ECONOMETRIC INSTITUTE

AN APPLICATION OF THE GENERALIZED INVERSE IN INPUT-OUTPUT AND MACROECONOMIC ANALYSIS

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REPORT 7909/E

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AN APPLICATION OF THE GENERALIZED INVERSE IN INPUT-OUTPUT AND MACROECONOMIC ANALYSIS*

by

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* Prepared for the Seventh International Conference on Input-Output Techniques, Innsbruck, Austria, April 1979. The author gratefully acknowledges the facilities provided by Erasmus University for this project. Thanks are also due to Mr. A.S. Louter for his help in computations.

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ABSTRACT.

The generalized inverse concept is applied to a dynamic model for the Netherlands that incorporates both input-output and macroeconomic relationships. The model is over-determined in that there are 34 equations for its 33 endogenous economic variables.
The implications for policymaking of either retaining or dropping a constraint on balance of payments are studied through the generalized inverse. When that constraint is excluded, the model reduces to the more familiar fully-determined situation, having 33 equations in as many variables. Numerical results relating to shifts in indirect taxes are given. It is seen that the response of the over-determined model, the 34 equation case, is radically different from that of the fully-determined case. In fact, a reversal of charges in, for example, unemployment, wage rate, and in prices of exports and private consumption is observed. The conclusion is then that familiar ideas about policymaking which are based on fully-determined models, may have to be revised thoroughly when an over-determined model is more appropriate.

Table of Contents.

1. Introduction.
2. An Over-Determined Economic Model Incorporating Input-Output and Macro Constraints.
   2.1 General
   2.2 Always Binding Constraints
   2.3 Approximate Behavioural Relationships
3. Policy Analysis with an Over-Determined Model.
4. Conclusion.
References.

1. INTRODUCTION.

The use of the important mathematical concept of the generalized inverse in quantitative economic analysis is our subject here. Strange though this might seem at first to those not familiar with it, this concept helps one to obtain a solution of an over-determined system of equations which has more constraints than the number of unknowns, the endogenous variables. As an example of the over-determined situation, we give (in sec. 3) a model for the Netherlands economy. It has 34 constraints describing input-output, technical, definition, behaviour and other relationships. But there are only 33 endogenous economic variables. More constraints could be regarded as appropriate, but here the implications of having just one more constraint than the number of endogenous variables is investigated. As could be expected, it turns out that one might have to revise rather much earlier ideas about policymaking. It is usual to base policy analysis on fully-determined models that have exactly the same number of constraints and endogenous variables. It is then possible that conclusions derived from such fully-determined models may not apply when a situation is governed by a greater number of constraints.

In order to keep the subject here in proper perspective, it is useful first to review certain relevant features of the existing methodology. Two broad streams of present methods could be distinguished. First, there is the class of the so-called optimization (programming, decisionmaking) models. In them equations and/or inequalities represent input-output, economic behaviour, legal, technical and other constraints applicable to the situation under study. Here again two sub-classes could be identified. In one the number of constraints exceeds the number of variables and in the other it falls short. In either, however, the solution for the endogenous and policy (instrument, decision) variables is obtained by mathematical optimization of an objective function, given the exogenous (data) variables. Four models from literature could be cited as examples here. In the dynamic input-output, trade and development model of Bruno et al. [1], there are over 800 constraints and about 750 variables. Their objective is maximization of the discounted sum of private consumption over the planning period plus the terminal capital stock. Similarly, in the general equilibrium trade model of the linear programming variety given in Evans [4], there are 196 constraints but only 191 variables.

In contrast, the dynamic multisector model for India developed by Manne and Weisskopf [6] has 228 inequality constraints governing its 236 non-negative variables. "A gradualist time path" is their maximand. And, a development planning model described in Tinbergen and Bos [10] is under-determined. For the R regions and H sectors case, it has $2R + 3RH + 2RH^2 + R^2H$ variables and $1 + 2R + 2RH + 2RH^2 + R^2H$ equations. It thus has $2H - 1$ degrees of freedom. For determining a plan, Tinbergen and Bos suggest that the degrees of freedom may be used to optimize an objective, such as the national income.

Two characteristics of the optimization models are of interest here. First, the number of constraints is usually different from the number of unknown (endogenous and decision) variables in them. In particular, the number of constraints is usually different from the number of endogenous economic variables. This is clearly so in programming models that have less constraints than the total of both endogenous and decision variables. Secondly, even when the decision variables in a programming model are assigned values arbitrarily, the endogenous variables would not be determined uniquely in general. They would still require the mathematical optimization of the objective function for obtaining their values. In other words, if the decision variables were given reasonable values from outside, there is usually a range of feasible values for the endogenous variables.

In contrast to the optimization models, there are the models of the classical type in which the number of constraints is exactly equal to the number of <u>endogenous</u> variables. The constraints usually take the form of equations; but this is, of course, not necessary. No doubt one could still use optimization in conjunction with such fully-determined models. However, the crucial difference from the previous class is that the endogenous variables are normally uniquely determined once the decision and exogenous variables are specified. Thus, optimization in such models is a secondary issue, related to selection of decision variable values. The econometric models reported so far appear to be of the "fully-determined" variety (cf. [12]). Even in some recent models that incorporate both input-output constraints and macroeconomic behaviour (e.g., [7], [8]), the number of constraints is kept equal to the number of endogenous variables.

As mentioned above, the model to be described in Section 3 is over-determined in that it has 34 constraints involving only 33 endogenous variables.

We will employ a generalized inverse or solution concept for solving this model when the exogenous and decision variables are specified. Before we consider other detials, it would be helpful if we first recapitulate in brief the main idea behind this approach.
For this, consider an over-determined system of equations

$$A\,x = b.$$

Here A is an (m×n) matrix such that m>n, x is the n-dimensional vector of endogenous variables and b is the given vector of m exogenous (and decision) variables. As the number of equations exceeds the number of unknowns (m>n) by assumption, this is an inconsistent system. We also assume that no equation is a dependent version of some others. No ordinary solution x for this system then exists that can satisfy all the equations. However, a generalized solution can be defined so as to meet some condition or criterion. Let A^t denote a generalized inverse for A. Then an approximate solution for x is

$$x^* = A^t\, b.$$

This approximate solution implies occurrence of residuals in the equation, so that

$$A\,x^* + r = b,$$

where r is the vector of residuals.

An alternative way of obtaining a "best" approximate solution is by selecting the residuals r so that they satisfy certain criteria. For example, one could obtain a least squares solution (the ℓ_2 norm) or a solution that minimizes the sum of absolute deviations (the ℓ_1 norm). In this report we employ the least-squares solution (the ℓ_2 norm) for an over-determined system of equations. It is to be remarked that in case of a fully-determined system when m=n, the generalized solution concept coincides with the ordinary solution.

Without going into further details, it is vital to note here that even though mathematical optimization may be used in solving both an over-determined model and a programming model, the two are conceptually quite different.

A description of the economic model employed in this study is given in 3. In Section 4 we examine the implications of its over-determined setup for policymaking. Some conclusions to be derived from this study are given in Section 5.

2. AN OVER-DETERMINED ECONOMIC MODEL INCORPORATING INPUT-OUTPUT AND MACRO CONSTRAINTS.

2.1 General.

The economic models we use in this study is based on statistical data for the Netherlands. It is small compared with some of the existing models that have many times more variables and constraints. There are 33 endogenous economic variables in it. It is also relatively simple. Only the behaviour of exports is represented through a complicated non-linear relationship. The other constraints are mathematically rather simple, although products of quantity and price are also present. There are, however, 34 constraints. That is, there is one more constraint than the number of endogenous variables. By excluding the constraint on balance of payments, the model reduces to the classical fully-determined variety. Both over- and fully determined cases can, of course, be handled in the same way through the generalized inverse. On the whole, it is believed that the model is fairly representative even though it is small. And, clearly a similar approach can be adopted in conjunction with larger models.

The Netherlands Central Bureau of Statistics (C.B.S.) publishes a 34 sector input-output table periodically. For this study there are, however only 4 sectors in the model, viz.,

Sector 1: The primary sector including agriculture, forestry and fishery.
Sector 2: Manufacturing.
Sector 3: Construction.
Sector 4: Services.

These four are obtained by aggregation of the sectors in the C.B.S. publications in the following manner:

Sector 1: Sector 1 of C.B.S.
Sector 2: Sectors 2 through 20 of C.B.S.
Sector 3: Sector 21 of C.B.S.
Sector 4: Sectors 22 through 34 of C.B.S.

In case of primary inputs, the sum of indirect taxes and subsidies is regarded as one single indirect taxes category here. There are then five primary demand items: imports, depreciation, indirect taxes (sum of C.B.S. indirect taxes and subsidies), wage incomes and other (nonwage) incomes. Similarly, in case of the final demand, only a slight modification of the C.B.S. categories is involved. The sum of C.B.S. gross private fixed investment and increase in stocks is taken as a whole representing gross private investment in the model here. This helps to keep the size of the model small. It is quite feasible to preserve the two categories separately, which is the case in other models of the Netherlands economy, such as in the so-called Vintaf - I and Vintaf - II models ([3], [5]). The five final demand categories are: exports, consumption, gross private investment, Government expenditure and Government investment.

The 33 endogenous variables of the model are listed in Table 1 and the 10 exogenous variables in Table 2. Their dimensions are also given. The symbol BDfl stands for a billion (1,000,000,000) Dutch guilders. Statistical data for these variables are found in the yearly Nationale Rekeningen (National Accounts) published by the Central Bureau of Statistics and in the Centraal Economisch Plan of the Central Planning Bureau. The National Accounts for 1977 also contains the input-output table for 1975. This table was aggregated in the manner described above.

As mentioned earlier, we consider 34 constraints in the model for analysis here. To present clearly the application of the generalized inverse, it is helpful to group the constraints in two different types: those that are always binding and those which may hold approximately, allowing residuals to occur. We now consider the constraints. In the description numerical values for parameters are given. While parameter estimation is an important aspect, it is not the main issue here. Formulation of an over-determined economic model, its solution through the generalized inverse and the implications of this approach for economic analysis are the major objectives. It would be improper to demand that estimation be considered first. Estimation is, in a way, the reverse of the solution process -in that one is given observations (solutions) and it is required to determine parameter values. Estimation then obviously depends upon the manner of obtaining a solution for a model. For use in this study, the parameters in behavioural equations were determined using an ℓ_2 (least squares) norm, excepting for exports, Eq (27) which is taken directly from Vintaf - II model [3].

2.2 Always binding constraints.

Constraints implied by accounting, definition and other such identities must hold completely. There are 26 such relationships in the model. When calculating a "best" approximate solution for the over-determined model, care is taken that they are conformed to.

Sector Outputs

The aggregated input-output table used has 9 rows. They correspond to:

1. Output of Sector 1.
2. Output of Sector 2.
3. Output of Sector 3.
4. Output of Sector 4.
5. Imports.
6. Depreciation.
7. Indirect taxes.
8. Wage income.
9. Non-wage income.

And its 9 colums correspond to:

1. Sector 1 inputs.
2. Sector 2 inputs.
3. Sector 3 inputs.
4. Sector 4 inputs.
5. Exports.
6. Private Consumption.
7. Gross private investment.
8. Government Consumption.
9. Government investment.

Expecting for rows 6 and 7, which are treated separately later, per unit coefficient for entries in other rows could be defined, such that

$f_{ij} = F_{ij} / F_j$, where Fij is the i, j-th entry in the flow matrix,

and F_j is the total for column j.

For sector j we have

$$x_j = f_{j1} x_1 + \dots + f_{j4} x_4 + f_{j5}b + f_{j6}c + f_{j7}i + f_{j8}g + f_{j9}ig,$$

$$j = 1, 2, 3, 4 \quad (1) - (4)$$

<u>Sector Inputs</u>

For sector j, the input relationship implies

$$p_j = f_{1j}\, p_1 + \dots + f_{4j} + f_{5j}\, pmg + \frac{\overline{DO_j}}{\overline{DO}}\,\frac{D}{x_j}$$

$$+ \frac{\overline{TKO_j}}{\overline{TKO}}\,\frac{TK}{x_j} + \frac{f_{8j}}{\overline{\ell_0}}\,\ell + \frac{\overline{ZO_j}}{\overline{ZO}}\, p_j, \qquad j = 1, 2, 3, 4 \qquad (5) - (8)$$

Here $\overline{DO_j}$ denotes the depreciation flow in sector j in the base (0) year for which the input-output flow table is being used, viz., 1975. Simil rly, $\overline{DO}$ denotes the total depreciation in the economy. An anologous interpretation holds for similar other terms in Eq. (5) - Eq. (8). The term $(f_{8j}/\overline{\ell_0}).\ell$ represents the current wage cost in sector j, $\overline{\ell_0}$ being the wage rate in the base year.

The following assumptions are involved in this relationship:

- The volume of imports is assumed to vary in proportion to the volume of sector input,
- The contribution of depreciation (D) and the indirect taxes (TK) is in the same proportion as in the base year.
- The labour requirement is proportional to the sector output, the proportion being fixed at the base year-level.
- The share of non-wage income has the same ratio to the (current) value of the sector output as in the base year.

<u>Price of endogenous final demand items.</u>

The price of exports is governed by:

$$pb = f_{15}\, p_1 + f_{25}\, p_2 + f_{35}\, p_3 + f_{45}\, p_4$$

$$+ f_{55}\, pmg + \frac{\overline{TKb}}{\overline{TKO}}\,\frac{TK}{b} \qquad (9)$$

Here too the assumptions are similar to those in (5) - (8). However, depreciation (D), wages (ℓ) and non-wage income (Z) do not play a role.

Similar equations arise for the price of consumption and the price of imports. But we do not write them in full here as they are completely analogous to Eq. (9).

$$pc = \dots. \tag{10}$$

$$pi = \dots. \tag{11}$$

Potential Production

Our treatment of the investment, depreciation and capacity expansion is rather simplistic. This is done in order to keep the model both small and simple. In recent work relating to the Netherlands economy, a far more sophisticated approach is employed (cf. [3], [5]). It may be remarked that this type of detailed analysis could also be incorporated in our study.

We use an aggregated measure of capacity in the economy as a whole; namely, the potential production (K). Positive net investment in a period leads to an increase in the potential production. Thus,

$$K_1 - K_0 = \frac{1}{k} [(I-D) / pi] + \frac{1}{k} ig \tag{12}$$

The symbol k denotes the capital-to-output ratio. We employ a value of k = 1 - 6 which is close to the value estimated in [ ].

A constraint on the balance of payments

As is recognized, the international transactions impose a constraint on the balance of payments of an economy. We represent this requirement in the form:

$$B = M + SOC \tag{13}$$

Here B is the value of goods and services exported and M is the value of corresponding imports. The variable SOC denotes an exogenously specified surplus of exports over imports. This figure is not found readily in the published national accounts. But it can be derived from other data. The distinction here is that it does not involve other income transfers with other countries in the current accounts.

The inclusion of this constraint in a model appears more realistic than its neglect. It is realized that over a long period the balance of payments situation of a country must remain within certain bounds and that it should not be allowed to assume arbitrarily large positive or negative values. In the model here, it is assumed that one has fairly good knowledge of the required surplus of current exports over imports.

In this report, as mentioned earlier, we examine the effect of including or excluding this particular constraint on policymaking. It will be seen that conclusions may be radically different in the two situations. In the classical fully-determined economic models, the balance of payments is usually allowed to vary freely. They therefore overlook the effects of this very important constraint.

Identities

The following equalities represent definition or accounting relationships. They do not require elaboration.

Total employment in all sectors

$$a = \sum_{j=1}^{4} (f_{8j} / \overline{\ell 0}) \cdot x_j \qquad (14)$$

Value of exports

$$B = b \cdot pb \qquad (15)$$

Value of consumption

$$C = c \cdot pc \qquad (16)$$

Value of gross investments

$$I = i \cdot pi \qquad (17)$$

Total wage income in the private sector

$$L = a \cdot \ell \qquad (18)$$

Quantity of imports of goods and services

$$m = f_{51} \cdot x_1 + \ldots + f_{54} \cdot x_4 + f_{55} \cdot b + f_{56} \cdot c + f_{57} \cdot i + f_{58} \cdot g + f_{59}\ ig \qquad (19)$$

The assumption here is that imports are proportional to sector productions as well as to final demand items.

Capacity Utilization

As we have but one measure of capacity for production, a single indicator of capacity utilization is considered:

$$q = (x_1 + x_2 + x_3 + x_4) \,/\, K \qquad (20)$$

Unemployment

$$U = P - a - ag - az \qquad (21)$$

Value of sector productions

$$X_j = x_j \cdot p_j, \qquad j = 1, 2, 3, 4 \qquad (22) - (25)$$

Non-wage Income

$$Z = \sum_{j=1}^{4} f_{9j} X_j \qquad (26)$$

2.3 Approximate Behavioural Relationships.

The constraints representing economic behaviour are allowed to admit residuals when an approximate solution to the over-determined model is being calculated. That is, unlike equations (1) through (26) given above which are always satisfied completely, the behavioural equations in the model are regarded as approximate relationships. There are eight such equations. They make use of the recent work on the Netherlands economy given in Vintaf - I and II reports [3], [5].

Quantity of Exports

The behaviour of exports is modelled completely analogous to that in the Vintaf - I and II models. In fact, even though our definitions of the relevant variables are somewhat different, we use parameter estimates reported in them. Only the constant term is adapted. The equation used is

$$\begin{aligned} \ln b = {} & 0.5 \ln b + 1.068 (\ln mw - 0.5 \ln mw_{-1}) \\ & - 0.846 \ln (pb/pmw) \\ & - 0.514 (\ln q - 0.5 \ln q_{-1}) + (5.356 - 0.5 \ln 1000.0) \end{aligned} \qquad (27)$$

This relationship implies an elasticity of 1.068 with respect to the reweighted world imports (mw). The elasticity of exports to relative national and international prices (pb/pmw) is -0.846. A distributed lag effect of the relative prices is assumed which gives this form to the relationship. Earlier work on the Netherlands indicates that capacity utilization (q) in the economy plays a role in export behaviour. The corresponding elasticity is -0.514.

Even though we use parameter values obtained elsewhere, this relationship still performs rather well in simulations generated with the model as a whole over the historical period 1973 - 77. In order to keep this report short, we do not describe the simulation results any further.

Value of private consumption in current prices

Like in other models of the Netherlands economy, the consumption behaviour is mainly governed by disposable wage and non-wage incomes of the same period. The equation used is

$$C = 0.98\ (L + LG - TL) + 0.35\ (T - TZ) + 2.45 \qquad (28)$$

The marginal propensity of 0.35 with respect to the aftertax non-wage income was taken from Vintaf - II report [3]. The other two parameters were then determined using past data.

Gross private investment

The investment variable in the model represents the sum of gross fixed investment and increase in stocks. This is rather different from the practice in other models relating to the Netherlands economy. However, for our purpose this aggregation does not seem to matter. The relationship we use is

$$I = 0.785\ [(ZP + ZD_{-1} + D + D_{-1})/2]$$

$$- 63.34\ [1 - (0.75\ q + 0.25\ q_{-1})] + 2.53 \qquad (29)$$

The sum of aftertax non-wage income and retained depreciation is regarded as the major determinant of private investment. And, as in [3], unused capacity leads to less investment.

Depreciation

Similar to in the Vintaf models, the total value added at factor costs is considered to determine depreciation:

$$D = 0.1095\ (L + LG + Z) + 0.069 \qquad (30)$$

Although in those two models depreciation is differentiated from actual scrapping of a part of the existing capital stock, we do assume that the variable D is also a measure of the decrease of the capital stock. This assumption enters into the equation for potential production, Eq. (12) as well.

Wage rate movements

The equation for movements in the wage rate is

$$(\ell - \ell_{-1}) = 2.582\ (pc - pc_{-1}) - 0.189\ (v/p) + 0.007 \qquad (31)$$

Here the assumption is that wage rate changes in response to changes in the price of private consumption and to the state of the labour market. This relationship is much simpler than in other models. The Vintaf - I and II models, for example, employ a more sophisticated relationship in which productivity changes also play a role.

Tax on wage income

This is taken to be a linear function of the wage income:

$$TL = 0.16\ (L + LG) - 2.2 \qquad (32)$$

Tax on non-wage income

This is also a linear function:

$$TZ = 0.36\ Z - 1.16 \qquad (33)$$

Labour Supply

The supply of labour is mainly determined by demographic factors, emigration and immigration. However, the state of the labour market also effects the availability of labour. The relationship used in the model is:

$$P = 0.36\ P_{-1} - 0.5\ U_{-1} - 1.16 \qquad (34)$$

The effect of unemployment, as in other models of the Netherlands economy, is to depress labour supply.

3. POLICY ANALYSIS WITH AN OVER-DETERMINED ECONOMIC MODEL.

The over-determined model presented enables both policy analysis and forecasting, like its more familiar fully-determined counterparts. Here we consider the policymaking problem. By means of an experiment relating to movements in indirect taxes (TK) we examine the implications of having one more constraint than the number of endogenous variables. The generalized "best" approximate solution concept is employed in the analysis. As mentioned earlier, we obtain approximate solutions that minimize the ℓ_2 norm. The experiment consists of determining the effects of excluding or including, in particular, the balance of payments constraint, Eg. (13). When this constraint is retained the model has 34 equations in 33 variables -and is over- determined. When, however, Eg. (13) is dropped, the more familiar fully-determined situation of 33 equations in 33 variables is obtained. The scheme of computations reported here is as follows:

1. The historical period 1973 through 1977 is chosen for analysis. In the so-called "control" solution, the exogenous variables are assigned their observed values over this period. The observed data for 1972 are used as starting values for the lagged endogenous variables. The five-period solution problem then involves $33 \times 5 = 165$ endogenous variables and $34 \times 5 = 170$ constraints. This enlarged over-determined system can be solved as a whole through the generalized inverse, giving the "control" solution. In constrast, the fully-determined situation when the balance of payments constraint, Eq. (13) is excluded, leads to solving $33 \times 5 = 165$ equations in as many unknowns. This system can also be solved by the same generalized inverse method. Only, now all the residuals are identically equal to zero. And, it could either be solved as a whole or recursively, year by year.
2. Similarly, two "disturbed" solutions are calculated, one for the over-determined and the other for the fully-determined case. For this, the indirect taxes (TK) are lowered by 1 BDfl during 1973, 74 and 75 from their corresponding observed value used in 1 above. The changes in "disturbed" solution from the respective "control" solution are calculated in each of the two cases. This particular type of "disturbed" situation, which means giving an impulse over 3 successive years and then bringing the level to original value, was chosen in order to facilitate direct comparison with the results of a similar analysis described previously in Vintaf - I report [5].

In the calculations, some other complexities of the model, however, need also to be taken into account. Not all the constraints in the model are linear. The generalized solution concept mentioned in Section 1 in the context of a linear system is therefore not readily applicable. But an approximate linear system can be derived by linearization of the non-linearities. For results presented here, the first, linear terms in the Taylor series expansion of non-linearities are retained.
This requires an initial guess for the values of endogenous variables around which linearization can be performed. The initial guess, as usual, is then successively improved provided the procedure is convergent. In our calculations convergence is achieved fairly quickly. Just in three iterations very good convergent values are obtained. Five iterations ensure even more improved solution valves.

Some interesting outcomes of this numerical policy analysis experiment are presented in two tables. The results of using the over-determined model as a whole with 34 constraints in 33 endogenous variables are shown in Table 3. When a fully-determined model is considered by excluding the balance of payments constraint, the values shown in Table 4 are obtained. A rather striking feature to be seen is that the effect of the same policy instrument can be radically different under the two regimes. To appreciate this it is helpful to examine first Table 4 which relates to the more familiar fully-determined situation. During 1973 the decrease in indirect taxes in the same year would have led to rises in exports (0.5 percent), private consumption (1.4 percent), gross private investment (2.8 percent), imports (1.3 percent) and capacity utilization (0.9 percent). And, it would have caused falls in the wage rate (-1.3 percent), export prices (-0.9 percent), consumption price (-1.4 percent). It would also have resulted in a decrease in unemployment by as much as 35,000 manyears. However, it would also have led to a balance of payments situation which did not conform to an exogenously determined level in general. That is why the corresponding entries are marked "inappropriate".
The response of this fully-determined system is in agreement with conventional analysis of this type. For example, the impact changes reported in a similar exercise with the Vintaf - I model [5] agree with those given here, except maybe for private investment. The model here indicates a rise in investment during 1973 whereas the Vintaf - I analysis would give a fall.

When we consider the over-determined situation in which the balance of payments is required to conform completely to exogenously specified levels, the response to policy change is altered radically. Instead of a reduction (- 35,000 manyears), an increase in unemployment (+ 41,500 manyears), for one, would be required to satisfy the additional constraint. Similarly, both private consumption and export price would have to register rises instead of falls (+ 1.3 and + 2.0 per cent, respectively, against - 1.4 and - 0.9 per cent). The volume of private consumption would nevertheless rise (0.3 per cent) as in the fully-determined situation, when the balance of payments was allowed to move freely. The response of investment and imports is likewise reversed (- 2.5 and - 1.4 per cent, respectively vis a vis + 2.8 and+1.3 per cent). The capacity utilization would also have fallen (- 1.3 per cent) instead of rising (+ 0.9 per cent). We do not comment on the effects on the potential production because the changes are rather small. Even then the signs are indicative of the direction of changes.

To be sure, in this analysis two polar cases of whether imposing a balance of payments constraint completely or not are considered. It is equally possible to envisage a balance of payments constraint which need not be met exactly, but only approximately, as is the case with the behavioural equations, Eq. (27)-(34). It is also permissible to attach different weights to residuals in several equations. In actual computations, this is easily taken care of.

To keep the description short, we do not elaborate further on the results of the policy analysis experiment. And it is important to remark that as the model is non-linear, it is not always easy to generalize and state what type of effects policy changes will have in different situations. As Tinbergen [9] explains, the cyclical situations play an important role in determining the response to instruments. Presence of non-linearity implies that a policy change may not result in similar effects in all circumstances. Finally, it may be remarked that response to other policy variables may be determined in similar fashion.

4. CONCLUSION.

A study of the application of the generalized inverse concept in economic analysis and forecasting has been continued in this report. Previously some aspects relating to formulation and solution of over-determined economic models were considered ([11], [12]). Here the objective has been to examine the implications for policymaking of having more constraints than the number of endogenous variables in a model. It is seen that the effects of changes in a policy variable in an over-determined model may be radically different from its effects in a fully-determined model. The results given here have been obtained by means of an economic model that incorporates both input-output and macroeconomic constraints and others. It has 34 constraints governing the 33 endogenous economic variables. By ignoring a constraint on the balance of payments, the model reduces to the classical fully-determined variety, having 33 equations for 33 unknowns. The model is based on statistical data for the Netherlands. Results relating to a decrease in the indirect taxes from their observed levels during three successive years are presented. The 33 equations, fully-determined case corroborates the conventional ideas about the effect of reduction in indirect taxes. For instance, it is seen that it leads to, in the first period, an increase in employment, exports, consumption, imports and capacity utilization, while resulting in a decrease in the wage rate, and in prices of private consumption and exports. However, the balance of payments in this case may well be inconsistent with what might be required. When a constraint on the balance of payments is included in the model, the 34 constraint case, the influence of an indirect tax reduction changes dramatically. Now the model leads to a state in which this is in fact a decrease in employment, exports, imports and capacity utilization, while there is an increase in the wage rate, and in prices of private consumption and exports. But now one is assured that the constraint on the balance of payments is observed completely. It has been remarked that a somewhat less exacting constraint could also be taken into account equally well through the generalized solution method. Clearly this approach offers greater flexibility for modelling and analysis.

Quantitative models in which constraints are not always equal in number to the endogenous variables, already exist in economics literature. The programming or optimization models could either be over- or under-determined. In contrast, the so-called econometric models have so far tended to be exclusively fully-determined, having an equal number of constraints and of endogenous variables. This report further demonstrates the usefulness of the generalized inverse concept in coping with over- and fully-determined situations. Its application for economic forecasting using the model described here, is considered in a companion report [13]. Although this model involves just one more constraint than the number of endogenous variables, the generalized inverse is equally applicable when there are many more constraints. This important and useful mathematical concept thus has much to offer for economic analysis and forecasting.

TABLE 1. THE ENDOGENOUS VARIABLES[a]

S.No.	Symbol	Variable
1	a	Total private employment (100,000 manyears)
2	B	Value of exports of goods and services (BDfl)
3	b	Quantity of exports of goods and services (1975 BDfl)
4	C	Value of private consumption (BDfl)
5	c	Quantity of private consumption (1975 BDfl)
6	D	Private depreciation (BDfl)
7	I	Value of gross private investment (BDfl)
8	i	Quantity of gross private investment (1975 BDfl)
9	K	Potential production (1975 BDfl)
10	L	Total wage income in the private sector (BDfl)
11	ℓ	Wage rate (10,000 Dfl per manyear)
12	m	Quantity of imports of goods and services (1975 BDfl)
13	P	Labour supply (100,000 manyears)
14	pb	Export price (Index, 1975 = 1)
15	pc	Price of private consumption (Index, 1975 = 1)
16	pi	Price of private investment (Index, 1975 = 1)
17	px_1	Price of Sector 1 output (Index, 1975 = 1)
18	px_2	Price of Sector 2 output (Index, 1975 = 1)
19	px_3	Price of Sector 3 output (Index, 1975 = 1)
20	px_4	Price of Sector 4 output (Index, 1975 = 1)
21	q	Capacity utilization
22	TL	Tax on wage income (BDfl)
23	TZ	Tax on nonwage income (BDfl)
24	U	Unemployment (100,000 manyears)
25	X_1	Sector 1 output in current prices (BDfl)
26	X_2	Sector 2 output in current prices (BDfl)
27	X_3	Sector 3 output in current prices (BDfl)
28	X_4	Sector 4 output in current prices (BDfl)
29	x_1	Sector 1 output in constant prices (1975 BDfl)
30	x_2	Sector 2 output in constant prices (1975 BDfl)
31	x_3	Sector 3 output in constant prices (1975 BDfl)
32	x_4	Sector 4 output in constant prices (1975 BDfl)
33	Z	Nonwage income (BDfl)

[a] BDfl stands for a billion (1,000,000,000) Dutch guilders.

TABLE 2. THE EXOGENOUS VARIABLES [a]

S.N.o.	Symbol	Variable
1	ag	Employment in the public sector (100,000 manyears)
2	az	Selfemployed (100,000 manyears)
3	g	Quantity of Govt. expenditure (1975 BDfl)
4	ig	Quantity of Govt. investment (1975 BDfl)
5	LG	Wage income of Govt. employees (BDfl)
6	mw	Reweighted world imports (1970 BDfl)
7	pmw	Price of competing exports (Index, 1970 = 1)
8	pmg	Price of imports (Index, 1970 = 1)
9	SOC	Exogenously specified current balance of payments surplus (BDfl)
10	TK	Indirect taxes (BDfl)

[a] BDfl denotes a billion (1,000,000,000) Dutch guilders.

TABLE 3. Over-Determined Model. Effects of a reduction in the indirect taxes during 1973, '74 and '75.[a]

No.	Variable	1973	1974	1975	1976	1977
1	Export quantity (b)	-1.4	-1.1	3.0	0.2	-0.3
2	Quantity of private consumption(c)	0.3	-0.6	1.8	-1.5	-0.5
3	Quantity of gross private investment (i)	-2.5	-1.4	4.7	0.6	1.2
4	Potential production (k)	-0.2	-0.2	-0.5	0.1	0.0
5	Wage rate (ℓ)	4.5	0.8	4.3	-1.4	-0.5
6	Quantity of imports (m)	-1.4	-1.0	3.4	-0.5	-0.5
7	Price of exports (pb)	2.0	0.1	-0.6	-0.5	-0.3
8	Price of private consumption (pc)	1.3	0.2	-0.6	0.0	-0.2
9	Capacity utilization (q)	-1.3	-0.7	2.4	-0.5	-0.3
10	Unemployment (U)	0.415	0.469	-0.463	-0.058	0.111

a. All figures are in percent changes, except for unemployment which is in 100,000 manyears.

TABLE 4. Fully-Determined Model. Effects of a reduction in indirect taxes during 1973, '74 and '75[a].

No.	Variable	1973	1974	1975	1976	1977
1	Export quantity (b)	0.5	-0.3	0.1	-0.3	-0.2
2	Quantity of private consumption(c)	1.4	-0.1	0.0	-0.8	-0.2
3	Quantity of gross private investment (i)	2.8	-0.1	-0.2	-1.8	-0.5
4	Potential production (k)	0.1	0.1	0.2	0.0	0.0
5	Wage rate (ℓ)	-1.3	0.4	0.2	1.1	0.0
6	Quantity of imports (m)	1.3	0.0	0.0	-0.7	-0.2
7	Price of exports (pb)	-0.9	0.3	0.1	0.8	0.0
8	Price of private consumption (pc)	-1.4	0.4	0.1	0.8	0.0
9	Capicity utilization (q)	0.9	-0.1	-0.1	-0.6	0.2
10	Unemployment (U)	-0.354	-0.158	-0.113	+0.147	+0.150
11	Balance of payments	Inappropriate level				

a. All figures are in percent changes, except for unemployment (U) which is in 100,000 manyears.

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