Netherlands school of economics
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A NOTE ON THEIL'S DEVICE FOR CHOOSING A BLUS BASE

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by C. Dubbelman

In order to choose one base from a set of bases for BLUS vectors in the linear model, Theil (1971), p. 218, recommends a procedure. As a by-product of a more general power comparison, we examined the usefulness of Theil's device in BLUS tests against either positive or negative autocorrelation. In our applications the device does not work.

We denote the model by $y = X\beta + u$, where X is a fixed $n \times k$ matrix with rank k, (we take $n = 15$ and $k = 3$) and $u \sim N(0,\sigma^2\Gamma)$ with

$$\Gamma = \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{14} \\ \rho & 1 & \rho & \dots & \rho^{13} \\ \rho^2 & \rho & 1 & \dots & \rho^{12} \\ \vdots & & & & \vdots \\ \rho^{14} & \rho^{13} & \rho^{12} & \dots & 1 \end{bmatrix}$$

The null hypothesis is $\rho = 0$, the alternative hypothesis is either $\rho = 0.7$ or $\rho = -0.7$.

Each BLUS vector estimates $n - k = 12$ elements of the 15-element vector u. Let $J'u$ be the 12-element vector of elements of u, which are estimated, then the BLUS vector is $(J'MJ)^{-\frac{1}{2}}J'My$, where $M = I - X(X'X)^{-1}X'$. The 15×12 matrix J consists of 12 columns of $I_{(15)}$, and the base of a BLUS vector is represented by the numbers of those three columns of $I_{(15)}$ which are not included in J. In accordance with the device, the class of admissible bases consists of four bases, namely (1,2,3), (1,2,15), (1,14,15), and (13,14,15). We denote the corresponding BLUS vectors by a, b, c, and d, respectively. Hence, each of a, b, c, and d estimates 12 successive elements of u; while a does not estimate the elements 1, 2 and 3; b does not estimate the elements 1, 2 and 15; and so on. Given a BLUS vector, let S denote the sum of the positive square roots of the eigenvalues of $X_0(X'X)^{-1}X_0'$, where X_0 is the 3×3 matrix consisting of those rows of X which are not included in $J'X$. The device tells us to adopt that base for which S is maximal. For instance, for $X = X_C$ we found $S = 1.324$ for a, $S = 1.529$ for b, $S = 1.522$ for c, and $S = 1.011$ for d, so that BLUS vector b is to be used in tests against autocorrelation. We arranged the vectors according to non-increasing order of their corresponding *S*-values, as follows.

BLUS vectors, arranged according to *S*

X	selected vector	remaining vectors		
X_C	*b*	*c*	*a*	*d*
X_D	*c*	*b*	*a*	*d*
X_K	*b*	*c*	*d*	*a*
X_S	*b*	*c*	*d*	*a*
X_T	*c*	*b*	*d*	*a*

The powers at significance level 0.05 of all four BLUS tests against autocorrelation are presented in the table below, in exactly the same arrangement as the BLUS vectors in the first table.

Powers of the BLUS tests against autocorrelation

X	selected vector	remaining vectors		
		$\rho = 0.7$		
X_C	0.388	0.399*	0.366	0.374
X_D	0.418	0.387	0.422	0.453*
X_K	0.375	0.357	0.353	0.423*
X_S	0.401	0.404	0.497*	0.480
X_T	0.295	0.292	0.361*	0.316
		$\rho = -0.7$		
X_C	0.552*	0.540	0.475	0.486
X_D	0.488	0.457	0.511*	0.503
X_K	0.448	0.418	0.435	0.585*
X_S	0.513	0.416	0.613*	0.553
X_T	0.410	0.450	0.591*	0.490

The maximum in each row is marked by an asterisk. The device is designed to choose a BLUS base such that it "leads to a power which is not too far below the attainable maximum". The number of asterisks in the first (and also in the second) column is strikingly low, however. And with respect to the row maxima, the average loss of power when adopting the device is 0.051 if $\rho = 0.7$ (and 0.088 if $\rho = -0.7$), while such simple devices as: do not estimate the first k elements of u (i.e. use a) or, do not estimate the last k elements of u (i.e. use d), lead to average losses of power of 0.025 and 0.019 if $\rho = 0.7$ (and 0.047 and 0.045 if $\rho = -0.7$), respectively, which is about half as much as the loss inflicted by Theil's device.

All five X-matrices contain a constant term column. The remaining two columns of X_D are data from Durbin and Watson (1951), Table 1, namely log real income per capita and log relative price of spirits, 1870-1884. The matrices X_C, X_K, X_S, and X_T are described in Dubbelman (1972), p. 427.

References

Dubbelman, C. (1972), A priori fixed covariance matrices of disturbance estimators. European Economic Review 3, 413-436.

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