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ON THE RELATIONSHIP BETWEEN REVANKAR'S AND LU-FLETCHER'S PRODUCTION FUNCTION

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by

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ABSTRACT

In this note it is shown that - contrary to common belief - the production function of Revankar (1971) is a special case of that of Lu and Fletcher (1968) provided that the latter is generalized so as to allow for an arbitrary degree of homogeneity.

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1. Introduction

It is commonly believed that the VES production function of Revankar (1971) and that of Lu and Fletcher (1968) are two distinct functions, see for instance Nadiri's article on producers theory in the Handbook of Mathematical Economics (1982).

Apparently this is true since the function of Lu and Fletcher is linear-homogeneous and that of Revankar allows for an arbitrary degree of homogeneity and because the relationship between the elasticity of substitution and the capital-labour ratio is in general non-linear in case of the function of Lu and Fletcher and linear for Revankar's function. However, if we slightly generalize the production function of Lu and Fletcher so as to allow for an arbitrary degree of homogeneity too, it can be shown that the function of Revankar is a special case of that of Lu and Fletcher.

In Section 2 we prove this assertion and we subsequently show that in that special case the non-linear relationship between the elasticity of substitution and the capital-labour ratio boils down to the linear one postulated by Revankar.

2. Proof of the assertion

Lu and Fletcher (1968) postulate the following relationship between the output-labour ratio, the wage rate and the capital-labour ratio:

(1) $$\log(V/L) = \log a + b \log w + c \log(K/L),$$

in which V: output (= value added),
w: wage rate,
K: capital, and
L: labour.

Assuming that perfect competition prevails on both product and factor markets and that the production function is linear-homogeneous they derive their function from (1). Allowing for an arbitrary degree of homogeneity, say h, the function reads:

(2) $$V = [\beta k^{-\rho} + \alpha\eta(K/L)^{-c(1+\rho)}L^{-\rho}]^{-h/\rho},$$

where β = a constant of integration,

(3) $\rho = (1-b)/b$,

$\alpha = a^{-1/b}$, and

(4) $\eta = (1-b)/(1-b-c)$,

[cf. Lu and Fletcher, eq (5)].
Setting:

(5) $\alpha = (1-\delta)\ \gamma^{-\rho}$, and

(6) $\beta = \delta\gamma^{-\rho}$,

(2) can be rewritten as:

(7) $$V = \gamma[\delta K^{-\rho} + (1-\delta)\eta(K/L)^{-c(1+\rho)}L^{-\rho}]^{-h/\rho},$$

i.c. eq. (6) of Lu and Fletcher (1968), where we introduced the arbitrary degree of homogeneity h.
The relationship between the elasticity of substitution σ and the capital-labour ratio is derived to be:

(8) $$\sigma = \frac{b}{1 - c[1 + 1/(\beta\alpha^{-1}(\frac{K}{L})^{A} - B)]}$$

[cf. Lu and Fletcher (1968), eq. (13)*], in which

(9) $A = (b + c - 1)/b$, and

(10) $B = c/(b + c - 1)$.

Next, we consider the following special case of the function of Lu and Fletcher:

(11) $c = 1$.

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* This equation contains a slight notational error: $\beta\ \alpha^{-1}[(K/L)^{A} - B]$ instead of the correct formulation in (8).

In that case it follows from (4) and (3) that

(12) $\eta = -\rho$

After substitution of (11) and (12) into (7) we get:

$$V = \gamma[\delta K^{-\rho} + (\delta-1)\rho(K/L)^{-(1+\rho)}L^{-\rho}]^{-h/\rho},$$

which yields after rearrangement:

(13) $$V = \gamma\,[(\delta-1)\rho]^{-h/\rho}\,K^{\frac{h(1+\rho)}{\rho}}\,[L + \frac{\delta}{(\delta-1)\rho}K]^{-h/\rho}.$$

Define the following reparametrization:

(14) $\alpha^* = h$

(15) $\rho^* = \frac{(\delta-1)\rho + \delta}{(\delta-1)\rho},$

(16) $\delta^* = -\frac{(\delta-1)}{(\delta-1)\rho + \delta},$ and

(17) $\gamma^* = \gamma[(\delta-1)\rho]^{-h/\rho},$

then obviously:

(18) $\frac{h(1+\rho)}{\rho} = \alpha^*(1-\delta^*\rho^*)$

(19) $\frac{\delta}{(\delta-1)\rho} = \rho^*-1,$ and

(20) $-h/\rho = \alpha^*\delta^*\rho^*.$

Substitution of (17) through (20) into (13) yields:

(21) $$V = \gamma^* K^{\alpha^*(1-\delta^*\rho^*)}[L + (\rho^*-1)K]^{\alpha^*\delta^*\rho^*},$$

which is equation (2.1) of Revankar (1968)*, where we added asterixes to the parameters in order to avoid confusion with the symbols of Lu and Fletcher.

The consequence of c = 1 on (9) and (10) is:

(22) $A = 1$ and $B = 1/b$.

Substitution of (11) and (22) into the relationship between the elasticity of substitution and the capital-labour ratio, i.c. (8), yields:

$$\sigma = 1 - \frac{b\beta}{\alpha}(K/L)$$

$$(23)\qquad = 1 - \frac{\delta}{(1+\rho)(1-\delta)}(K/L)$$

$$= 1 + \frac{\rho^* - 1}{1-\delta^*\rho^*}(K/L),$$

where the second line follows from (3), (5) and (6) and the third line from (15), (16) and (17).

The expression in the third line of (23) is the linear relationship of Revankar (1971), c.f. his equation (2.2).

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* One of the VES functions that Sato and Hoffman (1968) derive is the linear-homogeneous version of Revankar (c.f. eq. (10) of Sato and Hoffman).

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