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SMOOTHING THE AGGREGATE FIX-PRICE MODEL AND THE USE OF BUSINESS SURVEY DATA

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Abstract

The aggregate two-market fix-price model involves discrete regime switches that are grossly unrealistic. In order to derive a smoothed version of the model I aggregate firms in disequilibrium, using the distribution of notional supplies and demands in the population of firms as a weight function. In the aggregate this yields a mix of regimes that varies continuously over the cycle. Each of the regimes is characterized by the operation of a particular constraint. This allows us to make use of business survey results reporting on the proportions of firms operating capacity constrained, demand constrained, labour constrained or unconstrained. The model is applied to the Dutch manufacturing sector.

Contents

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1. INTRODUCTION

Recently there have been several attempts to implement empirical versions of the aggregate two-market fix-price (quantity rationing) model due to Barro and Grossman (1971, 1976) and Malinvaud (1977); in particular Arthus, Laroque and Michel (1981) and Vilares (1981) for France, Kooiman and Kloek (1980, 1981) for the Netherlands, Sneessens (1981) for Belgium, and Vilares (1982) for Portugal. In these highly non-linear models the explanation of the level of economic activity switches endogenously between a small number of alternatives, each of which represents a more or less pure case of supply constrained or demand constrained operations of the economy under consideration. From the applied point of view this has to be contrasted with the usual linear model with constant parameters, that implicitly assumes a constant role for supply side factors and demand side factors over the cycle. Although the allowance for regime switches in macroeconomic models may contribute to their structural stability, cf. Muellbauer (1978), it has been stressed by several authors that discrete jumps between all-or-nothing explanations from the supply side or the demand side may overshoot the case. It is rather unlikely that regime switches occur simultaneously for all sectors in the economy. In actual aggregate economies supply constrained and demand constrained operations will always coincide, albeit in varying proportions. This clearly calls for a smoothing approach which replaces the discrete switches of the aggregate fix-price model by a smoothly and gradually varying mix of supply side factors and demand side factors in explaining aggregate levels of transactions.

The aggregation-by-integration technique employed by Houthakker (1955/56), Johansen (1972), Sato (1975), Hildenbrand (1981) and others in deriving aggregate production functions seems to be a natural procedure for smoothing purposes. Although Ando (1971) and Batchelor (1977) contain similar approaches, Muellbauer (1978) was the first to show its usefulness in the context of models for markets in disequilibrium. Starting from a continuum of micro-markets in disequilibrium he obtains aggregate transactions as the mathematical expectation of the minimum of supply and demand, given the size distribution of supply and demand in the population of micromarkets. As a result aggregate transactions are obtained as a smooth function of aggregate supply and aggregate demand. The contributions of the supply side and the demand side vary with the proportion of markets in excess demand. Both Kooiman and Kloek (1979) and Malinvaud (1982a) have elaborated on Muellbauer's approach, and the present study essentially contains a further elaboration to the extent that the goods market and the labour market are now simultaneously dealt with.

The present approach deviates from the earlier ones in that the aggregation-by-integration technique is applied to firms in disequilibrium instead of markets in disequilibrium. This has the advantage that one can proceed from a single population of micro level entities from which both aggregate output and employment are obtained.[1] Spillovers naturally arise when considering the interaction effects between the output and employment decisions of individual firms. Aggregating over firms instead of markets we also easily obtain expressions for the proportions of firms whose operations are constrained by demand for goods, by supply of labour, or by the available amount of productive capacity. These proportions, defining the mix of regimes prevailing in the economy under consideration, are in principle observed from business surveys in all EEC countries. The present model allows us to exploit these business survey results in a natural way. The possibility to incorporate this type of unusual information in short run macroeconometric modelling constitutes a most promising aspect of the smoothing-by-aggregation approach to fix-price modelling, cf. Malinvaud (1982b).

The remainder of the paper consists of two parts. First, in Section 2, the model is presented and its main features are discussed. Second, in Section 3, the part of the model that relates to the business survey data is applied to the Dutch manufacturing sector.

2. THE MODEL

In this section I derive a smooth version of the aggregate two-market fix-price model, aggregating over firms in disequilibrium. There are three

1. Muellbauer (1978) discusses the treatment of goods and labour markets in section 6. As the populations of both types of micromarkets do not coincide he is forced to aggregate them separately. Incorporating spillovers between the goods and labour markets he has to assume "that firms on each labour market sell goods to a representative sample of goods markets, and that government and workers on each labour market buy a representative sample of goods" (p.26). This unrealistic orthogonality assumption is avoided once we shift attention from markets to firms in disequilibrium. There is a loss as well, in that we can no longer deal with consumers in a symmetrical way when concentrating upon firms instead of markets.

subsections. The first presents the model of the firm and discusses the aggregation procedure. In the second subsection I discuss the shape of the firm's effective goods supply and labour demand functions in view of the available business survey results. The final subsection contains a general discussion of the resulting model.

2.1. Aggregating over firms in disequilibrium

Our economy consists of a large number of production plants, each of them operating a single production technique. I shall use the term 'firm' to indicate these plants, although actual firms are often complicated conglomerates of such elementary production activities. Each firm combines equipment, labour, and probably material inputs to produce 'output'. During the period under consideration physical capital stock is fixed for each firm. There are no substitution possibilities as these will usually involve capital adjustments. Thus notional supply of goods y^s corresponds to the full capacity level of output and notional demand for labour ℓ^d to the associated level of full capacity labour input. At the beginning of each period the firm announces the price of its product and the wage it is prepared to pay. Presumably these will be based upon the perceived goods demand and labour supply schedules of the firm involved. We shall not analyse the price and wage decisions of our firms. During the period they will stick to the prices and wages they have announced. Thus the burden of adjustment falls entirely on quantities. This aspect of the model is of course a central feature of fix-price analysis.

The firm is now confronted with an (effective) demand y^d for its output and an (effective) supply of labour ℓ^s. Given these short run supplies and demands, and its available capacity, the firm then decides on its level of output y and on the amount of labour ℓ it will employ. When material inputs are available in sufficient amounts[2] the level of operations of each firm is determined by its capacity, by demand for its output, or by its supply of labour. Iwai (1974) shows in a more formal setting that profit maximization by the firm leads to the following minimum conditions for the levels of

2.The model derived in this paper is not well suited to describe situations where goods markets are persistently tight. Excess demands may spread out through the economy in that case due to a lack of material inputs. This could occur during wartime, and often characterizes the situation in developing countries and socialist economies. A balance of payments constraint rules out compensation by additional imports in the latter case. A lack of material inputs is a rather exceptional phenomenon in Western industrialized economies, that can safely be neglected.

transactions y and ℓ:

(1) $$y = \min\{y^s, y^d, y(\ell^s)\}$$

(2) $$\ell = \min\{\ell^d, \ell^s, \ell(y^d)\}$$

where $y(\ell^s)$ and $\ell(y^d)$ are the firm's effective output supply and labour demand functions respectively. We shall assume that these functions satisfy the following regularity conditions:

(3) $$\begin{array}{lcl} y(\ell^d) = y^s & ; & \ell(y^s) = \ell^d \\ y'(.) \geq 0 & ; & \ell'(.) \geq 0 \\ \multicolumn{3}{c}{y(\lambda) \geq \ell^{-1}(\lambda) \quad \forall\lambda \leq \ell^d} \end{array}$$

where the prime denotes first derivatives and λ is a dummy argument. These conditions imply figure 1, which shows the four possible regimes that result. The cc- (capacity constrained), dc- (demand constrained), lc- (labour constrained) and dlc- (demand and labour constrained) regimes correspond to the four regimes obtained in the aggregate two-market fix-price model with inventories, cf. Malinvaud (1977) or Muellbauer and Portes (1978).

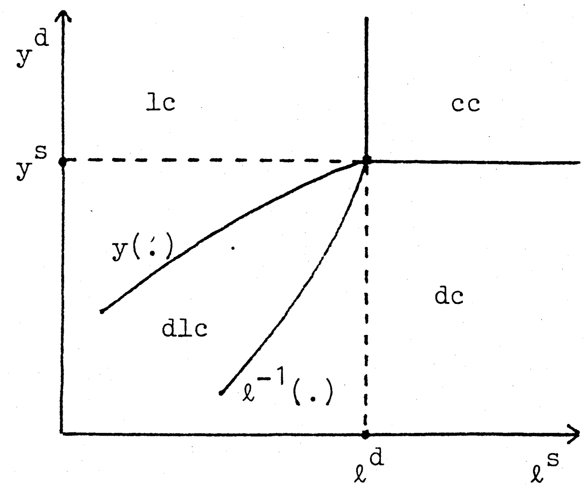

fig. 1: Available regimes for the firm

Table 1 gives the regime definitions and the levels of transactions y and ℓ associated with each of the regimes as they are obtained from the minimum conditons (1) and (2). Obviously both the levels of transactions and the

Table 1

Regime definitions and levels of transactions

Regime	goods market	labour market
cc	$y = y^s \leq y^d$	$\ell = \ell^d \leq \ell^s$
dc	$y = y^d < y^s$	$\ell = \ell(y^d) \leq \ell^s$
lc	$y = y(\ell^s) \leq y^d$	$\ell = \ell^s < \ell^d$
dlc	$y = y^d < y(\ell^s)$	$\ell = \ell^s < \ell(y^d)$

ruling regime are unique determined as functions of y^s, ℓ^d, y^d and ℓ^s. As soon as we know the distribution of these four supply and demand entities in the population of firms we can change variables according to these functions in order to derive the implied distribution of transactions and regimes. This in turn can be used to obtain aggregates by integration.

More precisely, let H(.) represent the (four dimensional) size distribution of y^s, ℓ^d, y^d and ℓ^s, i.e. H(a, b, c, d) gives the number of firms in the economy with $y^s \leq a \wedge \ell^d \leq b \wedge y^d \leq c \wedge \ell^s \leq d$, for all positive a, b, c and d. Aggregate supplies and demands are then obtained by summation or, for that matter, Lebesgue integration over firms:

(4) $$Y^s = \int y^s dH \; ; \; L^d = \int \ell^d dH \; ; \; Y^d = \int y^d dH \; ; \; L^s = \int \ell^s dH$$

where capital Y and L denote aggregates. Similarly aggregate output and employment follow from figure 1 as:

(5) $$Y = \int_{cc} y^s dH + \int_{dc+dlc} y^d dH + \int_{lc} y(\ell^s) dH$$

(6) $$L = \int_{cc} \ell^d dH + \int_{lc+dlc} \ell^s dH + \int_{dc} \ell(y^d) dH$$

Weighting[3] firm's degree of capacity utilization y/y^s by capacity shares y^s/Y^s

3. Alternatively one might consider weighting by output shares y/Y. This is less attractive as it invalidates expression (7). In preparing the aggregate business survey results the Dutch Central Bureau of Statistics employs a kind of value added shares that are updated now and then. As the different weights are highly correlated the differences will be minor in the aggregate.

the average degree of capacity utilization Q is directly obtained as:

(7) $$Q = Y/Y^s$$

whereas the correspondingly weighted proportions of firms[4] whose operations are capacity constrained, demand constrained or labour constrained, obtain as:

(8) $$P_i = 1/Y^s \int_i y^s dH \qquad i \in \{cc,\ dc+dlc,\ lc+dlc\}$$

In actual economies H(.) will obviously be discrete, with $\int dH = J$, J being the number of firms in the economy. To ease further interpretation we change variables according to

$$\tilde{y}^s := Jy^s \ ; \ \tilde{\ell}^d := J\ell^d \ ; \ \tilde{y}^d := Jy^d \ ; \ \tilde{\ell}^s := J\ell^s$$

and subsequently divide by J in order to arrive at the normalized distribution function:

$$G(\tilde{y}^s, \tilde{\ell}^d, \tilde{y}^d, \tilde{\ell}^s) := J^{-1}H(\tilde{y}^s/J, \tilde{\ell}^d/J, \tilde{y}^d/J, \tilde{\ell}^s/J)$$

In terms of G(.) equations (4) through (6) and (8) read as:

(4') $$Y^s = \int \tilde{y}^s dG \ ; \ L^d = \int \tilde{\ell}^d dG \ ; \ Y^d = \int \tilde{y}^d dG \ ; \ L^s = \int \tilde{\ell}^s dG$$

(5') $$Y = \int_{cc} \tilde{y}^s dG + \int_{dc+dlc} \tilde{y}^d dG + \int_{lc} \tilde{y}(\tilde{\ell}^s) dG$$

(6') $$L = \int_{cc} \tilde{\ell}^d dG + \int_{lc+dlc} \tilde{\ell}^s dG + \int_{dc} \tilde{\ell}(\tilde{y}^d) dG$$

(8') $$P_i = 1/Y^s \int_i \tilde{y}^s dG \qquad i \in \{cc,\ dc+dlc,\ lc+dlc\}$$

where the effective goods supply and labour demand functions have been correspondingly redefined as:

$$\tilde{y}(\tilde{\ell}^s) := Jy(\tilde{\ell}^s/J) \ ; \ \tilde{\ell}(\tilde{y}^d) := J\ell(\tilde{y}^d/J)$$

4. To be precise, P_i is the fraction of total productive capacity that is located at firms operating under regime i, cf. equation (8).

Equations (5'), (6'), (7) and (8') express the endogenous variables Y, L, Q and P_i as functions of the moments of the transformed size distribution G(.). Equation (4') shows that its means are equal to the aggregate supplies and demands Y^s, L^d, Y^d and L^s. Consequently the model can be summarized as:

$$
(9) \qquad \begin{aligned}
Y &= Y(Y^s, L^d, Y^d, L^s; \theta) \\
L &= L(Y^s, L^d, Y^d, L^s; \theta) \\
Q &= Y/Y^s \\
P_i &= P_i(Y^s, L^d, Y^d, L^s; \theta) \qquad i \in \{cc, dc+dlc, lc+dlc\}
\end{aligned}
$$

where the vector θ consists of the parameters of G(.) relating to its second and higher order moments. The first two equations of (9) define aggregate output and employment as smooth functions of aggregate supplies and demands, provided we approximate the (discrete) empirical distribution G(.) by some convenient continuous distribution function. The stability of this function crucially depends upon the stability of the underlying micro level distribution about the means, i.e., the stability of the parametervector θ.

2.2. Business survey data and excess capacity

One of the main objectives of the present study is to investigate the possibility to exploit in a systematic way certain business survey results in macroeconomic model building. Since the beginning of the seventies business surveys in the EEC countries have been harmonized. A representative sample of manufacturing firms is asked whether their operations have been constrained and, if so, whether it was due to a lack of demand, a lack of labour, a lack of capacity, or other causes.[5] Moreover they are asked to give an estimate of their average degree of capacity utilization during the past quarter. In figure 2 the answers are displayed for the Netherlands[6]; they are listed in Appendix A. The figure clearly shows the gradual, though sometimes quite

5. The present formulation reflects the phrasing of the Dutch business survey. It is different in some other countries. In France and Belgium, for instance, firms are first asked whether they would have been able to meet a larger demand than the one they actually experienced. If not so, they are subsequently asked what prevented them from producing more.
6. From the end of 1971 till the end of 1978 there have been three surveys a year, in January, May and October. From 1979 on the surveys are on a quarterly basis. For France, Germany, Italy and Belgium these series go back to 1963 or earlier. For the UK they are only available since 1977.

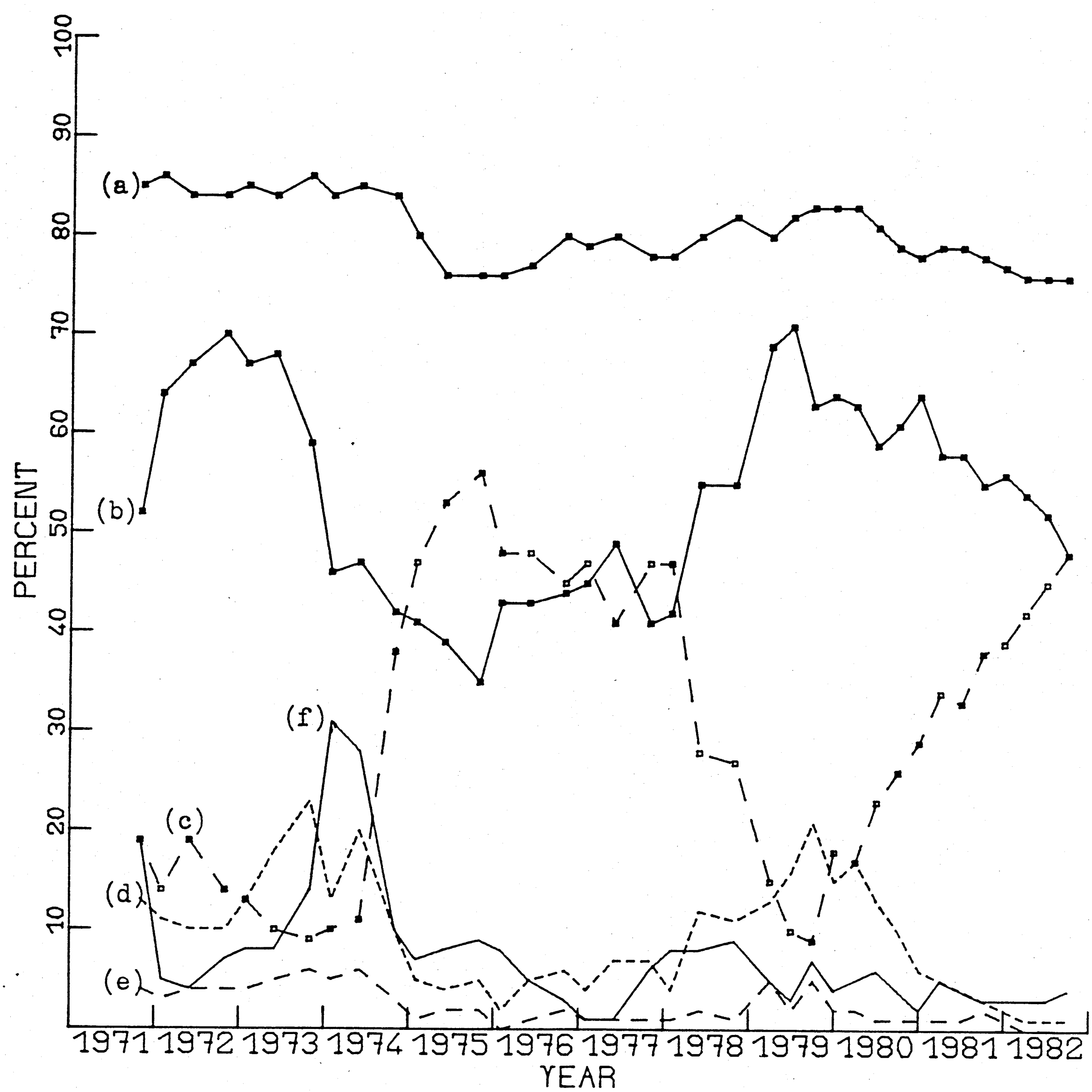

Figure 2. Business survey results for Dutch manufacturing 1972-1982 (excluding food processing, drinks, tobacco and oil refineries)

(a) degree of capacity utilization

(b) no constraints

(c) demand constrained

(d) labour constrained

(e) capacity constrained

(f) other constraints

considerable changes in the proportions of the constraints mentioned. According to our model, cf. equations (9), this variation originates in shifts in the balance of aggregate supply and demand, both on the goods and the labour markets. As such the business survey data displayed may help to inform us about the likely development over time of these, otherwise unobserved, aggregate supplies and demands, just as the aggregate levels of transactions Y and L do.

This however requires a redefinition of the regimes of our theoretical model as these do not correspond one-to-one to the categories of the business survey responses. The most important discrepancy concerns the lack of a sizeable unconstrained regime in our theoretical model: with a continuous distribution function G(.) the probability of being unconstrained, i.e., $y^d = y^s \wedge \ell^s = \ell^d$, is zero. Moreover being unconstrained implies producing at full capacity levels: $y = y^s \wedge \ell = \ell^d$. Quite to the contrary the survey results show that a very considerable number of firms report meeting no constraints.[7] The average degree of capacity utilization is never more than 86%, even with unconstrained production as large as 70%. This clearly implies that firms report being unconstrained even if capacity utilization stays well below 100%. This is also reflected in the small number of firms reporting to be capacity constrained: about 5% or less during the seventies. It substantiates the view that (considerable) excess capacity is the rule rather than the exception in manufacturing industries of Western economies. We are thus forced to modify the simple picture of the firm introduced in the preceeding subsection. In particular it implies that one way or the other a region will have to be defined below the full capacity point (y^s, ℓ^d) in figure 1, where firms report to operate unconstrained. As before the cc-region can then be reserved for those firms that do not succeed in meeting demand because they have used up all their available productive capacity, and consequently report to be capacity constrained.

An unconstrained regime can be created in a simple way by choosing a specific shape for the effective goods demand and labour supply functions y(.) and ℓ(.) in figure 1. These functions reflect the firm's employment policy in view of the uncertainty about future levels of demand and/or about the amount

7. This is not the case for those countries where the surveys do not include the possibility to indicate this category explicitly. Firms can only indicate to be unconstrained then by not filling in any of the constraints mentioned. This only incidentally occurs. It shows the crucial importance of the phrasing of the questions.

and quality of labour that will be available. An optimal investment policy of the firm may then involve the creation of some excess capacity as compared to mean expected demand, in order to avoid the costs associated with insufficient capacity, i.e., foregone profits. Similarly an optimal employment policy of the firm will as a rule involve some labour hoarding to occur due to adjustment costs. It is much too ambitious in the present context to try and derive the shape of the effective goods supply and labour demand functions from a formal treatment of these phenomena. Explicitly aggregating over firms, as we do, one is bound to end up with completely intractable results as soon as one introduces a more sophisticated treatment of the micro level.[8] So, in order to keep things simple, let us assume that an interval $[\tau, 1]$, $0 < \tau < 1$, exists of degrees of capacity utilization that the firm considers as 'normal'. We shall assume that the firm reports to operate unconstrained as long as demand falls within this region and, moreover, the available amount of labour supply is sufficient to meet such a level of demand. Also the firm will stick to its full capacity level of labour demand ℓ^d as long as it succeeds to operate at 'normal' levels of capacity utilization. In this situation labour hoarding will be maximal: the output elasticity of labour demand is zero. Only when demand falls below normal levels workers will be laid off or vacancies will not be refilled.

Similarly, let the interval $[\mu, 1]$, $0 < \mu < 1$, define 'normal' degrees of labour availability. We reinterprete the notional demand for labour ℓ^d as a kind of target demand for labour. We shall assume that the firm sets this target at such a level that it will be able to produce its full capacity output y^s as long as $\ell^s \geq \mu\ell^d$, i.e., when labour availability is within the normal range. Only when labour supply falls short of that range production will have to be cut back. It would not be wise for the firm to restrict its target demand for labour to the minimal level $\mu\ell^d$. Usually there will be vacancies due to normal turnover, and employees may fall ill. Stress would ultimately lead to a decrease in labour productivity, and output would fall below y^s.

The introduction of 'normal' ranges of capacity utilization and labour

8. The alternative, and more common, way to simplify matters is in terms of the distributional assumptions. The most drastic, and most frequently made, assumption is that all firms in the economy are identical or 'representative'. It is precisely this conventional approach that the present model seeks to avoid. Of course, this can only be achieved at a certain cost manifesting itself at some other place in the model.

availability implies a kinked form for the effective goods supply and labour demand functions y(.) and ℓ(.), as depicted in Figure 3, which replaces Figure 1. As such the approach chosen neatly fits in with the overall discrete

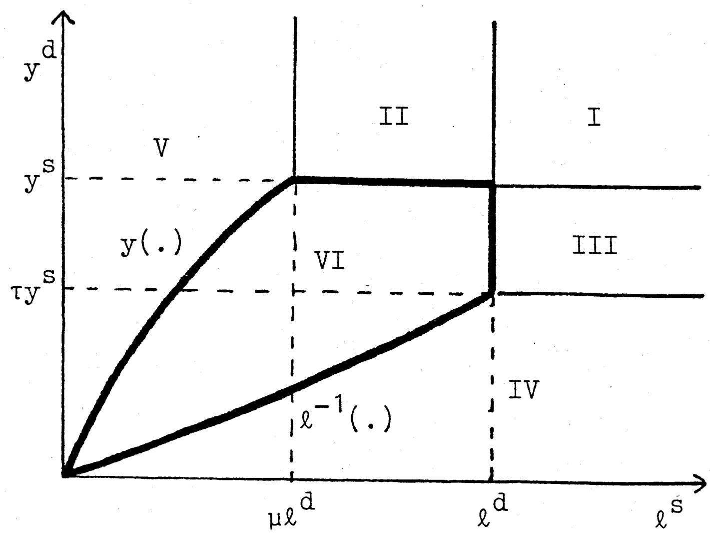

Figure 3. Kinked effective goods supply and labour demand

character of our micro level picture of the firm, i.e., the minimum conditions (1) and (2). In figure 3 six regions can be distinguished, that are labeled I through VI. Applying minimum conditions (1) and (2) we easily obtain the following aggregate output and employment equations, replacing equations (5') and (6'):

$$(5'') \qquad Y = \int_{I+II} \tilde{y}^s dG + \int_{III+IV+VI} \tilde{y}^d dG + \int_{V} \tilde{y}(\tilde{\ell}^s) dG$$

$$(6'') \qquad L = \int_{I+III} \tilde{\ell}^d dG + \int_{II+V+VI} \tilde{\ell}^s dG + \int_{IV} \tilde{\ell}(\tilde{y}^d) dG$$

Surprisingly, the number of integrals[9] in these expressions does not increase, despite the increase in the number of regions when passing from Figure 1 to Figure 3.

When it comes to the regime classification according to the business survey categories we have to reconsider the dlc-region of Figure 1, which corresponds to region VI in Figure 3. According to our model firms are

9. When Figure 3 is employed in the context of a discrete switching model the joint density function of y and ℓ involves six integrals, each of them corresponding to one of the regions in the figure.

simultaneously demand and labour constrained in this region. Although the business surveys explicitly allow for the indication of more than one constraint at the same time, firms only incidentally do so. Consequently the available data reject the existence of a sizeable regime where both constraints coincide. In choosing what constraint the firm actually reports I shall assume that it is primarily output oriented. As a rule a firm will have good knowledge of the existing demand for its products, whereas its potential supply of labour is only observed as soon as it effectively engages in search activities in the labour market. Consequently the firm will only report to be labour constrained when the amount of labour it has been able to contract was insufficient to sustain a level of output that would otherwise have been feasible. Together with the remarks made earlier when introducing the unconstrained regime this implies regime definitions according to Figure 4, where

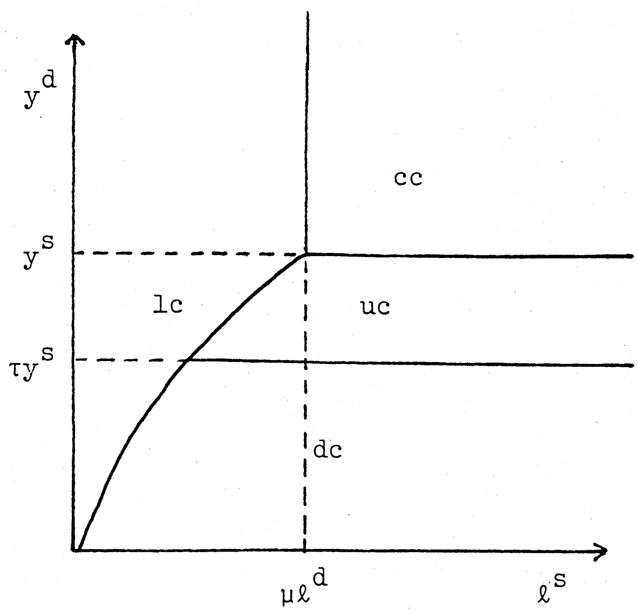

Figure 4. Modified regime definitions

'uc' denotes the <u>unc</u>onstrained regime. Equation (8') for the proportions of firms in each of the regimes correspondingly changes into:

(8'') $$P_i = 1/Y^s \int_i y^s dG \qquad i \in \{cc, dc, lc, uc\}$$

I have neglected the group of <u>other constraints</u> so far. One may guess that these have mainly to do with shortages in material inputs in production and with occasional breakdowns of machinery. Less than 10% reports to be in this group, with the exception of the period of the first oil crisis when figures up to 30% occur. One may also guess that this group will frequently be involved when more than one constraint at the same time is mentioned. As the group of other constraints cannot properly be dealt with in the theoretical model, the best thing to do seems to be to neglect it altogether, and to rescale the other groups in order to have them add to unity. Clearly the four proportions P_i in equation (8'') add to unity as well since they derive from a partitioning of the sample space associated with G(.).

Obviously the general representation (9) of the model still applies, albeit that the equations for output, employment and for the proportions P_i now represent (5''), (6'') and (8''). We also have to expand the vector θ with two additional parameters τ and μ, locating the kinks in the effective goods supply and labour demand functions of our firms. In the next subsection we shall discuss some general features of the model.

2.3. Means and dispersion about means

The model that we have derived in this section is still incomplete: in fact it only provides us with a framework for an economic model. What essentially it does is to relate unobserved theoretical entities, i.e., aggregate supplies and demands, to observable economic variables: aggregate levels of transactions, the average degree of capacity utilization and the prevailing mix of regimes. In the terminology of state-space models the equations of our model constitute a non-linear set of measurement equations for the (state) variables Y^s, L^d, Y^d and L^s. In order to obtain a proper economic model these equations have to be supplemented with a set of equations describing the behaviour over time of these state variables. In other words, we need a set of aggregate supply and demand equations in order to complete the model. In the context of our model the aggregate supplies and demands figure as means of the underlying micro level distribution G(.). They represent "common factors that influence all sectors (i.e., our firms, p.k.) simultaneously, although in different fashions" [Malinvaud (1982a), p.243]. It is in specifying these additional supply and demand equations that the main body of economic theory can come into play. There does not seem to be any difference in this respect with traditional econometric model building.

The underlying micro level distribution is not only present through its means Y^s, L^d, Y^d and L^s, but also through the parameter vector θ relating to all its higher order moments. The explicit appearance of dispersions about the means creates some interesting possibilities for modelling, that are absent from traditional approaches concentrating on means only. One could for instance think of the increase in frictions in the labour market that occurred in the Western European economies during the late sixties and seventies, as was evidenced by the outward shift of the UV-curves. In the present model this can easily be incorporated as a decrease in the correlation between the supply and demand for labour. The example makes clear that it is certainly not always warranted to assume stochastic stability of the Tobin (1972) type, i.e., a constant parameter vector θ. Ideally one would like to endogenize such changes in the micro level dispersion by making θ dependent upon a suitably chosen set of explanatory variables, compare Malinvaud (1982a), p.243. The task to specify such additional equations for the elements of θ is far from simple, however, as economic theory does not provide much guidance as to what causes them to move. Restricting oneself to second order effects only, as one is likely to do in empirical work, the co-variances of G(.) will pick up most of the consequences of misspecification of the micro level. It was stressed before that one has to proceed from a highly stylized picture of the micro level in order to end up with tractable results in the aggregate. Consequently one has to neglect several important phenomena that prevail in actual production. We have neglected the constraints in production that may be due to insufficient material inputs. We have not incorporated the possibility of multiple labour inputs or the joint production of several commodities. We have not dealt with spillovers as between firms, that may occur within the period when unsatisfied consumers try another firm that provides a comparable opportunity to work, or produces a comparable commodity to buy. Similarly concern-management may shift workers from one plant to the other aiming at a reduction of the internal dispersion of excess supplies with respect to its various production activities. Even at the micro level imperfections may exist due to incomplete information, invalidating the minimum conditions (1) and (2). Jointly these phenomena contribute to determine the overall efficiency of the matching process of supply and demand on the goods and labour markets. The co-variances of our model represent the net effect of quite a number of such factors, and will thus be difficult to model in a satisfactory way. It may also be difficult to identify these second order effects from aggregate data alone. The application of the model that we discuss in the next section suggests that this may be a serious problem indeed.

Additional economic content can finally enter the model via the parameters μ and τ deriving from the approximation to the firm's employment policy under uncertainty that we have introduced in the preceeding subsection. These parameters are likely to depend upon several factors that are not constant over time, in particular expectations about future demand and probably also about prices and financial resources. In practical applications one would like to incorporate these effects by specifying appropriate equations for μ and τ.

To conclude our general remarks on the model that we have derived let us compare the output and employment equations (5') and (6') obtained from the aggregation approach to those resulting from the following discrete switching model:

$$\tilde{y} = \min[\tilde{y}^s, \tilde{y}^d, \tilde{y}(\tilde{\ell}^s)]$$

$$\tilde{\ell} = \min[\tilde{\ell}^d, \tilde{\ell}^s, \tilde{\ell}(\tilde{y}^d)]$$

$$\tilde{y}^s = Y^s + \varepsilon^s \ ; \ \tilde{\ell}^d = L^d + \eta^d$$

$$\tilde{y}^d = Y^d + \varepsilon^d \ ; \ \tilde{\ell}^s = L^s + \eta^s$$

When the error terms ε^s, η^d, ε^d, η^s have zero means and a joint distribution equal to G(.), except for those means, then $\tilde{y}^s$, $\tilde{\ell}^d$, $\tilde{y}^d$ and $\tilde{\ell}^s$ have G(.) as their joint distribution. As figure 1 applies to this model as well we obtain the <u>expectations</u> of $\tilde{y}$ and $\tilde{\ell}$ as the right-hand sides of equations (5') and (6'). So $Y = E\tilde{y}$ and $L = E\tilde{\ell}$, a property of the model that was already noticed by Muellbauer (1978). When it comes to matters of estimation it implies that the two models are observationally equivalent as far as the first moments of the output and employment equations are concerned. This is a quite surprising result in view of our purpose to develop a more realistic alternative to the aggregate switching model. In a sense we are back to where we started.

However, things may not be that serious. The aggregate switching model has no natural counterparts for our equations for the proportions P_i. In that model these are either unity or zero. Moreover, the identification problem can easily be solved. As always this has to be achieved by the introduction of prior information. Here this prior information is at the same time trivial and unusual. The facts are simply these: aggregate regime switches do not occur in the actual economies we intend to deal with.

3. AN APPLICATION

In this section I shall apply the model developed in the preceeding section to the Dutch business survey data displayed in figure 2. For reasons to be indicated below I shall skip the output and employment equations and estimate the equations relating to the business survey data separately. It will be shown that, under the assumptions that we make, the equations for Q and the proportions P_i only involve ratios of the aggregate supplies and demands Y^s, L^d, Y^d and L^s, in particular the relative aggregate excess demand for goods $\log(Y^d/Y^s)$ and the excess supply of labour $\log(L^s/L^d)$. Instead of substituting aggregate excess demand equations for these entities I treat them as free parameters that are estimated for each observation. In doing so the business survey results are translated into time series for the average pressure of demand on the goods and labour markets, which may be useful for other applied work.

3.1. The lognormal case

In working out our model equations we have to specify the distribution function G(.) and the effective goods supply and labour demand functions $y(\ell^s)$ and $\ell(y^d)$, in particular the part to the left of the kinks in figure 3. With respect to the former I shall assume that it is lognormal with means equal to the aggregate supplies and demands, as in equation (4'), whereas I take the latter to be linear through the origin. Given these assumptions equations (7), (5'') and (8'') can be worked out to yield:

$$P_{cc} = \int_0^\infty \int_0^\infty n(A,\ B)dAdB$$

$$P_{dc} = \int_{-\infty}^{\log\tau} \int_A^\infty n(A,\ B)dBdA$$

$$(10)\qquad P_{lc} = \int_{-\infty}^{0} \int_B^\infty n(A,\ B)dAdB$$

$$P_{uc} = \int_{\log\tau}^{0} \int_A^\infty n(A,\ B)dBdA$$

$$Q = \int_0^{\infty}\int_0^{\infty} n(A, B)dAdB + \int_{-\infty}^{\infty}\int_A^{\infty} e^A n(A, B)dBdA + \int_{-\infty}^{\infty}\int_B^{\infty} e^B n(A, B)dAdB$$

Here A and B represent micro level relative excess supplies and demands

$$A := \log \tilde{y}^d - \log \tilde{y}^s$$

$$B := \log \tilde{\ell}^s - \log \tilde{\ell}^d - \log \mu$$

whereas n(A, B) is a normal density function[10] with co-variances σ_A^2, σ_B^2, $\rho_{AB}\sigma_A\sigma_B$ and mean vector
$(\mu_A \vdots \mu_B) := \left(\log(Y^d/Y^s) - \frac{1}{2}\sigma_A^2 \vdots \log(L^s/L^d) - \frac{1}{2}\sigma_B^2 + \text{const.}\right)$.
Appendix B presents the main steps to be taken in deriving (10). In addition to (10) the equation for Y clearly obtains as Y^s times the equation for Q. A similar expression can be derived for L with L^d replacing Y^s and only slightly different integrals. The expressions obtained for Q and the proportions P_i are zero homogeneous in the pairs (Y^s, Y^d) and (L^d, L^s), whereas Y is linearly homogeneous in (Y^s, Y^d).[11] It implies that business survey data on Q and P_i only contain information about the relative aggregate excess demand for goods and excess supply of labour, not on the levels of the aggregate supplies and demands themselves, that can only be identified from the levels of aggregate transactions Y and L. This allows us to take the following short-cut. Suppose we do not substitute aggregate supply and demand equations for Y^s, L^d, Y^d and L^s, but instead treat them as free parameters that have to be estimated for each observation separately. Then we can always adjust the levels of Y^s, L^d, Y^d and L^s such that the observations on Y and L are exactly reproduced, for any given value of the relative excess demands $\log(Y^d/Y^s)$ and $\log(L^s/L^d)$. Consequently there is zero information in the time series on Y and L with respect to these excess demands, and we can skip the output and employment equations. We are then left with the problem to estimate the time series for the relative aggregate excess demands from the information in the business survey data alone. Following this approach we estimate model (10) with time

10. It is not the normal density function of A and B because μ_B is not the marginal expectation of B. The additional constant involves log μ and the covariance of B and log $\bar{y}^d$.
11. These homogeneity properties depend upon the linear homogeneity of the $y(\ell^s)$ and $\ell(y^d)$ functions. Minimum conditions (1) and (2) are linearly homogeneous as well then. This property is preserved under the aggregation procedure.

dependent means $\mu_A(t)$ and $\mu_B(t)$, i.e.,

$$\begin{aligned} &P_i(t) = P_i(\mu_A(t), \mu_B(t); \sigma_A, \sigma_B, \rho_{AB}, \tau) + \varepsilon_i(t) \quad i \in \{cc, dc, lc, uc\} \\ (11) \quad & \\ &Q(t) = Q(\mu_A(t), \mu_B(t); \sigma_A, \sigma_B, \rho_{AB}, \tau) + \varepsilon_q(t) \end{aligned}$$

where time indices and error terms have been added. The means $\mu_A(t)$ and $\mu_B(t)$ represent $\log(Y^d(t)/Y^s(t))$ and $\log(L^s(t)/L^d(t))$ except for an additive constant. As a first approximation I shall assume that σ_A, σ_B, ρ_{AB} and τ are constant over time.

3.2. Estimation

Model (11) was estimated by nonlinear least squares.[12] Computational details, both with respect to the numerical evaluation of the two-dimensional integrals and the numerical optimization procedure employed, are listed in appendix C. The choice of the optimization criterion can be rationalized by the assumption that the errors are identically and independently normally distributed with zero means, except for the complication that is due to the adding-up restriction on the proportions P_i. As with complete systems of demand equations it implies singularity of the error covariance matrix. Taking the errors involved in the singularity to be equicorrelated it can easily be shown, cf. Deaton (1974), that the resulting log-likelihood function is

$$L \propto -2\log \sum_{t=1}^{T} \{\varepsilon_{cc}(t)^2 + \varepsilon_{dc}(t)^2 + \varepsilon_{lc}(t)^2 + \varepsilon_{uc}(t)^2 + \varepsilon_q(t)^2\}$$

which shows that maximizing the likelihood function amounts to minimizing the joint sum of squared residuals. Obviously the estimates obtained for the means $\mu_A(t)$ and $\mu_B(t)$ are not consistent as additional observations do not contain any additional information with respect to these parameters. As a consequence the estimates of σ_A, σ_B, ρ_{AB} and τ will be inconsistent as well due to the non-linearity of the model.[13] Asymptotic standard errors can therefore not be obtained from the inverse Hessian matrix of the log-likelihood function.

Conditionally on $\theta := (\sigma_A\ \sigma_B\ \rho_{AB}\tau)$ the means $\mu_A(t)$ and $\mu_B(t)$ can easily

12. In estimating the model we skipped the observations where the proportion of firms reporting to meet 'other constraints' was larger than 10%, compare the remark at the end of section 2.2. There are 33 remaining observations.
13. I owe this observation to Christian Gourieroux.

be estimated: optimal values are obtained for each observation t separately by solving a simple nonlinear least squares problem with five residuals and two unknowns. With respect to the elements of θ itself it appeared to be impossible to obtain satisfactory estimates for the shape and orientation of the micro level distribution G(.), i.e., the ratio σ_B/σ_A and the correlation coefficient ρ_{AB}, due to near under-identification. Minimization runs did not properly converge and ended up with widely different values for these parameters at only slightly different values of the log-likelihood function. On the other hand, the values obtained for σ_A and τ were very stable over different runs. The standard deviation σ_A of micro level excess demand for goods was always obtained as close to 10%, the 'best' value being 9.56 %. With goods markets in equilibrium on average, i.e. $Y^d = Y^s$, it implies that 20% (5%) of our firms experiences an excess supply or demand for output of about 13% (20%) or larger, which is not unreasonable. The parameter τ is estimated as .774, implying that firms report to operate unconstrained as long as capacity utilization stays above 77%. This confirms our prior notion that degrees of capacity utilization of about 80% are considered to be normal in manufacturing.

In figure 5 the values for $\mu_A(t)$ and $\mu_B(t)$ associated with the 'best' minimum obtained have been displayed. For comparison I have included a time series on industrial unemployment as an independent indicator of the pressure of demand in the manufacturing sector. The unemployment series refers to schooled industrial workers only, as it is probably a lack of schooled production workers that induces employers to report being labour constrained. The estimated time series for μ_B closely reproduces the overall pattern of the unemployment series, which is a nice result in view of the fact that it is based on an entirely different source. The result gives support to the common practice in applied work to use unemployment figures as a pressure of demand indicator. There seems to be a time lag of about three quarters, however, before the actual developments in the labour market are reflected in the unemployment figures. This supports the view that adjustments in the labour force are slow in the Netherlands, probably due to administrative procedures. The time series obtained for μ_A is negatively correlated with the μ_B series, implying that an excess demand for goods coincides on average with an excess demand for labour. The cyclical variation in μ_A is slightly less pronounced than the variation in μ_B. Aggregate excess supply of goods is between 13% (1973, 1979) and 28% (1975). The explosive behaviour of μ_B in the eighties

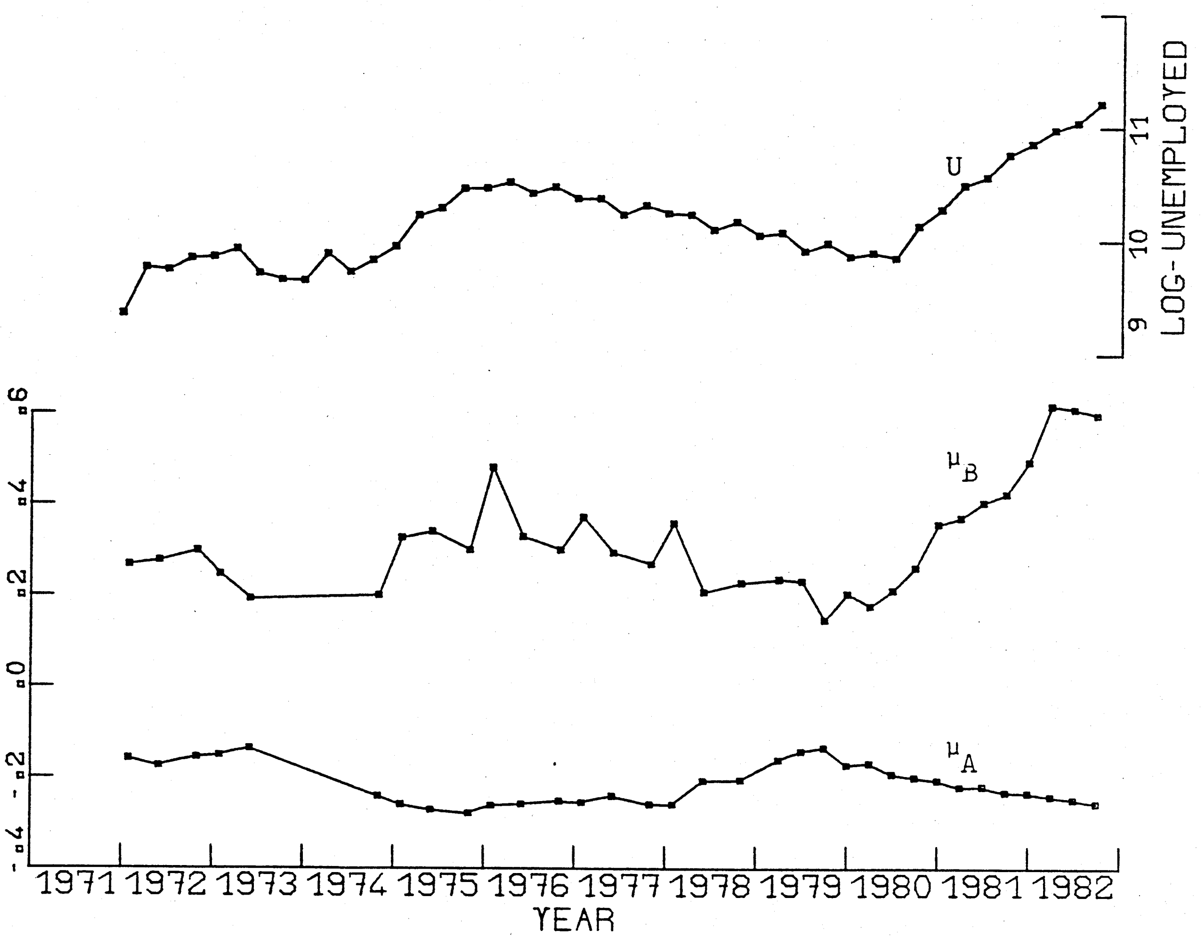

Figure 5. Optimal values for $\mu_A(t)$ and $\mu_B(t)$ compared to the number of unemployed industrial workers U.

is not reflected by a comparable shift in μ_A, probably because of a sharp reduction in productive capacity through increased scrap of older equipment and through bankruptcy. It clearly demonstrates that the use of unemployment figures as a proxy for the pressure of demand in goods markets is problematical.

Checking the sensitivity of the results obtained for μ_A and μ_B with respect to the value of σ_B/σ_A and ρ_{AB} that could not properly be estimated, it turns out that $\mu_A(t)$ is largely unaffected, whereas $\mu_B(t)$ shows up rather sensitive. Comparing several alternative series for the latter, at near-optimal points, it appears that it is mainly, though not entirely, a difference in level. The time shape is largely unaffected again. As the level of $\log(L^s(t)/L^d(t))$ cannot be identified from $\mu_B(t)$ anyhow, due to the presence of an unknown additive constant, the sensitivity of the level of the $\mu_B(t)$ series does not introduce an additional problem.

3.3. Discussion

It is well established in the theory of aggregation that explicit aggregation of microeconomic behavioural relationships results in very complicated aggregate expressions, except for certain highly restrictive cases. The usual way to proceed in macroeconomic analyses is to formulate aggregate relationships in terms of means only, i.e., working with first order approximations of the 'true' expressions. One of the interesting aspects of the explicit aggregation approach employed in deriving model (9) is that higher order moments of the underlying distribution come in as well. One may expect from beforehand that these second and higher order aggregation effects will be difficult to estimate from aggregate information alone, however. The present application of the model confirms this view. It shows that the information in the business survey data employed is too weak to properly identify the second moments of the excess demand distribution over firms, except for the scale σ_A, which can be shown to be essentially determined by the information in the capacity utilization series Q(t).

Two specific reasons can tentatively be advaned in order to account for the lack of identification of these second moments. The first is overparametrization. Having five observations each time period, one of them essentially gets lost because it depends on the others due to the adding-up restriction on the four proportions P_i. So effectively we start with 4T degrees of freedom.[14] The two time-dependent means $\mu_A(t)$ and $\mu_B(t)$ use half of them. The remaining 2T are sufficient in principle to identify the four time-independent parameters σ_A, σ_B, ρ_{AB} and τ, but in practice there may be insufficient independent variation in the data. When most of the information could be summarized in, say, 2 principal components, the remainder being largely (bivariate) noise, it may be very difficult to infer the shape of the underlying excess demand distribution from these data. Inspecting figure 2, one may guess that this is one of the causes of the problem. The second reason

14. This is not true in a strict sense. Barten (1969) shows that we can leave out one of the dependent equations when we proceed from an unrestricted error covariance matrix. In that case the statement in the main text would be true. We however proceed from a severely restricted covariance matrix, which entails the use of all equations. This shows that the dependent equation still contains additional information in that case. It has to do with the imposed covariance restrictions that have to be satisfied by the residuals of this equation as well (on average). One may guess, however, that it only contributes marginally to the identification of the model parameters.

has to do with the kind of information the business surveys provide. In estimating the second moments of n(A, B) we cannot dispose of a random sample of observations on A and B. The available business survey data only refer to masses according to some specific partitioning of the sample space. As in the case of discrete dependent variables the information content is much weaker in the latter case. So even with richer data the number of observations might still be too small to properly identify the shape of n(A, B).

Despite the lack of identification of the second moments the present application shows that it is possible to extract sensible information from business survey data. The close correspondence between the series obtained for the aggregate relative excess supply of labour, $\mu_B(t)$, and unemployment data, cf. figure 5, makes clear that on average the business survey responses do reflect actual developments in the economy. Economists, contrary to other social scientists, generally seem to mistrust interview-type data, that in the best possible case only represent subjective views. They prefer to think that they can dispose of more reliable data that objectively reflect the economic processes going on. The two types of data may largely supplement one another, however. The usual national accounts based data invariably reflect realisations, whereas interview data can be directed towards the observation of intentions, dispositions, expectations, and the like. The question always remains, of course, whether the interview data do indeed reflect the kind of phenomena one intends to observe. Subjecting the theory to this type of data one always tests the joint hypothesis of the correctness of the theory and the appropriateness of the interview questions. The same problem arises with the usual aggregate time series data, however. There one always jointly tests the correctness of the theory and the 'correspondence rules' relating the theoretical entities to the observed realisations. As Hendry and Spanos (1980) forcefully argue the relationship between the two may not be as direct as one often assumes: economic theory largely deals with unobserved intentions and equilibrium solutions, whereas the data reflect dynamic adjustment paths. Thus, in addition to common practice in applied macroeconomic modelling there seems to be scope for the use of interview type data that are deliberately devised such that they reflect the relevant theoretical entities as closely as possible.[15]

Finally, it should be noticed that the present application only involves

15. For a comparable view in the context of modelling expectations see Pesaran and Gulamani (1982).

the less interesting part of the model developed in section 2. The equations relating to the business survey results are hardly interesting by themselves. Their only role is to help identifying[16] the parameters of the output and employment equations that constitute the central part of the model. Consequently it remains to establish the empirical usefulness of the present approach towards modelling aggregate output and employment. This, however, would require the estimation of a more fully developed version of the model, as discussed in section 2.3, which is beyond the scope of the present paper. It clearly ranks high as a priority for further applied work.

16. It may be an important role, though. The output and employment equations taken apart are observationally equivalent to the transactions equations of a discrete switching model, as it was explained at the end of section 2. In trying to estimate such a model for the Netherlands we met serious identification problems with respect to the regime distribution, compare Kooiman and Kloek (1980, 1981). In fact the present model mainly originates from the need to be able to include additional information on the regime distribution.

Appendix A

The data

Month	Q	PUC	PDC	PLC	PCC	POC	SUM
7110	85.	52.	19.	13.	4.	19.	107.
7201	86.	64.	14.	11.	3.	5.	97.
7205	84.	67.	19.	10.	4.	4.	104.
7210	84.	70.	14.	10.	4.	7.	105.
7301	85.	67.	13.	13.	4.	8.	105.
7305	84.	68.	10.	18.	5.	8.	109.
7310	86.	59.	9.	23.	6.	14.	111.
7401	84.	46.	10.	13.	5.	31.	105.
7405	85.	47.	11.	20.	6.	28.	112.
7410	84.	42.	38.	10.	3.	10.	103.
7501	80.	41.	47.	5.	1.	7.	101.
7505	76.	39.	53.	4.	2.	8.	106.
7510	76.	35.	56.	5.	2.	9.	107.
7601	76.	43.	48.	2.	0.	8.	101.
7605	77.	43.	48.	5.	1.	5.	102.
7610	80.	44.	45.	6.	2.	3.	100.
7701	79.	45.	47.	4.	1.	1.	98.
7705	80.	49.	41.	7.	1.	1.	99.
7710	78.	41.	47.	7.	1.	6.	102.
7801	78.	42.	47.	4.	1.	8.	102.
7805	80.	55.	28.	12.	2.	8.	105.
7810	82.	55.	27.	11.	1.	9.	103.
7903	80.	69.	15.	13.	5.	5.	107.
7906	82.	71.	10.	16.	2.	3.	102.
7909	83.	63.	9.	21.	5.	7.	105.
7912	83.	64.	18.	15.	2.	4.	103.
8003	83.	63.	17.	17.	2.	5.	104.
8006	81.	59.	23.	13.	1.	6.	102.
8009	79.	61.	26.	10.	1.	4.	102.
8012	78.	64.	29.	6.	1.	2.	102.
8103	79.	58.	34.	5.	1.	5.	103.
8106	79.	58.	33.	4.	1.	4.	100.
8109	78.	55.	38.	3.	2.	3.	101.
8112	77.	56.	39.	2.	1.	3.	101.
8203	76.	54.	42.	1.	0.	3.	100.
8206	76.	52.	45.	1.	0.	3.	101.
8209	76.	48.	48.	1.	0.	4.	101.

Source: Central Bureau of Statistics, Conjunctuurtest, The Hague

Appendix B

The derivation of (10)

In this appendix I list the main steps to be taken in deriving (10). First we transform to the normal distribution of $\log \tilde{y}^s$, $\log \tilde{\ell}^d$, $\log \tilde{y}^d$ and $\log \tilde{\ell}^s$, which has covariance matrix $\Sigma = \{\sigma_{ij}\}$ and mean vector $(\log Y^s - \frac{1}{2}\sigma^2_{11} \vdots \log L^d - \frac{1}{2}\sigma^2_{22} \vdots \log Y^d - \frac{1}{2}\sigma^2_{33} \vdots \log L^s - \frac{1}{2}\sigma^2_{44})$. Second we transform to the new set of variables:

$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} := \begin{bmatrix} \log \tilde{y}^d - \log \tilde{y}^s \\ \log \tilde{\ell}^s - \log \tilde{\ell}^d - \log \mu \\ \log \tilde{y}^s \\ \log \tilde{\ell}^d \end{bmatrix}$$

Now using $\log \tilde{y}(\tilde{\ell}^s) = \log \tilde{y}^s + \log \tilde{\ell}^s - \log \tilde{\ell}^d - \log \mu$ it is easy to check that we obtain:

$$P_{cc} = 1/Y^s \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} e^C n(A, B, C, D)\, dA\, dB\, dC\, dD$$

$$P_{dc} = 1/Y^s \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\log\tau} \int_{A}^{\infty} e^C n(A, B, C, D)\, dB\, dA\, dC\, dD$$

$$P_{1c} = 1/Y^s \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{0} \int_{B}^{\infty} e^C n(A, B, C, D)\, dA\, dB\, dC\, dD$$

$$P_{uc} = 1/Y^s \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{\log\tau}^{0} \int_{A}^{\infty} e^C n(A, B, C, D)\, dB\, dA\, dC\, dD$$

$$Q = P_{cc} + 1/Y^s \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{0} \int_{A}^{\infty} e^{A+C} n(A, B, C, D)\, dB\, dA\, dC\, dD$$

$$+ 1/Y^s \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{0} \int_{B}^{\infty} e^{B+C} n(A, B, C, D)\, dA\, dB\, dC\, dD$$

where $n(A, B, C, D)$ is the joint normal density function of A, B, C and D. Obviously, as $e^C = \tilde{y}^s$ and $\int \tilde{y}^s dG = Y^s$, the four proportions add to unity.

The integrals can be worked out as follows. First we perform the integration with respect to D. As the inner integrations do not involve D, the outer integrals simply disappear, while we are left with the marginal density n(A, B, C). Then reversing the order of the integrations we obtain inner integrations of the form:

$$\int_{-\infty}^{\infty} e^{C} n(A, B, C)dC = n(A, B) \int_{-\infty}^{\infty} e^{C} n(C|A, B)dC = e^{\mu_{C|A,B} + \frac{1}{2}\sigma^2_{C|A,B}} n(A, B)$$

which can be worked out using formulas for the conditional mean and variance $\mu_{C|A,B}$ and $\sigma^2_{C|A,B}$. Completing the squares in the exponent we finally obtain (10).

Appendix C

Computational procedures

C.1. Numerical quadrature

In following Johnson and Kotz (1972), chapter 36, the two-dimensional normal integrals occurring in (10) were first transformed to the standard form:

$$L(h,k; \rho) = \int_{h}^{\infty} \int_{k}^{\infty} n_s(\theta,\xi; \rho)d\theta d\xi$$

where $n_s(., .; \rho)$ is the bivariate normal density function with zero means, unit variances and covariance ρ. To evaluate $L(h, k; \rho)$ I employed the following equality, cf. Johnson and Kotz (1972), p.99, eq. (36):

$$L(h, k; \rho) = \int_{-\infty}^{\infty} \Phi\left(\frac{c_1 t - h}{\sqrt{1 - c_1^2}}\right)\Phi\left(\frac{c_2 t - k}{\sqrt{1 - c_2^2}}\right)z(t)dt$$

where $z(.)$ and $\Phi(.)$ are the standard normal density and distribution functions respectively, and c_1 and c_2 are arbitrairy constants that have to satisfy $0 \leq c_i \leq 1$; $i = 1,2$ and $c_1 c_2 = \rho$. NAG-routine S15ABF was used to evaluate $\Phi(.)$, whereas the remaining integrations of the type $\int_{-\infty}^{\infty} f(t)z(t)dt$ were performed by means of quadrature routine D01BBF, option D01BAW with normal weights. In order to check the overall correctness of the computer program I also computed values of Q and P_i by means of Monte Carlo integration. Sampling from the bivariate normal distribution of A and B minimum condition (1) and figure 4 were applied to each drawing in order to obtain the value of $\bar{y}/\bar{y}^s$ and the ruling regime. Aggregates are then obtained by simple averaging. Stepwise increasing the number of drawings (up to 10^6) the results clearly converged to those from the numerical quadrature program, which seems to be a fairly strong check on the correctness of the latter procedure.

C.2. Optimization procedure

The estimation of model (11) involves the solution of a 2T + 4 = 70 dimensional numerical optimization problem. Fortunately the structure of the problem can be exploited to the extent that the following numerical concentration procedure can be applied. Conditionally on a given set of values for the four time-independent parameters σ_A, σ_B, ρ_{AB} and τ finding the minimal sum of squares with respect to the 2T means $\mu_A(t)$ and $\mu_B(t)$ requires the solution of T separate optimization problems, one for each observation. Accordingly a nested optimization procedure can be employed where the outer loop iterates on σ_A, σ_B, ρ_{AB} and τ and the inner loop consists of T independent nonlinear minimizations of sums of five squared residuals with respect to two unknowns. The latter problems can efficiently be solved by means of a Gauss-Newton procedure, since the Jacobi-matrix of the residuals with respect to the two means can easily be derived and programmed. Using NAG-library's E04GDF for this purpose it takes about 30 seconds cpu on a DEC-20 (double precision) to determine the optimal values of the 2T means in the inner loop. This is still a considerable amount in view of the fact that it has to be repeated each time the outer loop chooses new values for σ_A, σ_B, σ_{AB} and τ.

In the outer loop I started with the quasi-Newton routine E04CGF of the NAG-library. As no proper convergence could be obtained, I switched to the more robust Simplex routine E04CCF halfway. The final point reported in the main text was obtained from a run starting with this routine and ending with a version of the Complex Search method due to Box (1965).

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