q_t of the different types of capital

¹¹⁾ The numbering is ours; Klein's numbers are (1) and (2), respectively.

services t ($= 1, \dots, s$), respectively. For reasons of simplicity Klein assumed in his example that perfect competition prevails at the micro level. This means:

$$(9) \quad \frac{\partial x_{i\alpha}}{\partial n_{j\alpha}} = \frac{w_j}{p_i}$$

and

$$(10) \quad \frac{\partial x_{i\alpha}}{\partial z_{t\alpha}} = \frac{q_t}{p_i}$$

for $\alpha = 1, \dots, A$, $i = 1, \dots, m$, $j = 1, \dots, r$ and $t = 1, \dots, s$. This in itself is not sufficient for the choice of (7) and (8). Where Klein himself justifies this choice by the fact that his macro model is simple and manageable, we presume that, in addition, at the same time he attempted to reconcile the generally adopted practice of considering the behaviour of groups of individual decision units as similar to the behaviour of the individuals themselves, with mathematical rigour. In spite of his own criticism quoted above, he, too, was convinced, at that time, of the usefulness of this strong kind of representativity as a tool of economic analysis. Furthermore, he was of the opinion that linking the macro relation F explicitly with the micro production functions F_α was the only method to aggregate technological circumstances without spoiling them with other features, in particular those rooting in the behaviour of the economic units; moreover, his relation F was, in his opinion most suited for analysing consequences of technological change. So the micro models consisting of the relations (5), (9) and (10) and the macro model (6), (7) and (8) are completely analogous. In other words, the micro model(s) as well as the macro model are given; this was not new in macroeconomic analysis at that time (we shall discuss this further in section 5). The new element of Klein's article was the requirement of consistency of aggregation. The consequence of this is that, then, all degrees of freedom are exhausted which means that the mathematical form of the aggregates X , N and Z as functions of the $\{x_{i\alpha}\}$, $\{n_{j\alpha}\}$ and $\{z_{t\alpha}\}$, respectively¹²⁾, completely depends on the form of the given micro and macro models and, therefore, as a rule, will not be of a

12) Of course, the mathematically rather arbitrary choice of the aggregates and their arguments contributes to the above mentioned absence of freedom.

well-known type such as a sum or an unweighted arithmetic average. In the case of Cobb-Douglas micro and macro production functions, for instance, the aggregates are geometric averages.

3.2 The reactions were immediate. In the same Econometrica volume of 1946 there were articles by Pu and May and a reaction by Klein; a year later a second article by May appeared and in 1948 Nataf's famous study was published¹³⁾; they will be referred to by capital letters as indicated in footnote 13. Several points were touched upon. First, the mathematical shape of the aggregates as being mostly highly unusual (Pu). Second, the reasonability of imposing the macro relations a priori (Pu, May). Third, the possibility of alternatives; they stemmed from questions as: which kinds of relations do we aggregate and to what extent (Pu, May)? Fourth, the conditions for consistent aggregation (Nataf).

In C (pp.299-300) Pu considers Klein's Cobb-Douglas aggregates as constructions that 'certainly have no connection whatsoever with the economic variables which we are actually interested in, the total volume of employment and the total quantity of capital' and he qualifies them as 'monsters that are completely void of any economic significance.' We believe that Klein (D, pp. 310-311) made a good point in remarking that there is nothing 'sacred' about a sum and that it is not the first task of scientific theorists to make things familiar to the layman; very properly he states: 'Any macroeconomic theory which will enable us to make people happier through an analysis of the interrelationships between aggregates of income, employment, output, etc., is a good theory regardless of the specific form of the aggregates.'

13) The precise titles of all the articles in the debate are:

- A. L.R. Klein, "Macroeconomics and the Theory of Rational Behavior," Econometrica, 15 (1946), pp.93-108;
- B. K. May, "The Aggregation Problem for a One-Industry Model," Econometrica, 15 (1946), pp.285-198;
- C. S.S. Pu, "A Note on Macroeconomics," Econometrica, 15 (1946), pp.299-302;
- D. L.R. Klein, "Remarks on the Theory of Aggregation," Econometrica, 15 (1946), pp.303-313;
- E. K. May, "Technological Change and Aggregation," Econometrica, 16 (1947), pp.51-63;
- F. A. Nataf, "Sur la Possibilité de Construction de Certains Macromodèles," Econometrica, 17 (1948), pp.232-244.

The second point, that of imposing the macro function a priori, is certainly something that can be considered as a typical trait of economic scientists. There is among them a widespread belief that economic complexes behave in the same way as do the individual parts that constitute the complex in question. We will discuss this opinion in more detail in section 5 when discussing representativity. Pu and May were unambiguously against it. Both are of the opinion that a macro theory should be the result of looking for relations between (well-known) aggregates of features of micro elements on the basis of a theory of the behaviour of these elements. May (D, p.59) gives as an example that Boyle's law in physics on the relation between volume (V) and pressure (P) of a gaseous mass of a given temperature, $PV = \text{constant}$, is a macro theory that is not in the least similar to the (micro) laws of behaviour of the gaseous particles. He adds: 'In any case, since the economy is hardly less complicated than a gas, it seems rash to insist in advance on the exact character of the aggregate implications of micromodels.' Klein, however, had to impose macro relations a priori because, apparently, he was highly attached to his "purely technological" macro production function (6) as a part of the macro model; we saw this already above. In order to complete the macro model, he had to impose further relations on his aggregates (D, p.305). Moreover, he was very doubtful about the possibility of deriving a relatively simple macro theory relating aggregates of the types supplied by statistical offices, given the micro theory (D, p.311). So it seemed wise to keep matters in his own hand by postulating the macro model in the hope that the implied aggregates become such that they can reasonably well be approximated by published ones.

3.3 The third point deals with alternatives to the problem of aggregation rather than with criticisms on Klein's procedure itself. Pu (C, pp.300-301) argues that Klein's apparent insistence on aggregates that are independent of the distribution of the micro quantities over the individuals is too strong because the micro quantities are, according to the theory, distributed in a particular way, other distributions being not relevant. This means that only the variables of some subset of the whole domain of explanatory variables have to be aggregated which weakens the requirement of consistency. As an example¹⁴⁾ he discusses firms α ($= 1, \dots, A$) with production functions $x_\alpha =$

14) May, in fact, gives an example in the same line in which two kinds of labour are combined into only one aggregate.

$f_{\alpha}(n_{\alpha}, z_{\alpha})$, where the notation is taken from Klein (A, see above). As aggregates he simply takes $N = \sum_{\alpha} n_{\alpha}$, $Z = \sum_{\alpha} z_{\alpha}$ and $X = \sum_{\alpha} x_{\alpha}$. The pattern of distribution of the n_{α} and the z_{α} is determined by the marginal-productivity equations

$$(11) \quad \frac{\partial x_{\alpha}}{\partial n_{\alpha}} = \frac{\partial x_{\beta}}{\partial n_{\beta}}, \quad \frac{\partial x_{\alpha}}{\partial z_{\alpha}} = \frac{\partial x_{\beta}}{\partial z_{\beta}}$$

for $\alpha, \beta = 1, \dots, A$. The micro production functions together with the definitions of N and Z and the relations (11) form $3A$ independent equations in $3A + 2$ unknowns. Hence all x_{α} , n_{α} and z_{α} can be solved in terms of N and Z . Putting the solutions for x_{α} in $X = \sum_{\alpha} x_{\alpha} = \sum_{\alpha} f_{\alpha}(n_{\alpha}, z_{\alpha})$ leads to an aggregate relation

$$(12) \quad X = F(N, Z)$$

which in its optimum has the same (macro) marginal productivities as the micro relations have in their optima. It can not be denied that (12) is a production function in the sense that it relates macro output to macro inputs, given the technological and behavioural as well as possible other characteristics that prevail in the economy in question. Klein does not react on Pu's suggestion leading to (12). He insists on the importance of analyzing technological change which is, in his opinion, only possible by using his aggregate production function¹⁵). Theil (1954, p.13), however, generalizes Pu's idea (for linear relations) in introducing his "auxiliary equations" which are (in the notation used above) simply equations expressing the n_{α} and the z_{α} in the aggregates N and Z . The solution of n_{α} and z_{α} in N and Z as derived from, a.o., (11) form an example of a system of auxiliary equations, but, according to Theil, such a system does not necessarily have an economic meaning and hence Theil does not further elaborate Pu's idea. Green in his much cited

15) May (D, p.63) argues that there are no purely technological (micro) production functions at the firm level. He mentions time schedules, worker morale and management habits as non-technological elements. We can add to this that, moreover, these element can differ per firm. If in such a case the micro production functions are of the Cobb-Douglas type with common degree of homogeneity and the aggregates are taken as unweighted geometrical averages, then the macro production function can be distorted in the sense that, in fact, its degree of homogeneity differs from that of the micro relations; see, e.g., Van Daal (1980).

Aggregation in Economic Analysis (1964) works out the idea (only citing May, however) in his chapter 6 entitled "Degrees of Freedom Restricted: Optimal Conditions of Exchange and Production" and he discusses the matter in the subsequent chapter; see also Van Daal and Merkies (1984, ch.2)

Another alternative to Klein's approach is first reducing the micro model to a set of relations such that each endogenous variable is expressed in the exogenous ones and, subsequently, aggregating these reduced form equations choosing the form of the aggregate a priori (see the second point above). May as well as Pu advocate this and we met this approach already in Cournot's Recherches (see also footnote 4). In Klein's example we then have to aggregate the following supply and demand equations that result from optimizing behaviour:

$$(13) \quad x_{i\alpha} = x_{i\alpha}(p_{1\alpha}, \dots, p_{m\alpha}; w_{1\alpha}, \dots, w_{r\alpha}; q_{1\alpha}, \dots, q_{s\alpha}),$$

$$(14) \quad n_{j\alpha} = n_{j\alpha}(p_{1\alpha}, \dots, p_{m\alpha}; w_{1\alpha}, \dots, w_{r\alpha}; q_{1\alpha}, \dots, q_{s\alpha}),$$

$$(15) \quad z_{t\alpha} = z_{t\alpha}(p_{1\alpha}, \dots, p_{m\alpha}; w_{1\alpha}, \dots, w_{r\alpha}; q_{1\alpha}, \dots, q_{s\alpha})$$

for $i = 1, \dots, m$, $j = 1, \dots, r$ and $t = 1, \dots, s$ over $\alpha = 1, \dots, A$. Klein chooses to combine all $x_{i\alpha}$ for $i = 1, \dots, m$ into only one aggregate X of output and similarly for the $n_{j\alpha}$ and the $z_{t\alpha}$, but that is not necessary, of course.

May (E, p.58) gives an example of this procedure by studying the macro consequences of technological change. He finds that a technological change in the two sectors of his micro model such that output increases in both industries with the same percentage λ , turns out to result in an increase of output in the aggregate one-sector model with more than λ per cent. This shows once more that analogy reasoning à la Klein has its drawbacks. Furthermore, it may be possible that some parameters in the macro model (however aggregated from the micro model) are exclusively corresponding to micro parameters that express technological features. In such a case the analysis of the consequences of technological change is straightforward. Hence, in our opinion Klein's anxiety in this point is somewhat too extreme.

It is striking that Klein, when it comes to the (empirical) point, in his famous Economic Fluctuations in the United States, 1921-1941 (1950, pp.13-14) aggregates in the way described just above which means that he aggregates relations that are the result of optimization processes performed by households and firms, e.g. consumption and investment functions. It may be that this change of attitude can be justified by recalling the well-known adage "different models for different purposes" (here models for theory versus models for practice). Anyway, we are denied empirical illustration of his ideas set out in A and D. On the other hand, Klein (1950, pp.58-80) has provided us with the Klein I model, which, apart from being a toy for every econometrician, is a nice example of explicit aggregation of micro consumption, investment and wage functions.

The fourth point of the debate came somewhat later on the scene (1948). It deals with the technical question on the possibilities of consistent aggregation. Nataf (F) discovered that for consistent aggregation it is necessary and sufficient that all functions (the micro, macro and aggregation functions) are additively separable. This is the subject of the next section.

3.4 We end this section with some remarks. The Econometrica debate on aggregation, in spite of its importance, seems only known to a few specialists. The scholar texts and the textbooks on the history of economic science do not refer to it at all with only one noteworthy exception as far as we know; see Blaug (1978, pp.491-494). Although the debate remains absent in most textbooks on economics and econometrics, they usually deal with the problem of aggregation itself in more or less detail. The attention it receives goes from only a few lines to a separate chapter; see e.g., chapter 20 in Allen's Mathematical Economics, chapter 4 of Theil (1975), chapter 3 of Barnett (1981) and chapter 3 of Pearce (1964). Monographs on aggregation are scarce. The first one that appeared is by Theil (1954); it is confined to linear aggregation in the sense that the aggregates are sums and that the relations to be aggregated are linear. Green's Aggregation in Economic Analysis (1964) is much more general; it is a somewhat esoteric but, nevertheless, frequently cited book. There are even quite a lot of researchers that apparently are of the opinion that citing Green (1964) gives them an excuse for their failure of not solving themselves their own aggregation problems properly. Other monographs on aggregation are Pokropp (1972) and the

one recently written by the present authors (1984).

4. CONSISTENCY: NATAF'S THEOREM

4.1 Though coming last into the discussion, the consistency problem is in fact the "question préalable" in the debate on aggregation. This matter has to precede all other questions as it concerns the mere possibility of consistent aggregation. It must be acknowledged that Klein, indeed, made the first step. He remarked that his aggregates X , N and Z as functions of the $\{x_{i\alpha}\}$, $\{n_{j\alpha}\}$ and $\{z_{t\alpha}\}$, respectively, are not independent of each other because all these micro variables have to satisfy the micro production functions. Requiring, furthermore, the existence of an aggregate production function $F(X, N, Z) = 0$ means a rank condition on the matrix whose columns are the gradients of X , N and Z with respect to a set of independent micro variables; it does not matter which set, see, e.g., Burkill and Burkill (1970, pp.230 ff.) or any other advanced textbook on mathematical analysis. From this rank condition Klein derives a system of differential equations (A, p.99) that the aggregates X , N and Z have to satisfy, given the micro relations. His Cobb-Douglas aggregates indeed satisfy this system, but that does not contribute to making more transparent that system.

It was Nataf who provided us with this transparency in solving Klein's system of partial differential equations; his results are reported on page 243 of F. It turns out that Klein's micro production functions have to be of the form

$$(13) \quad \theta_{\alpha}(x_{1\alpha}, \dots, x_{m\alpha}) = \chi_{\alpha}(H_{\alpha}(n_{1\alpha}, \dots, n_{r\alpha}) + I_{\alpha}(z_{1\alpha}, \dots, z_{s\alpha}))$$

for all $\alpha = 1, \dots, A$, where θ_{α} , χ_{α} , H_{α} and I_{α} are unspecified functions of m , one, r and s variables, respectively. Aggregation, now, is simple. We define the aggregate X by

$$(14) \quad X = \sum_{\alpha=1}^A \chi_{\alpha}^{-1}(\theta_{\alpha}),$$

where we assume the χ_{α} to be invertible and we omit from now on the micro arguments of the functions. Furthermore, we define

$$(15) \quad N = \sum_{\alpha=1}^A H_{\alpha}, \quad Z = \sum_{\alpha=1}^A I_{\alpha}.$$

From (13), (14) and (15) one simply derives the macro relation

$$(16) \quad X = N + Z.$$

Nataf's solution of the consistency problem clearly demonstrates the two elements of this aggregation problem. The first one is an aggregation over goods and factors within each firm's production function: for each α the $\{x_{i\alpha}\}$ are aggregated into θ_{α} , whereas the $\{n_{i\alpha}\}$ and the $\{z_{i\alpha}\}$ boil down into H_{α} and Z_{α} , respectively. The second element is the aggregation over individuals resulting in (16), the subject of this paper; the θ_{α} , N_{α} and Z_{α} are now considered as single goods and factors. The remainder of this section is devoted to the latter kind of aggregation.

4.2 We start with reformulating the problem for micro functions with an arbitrary number M of variables that each apart have to be aggregated over the individual decision units. Note that it is completely irrelevant whether the functions involved are production functions, demand functions, supply functions or whatever. In the notation of (1) and (2) of section 1, we have J micro relations f_j , one for each individual, with arguments x_{jm} , where $m = 1, \dots, M$ and M denotes the number of explanatory variables. We now ask ourselves whether there exist aggregation formulae g_m that aggregate the x_{jm} over j into the aggregate x_m , hence

$$(17) \quad x_m = g_m(x_{1m}, \dots, x_{Jm}),$$

for all $m = 1, \dots, M$ as well as an aggregation rule G aggregating the micro dependent variables into the aggregate y , in other words

$$(18) \quad y = G(y_1, \dots, y_J),$$

such that there exists a macro function F with

$$(19) \quad y = F(x_1, \dots, x_M),$$

or, putting all things in one relation,

$$\begin{aligned}
 (20) \quad & F \{g_1(x_{11}, \dots, x_{J1}), \dots, g_M(x_{1M}, \dots, x_{JM})\} = \\
 & = G\{(f_1(x_{11}, \dots, x_{1M}), \dots, f_J(x_{J1}, \dots, x_{JM})\} = \\
 & = H(x_{11}, \dots, x_{jm}, \dots, x_{JM}) = y.
 \end{aligned}$$

The function H of all JM micro variables is called the "atomistic macro function." Relation (20) can be illustrated by means of the following scheme:

$$\begin{array}{ccccccc}
 & & & & f_1 & & \\
 & & & & \rightarrow & & y_1 \\
 x_{11}, & \dots, & x_{1m}, & \dots, & x_{1M} & & \\
 \cdot & & \cdot & & \cdot & & \cdot \\
 \cdot & & \cdot & & \cdot & & \cdot \\
 \cdot & & \cdot & & \cdot & & \cdot \\
 x_{j1}, & \dots, & x_{jm}, & \dots, & x_{jM} & \xrightarrow{f_j} & y_j \\
 \cdot & & \cdot & & \cdot & & \cdot \\
 \cdot & & \cdot & & \cdot & & \cdot \\
 x_{J1}, & \dots, & x_{Jm}, & \dots, & x_{JM} & \xrightarrow{f_J} & y_J \\
 & & & & & & \\
 \downarrow g_1 & & \downarrow g_m & & \downarrow g_M & & \downarrow G \\
 & & & & F & & \\
 x_1, & \dots, & x_m, & \dots, & x_M, & \rightarrow & y ;
 \end{array}$$

see also Pokropp (1972, p.31) and Van Daal and Merckies (1984, ch.1).

It is immediately clear that if we put plusses (+) between all the micro variables of scheme (21) and equality signs instead of arrows, we have a very simple example of consistent aggregation. Likewise, consistent aggregation is possible if we replace in this example all variables by transformations of them, i.e. if we have micro functions that are additively separable, i.e.

$$(22) \quad y_j = k_j \{ \phi_{j1}(x_{j1}) + \dots + \phi_{jM}(x_{jM}) \}$$

where the k_j are invertable functions of only one variable, and if the aggregation formulae look like

$$(23) \quad x_m = h_m \{ \phi_{1m}(x_{1m}) + \dots + \phi_{Jm}(x_{Jm}) \}$$

(m = 1, ..., M) and

$$(24) \quad y = N \{ k_1^{-1}(y_1) + \dots + k_J^{-1}(y_J) \}.$$

Scheme (21) then takes the form:

$$(25) \quad \begin{array}{ccccccc} \phi_{11}(x_{11}) & + & \dots & + & \phi_{1M}(x_{1M}) & = & k_1^{-1}(y_1) \\ & + & & + & & & + \\ & \cdot & & \cdot & & & \cdot \\ & + & & + & & & + \\ \phi_{J1}(x_{J1}) & + & \dots & + & \phi_{JM}(x_{JM}) & = & k_J^{-1}(y_J) \\ & \parallel & & \parallel & & & \parallel \\ h_1^{-1}(x_1) & + & \dots & + & h_M^{-1}(x_M) & = & N^{-1}(y). \end{array}$$

In the latter scheme all micro, macro and aggregation functions are additively separable. It can be proved that this is not only sufficient but also necessary for consistency; we call this Nataf's theorem. Nataf proved this already for Klein's case $M = 2$. It is not very practical to use Nataf's technique for arbitrary values of M , because that would mean the analysis of the consequences of requiring that a $JM \times (M+1)$ -matrix of $M + 1$ gradient vectors has rank M .¹⁶⁾ Therefore, another strategy of proof of the necessity of the scheme (25) has to be adopted. A short outline of this follows now; see also Van Daal and Merkies (1981 and 1984, ch.1).

Our proof, as others', leans heavily on two separation lemmas that go back to Leontief (1947). The first lemma states that a function f which maps a vector $(z_1, \dots, z_N)' \in \mathbb{R}^N$ into the real line, is additively separable if and only if for all $i, j = 1, \dots, N$ the ratios $(\partial f / \partial x_i) / (\partial f / \partial x_j)$ depend only on x_i and x_j . The second lemma deals with what we call partial weak separability of a function. The function f is said to be partially weakly separable with respect

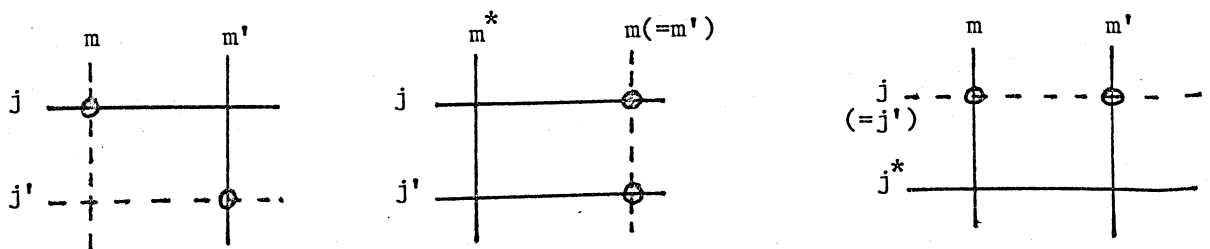
16) In Nataf's case the rank requirement resulted in the vanishing of a number of 3×3 determinants with many zeroes. From this Klein's (A, p.99) system of differential equations could easily be derived.

to a subset S of its variables, say (for notational convenience), the first K ($< N$), if there are functions ϕ and ψ such that $f(z_1, \dots, z_N)$ can be written as $\phi\{\psi(z_1, \dots, z_K), z_{K+1}, \dots, z_N\}$ for all z_1, \dots, z_N from the domain of f .¹⁷⁾ The second lemma states that a function f is partially weakly separable with respect to S if and only if for all i and j from S the ratios $(\partial f / \partial x_i) / (\partial f / \partial x_j)$ depend only on variables from S . New proofs for these two lemmas, that are much simpler than the existing ones, are provided by the present authors (1984, ch.1, Appendix B). The following two corollaries of these two lemmas appear to be useful: (i) if f is partially weakly separable with respect to each pair (i, j) for $i, j = 1, \dots, N$ then f is additively separable, (ii) if f is partially weakly separable with respect to S_1 and S_2 , where $S_1 \cap S_2 \neq \emptyset$, then f is partially weakly separable with respect to $S_1 \cap S_2$, as well as $S_1 \cup S_2$. These truths are easily verified; see also Gorman (1968).

Now we can prove the necessity of (25). First, the additive separability of the atomistic macro function H of (20) can be proved as follows. Given that (20) holds good, H is apparently partially weakly separable with respect to each "row" and each "column" of variables x_{jm} of the scheme (21) and hence, by the second corollary, it is, in particular, partially weakly separable with respect to each collection of variables x_{jm} of one or two of the first J rows and one of the first M columns, or of one or two such columns and one row of the scheme. Consequently, H is partially weakly separable with respect to each set $(x_{jm}, x_{j'm'})$ of variables because such a set can always be found as the intersection of two collections as mentioned above¹⁸⁾. Because of the first corollary, H is additively separable. Second, the additive separability of the other functions follows can now easily be proved.

17) In our monograph (1984) we use the term "quasi-separability" (p.28) but now we are of the opinion that the present term is more in line with related terms in the literature.

18) This can be illustrated as follows (---- = collection 1, - - - = collection 2):



4.3 The proof above requires differentiability of all the functions involved (to be precise, the functions have to be twice continuously differentiable because we needed that for the proofs of the lemmas). Green (1964, pp. 36 ff.) was the first who proved Nataf's theorem for arbitrary M . He assumed that the functions be thrice differentiable¹⁹⁾ and started with proving the additive separability of the functions F and G of our scheme (20) from which the additive separability of the f_j and the g_m functions follows. In fact, he left undone the proof of the atomistic macro function's additive separability. Green's proof contains some elements that can also be found in Nataf's. It is likewise very much complicated. Gorman (1968) dropped the assumption of differentiability of the functions involved but imposed monotonicity instead ("increasingness" in the case of production functions) and continuity. With these assumptions he succeeded in proving Nataf's theorem. Pokropp showed that the continuity of the micro functions to be aggregated is also inessential, provided that they are "separable" in a generalized sense; see Pokropp (1972, p.33 ff). Somermeyer and Van Daal (1978) presented an alternative proof in terms of finite differences, without assuming continuity either.

5. REPRESENTATIVITY

5.1 The strongest kind of representativity we met already in discussing Klein's Econometrica articles A and D; namely complete analogy between the micro and the macro model. Klein may know himself backed up by one of the most influential authors since the thirties in the person of J.R. Hicks. The latter author writes in both editions of his Value and Capital (1946 (1939), XX.1): 'The transition [from micro to macro level] is made by using the simple principle (...) that the behaviour of a group of individuals, or group of firms, obeys the same laws as the behaviour of a single unit.' Until now, many economists are apparently convinced of the truth of this quotation. Note that representativity and consistency are separate notions that need not to coincide. One can, of course, investigate whether there is representativity

19) Some details of Green's proof, however, can be changed such that only the existence of continuous non-zero first derivatives is sufficient for his proof; this makes the conditions just as severe as Nataf had to impose.

(analogy) given consistency, i.e. within the framework of scheme (20). In that case analogy means that F and all the functions f_j in (20) have to be of the same type. Alternatively, however, representativity can also be presupposed without explicit requirement of consistency. The latter approach prevails in macroeconomics. At best one takes for granted (or one hopes) that not bothering about consistency is rather harmless; the following cases are examples of this approach.

In growth models analogy reasoning is the order of the day. In contradiction to the problem of aggregating over sectors, aggregation over individuals is hardly seen as a problem. Mostly aggregate profit functions, in the form of $F(K, L) - WL - QK$ for the one-sector case, are posited and maximized over the whole economy. Famous examples are Solow's seminal article (1957) on technical change and factor productivity, Allen's textbook (1967) on macroeconomics and Burmeister and Dobell's, technically advanced, book (1970) on economic growth. Also in empirical econometric model building analogy reasoning is frequently used. To stay near home, we only mention two Dutch models that until recently were in use for practical purposes, namely Driehuis' quarterly model (1972) and Den Hartog and Tjan's vintage model (1980). The lists can be extended ad libitum with similar examples.

Furthermore, we encounter analogy reasonings in demand analysis (for consumption goods as well as for production factors). Sometimes a microeconomic demand system is simply confronted with macro data without any justification at all (e.g. Philips (1983, p.101)). Sometimes the existence of a representative consumer, whose consumption and income are just the average amounts over the whole economy, is taken for granted. If in the latter case an estimated demand system does not pass all the tests on the integrability conditions, the representative consumer is considered not to "behave himself rationally"; it happens that, then, even all his real brothers and sisters of flesh and blood are likewise accused of the same irrationality; see, e.g., Christensen, Jorgenson and Lau (1975, p.381)²⁰). In this respect we must also

20) In January 1983 at the occasion of a conference in Rotterdam (Econometric Institute, Erasmus University) Professor Jorgenson admitted, during a very successful lecture on the econometrics of general economic equilibrium, that this assertion was too bold. A better conclusion, he said, would have been that there exists no representative consumer (as mentioned above).

mention the recently developed methods of non-parametric demand analysis: aggregate consumption data are subjected to revealed-preference tests as if these data are related to one single person; see e.g. Afriat (1967, 1973, 1977), Maks (1980, pp.29-32) and Varian (1982, p.965).

5.2 The question that arises is: how came economists to reasoning by analogy? One of the first authors who was very definite on the subject of analogy is Jevons, as we have seen above where we discussed his trading body. He defends his analogy approach in economic analysis by means of an analogy reasoning of a higher order, so to speak. To make this clear, we have to refer to Jevons' Principles of Science (first edition 1874, second edition 1877). In chapter XVI of that book, entitled "The method of means" he develops the notion of a "fictitious mean". Such a mean is an average that, though not always really existing, 'is of the highest scientific importance, as enabling us to conceive in a single result a multitude of details.' As an example he mentions the point of gravitation of a gravitating body. The behaviour of the aggregate consisting of all the 'particles' of the body, i.e. the behaviour of the whole body, 'will be exactly represented by the behaviour of this heavy point.' Two or more bodies, whether connected or separate, may also be conceived as having one centre of gravity. So the whole entity of all points of the bodies acts as if it were only one point (with respect to gravitating behaviour, of course). This example and others were for Jevons sufficient to construct by analogy the fiction of his trading body. Nevertheless, as said above, it remains an analogy reasoning on analogy reasoning: what can be said of physical sciences can likewise be said of the science of economics according to Jevons. Parenthetically, we remark that in science there are just as well examples where Jevons' fictitious mean does not exist. We already mentioned Boyle's law to stress the risks of analogy reasoning. Moreover, as Boyle's law and its generalizations work in practice, we see that Jevons' method of the fictitious mean is unnecessarily restrictive for deriving macro laws.

We are not aiming at historical or methodological completeness in this paper. So we take a big step and come to Hicks whom we already quoted. That quotation can be considered as a conclusion. In section II.5 of Value and Capital Hicks makes plausible that 'Market demand has almost exactly the same properties as individual demand.' He argues that for each individual the effect of a price change can be decomposed into a substitution effect and an income effect. The

former effects (as far as regarding changes of the quantity of a good in response to its own price) are either all negative or all positive; hence the group effect, being the sum of the individual effects, is likewise negative or positive. The individual income effects, however, are not quite reliable in direction according to theory; the group income effect will usually be negligible, Hicks remarks, if the group as a whole spends a small proportion of its total income upon the commodity in question. That is all. It seems that there is some contradiction between Hicks' expositions regarding the similarity between individual and group demand behaviour as stated in Value and Capital and his proof in A Revision of Demand Theory (1956, pp.55-57), of the possibility of "irrational" community behaviour resulting from individual rationality: one can always construct examples of individually rational decisions (i.e., optimal points on budget hyperplanes) such that after a change of prices and/or incomes the new individually optimal points are such that the two aggregate decision points (before and after the change of prices and/or incomes) do not pass the revealed preference tests in spite of the fact that all individual components of these aggregate points do pass the tests. This is, of course, simply a consequence of Arrow's "Possibility Theorem" stating the impossibility, in general, of rational group behaviour as an aggregation result of rational individual behaviour (Arrow (1963, pp.59,97)). It can be argued, however, that the above inconsistency may disappear if the determinants of the individuals' preference schemes do not differ too much, i.e. if we mitigate Arrow's "condition of the unrestricted domain"²¹⁾.

Because of the symmetry between producer theory and consumer theory, it is not amazing that Hicks states that a group of firms, operating in the same market, 'must obey the same laws as the single firm' (section VIII.3). Here there is a complication viz. that firms may leave or enter the industry in question. Hicks, however, argues that it is implausible that this complicates matters such that the direction of the total change, due to a change in (relative) prices, is affected. The difference between consumers and producers with respect to entry or leaving is, in fact, only a gradual one; it takes time for

21) An example of such a mitigation is the case of "single-peaked" preferences for all individuals: for each individual the alternatives can be ordered in preference according to some one-dimensional criterion in a unimodal way (Arrow (1963, pp.75-77)); see Van Daal and Merckies (1984, p.117) for a suggestion to generalize this concept.

new consumers to enter a market, because before entering they have to be born, but new firms can be founded overnight.

Another attempt to justify analogy reasoning stems from Dresch (1938). He proves that, under conditions of perfect competition, aggregates of micro inputs and outputs of the form of Divisia indices indeed obey relations that are analogues of the micro (marginal) equilibrium conditions for profit maximization. He does not prove, however, that there exists an acceptable macro production function that links these Divisia aggregates without violating the macro equilibrium conditions. In our opinion, this is in general impossible.

A successful attempt to derive micro and macro models that are analogous to each other and at the same time consistent (in aggregation) with each other is Muellbauer's (1975, 1976) work on representative consumer models. It appears that a representative consumer only exists if the individual consumers' preference schemes are very similar, namely such that they result in demand equations that express all budget shares, as linear functions of a Box-Cox transformation η of income²²⁾ plus a term which averages out to zero over all individuals, the intercepts and coefficients of the linear functions being functions of prices that can differ per commodity but not per individual. The representative individual's demand system, then expresses community's budget shares as the same linear functions as above of "representative income" which is some generalized harmonic mean of the individual incomes the form of which depends on η . Hence Muellbauer's representative consumer is by no means a "mean" person. See also Pearce (1964, ch.3), however, for an alternative approach; he introduces an "average" consumer as the representative at the cost of assuming constancy of the income distribution over time.

Muellbauer's results, however, indicate clearly that the requirement of representativity, in addition to consistency, is unnecessarily restrictive as it is possible to derive macro models that are the result of consistent aggregation of micro models over individuals and that have Muellbauers representative consumer models as special cases; see on this point the present

22) If y denotes income then the Box-Cox transformation $\eta(y)$ is defined as $(y^a - 1)/a$ if $a \neq 0$ and as $\log y$ if $a = 0$; a is a parameter to be chosen or to be estimated.

authors (1984, section 7.3).

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5.3 Altogether, it remains a matter of taste, or of sheer necessity, whether one adopts analogy reasoning. In this respect we have to bear in mind that the behaviour of an individual decision unit such as a household or a firm is itself already the result of a considerable process of aggregation; going from that individual's behaviour to group behaviour is perhaps a relatively small step in the whole way of aggregation of the smallest "cells" of the individuals to the whole group. May's example of Boyle's law is, therefore, not quite fair; he compared two extremes, namely the behaviour of the highly aggregated whole gaseous mass as expressed in terms of the aggregates temperature, volume and tension on the one hand and, on the other hand, the gas molecules with their impulses, a case of extreme disaggregation, which is much simpler than most economic examples as he himself admits. Moreover, Boyle's law and its refinements, the laws of Gay-Lussac and Van der Waals, are discovered in first instance as macro laws in an empirical way. Similarly, macroeconomic relationships can be (and have been) found and verified (or falsified) in an empirical way without too much appeal on micro behaviour in the first instance. We already mentioned Tinbergen's models (1936, 1939) as good examples of such procedures; simple economic reasoning (such as, e.g., income and consumption are positively related) combined with carefully selected units of measurements of variables mostly running in relative changes. In this respect it is interesting to ascertain that Klein apparently left his ideas on strict representativity to become more and more eclectic over time; see Lindbeck (1985, p.45).

5.4 This brings us to a less extreme notion of representativity viz. the similarity between micro and macro theories with respect to some aspects only. A well-known example is the case of absence of money illusion in consumption behaviour: the individuals are supposed not to react in real terms on a simultaneous change in the same proportion of prices, wages and monetary variables such as income and wealth. This means that an individual's consumption function must be such that real consumption is a zero-homogeneous function of prices, wages, income and wealth. For that reason one sometimes imposes the same zero-homogeneity on the macro consumption function. A well-

known example is Branson and Klevorick's (1969) macro consumption function. Using U.S. quarterly series from 1955-I to 1965-IV they estimated a log-linear macro consumption function which significantly turned out not to be zero-homogeneous in the sense mentioned above. This was for them reason enough to conclude that consumers suffer from money illusion. Apart from the question about the fundamental possibility anyway of deriving micro conclusions from macro results, one can in this case easily show that the macro consumption function's deviation of zero-homogeneity could be an "aggregation bias" arising from aggregating micro consumption functions each showing complete absence of money illusion but having parameter sets that can differ per individual; see, e.g., Van Daal (1980).

5.5 We end this section by discussing whether we do need representativity in some form. If it is the case that a macro model can be considered as the result of consistent aggregation of micro models over individuals then, in our opinion, the additional requirement of some kind of representativity is superfluous. It can even be argued that, then, requiring representativity is harmful in the sense that a false impression is raised about the individuals' behaviour from macro data only. We saw this already above where we discussed Christensen, Jorgenson and Lau's (1975, pp.381) claim that their results 'make possible an unambiguous rejection of the theory of demand'.

If the macro model is not the result of consistent aggregation of the micro models, then representativity can be imposed if one believes to have reason for that; it happened more than once that, then, even useful models were found. It is obvious, however, that the device of representativity for building macro models is not without danger if followed blindly. Being aware of what one is doing and a good intuition are prerequisites for avoiding mistakes.

6. CONCLUDING REMARKS

Macroeconomic analysis may be founded upon analogy reasoning or upon explicit aggregation from microeconomics. In the development of economic doctrines there has not always been a clear distinction between the notions of consistency and representativity in discussing a macro model's connection with micro theory. We have shown how these concepts have been mixed in the older

clarified. In our opinion the concept of representativity is completely superfluous if a macro model is the result of consistent aggregation of individuals' micro models. Although the micro models' realism could be questioned, the introduction of representativity, however, does not help to make its aggregate, i.e. the macro model, more realistic; a good example of a consistently aggregated macro model, which does not reflect some representative's behaviour, is Somermeyer and Bannink's (1972, chapters 5 and 7) macro consumption model.

The reverse may be different. If a model is directly stated at the macro level and one is interested in general statements only, e.g. in welfare theory, it may be appropriate to introduce a representative consumer and/or producer (perhaps to the extreme analogy). In such a case, however, there remains a need for showing that this "representative's behaviour" is consistent with actual individual behaviour. To charge simply the "representative" with some particular behaviour, only helps in the sense that it shows that there is at least one possibility, viz. that where there is only one individual, with the indicated behaviour.

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