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MONEY ILLUSION AND AGGREGATION BIAS

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by

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ABSTRACT

This note shows that what often has been called money illusion in macro consumption functions can well be due to aggregation bias.

Contents

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0. Introduction

The well-known "homo economicus"[1] does not suffer from money illusion: maximizing the utility he derives from consuming commodities he buys them such that the implied demand functions are zero-homogeneous in prices, money income and money wealth; see, for instance, Patinkin (1965), p. 404. There are, however, examples of empirical per capita (hence macro-)demand functions that do not show this zero-homogeneity; see, for instance, Bransson and Klevorick (1969). Many economists consider this phenomenon as evidence of money illusion. It can, however, also be due to aggregation bias if the "rational" consumers' consumption functions have different parameters. In section 1 we specify these consumption functions. In section 2 we describe briefly the concept of aggregation bias. In section 3 we prove our statement.

1. A consumption function

We confine ourselves to the log-linear models found in economic literature. The seminal paper on this subject is by Bransson and Klevorick (1969). A paper dealing with money illusion within the context of a supply of labour model is Fair (1971); our reasoning applies to this study as well.

Bransson and Klevorick introduce the following macro model:

$$(1.1)\quad \left(\frac{C}{P}\right)_t = b_0 \left(\frac{Y}{P}\right)_t^{b_1} \left(\frac{W}{P}\right)_t^{b_2},$$

where:

C_t = per capita consumption in period t

Y_t = per capita non property income in period t

W_t = per capita "consumer net worth" (wealth) in period t

all assumedly positive and measured nominally, and where

P_t = the price level of consumer goods.

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[1] See Samuelson (1947), p. 90.

Writing c, y and w for real consumption, labour income and wealth per capita, respectively, the following relationship is introduced:

$$(1.2)\quad c_t = b_0 y_t^{b_1} w_t^{b_2} P_t^{b_3}.$$

The generally accepted idea is now that money illusion[2] is present when, running appropriate regressions on c, y, w and P, the exponent b_3 turns out to be significantly positive (according to some statistical criterion).

Using U.S. quarterly series from 1955-I to 1965-IV Bransson and Klevorick estimated some dozens of equations based on (1.2); inter alia, they experimented with several kinds of lag structures and with different price variables. They convincingly showed that in most cases the price parameter b_3 (or, in the case of lags, the sum of the concerning exponents) was significantly positive according to the most severe statistical criteria.

The choice of equation (1.2) for investigating the phenomenon of money illusion is easily understood from an econometric point of view: econometricians try to formulate null-hypotheses that are as simple as possible.

The question arises, however, what kind of micro model(s) can be considered as underlying (1.2). Since the aforementioned authors concluded that money illusion (in the Patinkin sense; that is $b_3 > 0$) is not absent they must have had in mind some micro model, for money illusion is an individual trait.

One can only make a guess with respect to Bransson and Klevorick's micro model. An obvious guess is an "analogue" micro model for the j-th individual:

$$(1.3)\quad C_{jt} = \beta_{0j} y_{jt}^{\beta_{1j}} w_{jt}^{\beta_{2j}} P_t^{\beta_{3j}},$$

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2 For short we shall mean with this term the phenomenon of money illusion à la Patinkin (1965) occurring in isolation as well as combined with the phenomenon of "price expectation"; see, for instance, Driehuis (1972), p. 69.

where $j = 1, \ldots, J$. In this case, however, the consumers (or at least one of them) are (is) suspected beforehand of money illusion. Within the context of the models (1.2) and (1.3) one cannot answer the question whether observed derivations from theoretically required homogeneities can be caused by other circumstances.

Below we show that individual rationality (that is: no individual suffers from money illusion) can be compatible with absence of the corresponding homogeneity of the macro consumption function. Therefore we base ourselves on the constant elasticity model:

$$(1.4) \quad \frac{C_{jt}}{P_t} = \alpha_{0j}\left[\frac{Z_{jt}}{P_t}\right]^{\beta_{1j}} \left[\frac{W_{jt}}{P_t}\right]^{\beta_{2j}},$$

where:

C_{jt} = individual j's consumption in period t,

Z_{jt} = his income in period t,

W_{jt} = his "net worth" (money wealth), measured at the beginning of period t,

all measured as nominal amounts and assumedly positive.

P_t = price level of consumption goods in period t.

These magnitudes are supposed to be adequately measured for $j = 1, \ldots, J$ and $t = 1, \ldots, T$.

We consider (1.4) as the (approximate) result of maximization of a lifetime utility function subject to a lifetime budget restriction. The possible effect of the rate of interest has been disregarded for convenience's sake; also other refinements, such as introducing lags, have been passed by. These extensions do not influence our arguments.

The model can be rewritten as:

$$(1.5) \quad \log C_{jt} = \beta_{0j} + \beta_{1j} \log Z_{jt} + \beta_{2j} \log W_{jt} + \beta_{3j} \log P_t,$$

with

$$(1.6) \quad \beta_{1j} + \beta_{2j} + \beta_{3j} = 1.$$

The relation (1.6) expresses the absence of money illusion.

2. Aggregation bias[1]

Let be given the model:

$$(2.1)\quad y_j = X_j\beta_j + u_j \qquad (j = 1, \ldots, J)$$

where $y_j' = (y_{j1}, \ldots, y_{jT})$ is a vector of dependent variables, $u_j' = (u_{j1}, \ldots, u_{jT})$ a vector of disturbances, $\beta_j' = (\beta_{1j}, \ldots, \beta_{Kj})$ a coefficient vector and X_j is a T×K matrix of non-stochastic explanatory variables with columns $x_{kj} = (x_{kj1}, \ldots, x_{kjT})'$ for $k = 1, \ldots, K$. About the disturbances we make the well-known assumption $Eu_j = 0$.

We define the following per capita figures:

$$(2.2)\quad \bar{y} = \frac{1}{J}\sum_{j=1}^{J} y_j, \qquad \bar{X} = \frac{1}{J}\sum_{j=1}^{J} X_j$$

and we ask ourselves (as Theil did): what are the consequences of committing the specification error of running a regression of $\bar{y}$ on $\bar{X}$, assuming that (2.1) is the "right model"? This yields the regression coefficient vector $a = (\bar{X}'U\bar{X})^{-1}\bar{X}'Uy$, where U is some positive definite matrix.

This question coincides with that arising from the assumption that at the same time the model:

$$(2.3)\quad \bar{y} = \bar{X}\alpha + v$$

(with v a T-dimensional distrubance vector having zero expection) applies and we enquire into the relation between the vector α and the micro parameters β_j.

Evidently, the generalized least squares estimate of the K-dimensional parameter vector α is (again):

$$(2.4)\quad a = (\bar{X}'U\bar{X})^{-1}\bar{X}'U\bar{y},$$

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[1] The main references regarding this subject are Theil (1954) and Kloek (1961). The hasty reader is referred to Theil (1971), pp. 556-562. Our exposition differs slightly from that of these authors.

where U is the inverse of (an estimate of) the covariance matrix of the disturbances. Our assumptions on the disturbances and on the explanatory variables allow us to state:

$$(2.5)\quad \alpha = Ea = E\{(\bar{X}'U\bar{X})^{-1}\bar{X}'U\bar{y}\} =$$
$$= (\bar{X}'U\bar{X})^{-1}\ \bar{X}'U\cdot\frac{1}{J}\cdot\sum_{j=1}^{J} X_j\beta_j = \sum_{j=1}^{J} W^j\beta_j,$$

where

$$(2.6)\quad W^j = \frac{1}{J}(X'UX)^{-1}X'UX_j.$$

Hence our assumption of different corresponding parameters $\beta_{k1}, \ldots, \beta_{kJ}$, of $k = 1, \ldots, K$, leads us to the conclusion that in general the G.L.S. estimator of the parameters of the macro model (2.4) does not yield unbiased estimates of the averages of the corresponding micro parameters β_{kj}.

About the matrices of weights W^j two observations appear to be in order. First, because of the definition of $\bar{X}$, the matrices W^j add up to the identity matrix I; this yields:

$$(2.7)\quad \sum_{j=1}^{J} w^j_{kk'} = \delta_{kk'},$$

where $\delta_{kk'}$ is the well-known Kronecker delta.
Second, we refer to the case where all the matrices X_j have identical k-th columns for one or more values of k. Often this applies to the intercept (all entries being 1) but also the k-th explanatory variable may be independent from individual characteristics (e.g., the price level or the interest rate). In that case:

$$(2.8)\quad w^j_{k'k} = \frac{1}{J}\cdot\delta_{k'k}$$

for $k' = 1, \ldots, K$ and $j = 1, \ldots, J$. This can easily be concluded from the fact that, then, the matrices $(\bar{X}'U\bar{X})^{-1}\bar{X}'U\bar{X} = I$ and $(\bar{X}'U\bar{X})^{-1}\bar{X}UX_j$ have identical k-th columns.

3. Illusion of money illusion

We rewrite the model (1.5) as follows:

$$(3.1)\quad y_{jt} = \beta_{0j} + \beta_{1j}x_{1jt} + \beta_{2j}x_{2jt} + \beta_{3j}x_{3t} + u_{jt}$$

where the meaning of the symbols is obvious. Notice that x_{3t} (the price level) is (supposed to be) the same for all individuals. Further we remember that:

$$(3.2)\quad \beta_{1j} + \beta_{2j} + \beta_{3j} = 1.$$

Because of (2.8) for k = 1 and k = 4:

$$(3.3)\quad W^j = \begin{bmatrix} \frac{1}{J} & w^j_{01} & w^j_{02} & 0 \\ 0 & w^j_{11} & w^j_{12} & 0 \\ 0 & w^j_{21} & w^j_{22} & 0 \\ 0 & w^j_{31} & w^j_{32} & \frac{1}{J} \end{bmatrix}$$

Hence:

$$(3.4)\quad \alpha_1 = \frac{1}{J}\sum_j \beta_{1j} + \sum_j (w^j_{11} - \frac{1}{J})\beta_{1j} + \sum_j w^j_{12}\beta_{2j},$$

$$(3.5)\quad \alpha_2 = \frac{1}{J}\sum_j \beta_{2j} + \sum_j w^j_{21}\beta_{1j} + \sum_j (w^j_{22} - \frac{1}{J})\beta_{2j},$$

$$(3.6)\quad \alpha_3 = \frac{1}{J}\sum_j \beta_{3j} + \sum_j w^j_{31}\beta_{1j} + \sum_j w^j_{32}\beta_{2j}.$$

From this we conclude:

$$(3.7)\quad \alpha_1 + \alpha_2 + \alpha_3 =$$

$$= 1 + \sum_j (w^j_{11}+w^j_{21}+w^j_{31}-\frac{1}{J})\beta_{1j} + \sum_j (w^j_{12}+w^j_{22}+w^j_{32}-\frac{1}{J})\beta_{2j}.$$

The second and third term of the right-hand member of (3.7) as well as their sum are in general non-zero, albeit that the sums $\sum_j (w^j_{1k}+w^j_{2k}+w^j_{3k}-\frac{1}{J})$ of the weights of the β_{1j} (for k=1) and the β_{2j}(k=2), are zero.

Consequently, what often has been considered as money illusion in empirical macro consumption functions of this kind may also be due to aggregation bias (or, perhaps, a combination of both phenomena).

The problem often is still more confused because in empirical analysis the aggregates are arithmetic means whereas the means we considered above are geometric ones. One can argue that using arithmetic means and assuming other micro models as we have in mind give rise to macro models as ours; this, however, causes still more aggregation problems.

4. Conclusion

We have seen that significant absence of zero homogeneity (in a well-defined statistical sense) in macro consumption functions, relating real consumption to real income and real net worth, does not need to be due to money illusion. Conversely, in case of zero homogeneity one can not conclude that there is no money illusion.

A case in which within the context of the models considered above conclusions on money illusion can rightly be made is the case of all individuals having identical corresponding parameters; this seems to be a very unrealistic case.

The aggregation bias in (3.7) can be considered as a sum of covariances, as has been pointed out by Theil (1954). This and some economic arguments on dependencies can perhaps give a rough a priori indication of sign and magnitude of the bias; see Allen (1956), Ch. 20.

Once again, the preceding considerations call for more micro data on consumption, income, wealth etc. preferably in "panel form", and for more sophisticated aggregates, such as geometric or generalised harmonic means, as well as for more research on micro (consumption) functions.

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