ECONOMETRIC INSTITUTE

ON THE DIOPHANTINE EQUATION $(2y^2-3)^2 = x^2(3x^2-2)$

in connection with

THE EXISTENCE OF NON-TRIVIAL TIGHT 4- DESIGNS

R.J. STROEKER

REPORT 7934/ M

(additions and corrections to report 7930/ M)

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Because of a serious lack of motivation, the introduction to Report 7930/M: THE DIOPHANTINE EQUATION $(2y^2 - 3)^2 = x^2(3x^2 - 2)$, from now on referred to as [R], has been thoroughly changed. The revised form of this introduction now reads:

1. INTRODUCTION *)

The object of this paper is to fill the final gap in the proof of Noboru Ito's theorem [4] on the existence of non-trivial tight 4-designs. To this end, we prove the following theorem:

<u>THEOREM</u> The Diophantine equation

$$(2y^2 - 3)^2 = x^2(3x^2 - 2) \tag{1.1}$$

has precisely two solutions in non-negative rational integers x and y, namely $x = y = 1$ and $x = y = 3$.

We follow the notation of [4]. Let ν, k, t and λ be positive rational integers, subject to $\nu > k \geq t$. A t-design on ν points with block size k and index λ or, for short a t-(ν,k,λ) design, is a pair $(X,\mathfrak{B})$, where X is a finite set of points and $\mathfrak{B}$ a family of subsets of X (the blocks) such that:

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*) In this new version of the introduction, we make use of the REFERENCES in the new (enlarged and rearranged) form.

(i) $|X| = \nu$

(ii) $|A| = k$ for all $A \in \mathfrak{A}$

(iii) for each t-subset T of X, there are exactly λ blocks A containing T.

If $\mathfrak{A}$ consists of all the k-subsets of X, then $(X,\mathfrak{A})$ is called trivial. Moreover, a t-design is tight if, roughly speaking, the number of blocks is minimal. In case $t = 4$, this minimal number is $\frac{1}{2}\nu(\nu-1)$. Tight t-designs with $t \geq 4$ seem to be very rare.

Now Ito's theorem ([4], p.493) asserts that the only non-trivial tight 4-designs are the well-known 4-(27,7,1) and 4-(23,16,52). However, in the proof given, a host of errors occurred, some of which seemed irrepairable (cf.[5]). In the recent paper [2] the gap in the proof of Ito's theorem is filled up to at most a finite number of tight 4-designs resulting from the integer solutions of equation (1.1) (cf.[2], p.42).

Our theorem shows that the only tight 4-designs resulting from the solutions of (1.1) are trivial.

We return to equation (1.1). In the next section we shall reduce the problem of solving (1.1) to an equivalent but easier to handle problem. More precisely, in section 2 we show the relation between solutions of (1.1) and units of a certain type in a given quartic number field K. This number field is investigated in section 3, a particular sequence of algebraic integers of K is considered in section 4 and the last section is devoted to the completion of the proof of the theorem.

On page 3 of [R], section 3, line 5 from the bottom of the page, "cyclotomic units" should be replaced by "roots of unity".

The references on page 7 of [R] are extended to:

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