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EVIDENCE ON THE SLUTSKY CONDITIONS FOR DEMAND EQUATIONS

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Contents

1. INTRODUCTION

The modern theory of consumer demand has been in the core of economic theory from its very beginning around the 1870's. Somewhat earlier (1857) Engel [6] published his study for the Kingdom of Saxony, which marks the start of systematic measurement of consumer behaviour. Both theory and measurement have developed enormously nice their beginning. Remarkably enough, the links between these two branches of study of consumer demand have remained rather weak. Nobody will deny the importance of those classics on theory and measurement of consumer demand as the monumental monographs by Schultz [15], Wold [22] and Stone [17], but even there the relation between the exposition of the theory and the derivation of the empirical results is frequently superficial only. In this connection we can quote Cramer [4]:

> "The pure economic theory of consumer behaviour has recently developed into an elaborate, precise and highly technical piece of abstract reasoning. We do not believe that in its present form the theory can contribute much to empirical research: it is insufficiently specific to yield fruitful hypotheses, and not well enough established in fact to be applicable to observed phenomena."

This is a rather pessimistic statement. Is it really true that the properties of demand equations derived from economic theory cannot be confronted with empirical evidence in any meaningful way? This question can be answered at many different levels and the answer is not necessarily positive. There is some truth, however, in another statement by Cramer, that "the more sophisticated proportions of pure theory, like the Slutsky condition, which appeal less readily to common sense and introspection, have rarely been fruitful in empirical work," as long as one means by fruitful: subjected to empirical research. The Slutsky condition, or rather conditions, have been used on several occasions as effective restrictions on empirical demand equations, as is obvious from the work on linear expenditure systems by Stone [18] and Leser [12].

In this article we will try to present some evidence on the Slutsky conditions for demand equations. These conditions have never been subjected to a formal test. This is rather astonishing considering the fact that the Slutsky conditions are the cornerstones of modern demand theory. Such a situation is not uncommon in economics, however. Many statements are generally accepted as being a true description of behaviour without a thorough questioning of their empirical validity. How many economists have for practical applications of the theory of the firm assumed constant returns to scale? Only recently deviations from this state of long-run equilibrium are being considered.

First, we will give a brief outline of the current state of the classical theory of consumer demand. We will formulate theoretical restrictions on the demand equations, which together will be denoted by the term Slutsky conditions. Next, we will specify a type of demand equation which allows us to test these conditions. That is, the Slutsky conditions are not to be imposed on the estimation procedure, so that we can check whether the resulting estimates are in accordance with these conditions. Finally, we will present results which refer to a four-equation system of consumer demand in the Netherlands in the periods 1922-1939 and 1949-1961. To group all commodities in four composites means a high level of aggregation, which however has no serious consequences for the purpose of this paper. Without aggregation we would have many more explanatory variables in our system than we have observations, which would make estimation in the usual sense impossible. Therefore, aggregation is essential and not only a means of presenting data in a condensed way.

2. AN OUTLINE OF DEMAND THEORY

As is well-known a number of axioms on the ordering of preferences (see for instance Debreu [5]) lead to the possibility of formulating a differentiable utility function

$$u = u(q_1, \dots, q_n) \qquad (2.1)$$

where u is utility or satisfaction derived from a certain bundle of quantities of n commodities and q_i ($i = 1, \dots, n$) is the quantity purchased of commodity i in a certain time interval. We will assume that the utility function has only positive first derivatives in the relevant range, meaning absence of satiation, and that it possesses second derivatives, which are continuous functions of their arguments. The axiom of rationality says that the consumer will maximize his utility. His resources are given and limited. This is expressed by the budget constraint, viz.:

$$\sum_i p_i q_i = m \tag{2.2}$$

where p_i (always positive) is the price of commodity i and m is total expenditure or, loosely said, income. Utility maximization under the budget constraint leads to the famous second law of Gossen

$$\frac{\partial u}{\partial q_i} = \lambda p_i \qquad (i = 1, \ldots, n) \tag{2.3}$$

At the left-hand side of (2.3) we have the marginal utility of commodity i, while λ, mathematically speaking a Lagrangean multiplier, can economically be interpreted as the marginal utility of income.

Equations (2.2) and (2.3) are n + 1 equations in 2(n + 1) variables: the n quantities of the commodities, the n prices, λ (the marginal utility of income) and income itself. For a given set of numerical values for income and the prices we can find a unique solution for the quantities of the n commodities, which form together the optimal bundle, and for λ, if the second order conditions for a constrained maximum are fulfilled. We will now concentrate on the resulting demand equations, which in their most general form can be written as

$$q_i = q_i(m, p_1, \ldots, p_n) \qquad (i = 1, \ldots, n) \tag{2.4}$$

In the course of development of the theory of demand a number of restrictions on these equations have been formulated. Some of these restrictions are almost trivial; others are not obvious at all. These restrictions are mainly properties of the partial derivatives of (2.4) with respect to income and the prices or can be formulated as such. First of all we have the <u>adding-up property</u>

$$\sum_i p_i \frac{\partial q_i}{\partial m} = 1 \tag{2.5}$$

which implies that an increase in total expenditure is completely allocated to all commodities in the budget. Secondly, we have the break-down of the partial derivatives with respect to the price in two components:

$$\frac{\partial q_i}{\partial p_j} = k_{ij} - \frac{\partial q_i}{\partial m} q_j \tag{2.6}$$

of which the first represents the well-known substitution effect and the second term corresponds with the income effect. These names have been coined by Allen [1, 2] and Hicks [9, 10]. The break-down itself has first been proposed by Slutsky [16]. This decomposition is not a real restriction in itself. Its importance lies in the restrictions on the k_{ij}, the substitution terms. First of all the substitution terms for one commodity multiplied by the appropriate prices add up to zero:

$$\sum_j k_{ij} p_j = 0 \tag{2.7}$$

which we call the homogeneity property, since it ensures in combination with (2.6) that an equal relative increase in all prices and income leaves the amounts of commodities purchased unchanged. It implies the absence of a monetary veil. There are n of these restrictions, for each demand equation one. An even more powerful set of restrictions is the one provided by the symmetry property

$$k_{ij} = k_{ji} \tag{2.8}$$

We have $n \times n$ k_{ij}-terms, because we have n commodities in the bundle and n prices. The symmetry property is only non-trivial for those k_{ij}-terms, where $i \neq j$, i.e. for the substitution parts of the partial derivative with respect to another than the own price. There are $n^2 - n$ of these k_{ij}-terms. The symmetry property reduces the number of "independent" k_{ij}-terms ($i \neq j$) by one half. Finally we have perhaps the most important restriction of all: the negativity property for the substitution part of the own price derivative, viz.:

$$k_{ii} < 0 \tag{2.9}$$

The consequence of this restriction is that in the case where the income derivative in (2.6) is positive or zero, i.e. when the commodity in question is not an inferior commodity, increase in the price of this commodity leads to a decrease in the quantity bought. This provides the analytical justification of the intuitive notion of the negative relationship between quantity and price.

The properties (2.5), (2.7), (2.8) and (2.9) will be called together the Slutsky conditions, since we can attribute to this mathematical economist the honour of having been the first to state them explicitly. It may be remarked in passing that the Slutsky conditions are not dependent on the choice of a particular utility function. They follow from all utility functions which describe a preference ordering. In this sense these properties are ordinal.

The decomposition (2.6) enables us to form the following expression for the differential of the demand equations

$$\text{(2.10)} \qquad dq_i = \frac{\partial q_i}{\partial m}\, dm + \sum_j \frac{\partial q_i}{\partial p_j}\, dp_j =$$

$$= \frac{\partial q_i}{\partial m}\,(dm - \sum_h q_h dp_h) + \sum_j k_{ij} dp_j$$

where h, i, j = 1, ..., n. This is the differential of the demand equation for the individual consumer. Adding up the demand for one commodity over all consumers considered, the same formulation with the same properties for the partial derivatives can be retained for the aggregate demand equation, as long as the price changes are the same for all decision units. Usually the variables in the aggregate demand equation are arithmetic averages over the whole population or in more familiar terms demand is demand per capita and income is income per capita.

One might argue that this type of explanation of demand behaviour is remote of what happens on the market place. A number of axioms are used, assumptions are made and more will be made in the sequel, which are at best a very severe smoothing of actual motivations and conditions. This is true for many econometric experiments. However approximate the line of reasoning presented here may be to actual decision making, we can always check whether the results of this theory are rejected by the empirical data or not. If not, one cannot invoke the argument that the procedure sketched above does not describe actual behaviour.

3. THE SPECIFICATION OF THE DEMAND EQUATIONS

We will choose that form of demand equation which enables us to shed some empirical light on the Slutsky conditions. The properties of the estimated coefficients should be readily comparable with the Slutsky conditions. The collective version of (2.10) i.e. the one with quantity demanded per capita and income per capita will be our starting point. Using the equality $dx = x\,d\log x$, where log denotes natural logarithms, it can be shown that (2.10) is equivalent with

$$\text{(3.1)} \qquad w_i d \log q_i = B_i(d \log m - \sum_k w_k d \log p_k) + \sum_j S_{ij} d \log p_j$$

where $w_i = p_i q_i/m$, the share of expenditure on commodity i in total expenditure, $B_i = p_i(\partial q_i/\partial m)$ and $S_{ij} = (1/m)p_i p_j k_{ij}$. The term in parentheses is the logarithmic change in real income. From (2.5) follows the transformed adding-up property

$$\text{(3.2)} \qquad \sum_i B_i = 1$$

while the new version of the homogeneity property is given by

$$(3.3) \qquad \sum_j S_{ij} = 0$$

The transformation from k_{ij} to S_{ij} leaves the symmetry untouched, hence

$$(3.4) \qquad S_{ij} = S_{ji}$$

Finally, because prices and income are strictly positive

$$(3.5) \qquad S_{ii} < 0$$

which is the new version of the negativity property. Equations (3.2) - (3.5) form the new set of Slutsky conditions. We will treat the B_i and S_{ij} as constants.

The observations on the variables refer to discrete amounts of time, say quarters or years. Therefore, we have to replace in (3.1) the differentials by finite differences, showing the changes in the logarithms of quantities, income and prices from one period to the other. Apart from the introduction of some approximation errors this causes no complication. We have, however, still to decide upon the period for which the value share w_i is evaluated. We can take w_i at its value in the earlier period or at its value in the current one. Both alternatives involve asymmetry. Following Theil [21], we choose the arithmetic average because of its symmetry. We define

$$(3.6) \qquad \bar{w}_{it} = \tfrac{1}{2}(w_{i,t} + w_{i,t-1})$$

where t denotes the current period of time. Upon changing to finite differences and replacing w_i by $\bar{w}_{it}$ in (3.1) we obtain

$$(3.7) \qquad \bar{w}_{it}\Delta \log q_{it} = B_i\Delta \log \bar{m}_t + \sum_j S_{ij}\Delta \log p_{jt}$$

where

$$(3.8) \qquad \Delta \log \bar{m}_t = \Delta \log m_t - \sum_h \bar{w}_{ht}\Delta \log p_{ht}$$

is the approximate change in real income.

It can be shown that with a reasonable degree of approximation

$$(3.9) \qquad \sum_i \bar{w}_{it}\Delta \log q_{it} = \Delta \log \bar{m}_t$$

One of the consequences of (3.9) is that the sum of the left-hand sides of the demand equations is equal to the value of a variable appearing in all equations on the right. This leads to the fact that for any sample the adding-up property holds identically apart from an approximation error and that the sum over i of the S_{ij}-coefficients is approximatively zero. If one has determined the values of the B_i- and S_{ij}-coefficients for n - 1 equations, the values of the coefficients of the remaining equation follow immediately.

When using classical types of estimation procedures like least-squares we need a number of observations which exceeds or equals the number of explanatory variables. The number of different commodities in any bundle is in principle extremely large. However, statistical information allows usually for a limited degree of detail and is restricted to a relatively small number of composite commodities, say one hundred or so. Even that number exceeds easily the number of observations, since consistent time series of economic phenomena are mostly very short. Therefore, the number of variables per equation has to be reduced by means of aggregation over commodities. In our case, for instance, we will distinguish four composites: food, pleasure goods, durables, and other commodities and services, shortly named remainder. By means of appropriate definitions of quantity and price index numbers and a not too restrictive assumption[1] on actual price behaviour we arrive at the demand equation for the g-th composite commodity

$$(3.10) \qquad \bar{w}_{gt}\Delta \log q_{gt} = B_g \Delta \log \bar{m}_t + \sum_{g'} S_{gg'} \Delta \log p_{g't}$$

Here

$$B_g = \sum_{i\in C_g} B_i \qquad S_{gg'} = \sum_{i\in C_g} \sum_{j\in C_{g'}} S_{ij}$$

The B_g and $S_{gg'}$ fulfill the Slutsky conditions, at least in theory.

[1] The definitions used are

$$\Delta \log q_{gt} = \sum_{j\in C_g} \bar{\omega}_{jt}\Delta \log q_{jt}; \; \Delta \log p_{gt} = \sum_{j\in C_g} \bar{\omega}_{jt}\Delta \log p_{jt}$$

where $\bar{\omega}_{jt} = (w_{j,t} + w_{j,t-1})/(w_{g,t} + w_{g,t-1})$, i.e. the average value share of j divided by the average share of expenditure on the composite g - to which j belongs - in total expenditure. The relevant assumption is

$$\sum_{j\in C_g} (S_{ij} - S_{ig}\bar{\omega}_j)(\Delta \log p_{jt} - \Delta \log p_{gt}) = 0$$

with S_{ig} being the sum of all S_{ij} for which j belongs to g. This is true if for all $j \in C_g$, $\Delta \log p_{jt}$ equals $\Delta \log p_{gt}$ (parallel price movement - the Hicksian condition for aggregation). It is also true if $S_{ij} = S_{ig}\bar{\omega}_j$ $(j \in C_g)$. Furthermore it holds if there is no covariance between fluctuations of the S_{ij} around $S_{ig}\bar{\omega}_j$ and fluctuations of $\Delta \log p_{jt}$ around $\Delta \log p_{gt}$. There is no reason to expect the opposite of this last proviso. As far as any of these conditions is only approximately true an error is involved in the transition from (3.7) to (3.10). This error is treated as a disturbance (see Section 4).

In the next section we will turn to the question whether they also do in reality.

4. THE ESTIMATES OF THE COEFFICIENTS OF THE DEMAND EQUATIONS[2]

We will add to the demand equation (3.10) for composite g an intercept coefficient and a disturbance term. We now have

$$(4.1) \qquad \bar{w}_{gt}\Delta \log q_{gt} = A_g + B_g\Delta \log \bar{m}_t + \sum_{g'} S_{gg'}\Delta \log p_{g't} + u_{gt}$$

The A_g represents the effect of a trendlike shift in preferences, while the u_{gt} as a disturbance term acts like a catch-all for the effects of factors other than trends, real income or prices and furthermore takes care of all observational and approximation errors. Since

$$(4.2) \qquad \sum_g \bar{w}_{gt}\Delta \log q_{gt} = \Delta \log \bar{m}_t$$

at least approximately, the sum of all trend terms and disturbance terms as well as the sum over g of all $S_{gg'}$ coefficients is identically equal to zero, again only to a reasonable degree of approximation. This means that if values for the coefficients of all but one of the demand equations of type (4.1) are given, those for the last equation can be found immediately.

First, we will estimate the coefficients of (4.1) without the use of the restrictions on their values implied by the Slutsky conditions. The method of single-equation least-squares is under the assumptions of homoscedasticity (constant variance) and intertemporal independence the optimal procedure. The data used are time series on expenditure and prices of private consumption in the Netherlands, based on various published and unpublished sources. From the detailed information 16 basic series have been constructed covering the years 1922-1939 and 1949-1961. These 16 groups have again been aggregated to four composites: food (groceries, dairy products, vegetables and fruits, meat, fish and bread), pleasure goods (confectionery and ice-cream, beverages, tobacco products), durables (clothing and other textile, footwear, household durables, other durables) and remainder (water, light and heating, rent, other commodities and services, for the greater part being services). The estimates for the coefficients with their standard errors are specified in Table 1.

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[2] The author is obliged to Mr. P.C. Kroes for his research assistance.

TABLE 1. UNCONSTRAINED LEAST-SQUARES ESTIMATES[a]

Composite	A_g	B_g	$S_{gg'}$				R^2
1. Food	.0098 (.1337)	14.61 (3.85)	-10.39 (2.78)	-1.23 (6.27)	3.71 (2.70)	12.23 (6.28)	.636
2. Pleasure Goods	.0068 (.0520)	8.57 (1.50)	2.27 (1.08)	-2.28 (2.44)	.09 (1.05)	1.14 (2.44)	.689
3. Durables	-.2816 (.1260)	50.92 (3.63)	5.80 (2.63)	-6.05 (5.91)	-5.59 (2.54)	2.27 (5.92)	.916
4. Remainder	.2651 (.1205)	25.81 (3.47)	2.56 (2.51)	9.47 (5.65)	1.57 (2.43)	-15.51 (5.66)	.719

[a] The A_g, B_g and $S_{gg'}$ have all been multiplied by 100. Standard errors are given in brackets.

In commenting upon these results it can be said that the correlation coefficients are not very low for such a relatively simple model. As is easily checked the A_g sum up to almost zero, the B_g to almost one. The vertical sums of the $S_{gg'}$ are farther from zero, which is the consequence of the approximation involved in (4.2). These sums are relatively very small, however. Division of the B_g by the corresponding value shares for a certain year gives the values of the income elasticities. Using for this purpose the year 1961 we find the following income elasticities: .55 for food, .85 for pleasure goods, 1.87 for durables and .72 for remainder. These results are in fair accordance with a priori expectations concerning income elasticities. All the S_{gg}-coefficients are negative. Therefore, the negativity property holds also for the estimates! What about the homogeneity condition? The horizontal sums (multiplied by 100) of the $S_{gg'}$ in Table 1 are 4.33 (3.10) for food, 1.22 (1.20) for pleasure goods, -3.57 (2.92) for durables and -1.91 (2.80) for remainder. Standard errors of these sums (also times 100) have been given in parentheses. These sums are not zero. However, these standard errors are so large in a relative sense that we may conclude that the empirical data used here are not inconsistent with the homogeneity property. That is, we have not been able to find with any worthwhile precision the effect of a monetary veil. Finally we can ask whether the results obey the symmetry condition. The differences between the hypothetically equal coefficients, together with their standard errors, are presented in Table 2.

TABLE 2. DEVIATIONS FROM SYMMETRY[a]

$S_{12} - S_{21}$	-3.50 (6.40)	$S_{14} - S_{41}$	9.67 (6.56)
$S_{13} - S_{31}$	-2.09 (3.20)	$S_{24} - S_{42}$	-8.33 (5.72)
$S_{23} - S_{32}$	6.15 (5.90)	$S_{34} - S_{43}$	.70 (6.37)

[a] All values multiplied by 100.

Also here the standard errors are in a relative sense so high that there seems no reason to reject the empirical validity of the symmetry condition. The Slutsky conditions are not to be rejected as unrealistic on the basis of this experiment.

A few comments on the trend terms. They seem negligible for food and pleasure goods, but not for durables and remainder. In the latter cases the estimates imply a downward trend of about one percent annually for demand for durables and an upward trend of around 3/4 percent annually for demand of remainder. The negative trend for durables seems to conflict with prior notions in this respect. A possible explanation of this result is that inventories of durables have increased considerably over time. Existence of inventories reduces the need for additional purchases. This seems to be borne out by the well-known experience that the durables market suffers severely when the increase of real income slows down or even turns to be negative; frequently this is a more severe set-back than can be explained from the income term only.

The testing procedures discussed above are not independent. Given the fact that the vertical sums of the $S_{gg'}$ in Table 1 are automatically almost zero, the validity of the symmetry properties implies the validity of the homogeneity condition. Furthermore, one is not really interested in the extent to which each individual coefficient or subset of coefficients meets the relevant Slutsky condition, but rather in the overall picture. Do the Slutsky conditions hold in general? To answer this question we have constructed a composite null hypothesis. It consists of two subsets: one is that all six symmetry relations hold simultaneously, the other that there are not trendlike shifts i.e. three out of the four A_g are zero; the remaining one is then zero too. Under this null hypothesis and the assumption of a multivariate normal distribution for the disturbances a F statistic can be constructed with 9 and 69 degrees of freedom (see Roy [14] p. 82). We have 9 degrees of freedom on the one hand, because there are 9 independent relations in the null hypothesis. We have 69 degrees of freedom on the other hand, since there are three independent equations in our system, each with 6 coefficients, while the number of observations is 29. The covariance matrix of the disturbance terms for the different equations is not known; it has to be estimated from the sample also. Our statistic is therefore only asymptotically, i.e. for an infinite number of observations, equal to the true F-value. The importance of our result: F = 1.50 should not be overstressed, therefore. However, $\hat{F}$ = 1.50 is sufficiently low[3] to conclude, that the composite null hypothesis is not flagrantly inconsistent with the empirical data. If the null hypothesis had only consisted of the symmetry relations, our $\hat{F}$ would have been more favorable for the null hypothesis than is the case now.

It is of some interest to determine the estimates when the null

[3] F = 1.50 corresponds with an upper percentage point of slightly less than 17 percent.

hypothesis is imposed on the estimation procedure. Since the symmetry condition involves coefficients of different equations, all equations have to be estimated simultaneously. As Zellner [23] has pointed out a generalized least-squares procedure of joint estimation produces efficient estimates. This method requires the inverse of the covariance matrix of the disturbance terms. In our case, however, this covariance matrix is singular and cannot be inverted, because the disturbance terms are functionally linear dependent. By using the generalized inverse of the covariance matrix[4] in question this problem was solved in an optimal way. The estimates derived under the constraints implied by the null hypothesis to which has been added the constraint that all vertical sums are exactly one (in the case of the B_g's) or zero (in the case of the $S_{gg'}$-coefficients) are presented in Table 3.

TABLE 3. CONSTRAINED JOINT LEAST-SQUARES ESTIMATES[a]

Composite	B_g	$S_{gg'}$				R^2
1. Food	17.63 (3.73)	-10.64 (3.20)	2.19 (1.20)	4.37 (2.41)	4.08 (2.51)	.579
2. Pleasure Goods	8.84 (1.67)	2.19 (1.20)	-2.81 (2.32)	.26 (1.17)	.36 (2.13)	.673
3. Durables	48.53 (3.51)	4.37 (2.41)	.26 (1.17)	-4.47 (2.75)	-.16 (2.04)	.888
4. Remainder	25.00 (3.50)	4.08 (2.51)	.36 (2.13)	-.16 (2.04)	-4.29 (3.26)	.641

[a] The B_g and $S_{gg'}$ have all been multiplied by 100. Standard errors are given in brackets.

To produce Table 1 4 × 6 coefficients have been estimated. If (4.2) would have held exactly, the coefficients of one equation could have been found from the values of the coefficients of the other equation. If one ignores the small approximation error we have in Table 1 only 3 × 6 coefficients which had to be estimated by least-squares. Only 9 coefficients in Table 3 have been estimated directly. The symmetry conditions have reduced the number of direct estimates by 6. The deletion of the trend terms accounts for the other three. In general, leaving trend terms aside, we have $(n - 1) \times (n + 1) = n^2 - 1$ coefficients to estimate when the Slutsky conditions are not imposed. The Slutsky conditions and in particular the symmetry conditions reduce this number by $\frac{1}{2}(n^2 - n)$ coefficients.

[4] The derivation of the generalized inverse is in our case relatively simple. To each element of the covariance matrix a small positive number, say α, divided by the number of equations, say G, was added. From the elements of the inverse of this matrix $1/G\alpha$ was subtracted to obtain the generalized inverse needed.

5. CONCLUDING REMARKS

The results presented in the preceding section seem to justify the use of the Slutsky conditions for the estimation of demand equations. The imposition of the Slutsky conditions in the way presented here is not a simple matter for somewhat larger and hence more interesting systems of demand equations than the one discussed here. If our system contains 20 equations 209 coefficients have to be estimated jointly. Even for large computers this might prove to be a formidable task. Furthermore, the probably high degree of multicollinearity of the time series of the prices might frustrate any attempt at getting reliable results even if the capacity of the computer is large enough to invert a matrix of 209 × 209. We need additional constraints on the estimation procedure. These can be provided by the use of prior notions about the interrelation of the commodities with respect to the satisfaction of wants like assumptions of want independence (Frisch [7], Houthakker [11]), utility trees (Strotz [19, 20], Gorman [8], Pearce [13]) or almost additivity (Barten [3]). The day is not yet close, however, when we have really detailed descriptions of consumer behaviour in the form of demand equations which comply with economic theory, that is, which satisfy the conditions put forward by Slutsky almost exactly fifty years ago.

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