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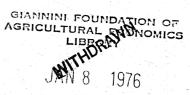
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ON THE-KALMAN FILTER AND THE ECONOMETRIC GENERAL LINEAR MODEL

by Michiel Hazewinkel

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In this note we show how a particular econometric estimator for the general linear model (the one best adapted to the noise present in the observations) arises as the limit of Kalman state estimators of a discrete dynamical system with trivial dynamics.

1. First consider a discrete dynamical system given by the equations

(1)

$$x_{k+1} = A_k x_k + B_k u_k + f_k + C_k w_k$$

$$y_k = H_k x_k$$

$$z_k = y_k + v_k$$

where x_k is an n-dimensional state vector; u_k are deterministic controls; f_k a deterministic forcing vector; w_k is random noise in the system; y_k is the p-dimensional output vector; z_k are the observations of the y_k corrupted by random noise v_k ; A_k , B_k , C_k , H_k are matrices of appropriate sizes, which are assumed to be known. One further makes the assumptions

(2)
$$E(w_k) = 0 = E(v_k), E(w_k w_l^T) = \delta_{kl}Q_k, E(v_k v_l^T) = \delta_{kl}R_k, E(w_k v_l^T) = 0$$

where E denotes expectation and T denotes transposition. The covariance matrices Q_k and R_k are also assumed to be known. Given all this, the Kalman filter with starting values $\hat{x}_0 = 0$, P_0 is given by the equations (cf. e.g. [2]).

(3)

$$P'_{k} = A_{k-1}P_{k-1}A_{k-1}^{T} + C_{k-1}Q_{k-1}C_{k-1}^{T}$$

$$P_{k} = P'_{k} - K_{k}H_{k}P'_{k}$$

$$K_{k} = P'_{k}H_{k}^{T}(H_{k}P'_{k}H_{k}^{T} + R_{k})^{-1}$$

$$\hat{x}'_{k} = A_{k-1}\hat{x}_{k-1} + f_{k-1} + B_{k-1}u_{k-1}$$

$$\hat{x}_{k} = \hat{x}'_{k} + K_{k}(z_{k}-H_{k}\hat{x}'_{k})$$

2. Now consider the general linear model as it is often used in econometrics

$$(4) z = Hx + v$$

(H is usually written X, and x is usually written β ; cf. [1] for a discussion of the linear model), where x is to be estimated from the observations z and v is random noise with E(v) = 0, $E(vv^{T}) = R$, where R is nonnegative definite.

We write the model as (a rather trivial) discrete dynamical system

(5)
$$x_{k+1} = x_k, y_k = Hx_k, z_k = y_k +$$

Now we are going to apply the discrete Kalman filter (3) to it starting with $\hat{x}_0 = 0$ and an arbitrary initial covariance matrix P_0 . Using a double induction one finds

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(6)
$$K_1 = P_0 H^T (HP_0 H^T + R)^{-1}, K_n = P_0 H^T (nHP_0 H^T + R)^{-1}$$

It follows from this that

(7)
$$K_n = K_{n-1} - K_n H K_{n-1}$$

and hence, using induction again, that the estimate for \hat{x}_n is equal to

(8)
$$\hat{\mathbf{x}}_{n} = n \mathbf{P}_{o} \mathbf{H}^{T} (n \mathbf{H} \mathbf{P}_{o} \mathbf{H}^{T} + \mathbf{R})^{-1} \mathbf{z}$$

3. We are interested in what happens as n goes to infinity. There are (at least) two interesting cases.

<u>Case A</u>: $HP_{O}H^{T}$ is nonsingular. (I.e. in any case less outputs than the dimension of the system). In this case the R in equation (8) can be neglected as n goes to infinity and we find the estimator

(9)
$$\hat{\mathbf{x}} = \mathbf{P}_{O}\mathbf{H}^{T}(\mathbf{H}\mathbf{P}_{O}\mathbf{H}^{T})^{-1}\mathbf{z}$$

<u>Case B</u>. R is nonsingular, $p \ge n$, rank(H) = n. This is what is usually assumed in the econometric general linear model. Premultiplication with $H^{T}R^{-1}H$ of the matrix in (8) gives

(10)
$$(H^{T}R^{-1}H)(nP_{O}H^{T})(nH^{T}P_{O}H^{T}+R)^{-1} = H^{T}R^{-1} - H^{T}(nHP_{O}H^{T}+R)^{-1}$$

Now

(11)
$$\lim_{n \to \infty} H^{T}(nHP_{O}H^{T}+R)^{-1} = 0$$

which is seen as follows. Because $rank(H^{T}) = rank(HP_{O}H^{T})$ it suffices to prove that $HP_{O}H^{T}(nHP_{O}H^{T}+R)^{-1}$ goes to zero. But

(12)
$$HP_{O}H^{T}(nHP_{O}H^{T}+R)^{-1} = n^{-1}I - n^{-1}R(nHP_{O}H^{T}+R)^{-1},$$

and one easily sees that the terms in $(nHP_OH^T+R)^{-1}$ are bounded independantly of n (e.g. by diagonalizing HP_OH^T). This proves (11). Using (10) and (11) in (8) it follows that in case B one obtains the limit estimator

(13)
$$\hat{\mathbf{x}} = (\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{H})^{-1}\mathbf{H}^{T}\mathbf{R}^{-1}\mathbf{z}$$

which is the econometric generalized least squares estimator with weighting matrix equal to $R = E(vv^{T})$.

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