ECONOMETRIC INSTITUTE

TOWARDS A MICRO-ECONOMIC THEORY ON
SPATIAL SHOPPING BEHAVIOUR:
THE EFFECT OF DISTANCE ON CONSUMER'S CHOICES

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Towards a micro-economic theory on spatial shopping behaviour:
the effect of distance on consumer's choices.

by

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Abstract.

This paper deals with spatial shopping behaviour of consumers in a
certain region. An attempt is made to formulate a theoretical framework
in such a way that the choice of mathematical representations of
variables in the model will become less arbitrarily than is usually the
case in this type of models. In this paper a first step is taken to such
a theory based on the ideas of Bacon (1971) by assuming that a consumer
optimizes an objective function which is more specific than the utility
function in logit or probit analysis and which gives us a better
opportunity to test the underlying hypothesis. Special attention is
given to economic variables. Furthermore the role of distance in the
choice problem of the consumer is emphasized, attraction areas of shops
are derived from the theory in case of a simplified regional structure,
and are compared with those derived from a disaggregate choice model
with a logit structure.

Contents

1. Introduction.

This paper deals with spatial shopping behaviour of consumers (or households) in a certain region.

Concerning this subject a substantial number of econometric models has been developed and applied in the last decade, with the purpose to describe and explain the distribution of the flows of cash from the population of consumers over the different shops or shopping centres in a specific region.

An important distinction that can be made between these different models is the level of aggregation. There are models operating immediately on a aggregated level using a functional relationship not directly derived from a theory describing individual behaviour. An example of an aggregate model is the one developed by Wilson (1981), based on earlier models of Huff (1964), Lakshmanan and Hansen (1965) and Harris (1965), "explaining" the distribution of the flows of cash from a population zone to various shopping centres in other zones.

On the other hand there are models, based on a ,theory of individual behaviour, operating on a micro level. Examples of these disaggregate models are: gravity models, entropy models and logit and probit models. These models have in common that they are based on disaggregate fenomena (locations or individuals) but differ in their basic theoretical set-up. Gravity and entropy models are founded on concepts analogous to physics and information theory, respectively, while logit and probit models are based on utility theory. For an ample enumeration and comparison of these models we refer to van Lierop and Nijkamp (1980). They draw, after the evaluation of the different models, the correct conclusion – at least in our opinion – that models like logit and probit, starting from a theory on individual behaviour, should be preferred.

But if we choose for these models and if we want to apply them on an aggregate level, the aggregation problem has to be solved. With respect to the logit model (read: stochastic simultaneous multinomial logit model) van Lierop and Nijkamp give an inventive aggregation method, which can handle model differences between classes of consumers, caused by the difference in characteristics of these consumers. It is however not very clear how this method can handle the difference in location of consumers and the implications of this factor on their behaviour.

Another problem, is the selection of the relevant variables and their mathematical representation in the functional relationships, describing the distribution of the flows of cash over the different shopping centres in the region. Although, there exists a certain consensus in the literature concerning the selection of variables, there is little argumentation (other than on empirical grounds) about the choice of the mathematical form, with which these variables should be incorporated in the model. Even the "better" theoretical models like logit and probit give no decisive answer on the specification of variables in the individual choice model. In these models one usually starts from a (stochastic) utility function, in which the relevant variables are included by a linear relationship; something that is difficult to attack in view of the abstract nature of the utility function. Econometricians, who try to formulate a model for a specific problem, with the purpose to give pronunciations which might have a substantial impact on real policies, are not happy with this simple assumption in view of the risk of inadequate predictions caused by misspecification.

For this reason it seems worthwhile to attempt to formulate a theoretical framework in such a way that the choice of mathematical representations of variables in the model will become less arbitrarily. In this paper a first step is taken to such a theory, based on the ideas of Bacon [1971] by assuming that a consumer optimizes an objective function which is more specific than the utility function in logit or probit analysis, and which gives us a better opportunity to test the underlying hypotheses.

In this set-up special attention will be given to economic variables like prices and assortment. These variables seem of a decisive character in the consumer's choice problem and do not show to full advantage in the existing models. Models which use only floorspace as the relevant shop characteristic (besides location and environment factors like parking facilities, etc.) are for planning purposes on a low aggregation level rather restrictive, since they are of limited use in questions, which are of importance to local authorities. We think of matters like the policy with respect to the settlement of new shops, the care for a well-balanced branch structure in the region, the effect of a changing infrastructure, consequences of extreme price competition or of trends towards mass retailing.

Besides these economic variables we emphasize the role of distance in the choice problem of the consumer, and explicitely formulate the way in which the consumer takes his (optimal) decision.

The order of the discussion is as follows: In order to demonstrate the relevance of a proper mathematical specification of variables in shopping models, we show in section 2 of this paper the difficulties that may arise with the representation of distance in aggregate choice models.

In section 3 we give a description of the individual spatial choice problem, while in section 4 the theoretical framework is constructed, in which this problem may be solved.

Section 5 deals with the mathematical formulation of the theory. The spatial choice problem is translated there into a mathematical programming problem, illustrated with an example of the solution method.

In section 6 we give attention to the attraction areas of shops, derived from the theory in case of a simplified regional structure.

In section 7 these attraction areas will be compared with those derived from a disaggregate choice model with a logit structure.

We end up this paper in section 8 with a summary of the main conclusions and some recommendations for further research.

2. Possible misspecification of distance in aggregate models.

In aggregate models, describing the distribution of sales in a certain region, the variable to be explained is defined by the proportion of spendings of inhabitants of living zone i to shops in zone j (which latter zoning system is usually replaced by a set of geographical points, in case the shopping centres are not aggregated). The basic set of explanatory variables consists of some measure of attractiveness of the shopping centre j (commonly represented by floor space) and a monotonously decreasing function of the travel time or travel costs linked up with a visit of consumers in zone i to shopping centre j. Examples of these types of models, having more or less the outlined structure, are those developed by Huff (1964), Lakshmanan and Hansen (1965), Harris (1965), Cullen (1969), Klaassen (1974) and Wilson (1981).

The travel time or travel costs are represented directly by an aggregate measure of distance or by an increasing function of it.

The general formulation of these models may be expressed by:

$$(2.1) \quad S_{ij} = \frac{A_j f(d_{ij})}{\sum\limits_{k=1}^{J} A_k f(d_{ik})} \qquad \begin{aligned} j &= 1, \ldots, J \\ i &= 1, \ldots, I \end{aligned}$$

with S_{ij}: the proportion of spendings of consumers in zone i to shopping

centre j. $(\sum\limits_{j} s_{ij} = 1)$

A_j: some measure of attractiveness of shopping centre j.

d_{ij}: (aggregate) distance from zone i to shopping centre j.

f: a monotonously decreasing function.

Because of the properties of f, the partial derivative of S_{ij} with respect to d_{ij} is strictly negative suggesting a relationship between S_{ij} and d_{ij} as pictured below in fig. 2.1 (all other variables being constant).
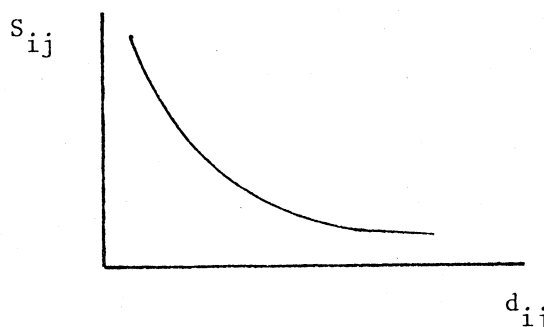
fig. 2.1.

Since zone i is representing a large number of consumers, the distance measure d_{ij} is an aggregate of the distances of the individual consumers in i to shoppig centre j. If we take d_{ij} to be the average distance[*] of consumers in i to shopping centre j (which is usually done in this type of models) there may rise some difficulties, especially with the proportion of spendings from i to shopping centre j, if this shopping centre is located in zone i.

[*] In some occasions one takes the distance from the centre of gravity of a zone to the shopping centre. The criticism against that aggregate will, however, be the same as we pass here on average distance.

This situation is further analysed below.

Suppose we have a region I, subdivided into 2 population zones, with 2 shopping centres, as pictured in fig. 2.2.a.
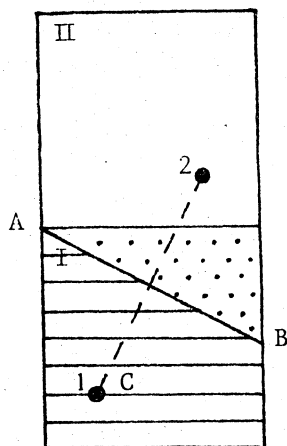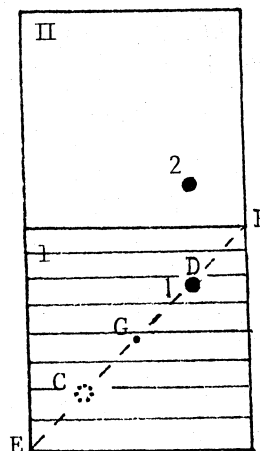

fig. 2.2.a   fig. 2.2.b

If we make the simple assumption that the consumer's are equally distributed over the entire region and that they all spend the same amount of money on shopping goods, and that the two centres are equally attractive, it is plausible then to assume that the difference in distance between the two centres for an individual consumer will be the decisive factor in choosing the centre he wants to visit.

With these assumptions it is possible to calculate the proportions S_{I1} en S_{I2} without the use of a specific model, since the dividing line of the attraction areas of both shops, is formed by the perpendicular bisector on the line of connection between shop 1 and 2, visualized by the line A-B in fig. 2.2.a. The people of zone I, buying in shop 1 are located in the shaded part, while the people buying in shop 2 are in the dotted part. The relative areas of these parts are in our case S_{I1} and S_{I2}, respectively. (in fig. 2.2.a $S_{I1} \sim .7$ and $S_{I2} \sim .3$)

If we look at the situation as pictured in fig. 2.2.b, where shop 1 is transferred from C to D in such manner that the average distance of the people in I to 1 in C is equal to the average distance to 1 in D, the result is that all the people in zone I will buy their goods in 1 and so, with shop 1 in D, $S_{I1} = 1$, $S_{I2} = 0$.

These outcomes can never be obtained by a model like (2.1), with d_{ij} average distance, since S_{I1} and S_{I2}, when shop 1 is in D, would have by this model the same values as in the case where shop 1 is in C, because of the equality of the average distances in both situations.

It can also be seen that the effect of average distance in (2.1), cannot realistically be measured by the decreasing function f, since the relationship of S_{I1} and d_{I1} is not even a function in our example. This is pictured in fig. 2.3 where values of S_{I1} have been calculated by the geometrical method for several values of average distance d_{I1}, in case shop 1 is moving along the diagonal E-F.
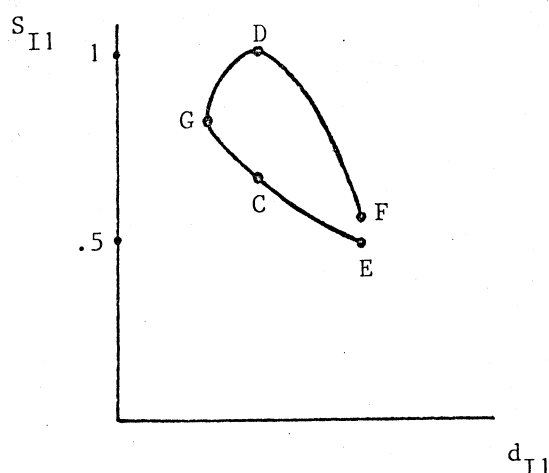
fig. 2.3

The characters in this figure denote the S_{I1}-d_{I1} combinations, which correspond to the locations of shop 1 running from the south-west corner E, via C and D to the north-east corner F. It is shown that on the way from the centre of gravity G towards D average distance has a _positive_ effect on S_{I1}.

Another property of the aggregate model (2.1), which gives rise to some criticism, is the characteristic known as the "independence of irrelevant alternatives" (Mc. Fadden, 1974), saying that the ratio of S_{ij} and S_{ik} is independent of the presence of other shopping centres than j and k.

Model (2.1) implies that:

$$\frac{S_{ij}}{S_{ik}} = \frac{A_j \ f(d_{ij})}{A_k \ f(d_{ik})}$$

and so the ratio $\dfrac{S_{ij}}{S_{ik}}$ is independent of all shopping centres other than j and k.

This property, already critisized with respect to individual choice models like logit, is in case we use average distance in (2.1), in general not valid, which we illustrate by means of the following example.

Suppose i is a zone as pictured in fig. 2.4. The consumers in this zone are equally distributed over the area and have to decide whether they spend their money (for everybody assumed to be the same amount) in shop 1 or shop 2. If we assume further that both shops are equal attractive, then distance is again the choice criterion for any individual consumer. As in the previous example the division line of the shop's attraction areas is the perpendicular bisector of the line of connection between both shops. The attraction areas are denoted by the areas A_1 and A_2.
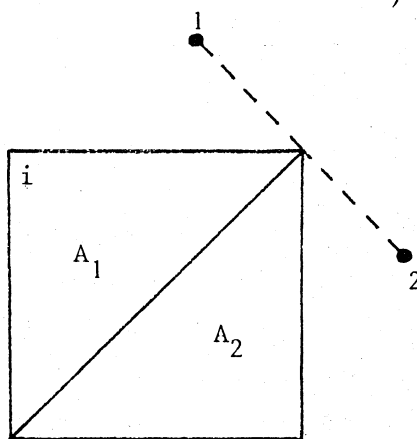

fig. 2.4

The geographical structure pictured in the figure is such that the following equation holds:

$$\frac{S_{i1}}{S_{i2}} = \frac{A_1}{A_2} = 1$$

Suppose further that there is a third shopping centre in the area, located as pictured in fig. 4.3, with the same attractiveness as the other shops.

We construct the attraction areas for either centre, by drawing the relevant perpendicular bisectors, yielding the attraction areas A_1, A_2 and A_3.
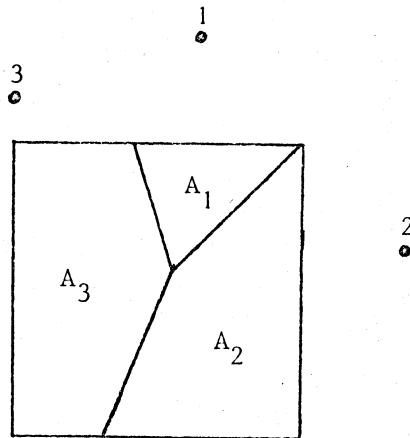
fig. 2.5

It is seen then that the ratio $\dfrac{S_{i1}}{S_{i2}} = \dfrac{A_1}{A_2}$ has become sustantially smaller than unity, which is contradictary to the "independence of irrelevant alternatives" property of the model (2.1).

From these two examples we may conclude that with a simple aggregate like average distance (or distance measured from the centre of gravity) model (2.1) may loose validity in certain situations, which may lead to an inadequate description of reality, due to misspecificaton of the functional forms in the model and/or the "wrong" choice of the aggregate measure of distance.

The specification errors of model (2.1) will become less dramatic, if regions are subdivided into small population zones (requiring a lot of data to estimate the model) or if a better aggregate distance measure is constructed, which in some cases might be possible, as is shown in Appendix A.

The criticism expressed here is of course at first sight only applicable to aggregate choice models but we will show in section 7 that also in case of disaggregate choice models, one should be careful with the selection of specific functional relationships.

It is therefore that we plead in favour of a disaggregate approach, based on a theory to describe individual behaviour, with the emphasis on the derivation of proper functional forms of variables in the ultimate model specification.

In the next sections we formulate a theory, that might be of help in this sense in order to analyse in which way distance, prices and assortment has to be incorporated in shopping models.

3. Description of the spatial choice problem.

The object of the next section is to formulate a micro-economic theory with the purpose to describe and explain the spatial distribution of the flows of cash in a certain region with respect to shopping goods, emphasizing the role of distance, prices and assortment in this matter. Since the distribution of the flows of cash, are determined by the decisions of individual consumers (or households) it is understandable that a proper description of individual behaviour will be the starting point of our analysis.

If we are interested in the shopping decisions of a consumer in a certain period of time we like to have answers on the following questions:

a. What are the type of goods a consumer wants to purchase in that period?

b. Which quantities of these goods are bought?

c. What is the amount of money a consumer spends on these goods?

d. Which shops are chosen for the purchase of these goods?

Looking at these fundamental questions and searching for a theoretical framework which might give us the requested answers, we may place the first three questions in a traditional utility framework, where it is assumed that the consumer purchases that combination of goods that gives him the highest utility given his income and prices of the goods. This means that if we know the consumer's utility function, his income and the prices of the goods, we are able to answer these questions by maximizing his utility function (assuming that this is a realistic

decision rule) constrained by his limited income. In that case only question d. remains unanswered.

In order to incorporate this spatial allocation problem into the previous utilty approach, it is necessary to bring in the specific nature of this decision problem. This implies that the difference in locations of the shops, the difference in assortment and prices among other relevant factors have to be formalized in the utility context. The way in wich this should be carried out is not easy, since the overall utility function to be optimized, should be specified in such a manner that all possible combinations of goods, differentiated by nature, prices and spatial availability, can be evaluated. It is obvious that this utility function will turn out to be very complex, in view of the large number (perhaps unmanageable) of variables, of which the mathematical specification can not be given a priori.

Because of this complexity, we give up this approach and treat the spatial allocation problem of goods in itself, assuming that it is allowed to consider it separately from the decisions the consumer has to make with respect to the budget allocation problem of the questions a. till c.

This assumption is certainly a restriction, since the selection of goods, a consumer wants to purchase, is supposed to be independent of the availability of these goods in his neighbourhood. The demand creating aspects of the supply side are therefore implicitly omitted.

The advantage of the separation is however that we may act as if the consumer has already allocated his budget (by maximizing his utility function given his income and say average prices of the goods in his region) and that we therefore may consider a number of variables as predetermined (exogenous) variables. Consequently we assume that we know the composition of the package of goods the consumer is intended to purchase in the chosen period of time, that we know the budget per category and the frequencies with which these budgets are spent. The underlying budget allocation model that should determine these variables will not be discussed in this paper.

The consumer's spatial choice problem may then be formulated as follows:
Suppose that consumer i purchases in a certain period a package of
goods, consisting of G_i categories and that good g_i ($g_i = 1, \ldots, G_i$) is
bought with frequency f_{ig_i} with a planned budget of b_{ig_i} per time.
Suppose further that there are J shopping centres in the region,
offering goods against prices p_{jg} ($j = 1, \ldots, J, g = 1, \ldots, G$).
The question to be answered (theoreticaly) is then: what are the
budgets E_{jg_i} consumer i spends on good g_i in shopping centre j in the
chosen period, taking the characteristics of j and its location with
respect to consumer i into account.
In the next section this problem will theoretically be formalized.

4. General theoretical framework.

4.1. Introduction.

The consumer's spatial choice problem formulated in the previous section
will be considered as a (for the time being) deterministic and discrete
decision problem in the traditional sense.
Suppose an individual i has to choose an alternative j from his
alternative set $A_i = \{j; j = 1, \ldots, J_i\}$. [1] Each alternative j is
characterised by a vector of attributes x_j.
The individual has a preference function U_i, which defines an ordering
on the alternatives by comparing the preference obtained by the
attributes of the various alternatives.
It is assumed that the individual choses the alternative which maximizes
this preference function. So alternative j is chosen by individual i if

$$U_i(x_j) \geqslant U_i(x_k) \text{ for all } k = 1, \ldots, J_i$$

(1) The alternative set is indexed by i, since the number and
    nature of the alternatives may differ among individuals.

From this it is seen that the decision problem for the individual is defined by:

1. the alternative set A_i containing a number of alternatives specific for individual i,

2. the vectors of attributes x added to the alternatives,

3. the preference function U_i, which is a function of the attributes x of the alternatives and which is supposed to be maximized.

This framework will be used as a starting point to attack the problem in hand and we will discuss the subsequent items in more detail in the next sub-sections.

4.2. The set of alternatives.

The set of alternatives for consumer i should in principle consist of all possible ways of purchasing the package of goods the consumer is intended to buy in a given period of time. The consumer is supposed to select the shops in his region he wants to visit and the combination of goods he wants to buy in each shop. These visits form the elements of the set of alternatives in his decision problem and will be denoted by $V(s, \{g_s\})$, in which s is the shop where the consumer buys the combination $\{g_s\}$ of goods. To give an example: $V(3, \{2, 3, 6\})$ is a visit of shop 3 where the goods 2, 3 and 6 are bought.

If all combinations of goods g = 1, ..., G are feasible and each shop s = 1, ..., S in the region can be reached, the consumer has an alternative set, which consists of $S \times (2^G - 1)$ alternative visits.[1]

(1)   To select k goods out of n can be done in $\binom{n}{k}$ different ways. So the number of combinations of goods that can be selected is given by $\sum_{k=1}^{n} \binom{n}{k} = 2^n - 1$.

4.3. The vector of attributes added to the alternatives.

The attributes of the alternatives we have to specify are those
variables which are supposed to be relevant to determine the preference
level of the alternative.
Since the consumer's alternative set consists of visits to shops with
the purpose to purchase a certain combination of goods it seems
appropriate to select those variables which tell us something about the
atractiveness of the shops and the goods they supply.
We make the following distinction between attraction factors

1. Properties of the shops concerning the general service level offered
   to the costumers; i.e.
   a. assortment
   b. prices and qualities
   c. size of the shop
   d. other factors like service and waiting time, clear arrangement of
      the products, atmosphere, advertising, etc.

2. Spatial properties
   a. situation of the shop in the region
   b. accessibility of the shop for different modes of transport.

3. Characteristics of the environment
   a. parking facilities
   b. additional facilities near the shop (gasstation, postoffice, etc.)
   c. atmosphere, sociability, etc.

Since we do not know exactly in advance the relative importance of all
these factors (which is already a selection), each factor should in
principle be incorporated in the decision problem of the consumer. But
as mentioned before in the introduction the main purpose of this study
is to formulate a theory in which the economic as well as the spatial
aspects of the decision problem are explicitly taken into account. It is
therefore that we will give more weight to that sort of variables than
to others. The last remark will become more clear in the next sections.

4.4. The preference function.

The preference function measures, in our context, the level of attractiveness of a certain visit to a shop, depending on the attribute vector of that visit.

To become more concrete about the preference function we firstly have to make some assumptions about the consumer's behaviour.

Ass 1. A consumer is intented to buy a known package of goods in the region in a certain known period of time.

Ass 2. Each good is purchased by the consumer in that period of time with a known frequency.

Ass 3. Each time the consumer buys a good he spends a known amount of money.

Now we introduce the following symbols:

f_{ig}: the frequency of buying good g by consumer i in the chosen period of time,

b_{ig}: the planned expenditure of consumer i on good g per time of purchase, valued at average prices of that good in the entire market,

p_{jg}: price of good g in shop j,

t_{ij}: travel costs related to a visit of shop j by consumer i.

Ass 4. The preference function of consumer i is supposed to have the following properties: A visit $V(s, \{g_s\})$ is preferred to a visit $V(s', \{g_{s'}\})$ by consumer i, in case that $\{g_s\}$ and $\{g_{s'}\}$ contain the same combination of goods (i.e. $\{g_s\} = \{g_{s'}\}$), if the "subjective" cost of $V(s, \{g_s\})$, denoted by $K_i(s, \{g_s\})$ is less than $K_i(s', \{g_{s'}\})$.

Bacon (1971) already considered cost as the criterion of the consumer's decision problem and the approach further discussed in this section is built upon the major ideas of his study.

It is supposed that the subjective cost of a visit can be expressed as follows:

$$(4.1) \quad K_i(s, \{g_s\}) = a_{is}\left(\sum_{g \in \{g_s\}} b_{ig} \frac{p_{sg}}{\bar{p}_g} + t_{is} \right)$$

where $\bar{p}_g$ is the average price of good g in the market and a_{is} is inversely related to the "attractiveness" of shop s to consumer i.

To give an interpretation of this expression we note that the term on the righthand side, within brackets, may be viewed as the total "cost" of visiting a shop, with the purpose of purchasing the $\{g_s\}$ combination of goods, consisting of the expenditure on that combination of goods valued against prices in shop s on the one hand, and travel cost on the other hand. This implies that if two shops are equally attractive, (i.e. $a_{is} = a_{is'}$), the consumer prefers a visit to that shop which offers the combination of goods against the least cost.

The related attraction factor a_{is} reduces the cost of the visit, if the shop is more attractive to consumer i, with respect to other factors than prices and travel cost.

Ass 5. The consumer i is supposed to maximize preference in the sense, that he selects that combination of visits, which enables him to purchase his planned package of goods in the chosen period of time with the lowest subjectve cost. So the decision problem of consumer i takes the form of finding the combination of visits out of the set of alternatives (consisting of $S \times (2^G - 1)$ visits; see section 4.2),which minimizes subjective cost, given the planned expenditure on the different goods and the intended frequencies, with which the goods are purchased.

In the next section we shall show that this decision problem can be formulated as a linear (integer) programming problem, which is mathematically more attractive than the "ad hoc" solution method suggested by Bacon.

5. Mathematical formulation of the individual's decision problem.

Assumption 5 of the previous section says that a consumer i will try to find that combination of visits which minimizes total subjective cost and which provides him his planned package of goods in the chosen period of time, taking into account the frequencies and budgets related to the purchase of the different goods in that period of time.

If we assume that all goods can be bought in each shop[1], then the total subjective cost of the combination of visits chosen by consumer i will be

$$(5.1) \quad C_i = \sum_s \sum_{\{g_s\}} f_{is\{g_s\}} K_i(s, \{g_s\})$$

with C_i: total subjective cost of the purchase of the complete package of goods by consumer i.

    $K_i(s, \{g_s\})$: subjective cost of the purchase of the combination of goods $\{g_s\}$ in shop s by consumer i.

    $f_{is}\{g_s\}$:   the frequency of buying the combination of goods $\{g_s\}$ in shop s by consumer i. (Note that there are $S \times (2^G - 1)$ such frequencies possible)

Consumer i is assumed to minimize C_i in (5.1) with respect to the frequencies $f_{is}\{g_s\}$ under the following restrictions:

$$(5.2) \quad \sum_s \sum_{\{g_s\}} f_{is\{g_s\}} = f_{ig} \text{ if } g \quad \{g_s\}; \ g = 1, \ldots, G$$

This constrained minimization problem of dimension $S \times (2^G - 1)$ with G constraints can be solved as a regular linear (integer) programming problem if the variables in the costfunction (4.1) are valuated.

Since the dimension of this problem may become substantially high if S en G appear to be large, any reduction of the problem can be viewed as being welcome.

In the next section we will show that with some elementary operations the problem may be reduced to a $(2^G - 1)$ dimensional linear programming system. And in order to avoid the complicated notation of this section and to make things more clear we will continue by means of a simple example.

(1) This assumption is not really a simplification of the problem since we may allow for shops with "incomplete" assortment by defining the prices of the missing goods unrealistic high enough to ensure that these goods will not be chosen.

5.1. Example.

Suppose there are 3 shops in the region, offering 3 sorts of different goods (i.e. S = 3, G = 3). A consumer (the index is omitted) wants to buy a package of goods in a certain period of time consisting of these 3 specific goods.

Firstly we define the set of alternatives for this consumer by distinguishing the different visits he can make:

Set of alternatives.

$$V(s, \{g_s\}) = \quad \begin{array}{l} (1)\ V(1, \{1\}) \\ (2)\ V(1, \{2\}) \\ (3)\ V(1, \{3\}) \\ (4)\ V(1, \{1, 2\}) \\ (5)\ V(1, \{1, 3\}) \\ (6)\ V(1, \{2, 3\}) \\ (7)\ V(1, \{1, 2, 3\}) \\ (8)\ V(2, \{1\}) \end{array}$$

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$$(21)\ V(3, \{1, 2, 3\})$$

As before we mean by $V(s, \{g_s\})$ a visit to shop s with the purpose of buying the combination of goods $\{g_s\}$. In this sense alternative (6) is a visit to shop 1 where the consumer buys the goods of category 2 and 3. By counting all the possible visits, the alternative set consists of 21 different visits.

Data.

Frequencies:

The consumer is intended to buy the three categories of goods with the following frequencies in the chosen period of time

$$f_1 = 1, \qquad\qquad f_2 = 3, \qquad f_3 = 5$$

Budgets:

Each time the consumer buys a good he spends the following budgets, valued at average prices:

$$b_1 = 25, \qquad\qquad b_2 = 15, \qquad b_3 = 5$$

Travelcost:

The travelcosts for the consumer of visiting the different shops are supposed to be:

$$t_1 = 12, \qquad\qquad t_2 = 10, \qquad t_3 = 7$$

Prices:

The consumer faces the following prices for the different goods in the shops, measured as price indices related to the average price level of each good in the market $(\bar{p}_1 = \bar{p}_2 = \bar{p}_3 = 100)$:

$p_{11} = 95$	$p_{21} = 103$	$p_{31} = 98$
$p_{12} = 98$	$p_{22} = 100$	$p_{32} = 1000$
$p_{13} = 97$	$p_{23} = 102$	$p_{33} = 92$

where p_{sg} = price index of good g in shop s ($p_{32} = 1000$ means that shop 3 is not supplying good 2).

Related attraction factors:

The factors a_{is} in eq. (4.1) which measure the inattractiveness of the shops are supposed to have the indexed values:

$$a_1 = 100, \qquad\qquad a_2 = 103, \qquad a_3 = 98$$

Subjective cost for each alternative.

With these data we are able to calculate the subjective cost of each visit $K(s, \{g_s\})$ in eq. (4.1), envisaged in the next table; with the shops vertically and the 7 different combinations of goods horizontally represented.

Table 5.1

values of subjective costs $K(s, \{g_s\})$.

	1	2	3	4	5	6	7
s = 1	3575	2670	1685	5045	4060	3155	5530
2	3682.25	2575	1555.3	5227.25	4207.55	3100.3	5752.55
3	3087	15386	1136.8	17787	3537.8	15836.8	18237.80

To give an example: The subjective cost of buying combination 4 (i.e. good 1 and 2) in shop 2 is calculated by the formula

$$a_2\{b_1 \frac{P_{21}}{\bar{P}_2} + b_2 \frac{P_{22}}{\bar{P}_2} + t_2\} = 103\{25 \frac{103}{100} + 15\frac{100}{100} + 10\} = 5227.25.$$

Reduction of the problem.

The 21 calculated values of the subjective cost of the different visits in the preceding table can be incorporated directly in the linear programming problem as formulated in eq. (5.1) and (5.2) yielding a problem of dimension 21. Fortunately this is not necessary since it may be noticed that minimizing total subjective cost implies that a visit to a certain shop with the purpose to buy a specific combination of goods will not be made if the subjective cost of this visit is higher than the subjective cost of a visit to another shop with the same purpose. The subjective cost of visits with the same purpose can be found in each column of table 5.1.

So before running into the linear programming problem we reduce the dimension by calculating the column-minima of table 5.1 and use these minima as the coefficients in our objective function.

We finally solve the linear programming problem:

$$\text{Min } C = f_{31} \times 3087 + f_{22} \times 2575 + f_{33} \times 1136.8 +$$

Subject to:
$$f_{14} \times 5045 + f_{35} \times 3537.8 + f_{26} \times 3100.3 + f_{17} \times 5530$$

$$f_{31} + f_{14} + f_{35} + f_{17} = f_1 = 1$$

$$f_{22} + f_{14} + f_{26} + f_{17} = f_2 = 3$$

$$f_{33} + f_{35} + f_{26} + f_{17} = f_3 = 5$$

The solution of this problem is reached by:

$$f_{35} = 1$$

$$f_{26} = 3$$

$$f_{33} = 1$$

This implies that the consumer buys

once the goods 1 and 3 combined in shop 3
three times goods 2 and 3 combined in shop 2
once good 3 in a separate visit to shop 3

6. Determination of attraction areas of shops.

6.1. Exposition.

In this section some implications of the theory formulated in the previous part of this paper will be considered.

The object will be to visualize the impact of the specification of travel costs and related attraction factors of the shopping centres on the basis of some simple examples.

We are especially interested in these variables because they are incorporated in many models (and in many ways), dealing with individual shopping behaviour and we like to know whether the implications of our theory differ from the existing models or not.

This comparison can be made if we look at the attraction areas of a shop, defined as the geographical areas wherein consumers are situated who are intended to buy in that particular shop.

In this section we will derive the attraction areas of shops from the theory in the previous sections, in case of a simplified regional structure.

6.2. Iso-preference lines.

Suppose we have 2 shops in a region with a population uniformly spread over the area. Moreover we assume that all individuals are "equal" in the sense that

- they have the same subjective cost function
- they have the same budgets to spend
- they are intended to buy the same package of goods

For simplicity we assume further that this package of goods is purchased once and that both shops supply these goods. So the only discriminating factors in the individual's decision problem are prices, travel costs and related attraction factors.

The subjective cost function introduced in section 4.4. has been expressed in general as:

$$(6.1) \quad K_i(s, \{g_s\}) = a_{is} \left(\sum_{g \in \{g_s\}} b_{ig} \frac{p_{sg}}{\bar{p}_g} + t_{is} \right)$$

Taking the above assumptions into account (6.1) can be simplified as follows:

$$(6.2) \quad K_{is} = a_s \left(b \frac{p_s}{\bar{p}} + t_{is} \right); \quad \begin{aligned} s &= 1, 2 \\ i &= 1, \ldots, I \end{aligned}$$

with K_{is} = subjective cost of buying the goods in shop s by consumer i

a_s = related attraction factor of shop s

b = budget spent on the goods

p_s = price level of the goods in shop s

$\bar{p}$ = average price level in the market

t_{is} = travel cost of consumer i, related to a visit of shop s

I = total numer of consumers in the region.

The travel cost is assumed to be a proportion of distance

$$(6.3) \quad t_{is} = \alpha d_{is}$$

with d_{is} = the distance between the location of consumer i and shop s

α = travel cost per unit of distance (α is supposed to be fixed for every consumer).

Insertion of (6.3) in (6.2) yields:

$$K_{is} = a_s(b \frac{p_s}{\bar{p}} + \alpha d_{is})$$
$$= a_s(\gamma p_s + \alpha d_{is})$$

with $\gamma = \dfrac{b}{\bar{p}}$.

Following Ass. 4 of section 4.4. we may say that a consumer i is indifferent for the two alternatives (i.e. buying the goods in shop 1 or in shop 2) if the subjective costs for both alternatives are equal; so if:

$$(6.4) \quad K_{i1} = a_1(\gamma p_1 + \alpha d_{i1}) = a_2(\gamma p_2 + \alpha d_{i2}) = K_{i2}$$

or rewritten if:

$$(6.5) \quad d_{i1} = \frac{1}{\alpha} (\frac{a_2 \gamma p_2 - a_1 \gamma p_1}{a_1}) + \frac{a_2}{a_1} d_{i2}$$

or

(6.6) $d_{i1} = \varepsilon + \delta d_{i2}$

with $\varepsilon = \frac{1}{\alpha} (\frac{a_2^{\gamma} p_2 - a_1^{\gamma} p_1}{a_1})$

with

$\delta = \frac{a_2}{a_1}$

From equation (6.6) we are able to derive the geometrical representation of the iso-preference line.

Firstly, we shall give the derivations for some extreme cases.

a. iso-preference line if $p_2 = p_1$ and $a_2 = a_1$.

This is the case where both shops can be viewed as being identical, since they do not differ in price and attractiveness. In this situation (6.6) becomes:

$d_{i1} = d_{i2}$

The iso-preference line is then a straight line which coincides with the perpendicular bisector on the line of connection of both shops. This situation is pictured in fig. 6.1, where the region boundary is visualized by the dotted line
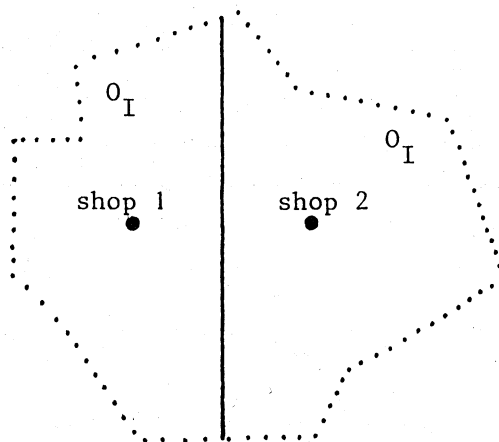

figure 6.1.

straight iso-preference line

With the assumptions made before, the marketshares of the shops equal

$$(6.7) \qquad m_1 = \frac{O_I}{O_I + O_{II}}$$

$$m_2 = \frac{O_{II}}{O_I + O_{II}}$$

with m_s = the marketshare of shop s

O_I = the surface of the area on the lefthandside of the line HH'

O_{II} = the surface of the area on the righthandside of HH'

b. iso-preference line if $a_2 \gamma p_2 = a_1 \gamma p_1$

In this case the price difference between shop 1 and 2 are compensated by the difference in attractiveness and (6.6) becomes

$$(6.8) \qquad d_{i1} = \delta d_{i2}$$

To derive the geometrical representation of the iso-preference line we place the location of shops in a rectangular coordinate system such that shop 1 is in the origin and shop 2 has the coordinates $(x_2, 0)$. The locations of the consumers which satisfy (6.8) are such that the x- and y-coordinates of these locations should hold the equation called for by (6.8); i.e.

$$(6.9) \qquad \sqrt{x^2 + y^2} = \delta \sqrt{(x-x_2)^2 + y^2}$$

After some straightforward manipulations we find for the iso-preference line the curve:

$$(6.10) \qquad (x + \frac{\delta^2}{1-\delta^2} x_2)^2 + y^2 = (\frac{\delta}{1-\delta^2} x_2)^2; \ \delta \neq 1$$

which is a circle with the center at $(- \frac{\delta^2}{1-\delta^2} x_2, 0)$ and
radius $|\frac{\delta}{1-\delta^2} x_2|$.

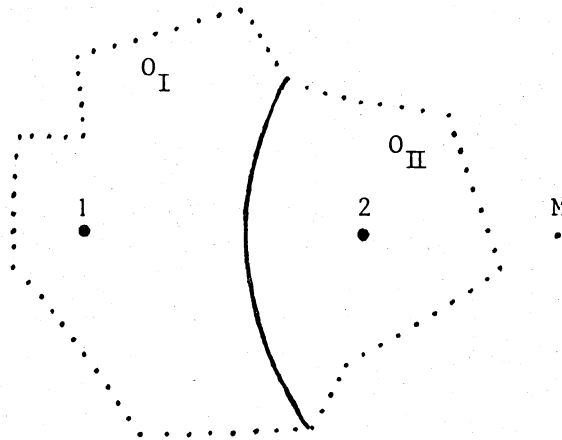In figure 6.2 the iso-preference line is pictured in case $\delta = 1.5$ and $x_2 = 3.75$.

figure 6.2

circular iso-preference line

The attraction area of shop 1 is in figure 6.2 denoted by O_I and of shop 2 by O_{II}

c. iso-preference line if $a_1 = a_2$

Here (6.6) becomes

(6.11) $d_{i1} = \varepsilon + d_{i2}$

With the choice of the same coordinates for shop 1 and 2 as in b. this may be written as

$$\sqrt{x^2 + y^2} = \varepsilon + \sqrt{(x-x_2)^2 + y^2}$$

which finally results in the curve:

(6.12) $\dfrac{(x - \frac{1}{2}x_2)^2}{\frac{1}{4}\varepsilon^2} - \dfrac{y^2}{\frac{1}{4}(x_2^2 - \varepsilon^2)} = 1; \quad |\varepsilon| < |x_2|$

which is a hyperbola, pictured in figure 6.3 (with $x_2 = 4$; $\varepsilon = 3$)
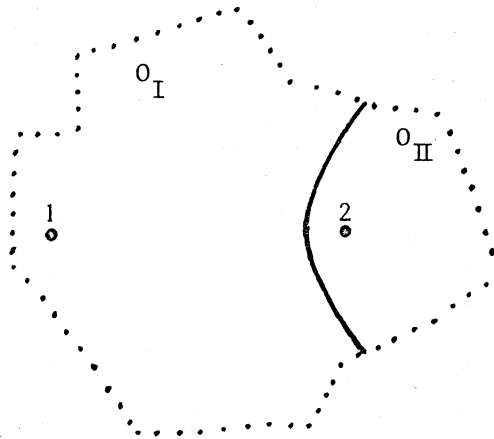
figure 6.3

hyperbolic iso-preference line

The three preceding examples are specific cases of the general relation (6.6).

In appendix A the derivation of the general geometric expression for the iso-preference line is given. It is shown there that the iso-preference line can be represented by:

$$(6.13) \quad \{x^2 - \delta^2(x-x_2)^2 + (1-\delta^2)y^2 + \epsilon^2\} = 4\epsilon^2(x^2+y^2)$$

which is known in the literature as the Limaçon of Pascal.

The different graphical forms of the iso-preference lines satisfying (6.13) are pictured in Appendix B.

7. Expected iso-preference lines in a stochastic utility approach with linear utility function.

The logit and probit-models widely used for the description of spatial consumer behaviour are based on maximizing an utility function with a stochastic component.

Suppose that a consumer i has to choose between two shops (1 and 2) with attractivity a_1 and a_2 respectively, and suppose further that the consumer buys a package of goods in that shop which has the largest contribution in utility to consumer i.

If the utility of shop j (j = 1, 2) for consumer i can be represented by:

(7.1) $U_{ij} = a_j - \beta d_{ij} + \varepsilon_{ij}$

with U_{ij}: the utility of buying in shop j for consumer i

   a_j: a measure of attractiveness of shop j

   d_{ij}: the distance between consumer i and shop j

   β: a positive distance friction parameter

   ε_{ij}: a stochastic term

In this utility function the non-stochastic part $a_j - \beta d_{ij}$ can be viewed as the "representative tastes" for shop j of all the consumers at the same distance of shop j, while ε_{ij} reflects the idiosyncracies of consumer i in tastes for shop j (see McFadden [1974])

Consumer i chooses shop j if the utility of shop j exceeds the utility of shop k, i.e. .

$U_{ij} > U_{ik}$

or

$a_j - \beta d_{ij} + \varepsilon_{ij} > a_k - \beta d_{ik} + \varepsilon_{ik}$; j = 1, 2; k ≠ j

It is noticed that if we would assume that the ε_{ij} are drawn independently for all i and j from a Gnedenko extreme-value distribution

with distribution function $F(\varepsilon_{ij}) = e^{-e^{-\varepsilon}}$, the choice probability of choosing shop j may be represented by:

$$P_j = \frac{e^{a_j} \cdot e^{-\beta d_{ij}}}{e^{a_j} \cdot e^{-\beta d_{ij}} + e^{a_k} \cdot e^{-\beta d_{ik}}}; \quad j = 1, 2; \quad k \neq j$$

which is the well-known logit formula in case of two alternatives (McFadden). We will however not run into the calculation of choice probabilities derived from specific distributions, since it is our interest to look at the iso-preference lines of this model and to compare them with the ones obtained in the previous section. Because of the fact that the utility function presented here is stochastic, the concept of indifference will be treated on the average. We therefore define the iso-preference line as the set of consumers having equal expected utility for both the shops, i.e.:

$$E(U_{i1}) = E(U_{i2})$$

or

(7.2) $\quad a_1 - \beta d_{i1} - a_2 + \beta d_{i2} = E(\varepsilon_{i2} - \varepsilon_{i1})$

By assuming that the expected value of the stochastic terms are equal, (7.2) becomes:

$$a_1 - \beta d_{i1} - a_2 + \beta d_{i2} = 0$$

or

(7.3) $\quad d_{i1} = \frac{a_1 - a_2}{\beta} + d_{i2}$

The geometrical representation of the consumers with locations according to this equation is a hyperbola (or a straight line if $a_1 = a_2$) if the assumptions for the simple case as stated in section 6.2 are met (compare (7.3) with (6.11)).

In appendix C we have seen that with the theoretical framework of section 4 we may have various geometrical forms of the iso-preference lines different from the hyperbola.

Our conclusion is therefore that the assumption about the specification of the utility function in (7.1) can work out very restrictively, and that although the stochastic utility framework seems to be a very general conception, the assumption of a linear utility function (in the literature commonly accepted as being useful) is not that harmless as it appears to be.

APPENDIX A. <u>Derivation of an aggregate model consitent with individual choice theory.</u>

In this Appendix it is shown that it is possible to derive the aggregate model (2.1), with a proper aggregate distance measure from the individual choice theory underlying the disaggregate logit model with a specific stochastic utility function.

Suppose that the utility U_{ij} of buying in shopping centre j of a consumer i can be expressed as:

(A.1) $U_{ij} = \log a_j - \beta \log d_{ij} + \varepsilon_{ij}$; $j = 1, \ldots, J$

in which a_j: is some measure of the attractiveness of shopping centre j

       d_{ij}: is the distance between i and j

       β: is a distance friction parameter

      ε_{ij}: is a stochastic term

      J: is the number of shopping centres in the region.

It is assumed that consumer i chooses that shopping centre, which offers him the greatest utility.

The probability p_{ij} that consumer i chooses centre j is:

$$P_{ij} = P[U_{ij} > U_{ik}; \quad k \neq j]; \; j = 1, \ldots, J$$

The choice probabilities may analitically be derived if we assume that the stochastic terms ε_{ij} are drawn uniformly and independently from a Gumbell (Gnedenko) distribution, yielding the logit formula:

(A.2) $P_{ij} = \dfrac{a_j d_{ij}^{-\beta}}{\sum\limits_{k} a_k d_{ik}^{-\beta}}$, $j = 1, \ldots, J$

Define Y_{zj} as the number of consumers in a certain zone z buying their shopping goods in shopping centre j. With the assumptions made on the ε_{ij} it is easy to verify that the mathematical expectation of Y_{zj}, denoted by $E(Y_{zj})$ equals:

(A.3) $E(Y_{zj}) = \sum\limits_{i\,z} P_{ij}$

$$= \sum\limits_{i\,z} \frac{a_j d_{ij}^{-\beta}}{\sum\limits_{k} a_k d_{ik}^{-\beta}}$$

If w_z is the average amount of money spent in z on shopping goods per capita, then $E(Y_{zj})w_z$ is the expected flow of cash from z towards j and if W_z is the total budget potential in z then

$$\frac{E(Y_{zj})w_z}{W_z} = \frac{E(Y_{zj})}{P_z}$$

is the expected proportion of the flows of cash from z to shopping centre j, with P_z the population size of zone z.

Replacing $\dfrac{E(Y_{zj})}{P_z}$ by S_{zj}, we can write S_{zj} as in equation (2.1) as:

(A.4) $S_{zj} = \dfrac{a_j \bar{d}_{zj}^{-\beta}}{\sum\limits_{k} a_k \bar{d}_{zk}^{-\beta}}$,

if we define aggregate distance $\bar{d}_{zj}$ as:

(A.5) $\bar{d}_{zj} = \left\{ \sum\limits_{i=1}^{P_z} \dfrac{d_{ij}^{-\beta}}{\sum\limits_{k} a_k d_{ik}^{-\beta}} \right\}^{-1/\beta}$

It is easily verified that $\bar{d}_{zj}$ is a consistent aggregate by substituting A.5. in A.4. in order to derive A.3.

We may conclude that in some cases an aggregate model like 2.1 can be consistent with individual choice theory, if variables are aggregated in the right way. It is seen from this example that the aggregate distance measure is some what more complicated than those commonly used in applied models.

Appendix B. Derivation of iso-preference lines for the general case.

In section 6.2. equation (6.6) gives the relation between the distances from a consumer i to both the shops in case the consumer is indifferent in his choice of either shop; i.e.:

(6.6) $d_{i1} = \varepsilon + \delta d_{i2}$

If we assume as before that shop 1 is located in the origin of a rectangular coordinate system and that shop 2 has the coordinates $(x_2, 0)$ and that the consumer satisfying (6.6) is represented by his coordinates (x, y), (6.6) becomes:

(B.2) $\sqrt{x^2 + y^2} = \delta\sqrt{(x - x_2)^2 + y^2} + \varepsilon$

Define: $a = x^2 + y^2$
$\qquad b = x_2(x_2 - 2x),$
then (B.2) can be written as:

$$\sqrt{a} - \delta\sqrt{a+b} = \varepsilon.$$

Squaring both sides of the equation yields:

$$(1 + \delta^2)a + \delta^2 b - \varepsilon^2 = 2\delta\sqrt{a(a+b)}$$

Squaring and rearranging terms results in:

(B.3) $\{(1 - \delta^2)a - \delta^2 b + \varepsilon^2\}^2 = 4\varepsilon^2 a$

Substitution of the definitions of a and b gives us the general expression of the iso-preference line:

(B.4) $\{x^2 - \delta^2(x - x_2)^2 + (1 - \delta^2)y^2 + \varepsilon^2\}^2 = 4\varepsilon^2(x^2 + y^2)$

This expression may be considered as a quadratic equation in y^2 with roots given by:

(B.5.1) $(1 - \delta^2)^2 y^2 = (\sqrt{Z_x} \pm \epsilon)^2 - (1 - \delta^2)^2 x^2;$

with $Z_x = \delta^2\{(1 - \delta^2)x_2(x_2 - 2x) + \epsilon^2\}$

This equation holds in case $\delta^2 \neq 1.$

With $\delta^2 = 1$, (B.4) becomes:

(B.5.2) $4\epsilon^2 y^2 = (x_2^2 - \epsilon^2) \{(x_2 - 2x)^2 - \epsilon^2\}.$

Appendix C. Graphical representation of the general iso-preference line.

The iso-preference line for the general case is determined by equation (6.13):

$$(6.13) \quad \{x^2 - \delta^2(x - x_2)^2 + (1 - \delta^2)y^2 + \varepsilon^2\}^2 = 4\varepsilon^2(x^2 + y^2)$$

From (B.5.1) and (B.5.2) in Appendix B we may derive the relation between y and x; yielding:

$$(C.1.1) \quad y^2 = \left\{\frac{1}{1-\delta^2}(\sqrt{Z_x} \pm \varepsilon)\right\}^2 - x^2; \quad \delta^2 \neq 1$$

with $Z_x = \delta^2\{(1 - \delta^2)x_2(x_2 - 2x) + \varepsilon^2\}$

$$(C.1.2) \quad y^2 = \frac{x_2^2 - \varepsilon^2}{4\varepsilon^2}\{(x_2 - 2x)^2 - \varepsilon^2\}; \quad \delta^2 = 1$$

For certain values of δ and ε we can compute the (x, y) combinations that suffice the former equations. By doing so, however, we will collect a number of inadmissible combinations, due to the fact that in the derivation of (6.13) in Appendix B the squaring of terms embezzles the original sign of δ and/or ε.
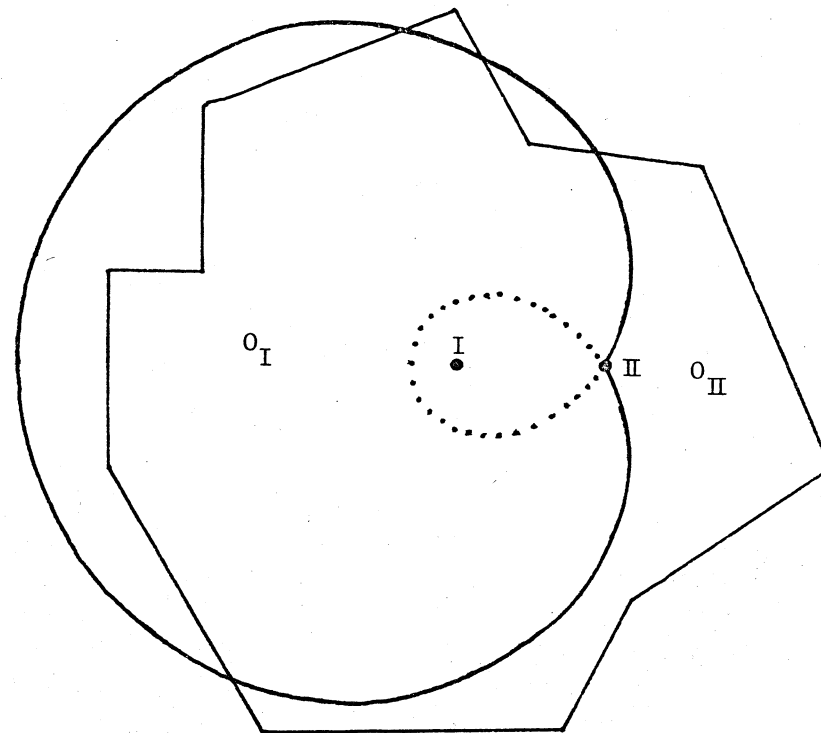
In the following table the admissible ranges of x-values are summarized for different ranges of δ and ϵ provided $x_2 > 0$:

$$
1. \quad
\left.
\begin{array}{c}
\delta > 1 \\[1em]
-\delta x_2 < \epsilon < x_2
\end{array}
\right\}
\quad
\frac{\delta x_2 + \epsilon}{\delta + 1} < x < \frac{\delta x_2 - \epsilon}{\delta - 1}
$$

$$
2. \quad
\left.
\begin{array}{c}
0 < \delta < 1 \\[1em]
-\delta x_2 < \epsilon < x_2
\end{array}
\right\}
\quad
-\frac{\delta x_2 + \epsilon}{1 - \delta} < x < \frac{\delta x_2 + \epsilon}{1 + \delta}
$$

$$
3. \quad
\left.
\begin{array}{c}
\delta > 1 \\[0.5em]
\epsilon < -\delta x_2 \\
\text{or} \\
0 < \delta < 1 \\[0.5em]
\epsilon > x_2
\end{array}
\right\}
\quad
-\frac{\delta x_2 + \epsilon}{1 - \delta} < x < \frac{\delta x_2 - \epsilon}{\delta - 1}
$$

Table C.1.

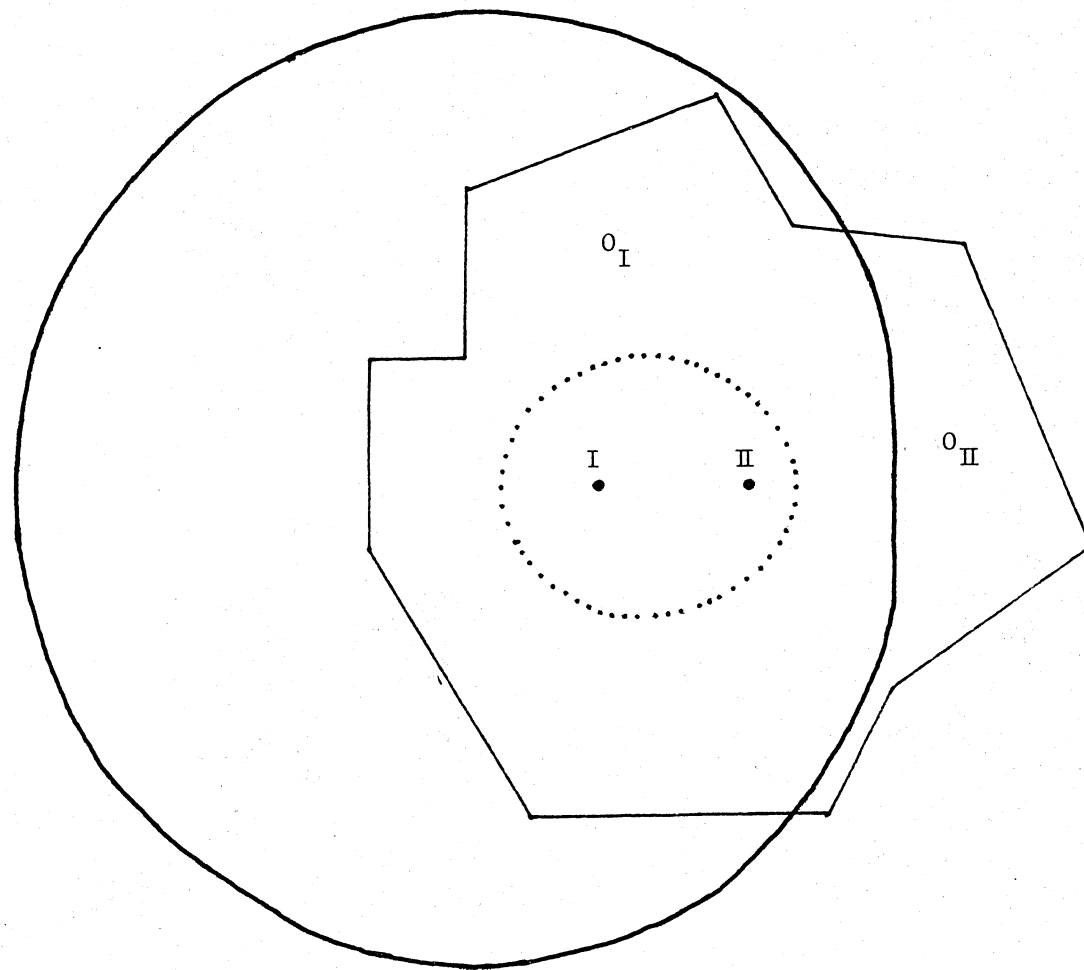Next we picture the iso-preference lines for seven different combinations of δ and ϵ, to visualize the various possibilities. The two shops are denoted by I and II and their attraction areas by O_I and O_{II} respectively (1 cm in the picture = 2 cm in reality). The dotted lines present the inadmissible combinations (x, y) derived from (C.1.1) and (C.1.2)
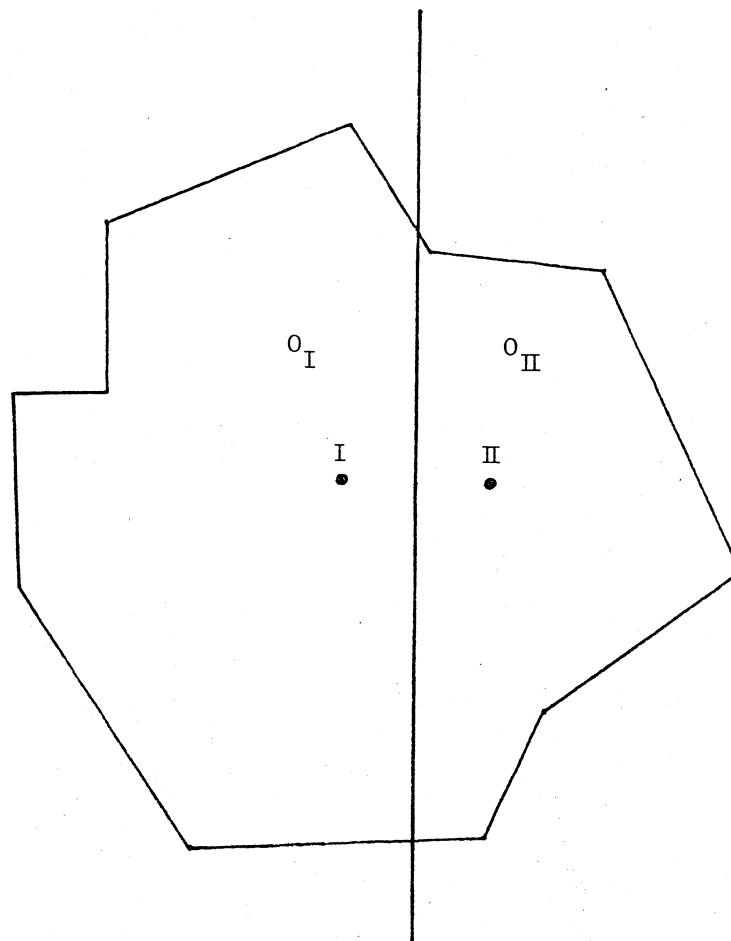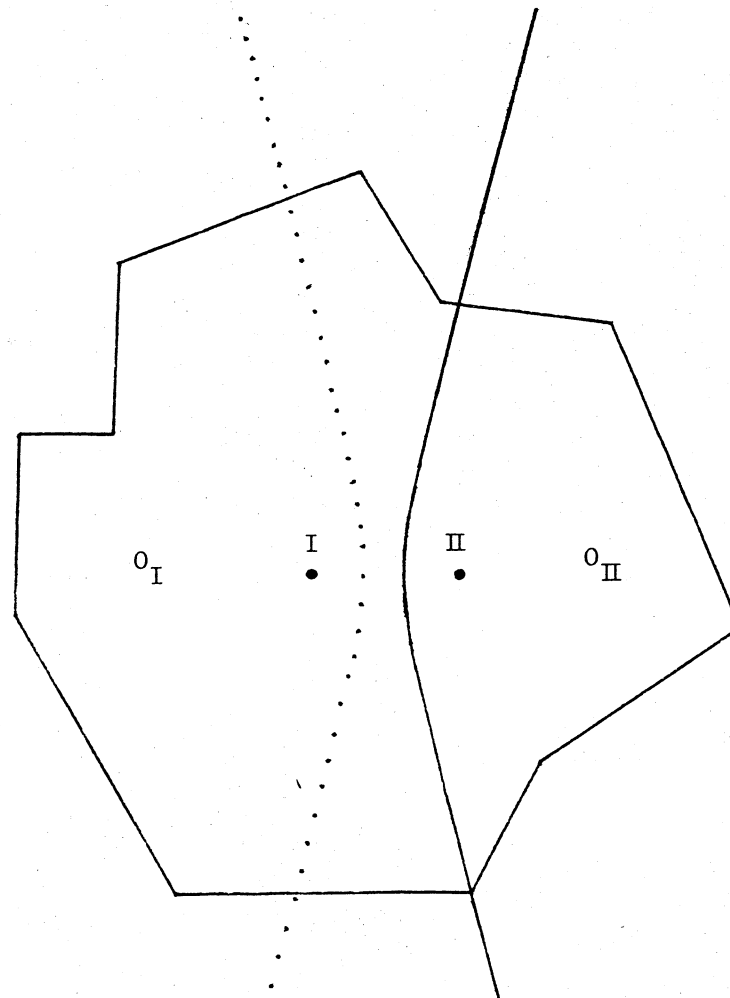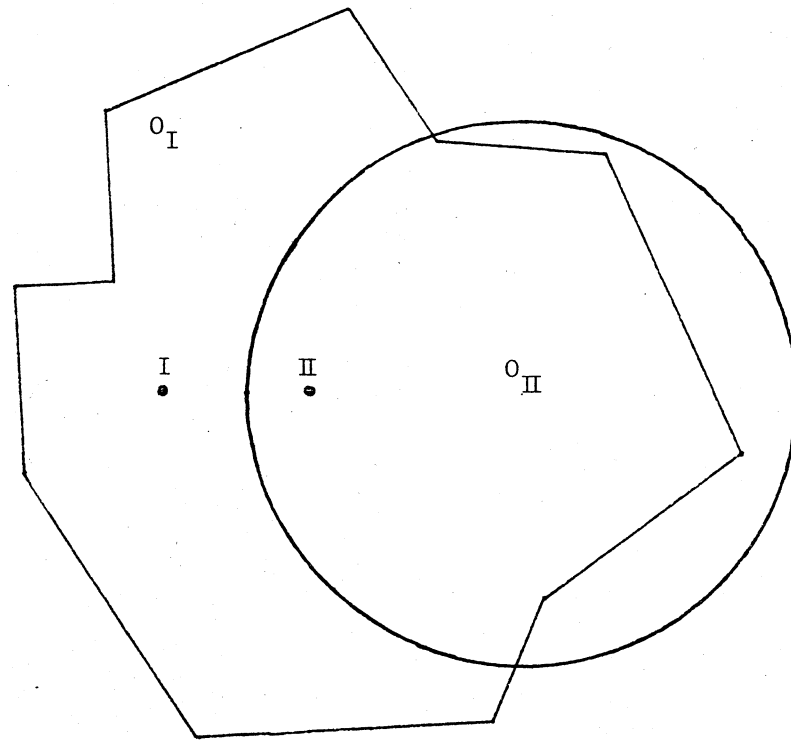
$\delta = .5 \qquad \varepsilon = 4$

$$\delta = .5 \qquad \varepsilon = 6$$
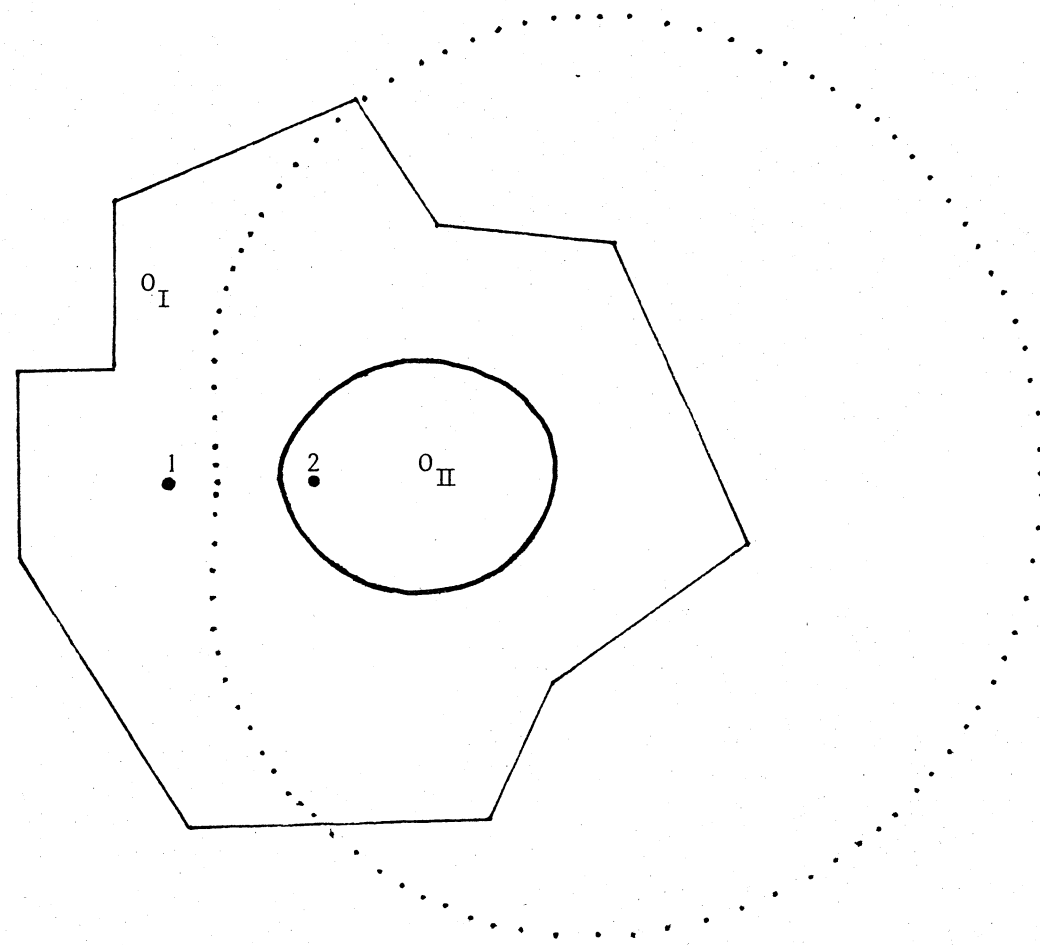
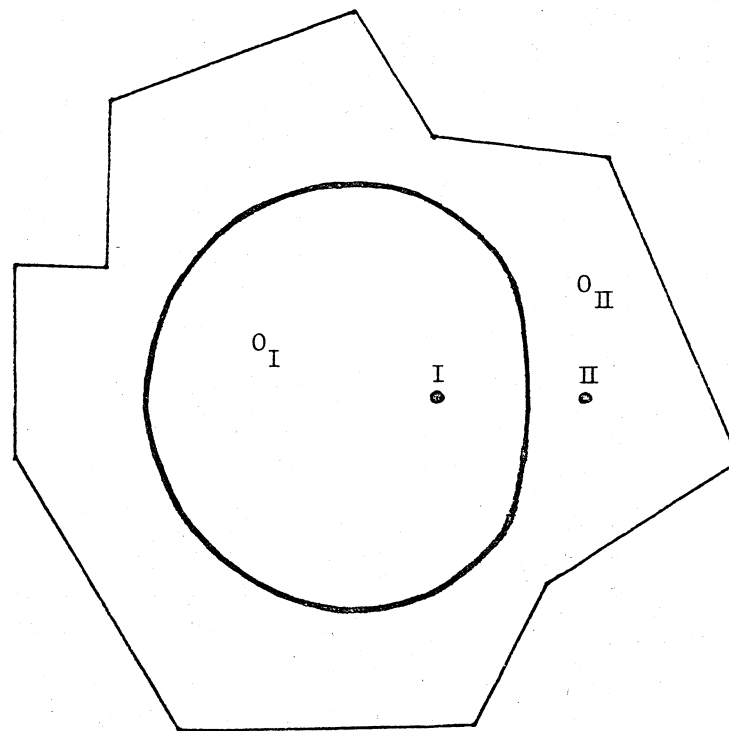$\delta = 1$ $\varepsilon = 0$

$$\delta = 1 \qquad \varepsilon = 1$$

$\delta = 1.3 \qquad \varepsilon = 0$

$$\delta = 1.3 \qquad \varepsilon = 2$$

$\delta = .5 \qquad \varepsilon = 2$

8. Conclusions and some recommendations for further research.

In the previous sections an attempt has been made to formulate a theory on spatial shopping behaviour of consumers in a certain geographical area.

The authors of this paper thought it necessary to search for a micro-economic theory to describe this phenomenon for the following reasons:

- Existing models on this topic, at least those which are memorized in sections 1 and 2, have in the authors' opinion very little theoretical foundation. This is apparent in case of the aggregate models, but also with respect to logit and probit models; although based on some utility concept, one may feel uncertain about the underlying assumptions and the ultimate mathematical form of the models.

- Because of the lack of a proper theoretical set-up of these models, specific micro-economic variables such as prices and assortment are not given the attention they deserve in view of the central position they have in the consumer's choice problem.

- The geographical impact on the distribution of shopping expenditures in a certain region, measured by some representation of distance is in the existing models undervalued, while the mathematical specification of distance in these models is at least questionable and while other important features as to the geographical structure of the region and the distribution of the population over the area are hardly incorporated.

- The existing models are therefore very limited and may not be adequate to answer realistic questions posed by decision makers acting in this field.

In this paper a micro-economic theory is presented in a simple form, which might be helpful in constructing econometric models describing spatial shopping behaviour, at last in offering a better theoretical justification for the mathematical formulation (or better approximation) of relevant economic and geographical variables. As an illustration of the usefulness of the theory we attacked the problem of analysing the rôle of distance in the consumer's choice problem by looking at the form of the theoretically derived iso-preference lines. An important conclusion of this exercise may be that it is not a simple problem to specify the effect of distance properly in an econometric model and that

more detailed research will be necessary on this topic. The approach followed here in this paper, might be useful in analysing the impact of prices and assortment on consumer's decisions, something that is challenging especially while such an analysis never has been carried out very deeply.

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