STOCHASTIC DOMINANCE: THE STATE OF THE ART IN AGRICULTURAL ECONOMICS

by

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In recent years the family of stochastic dominance techniques has become a very popular way to rank alternative risk management strategies consistent with the Expected Utility Hypothesis (EUH). As such, much of the success and the shortcomings of stochastic dominance can be related to its foundation in the EUH. This paper will attempt to survey the use of stochastic dominance in the field of agricultural economics, discuss theoretical bases, and describe some recent developments in the application of risk analysis with stochastic dominance. The paper is divided into several different sections. The theoretical foundations, the axioms of the EUH, and technique descriptions will be presented first. The second section will review the recent experience in agriculture to elicit risk preferences. This will be followed by a brief comparison of stochastic dominance with other methodologies such as E-V analysis, E-S_h analysis, Target Motad and Mean Gini Analysis. The final section will focus on some of the problems of implementing stochastic dominance and a review of selected works in progress designed to resolve some of these difficulties.

Much of the discussion will focus on the trade-offs between Type I and Type II errors. A Type I error will correspond to a situation where an inaccurate ranking of the alternative management strategies has occurred. That is to say that the analysis results in a conclusion that alternative A is preferred to alternative B when in reality B is preferred to A or there is no evidence to support preference of either alternative. A Type II error will represent the case where the analysis does not detect a preference between the two alternatives when in actuality the class of decision makers prefers one to the other. In stochastic dominance, this corresponds to a large efficient set. In most cases a reduction of the probability of one type of error is accompanied by an increase in the probability of the other type of error.

The Theoretical Foundations of Stochastic Dominance

As mentioned in the introduction, all stochastic dominance techniques are ways to rank alternative strategies consistent with the EUH. The EUH is one of the most commonly used models to guide decision making under uncertainty. Its strength lies in the framework it provides for rationally addressing choices. With origins in the works of Bernoulli, the French mathematician, its prominence emerged with the axiomatization presented by von Neumann and Morgenstern (Machina,1981). These axioms have been widely discussed and debated, but usually they can be depicted to consist of the following:

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1.) Transitivity : If there exists three lotteries, l_1, l_2, and l_3, and if l_1 is preferred to l_2 and l_2 is preferred to l_3; then l_1 is preferred to l_3.

2.) Continuity : If an individual has a preference for lottery l_1 over l_2 and l_2 over l_3; then there exists some unique probability, p, such that he is indifferent between receiving lottery l_2 and a super lottery with probability 1-p of receiving l_1 and probability p of receiving l_3.

3.) Independence : If lottery l_1 is preferred to lottery l_2, and there exists another lottery l_3; then a super lottery with l_1 and l_3 as prizes will be preferred to another super lottery with l_2 and l_3 as prizes as long as the probabilities of receiving l_1 and l_2 are equal.

Much of the debate of the utility of the EUH has centered on the Independence and Transitivity axioms. The Allais paradox, in particular, has challenged the Independence axiom. Several appraisals of the EUH have been published in the last several years. Schoemaker (1982) discusses the EUH in the context of the uses that can be made of it. He identifies several uses of the EUH: descriptive, predictive, prescriptive and postdictive. The overall evaluation of the EUH is complicated by the existence of a number of variants of the basic model. Schoemaker lists nine different variants and in addition, mentions four other recent developments. He concludes his appraisal with doubts that the EUH can be used as a general descriptive model of decision behavior due to the fact that people do not structure problems as holisitically as the EUH suggests, or process probabilistic information consistent with the EUH. It serves only slightly better as a predictive tool, but it may strengthen prescriptive analysis by circumventing some of the problems of unaided decision making. Concerns arise with the prescriptive use of the EUH from two areas: biases introduced by the axioms and persistent violations of the model.

A second appraisal of the EUH was that presented by Robison (1982). He concludes his appraisal by a statement that the evaluation of the EUH has been incomplete, but based on available evidence it can be surmised that the EUH is a useful but imperfect predictor of choice, with a Type I accuracy in the range of 60% to 70%. Problems with single argument specifications of risk attitudes are discussed.

The final appraisal to be reviewed here is the work of Machina (1982). He focuses on the problems encountered when the Independence axiom has been violated. Machina summarizes the evidence that a large proportion of decision makers frequently violate this axiom, but they do it in a systematic fashion. He identifies four common violations: the common consequence effect; the common ratio effect; oversensitivity to changes in small probability-outlying events; and the utility evaluation effect. He proposes a "generalized expected utility analysis" to accommodate these systematic violations of the Independence axiom. With this variant of the EUH, he sides with its defenders pointing out that it is a useful theory with "analytic power and the ability to generate refutable predictions and policy implications in a wide variety of situations" (Machina, 1985).

Table 1. Summary of Commonly Used Risk Aversion Coefficients

Study	Almost Risk Neutral	Strongly Risk Averse	Outcome Variable	Source of R(x)
1) Holt & Brandt	-0.005 to 0.005	0.02 to 0.04	Hog Prices ($/cwt)	Assumed based on Kramer & Pope
2) Meyer, 1977b	-	6.0	% Annual Return on Mutual Funds	Assumed based on C. E.
3) Cochran, 1985a	-	.0015	Annual Income from 10-acre Block	Elicited
4) Lemieux, et al.	-.00001 to .00001	.000015	After tax NPV (10 year)	Assumed
5) Tauer, 1985	-	.0002 to .0003	$100,000 farm Purchase	Assumed based on King Robison
6) Love & Robison	-.00001 to .0002	.0025 to ∞	After tax annual income	Elicited
7) Rister, et al.	-.00001 to .00001	.00004 to .00008	Annual Returns to Grain Storage	Assumed based on C. E.
8) Wilson & Eidman	-.0001 to .0001	.0002 to .001	After tax Annual Farm Income	Elicited
9) Zacherias & Grube	-.0000001 to .000001	.000042 to .0035	Annual Farm Income	Assumed Threshold
10) King & Oamek	-.00001 to .00001	.00005 to .0001	Annual Farm Income	Elicited
11) Danok, et al.	-	.1	Annual Farm Income	Assumed
12) King & Lybecker	-.0001 to .0001	.0003 to .0006	Annual Income from 1000 cwt dry beans	Assumed
13) Kramer & Pope	.000 to 0.00125	.02 to .03	Annual Farm Income	Assumed based on C. E.
14) King & Robison	-.0001 to .0001	.001	Annual Income	Elicited
15) Cochran, 1985b	-.0001 to .0001	.001	Annual Farm Income	Assumed based on Love & Robison; King & Robison; Cochran 1985a
16) Tauer, 1986	-.0001 to .001	.001 to ∞	Annual Farm Income	Elicited

Elicitation of Risk Preferences: Implications for Stochastic Dominance Analysis

The implementation of the EUH requires that both probability distributions and risk preferences be represented in the analysis. The use of stochastic dominance techniques focuses on the explicit representation of the probability distributions, but often risk preferences are only implicitly depicted. However, with Stochastic Dominance With Respect to a Function, SDWRF, (Meyer,1977) the risk aversion coefficients have to be explicitly provided by the researcher. Since SDWRF is becoming one of the most widely used of the stochastic dominance techniques, it becomes necessary to discuss the profession's experiences with identifying risk preferences and evaluate what are appropriate values for risk aversion coefficients.

Early attempts to elicit risk preferences employed several hypothetical gaming procedures to identify points of indifference between different lotteries or between gambles and certain incomes. From these points, single-valued utility functions were constructed which in turn were used to derive the Pratt/Arrow absolute risk aversion coefficients, commonly used in risk analysis. Young, et al. (1979) and Wilson (1985) have provided reviews of this work. Typically, due to the measurement problems inherent in elicitation procedures, the exact representations of the single-valued utility functions resulted in a high probability of a Type I error. Binswanger (1982) has developed a procedure to elicit preferences through experimental questioning with actual payoffs experienced by participants. However, Schoemaker (1982 p. 553) argues that there is "no evidence that suboptimal laboratory behavior improves when committing subjects financially to their decisions." In general, it appears that financial constraints have prevented much research in experimental elicitations of preferences in domestic agricultural settings.

To account for the myriad of measurement errors that can surface in elicitation and deriving single-valued utility functions, King and Robison (1981) have developed an interval technique to represent risk attitudes. Rather than represent preferences exactly, as with the single-valued utility function, the interval technique identifies upper and lower bounds on admissable risk aversion functions. By defining an interval on the preferences, researchers can allow for the possibility of measurement error or instability of preferences across decision settings and time. The wider the interval, the less precise the preference representation and the larger will be the probability of a Type II error. The narrower the interval, the more precise is the preference representation and the larger will be the probability of a Type I error.

A variety of studies have adopted the notion of representing risk preferences imprecisely with an interval. This facilitates the use of SDWRF, in addition, to having the advantage of allowing the researcher to weigh the relative costs of Type I as opposed to Type II errors. A summary of selected articles is presented in Table 1, displaying the intervals used to represent two loosely defined classes of preferences: almost risk neutral and strongly risk averse. Most of these studies have either defined the intervals by assumption or by using intervals in other studies as secondary data. It should be noted that the proper interpretation of the absolute risk aversion coefficients depends upon the definition of the outcome variable. Raskin and Cochran (1986) describe a procedure to approximate risk aversion coefficients, maintaining similar attitudes, when the outcome variables have been scaled. McCarl and Bessler (1986) also propose a procedure to estimate upper limits on appropriate risk aversion coefficients which will be discussed in a later section. From the interval elicitation efforts to date, it appears reasonable to expect that the preferences of a majority of farmers will be represented with the interval -.0002 to .0015, when measured at after tax net farm annual income levels (Love and Robison, 1984; Wilson and Eidman, 1983; Cochran, Robison and Lodwick, 1985; King and Oamek, 1983b; Wilson, 1985; Tauer, 1986). A large proportion could be represented by even a narrower interval. Different outcome variable specifications may require different interval representations.

It should be remembered that the selection of the preference interval will partially determine the likelihood of Type I and Type II errors. It is common to subdivide the interval designed to represent the entire class of decision makers into smaller subgroups so that the sensitivity of the strategy rankings to risk preferences can be identified. However, it should be recognized that there is little evidence to define proportions of decision makers that may be represented by each subgroup. Current status of preference elicitation is such that we are beginning to realize what preferences probably are not, but we still have much to learn about exactly what they may be.

STOCHASTIC DOMINANCE MODELS

There are many members of the family of stochastic dominance techniques. This section will briefly review some of the most commonly used techniques. First Degree Stochastic Dominance (FSD), Second Degree Stochastic Dominance (SSD), Third Degree Stochastic Dominance (TSD), Stochastic Dominance With Respect to a Function (SDWRF), and Convex Stochastic Dominance (CSD) will be discussed here.[1] Readers are referred to Bawa (1982), Whitmore and Findlay (1978), Zentner, et. al. (1981); King and Robison, (1984); Cochran, Lodwick and Raskin (1984), Kroll and Levy (1986b) and Anderson, Dillion and Hardaker (1977) for more complete discussions of the techniques.

FSD was one of the first efficiency criteria to be developed. It ranks alternative strategies consistent with the EUH for a class of decision makers who prefer more to less ($U'(x)>0$). This class should include all rational decision makers with preferences ranging from maxi-

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1 Vickson (1977) introduced an optimum criterion for decreasing absolute risk aversion utilities (DSD), citing both necessary and sufficient conditions for all DARA utility functions.

min to maxi-max. In terms of Pratt/Arrow absolute risk aversion coefficients, the interval stretches from $-\infty$ to $+\infty$. Due to the size of this preference interval, it is expected that the use of FSD would result in large Type II errors and small Type I errors. The FSD criterion can be formally stated as:

1.) Given two cumulative probability distributions, $F(x)$ and $G(x)$, associated with alternative management strategies, it can be shown that the expected utility of F is greater than G, if and only if,

$$[F(x) - G(x)] \leq 0, \text{ for all } x, \text{ and } [F(x)-G(x)] < 0 \text{ for some } x.$$

Graphically, the FSD criterion translates into a situation that dominance (greater expected utility) can be proven only if the two cumulative probability curves never cross and distribution F lies to the right of G at least one probability level.

The criterion of SSD is defined to represent a smaller preference interval. The class of decision makers represented by SSD includes all risk averse individuals including those using the rule of maxi-min. This criterion places two restrictions on admissable utility functions, $U'(x) > 0$ and $U''(x) < 0$, which results in possible Pratt/Arrow absolute risk aversion coefficients that range from 0 to $+\infty$. While the preference interval for SSD is smaller than the one for FSD, it may still produce large efficient sets (high Type II errors). It would generally be expected that SSD would generate lower Type II errors than FSD and due to the possibility that not all decision makers always are averse to risk, SSD may have higher Type I errors as well. Young (1979) in his review of preference elicitation efforts indicates that all studies which had not precluded the possibility risk loving behavior found at least some decision makers who exhibited such attitudes. Formally stated the SSD criterion can be expressed as:

2.) Given two cumulative probability functions, $F(x)$ and $G(x)$, associated with alternative management strategies, it can be shown that for all risk averse decision makers, the expected utility of F is greater than G, if and only if,

$$\int_{-\infty}^{x} [F(y) - G(y)]\,dy \leq 0 \text{ for all } -\infty < x < \infty$$
$$< 0 \text{ for some } x.$$

Graphically, the two probability curves may cross as long as the accumulated negative areas ($G > F$) remain greater than the accumulated positive areas (where $F > G$). This is the infamous left-hand tail problem and arises because of the inclusion in the preference interval of the maxi-min attitude. It also can increase the overall size of the efficient set.

By imposing an additional constraint on the set of admissable utility functions, a third criterion could be introduced. TSD requires three constraints: $U'(x)>0$, $U''(x)<0$ and $U'''(x)>0$. It represents the preferences of the class of decision makers with decreasing absolute risk aversion. It should be noted that TSD provides sufficient conditions for optimality for decreasing absolute risk aversion (DARA), but Vickson's (1977) DSD criterion suggests both necessary and sufficient conditions (Kroll and Levy, 1980b, p. 171).

Formally stated, dominance by TSD is observed when

3.) Given two cumulative probability functions, F(x) and G (x), associated with alternative management strategies, the expected utility of F is greater than G, if and only if

$$\int_{-\infty}^{x} \int_{-\infty}^{v} [F(y)-G(y)]\, dy\, dv \leq 0 \quad \text{and } E_F(x)>E_G(x) \text{ for all } x.$$

While FSD, SSD and TSD have preference intervals fixed by assumption, the next criterion to be discussed is much more flexible. With SDWRF the researcher can define a preference interval that is desired. This allows the researcher to weigh the trade-offs between Type I and Type II errors. If the selected interval has upper and lower bounds equal to $-\infty$ and $+\infty$, respectively; then the efficient set identified with SDWRF will be identical to that of FSD. If the selected interval is bounded by 0 and $+\infty$; then the strategy ranking will correspond to that of SSD. It is for this reason that SDWRF is often referred to as Generalized Stochastic Dominance (Meyer,1977b; Zentner et. al., 1981; Tauer,1985). It is also occasionally labelled the Meyer criterion.

The width of the preference interval is then related to the likelihood of Type I and Type II errors. Narrow intervals have high probabilities of Type I errors while wide intervals have high probabilities of Type II errors. King and Robison (1981) show that by narrowing the preference interval the incidence of Type II errors was lowered from 91% to 9% while the incidence of Type I errors were increased by the same change from 2% to 28%. The incidence of Type I and II errors for SSD were 2% and 93%, respectively.

Dominance can be displayed with SDWRF with the following procedure.

4.) Given two cumulative probability distributions, F(x) and G(x), associated with alternative management strategies, it can be shown that the expected utility of F is greater than the expected utility of G, if and only if, the utility function, $u_0(x)$ which minimizes

$$\int_{-\infty}^{\infty} [G(y) - F(y)]u'(y)dy,$$

subject to

$$r_1(y) < -u''(y)/u'(y) < r_2(y),$$

produces a positive value of the equation. A negative value would indicate that not all members of the class of decision makers would prefer F to G. SDWRF, like FSD and SSD, makes pairwise comparisons between distributions and hence requires that there be a consensus among all preferences in the interval. What the optimal control procedure of SDWRF accomplishes is to identify the preference least likely to prefer F to G. If it can be said that the least likely decision maker prefers F to G, than it can be concluded that all other preferences will have similar a ranking.

Whenever pairwise comparisons of alternative strategies are made, as is the case with FSD, SSD and SDWRF, a definite tradeoff exists between Type I and Type II errors. By adjusting the size of the preference interval, it is impossible to reduce the likelihood of a Type II error without increasing the probability of a Type I error. However, Convex Stochastic Dominance, CSD (Cochran, Robison and Lodwick, 1985; Cochran, Lodwick and Raskin, 1984; Fishburn, 1974; Meyer, 1979; Meyer, 1977b; Bawa, et. al., 1985; Lodwick, 1986a and 1986b) has the potential to reduce Type II errors without affecting the probability of incurring a Type I error. This potential arises from the avoidance of pairwise comparisons.

CSD ranks alternative strategies by comparing one distribution to a convex combination of alternative distributions. This procedure does not require a consensus of all preferences in the interval for dominance to be shown. The resulting efficient set contains as its elements only those alternatives which are the most preferred by at least one utility function in the preference interval. The reduction in the likelihood of Type II errors without increasing the probability of Type I errors is generated by the removal from the efficient set those strategies which would not be preferred by any individual in the represented class of decision makers, but for which there is no consensus as to which alternative is superior. The resulting reduction in the efficient set will depend upon the choice set examined and the heterogeneity in the preference interval.

The procedure commonly used to implement CSD is to extend an analysis using first one of the other stochastic dominance techniques to identify an efficient set for a preference interval and then to further refine the identified efficient set with CSD. Formally, CSD demonstrates that G(x) is an inferior strategy for FSD, SSD and SDWRF with the following equations:

4.) $\sum_{i=1}^{n} \lambda_i F_i(x) < G(x)$ for some $\lambda \wedge n$ for FSD.

5.) $\int_0^y \sum_{i=1}^{n} \lambda_i F_i(x)dx \leq \int_0^y G(x)dx \; \forall y \varepsilon [0,1]$

with at least one strict unequality for SSD.

6.) $\int_0^y [G(x) - \sum_{i=1}^{n} \lambda_i F_i(x)] U'_0(x)dx \geq 0$ for SDWRF

where $\sum_{i=1}^{n} \lambda_i = 1$ and $U'_0(x)$ is from the utility

function which minimizes the integral under the constraint of

$$r_1(x) \leq - U''(x)/U'(x) \leq r_2(x).$$

COMPARISONS BETWEEN STOCHASTIC DOMINANCE MODELS AND OTHER RISK ANALYSIS TECHNIQUES

There are, of course, alternative techniques other than stochastic dominance that can be used to rank alternative management strategies. Many comparisons appear in the literature that compare the performance of these techniques to the stochastic dominance procedures. The comparisons tend to focus on three distinct areas: theoretical or analytical foundations, memberships in efficient sets, and severity of sampling errors. Almost invariably these comparisons involve FSD, SSD and TSD -- rarely are SDWRF or CSD included. This review will restrict itself to summarizing the literature that discusses the latter two areas. Readers are referred to Fishburn, 1980; Tsiang, 1972; Bawa, 1978 and Hanoch and Levy, 1969 for discussions of the theoretical foundations of the different techniques. In addition, this study will only review comparisons involving FSD, SSD, TSD, SDWRF, E-V, E-S_h, Target MOTAD and (Extended) Mean Gini analyses.

It should be noted before we begin the remainder of the discussion that given normal distributions of outcome variables, rankings provided by SSD and E-V analysis should be identical (Hanoch and Levy, 1969). It is also commonly held that only if decision makers have quadratic utility functions or the probability functions are normally distributed will it be assured that the E-V set will contain the expected utility maximizing strategy (King and Robison, 1984; Robison and King, 1978; Samuelson, 1967; Tobin, 1958). However, Tsiang (1972) argues that only in pathological cases will E-V analysis exclude the preferred choice from its efficient set.

Efficient Set Membership

The first comparisons to emerge in the literature were those between FSD, SSD, TSD and E-V analysis, contrasting the memberships of efficient sets. Implicit in these early comparisons (and in most applied risk analysis) is the assumption that the probability distributions of the outcome variables are measured without error. Porter and Gaumnitz (1972) found that differences between SSD and E-V analysis were not as large as expected, with the most significant difference being that SSD eliminated E-V efficient portfolios with low means and variances. However, TSD reduced the SSD efficient set and tended to remove these portfolios as well. The major difference found between SSD and E-V efficient sets seems to arise for the more risk averse preferences represented by the two techniques. Of the 893 stocks examined by Porter and Guamnitz, 40 were SSD efficient, 39 were E-V efficient, 24 were both SSD and E-V efficient and 31 were TSD efficient. Levy and Sarnat (1970) uncovered similar relationships in the mutual funds market. By varying estimation procedures of the same data (monthly and quarterly estimates of returns), Porter (1973) produced similar results to Porter and Gaumnitz, finding that between 28% and 83% of the any SSD or TSD efficient set was also E-V efficient. Differences once again arose in the portfolios with low means and low variances. Fishburn (1980) offers a theoretical explanation of these observations.

Porter (1974) made a related comparison between SSD and $E\text{-}S_h$ analysis, concluding that there should be more consistency between these two than with SSD and E-V analysis. He proposes that generally the efficient set identified with $E\text{-}S_h$ analysis should be a subset of that identified with SSD. Bey (1979) in follow-up study found that by altering the target value over a wide range, the total $E\text{-}S_h$ efficient set will contain most members of the SSD efficient set and will provide a good approximation of SSD dominance.

Other procedures have been introduced by Fishburn (1977), Bawa (1978) and Holthausen (1982) to implement safety first rules consistent with stochastic dominance. These models use lower partial moments (semi-variances) defined at a target income or return. Bawa shows dominance of the nth order when

$$7.)\quad LPM_n(t;F)=\int_{-\infty}^{t} (t-y)^n dF(y) \leq LPM_n(t;G)=\int_{-\infty}^{t} (t-y)^n dG(y)$$

for all t, with at least one inequality, where LPM = lower partial moment; t = target income and G,F are cumulative probability functions.

N^{th} order LPM efficient sets will be subsets of n^{th} order stochastic dominant efficient sets. Empirical tests of LPM models can be found in Nantell, Price and Price (1982) and Atwood (1985).

Target MOTAD, a risk programming technique similar to Porter's $E\text{-}S_h$ analysis, has enjoyed a great deal attention in the field of agricultural economics. Tauer (1983) shows that Target MOTAD efficient strategies will also be SSD efficient but not necessarily vice versa.

Watts, Held and Helmers (1984) make similar claims. Bawa, Lindenberg and Rafsky (1979) also propose a mathematical programming procedure -- employing a grid system -- to approximate a SSD efficient set. McCamley and Kliebenstein (1986a) suggest a multiple Target MOTAD model and profess that it contains both necessary and sufficient conditions for SSD efficiency.

It should also be noted in passing that there exist several efforts to maximize expected utility directly. Readers are referred to the recent works of Lambert and McCarl (1984), Tew and Reid (1983) and Kroll, Levy and Markowitz (1984).

King and Robison (1984) compared strategy rankings for FSD, SSD, E-V, MOTAD, and SDWRF. They found that FSD was ineffective in discriminating between alternatives. The efficient sets of SSD, E-V analysis and MOTAD were identical even though the probability distributions were skewed and the latter two criteria had underlying assumptions that did not hold. SDWRF was used to allow the possibility of some risk preferring behavior. Efficient sets of SDWRF were identified for two preference intervals -- in one case the resulting efficient set was considerably smaller than the SSD efficient set while in the other case SDWRF produced only a slightly greater discriminating power.

Recently, Yitzhaki (1982) has demonstrated that the Gini's Mean Difference (MG) is a criteria that can be used to identify an efficient set which will be a subset of the SSD efficient set. Shalit and Yitzhaki (1984) introduce the Extended Mean Gini (EMG) which produces a larger efficient set than MG but all its members all still SSD efficient. Following Shalit and Yitzhaki (1985, p. 7) the EMG is defined as

$$\tau\,(v) = -v\mathrm{COV}\left[R_p,\ (1-F_p)^{v-1}\right]$$

where R_p = average return of strategy "i"
F_p = cumulative probability distribution of strategy "i"

When $v = 1$, risk neutrality is assumed; when $v \to \infty$, "maxi-min" behavior is assumed. Buccola and Subaei (1984) compare rankings of strategies with SDWRF, SSD, E-V, and EMG. They conclude that at weak risk aversion levels (r_2 = .0015 with outcome variables measured on a per acre basis) MG performs well compared with the stochastic dominance criteria. However, as risk aversion is increased, the EMG efficient set becomes a smaller and smaller subset of the SSD and SDWRF sets. Bey and Howe (1984) compare memberships of SSD, E-V, E-S_h, MG and EMG efficient sets. These authors found that the MG sets, on average, comprised only 19.4% of the SSD set, a poor approximation. MG efficient strategies tended to be E-V and E-S_h efficient as well. They caution that for MG and EMG to be useful, the class of decision makers that they represent must be more clearly articulated than "weakly risk averse". Such work appears to be in progress by Shalit and Yitzhaki (1985). Szmedra and Wetzstein (1986) have also applied MG and SSD, confirming previous results.

A final note to this section needs to be made before proceeding on to the section on sampling errors. Inclusion of an unpreferred strategy in an efficient set is an example of a Type II error. Exclusion of a preferred alternative from the efficient set results in a Type I error. Without an observed actual ranking of alternatives by the members of the specified class of decision makers it is difficult to identify which has occurred by simplying comparing memberships of the efficient sets of alternative criteria. A common speculation is that for very risk averse decision makers E-V analysis and MG procedures should have a higher incidence of Type I errors. For less risk averse individuals they may have a lower Type II error. These errors can arise from mathematical procedures inherent to the criteria (see Fishburn, 1980) or from sampling errors related to the data requirements of each criterion. It is difficult to conclusively resolve the issue in this manner. For that reason many of the studies using Monte Carlo techniques to examine sampling errors were designed.

Sampling Errors

More recently the literature has focussed on the implications of sampling errors for the accuracy of rankings of alternative criteria. Given that the probability distributions are generally constructed from sample data and that E-V analysis requires less data than stochastic dominance, these studies test the hypothesis that the presence of sampling errors may have differential impacts on the rankings dependent upon the criteria used.

Johnson and Burgess (1975) investigated the effects of sample size on efficient set membership for SSD and E-V analysis. They examined the accuracy of the rankings based on samples of 10, 20 ,30, 40 and 50 drawn from a population of two independent normal distributions. They concluded that two criteria performed relatively the same, with E-V analysis superior when population means were the same and the variances different, and SSD more accurate when the population means were different and the variances were identical.

Kroll and Levy (1980) performed a more complete examination of the issue when they studied the impacts of samples of 5, 10, 20, 30, 50, 70 and 100 drawn from correlated populations from normal, lognormal and uniform distribtuions. Their conclusions include: a) sampling errors of a Type II variety actually increased with sample size with FSD; b) percentages of accurate rankings are almost identical for SSD, TSD and E-V analysis and these percentages increase with sample size; c) in small samples (under 50) E-V analysis has an advantage since Type I erors are smaller; and d) sampling errors tend to decrease with an increase in the correlation coefficient.

Stein and Pfaffenberger (1983) examined potential tests of sampling errors of stochastic dominance techniques and concluded that there is no good way to test for stochastic dominance with small to moderate sample sizes. They proposed a moratorium upon testing for dominance until better methods are developed. In a related article, the same authors (Stein, Pfaffenberger and Kumar, 1984) suggested an analytical procedure to identify Type I errors for FSD when it is assumed that alternative strategies have been sampled from the same distribution of returns.

Pope and Ziemer (1984) examined with a Monte Carlo model the performance of parametric and nonparametric approaches to SSD and E-V analysis. They examined sampling errors when samples of 5, 10, 20, 30, 50 and 100 were drawn from normal, lognormal and gamma distributions. Their conclusions consist of: a) given normal distributions and small sample sizes, E-V analysis has a lower probability of correctly ranking dominant alternatives; b) E-V performs poorly relative to alternatives, given nonnormal distributions; c) empirical distributions perform favorably relative to Maximum Likelihood methods; d) probabilities of Type I errors do not appear to be different for any procedures and decrease with sample size; e) probabilities of Type II errors are generally higher for E-V analysis, especially in nonnormal cases and tended to be high except when extent of dominance and sample size were large; and f) the empirical distribution is probably the best candidate for estimation and E-V analysis does not appear to have general applicability.

Finally, Anaman and Boggess (1986) discovered that in the case of alternative marketing strategies in North Florida, sample sizes of less than 100 led to incorrect rankings. Strategies actually employed by farmers in the area were rejected as inefficient (Type I error) with small sample sizes and preference intervals above $r_1(x) = .0001$ (outcomes measured in terms of whole farm incomes). SDWRF was used in the study.

INNOVATIVE AND NEW APPLICATIONS OF STOCHASTIC DOMINANCE IN AGRICULTURAL ECONOMICS

Several recent applications have demonstrated innovations that could be of great use to agricultural risk analysis. Techniques have been developed that allow for threshold risk preferences to be identified (Hammond, 1974; Tauer, 1985); that measure the value of information for risk management (Bosch and Eidman, 1985; Schoney and McGuckin, 1983); that guide the construction of diversified strategies (McCarl and Knight, 1986; McCamley and Kliebenstein, 1986a); and permit rankings which define when transformations of existing random variables will produce a new random variable that dominates the original one (Meyer, 1984). These techniques will be discussed below.

Tauer discusses a procedure originally proposed by Hammond (1974) to identify threshold or break even risk aversion levels, in an analysis investigating the risk efficiency of life insurance policies. Since life insurance often produces the highest low income outcome, at some degree of risk aversion it will be preferred. The Hammond procedure allows the threshold value to be identified. For cumulative probability functions which cross at most one time and constant absolute risk aversion, Hammond shows that there exists, though not necessarily always unique, a risk aversion coefficient, a_0, that decision makers more averse than a_0 will prefer one distribution while decision makers less averse than a_0 will prefer the alternative. No threshold level, a_0, will exists if there is no dominance. The threshold value can be found by finding the a which maximizes $E(e^{-aw_i}/a)$, "an operation which, more often than not, should require little more than a table of moment-generating functions" (Hammond, 1974, p. 1059). Obviously, problems will arise in the advent that distributions that cross more than once or preferences can not be approximated by the negative exponential utility function. For many applications, nevertheless, it could prove to be a useful tool in place of trial and error searching for where dominance of a reference distribution may occur.

Bosch and Eidman (1985) demonstrate a procedure for measuring the value of information which takes into account nonneutral risk attitudes. This work is based on the article by Byerlee and Anderson (1982) that shows that in a risk efficiency context the value of information would be equal to the amount that a decision maker would be willing to pay in each state of nature and still remain indifferent between the previously dominant strategy and the next best alternative. This corresponds to finding a parallel shift in the cumulative probability distribution associated with the strategy, which will remove any dominance over strategies not using such information. The Bosch and Eidman procedure employs SDWRF and calculates the value of information, V_i by simultaneously satisfying the following inequalities:

$$8.)\ [G(x) - F(x - V_i)]U'(x)dx > 0$$

$$9.)\ [G(x) - F(x - V_i - Y)]U'(x)dx \leq 0$$

$$\text{subject to } r_1(x) \leq -U''(x)/U'(x) \leq r_2(x).$$

Initially V_i is set equal to 0 and then Y is iteratively increased by a small amount until V_i is maximized. Different values of information will likely be identified for different preference intervals and alternative strategies.

Schoney and McGuckin (1983) propose a similar procedure for SSD in determining their stochastic bid prices for alfalfa harvesting strategies. Their procedure also involves finding the parallel shift in the probability distribution of the dominant strategy that will force it into the same efficient set as the reference strategy.

The next innovation to be discussed involves the problems encountered when the choice set examined by stochastic dominance is not completely specified -- combinations of strategies should be analyzed in addition to the strategies themselves. This is often referred to as the diversification or portfolio building problem. McCarl and Knight (1986) offer a series of guidelines to determine when convex combinations of the strategies may need to be considered. They suggest that if option 1 dominates option 2, then option 1 will also dominate all convex combinations of options 1 and 2 when:

1.) SSD is used and the correlation coefficient is greater than or equal to the ratio of the standard deviation of option 1 to the standard deviation of option 2;

$\rho \geq \sigma_1/\sigma_2$;

2.) constant absolute risk aversion is present and the correlation coefficient satisfies $\rho \geq \sigma_1/\sigma_2 - (\mu_1-\mu_2)/(2\theta\sigma_1\sigma_2)$ where θ is the Pratt risk aversion coefficient.

McCamley and Kliebenstein (1986a) offer a multiple-target MOTAD model for examining the portfolio building problem. Based on Tauer's (1983) sufficiency condition and on necessary conditions of Dybvig and Ross (1982), they suggest a procedure to identify optimal mixtures that will be SSD efficient. This procedure promises the potential to resolve this problem. Russell and Seo (1980) also discuss this problem.

Meyer (1984) presents a transformation approach that permits a variety of risk efficiency questions to be analysized in a useful and easy faishon. He defines domination to occur in a second degree sense if

10.) $\int_0^y k(x)\, dF(x) > 0$ all $y \in [0,1]$ where $k(x) = t(x) - x$.

The transformation, $t(x)$, can be solved for, if the original random variable, $F(x)$, and the preference are known. This is a "how" question of how things must be changed to be preferred. The original random variable could also be determined if the transformation and preferences are known. Likewise if transformations and original random variables are known, the class of decision makers benefiting from the transformation could be inferred.

PROBLEMS AND CONCERNS FOR THE USE OF STOCHASTIC DOMINANCE

Despite its wide spread popularity, there are several areas which still pose problems for the use of stochastic dominance as an applied risk analysis technique. Before this discussion is complete, concerns must be expressed about these problem areas. Given that few new developments have emerged in the recent past, perhaps a recognition of the remaining problems may foster more theoretical developments. General areas of concern to this author lie with five major topics: a) generation of probability distributions; b) selection of preference intervals and scalings of outcome variables; c) diversification issues; d) the lack of statistical tests for differences in expected utility; and e) the validity of the EUH.

Generation of Probability Distributions

The discussion of the problems in generating the probability distributions should be prefaced with a reminder of the three major uses of the EUH (and inherently stochastic dominance) identified by Schoemaker (1982). He articulates three dominant uses; the descriptive, the predective and the prescriptive. Errors in representing the probability distributions have different impacts, depending on the use. One common source of data for probability estimation is historical records.

If the use is either descriptive or predictive, problems arise because there may be wide differences between the subjectively assessed probabilities of the decision makers the historical ones. Lee, Brown and Lovejoy (1985) describe such problems with predictions on reduced tillage practices in Indiana. Hanemann and Farnsworth discuss a similar problem with adoption of IPM practices. Skees (1986) presents data on differences between subjective and historical distributions and discovers a tendency on the part of farmers to underestimate risk. Farmers in his study generally overestimated means and underestimated variances. Predictions and descriptions may be biased by these differences if subjective probabilites are not used, but it might be necessary to use historical estimates in prescriptive work. Bessler (1980) examines differences between subjective and historical yield distributions.

Even in prescriptive analyses problems with probability estimation will be encountered. Distributions based on historical data must be carefully examined to remove trends or other phenomena which would detract from their ability to accurately represent the outcome distribution of decisions under study. It should be recognized also that not all decision makers will share the same historical or "objective" distribution. Capstick and Cochran (1983) show wide differences between historical yield distributions among farms and between individual farms and their respective county distributions. This should come as no surprise since it is generally recognized that not all farms have bad outcomes in the same season. For these cotton farms in Arkansas a definite tendency for the county distributions to underestimate the risk faced by individual farms was detected.

Additional problems for prescriptive use arise since another common source of data for probability estimation comes from experimental farm data. Once again the question has to be raised of how representative these data are. This concern is also relevant for predictive work. Occasionally, distributions are constructed from cross sectional data and then used as if they were representative of longitudinal risk for a single decision maker. The distributions are likely to be quite different from one another and different rankings of management strategies may occur.

The last source of data frequently used in stochastic dominance analysis comes from simulation models. Without proper validation of the model it may be unknown how representative the ensuing distributions are. When the rankings are to be used in either a predictive or prescriptive mode, it may be necessary to validate the models for more than one decision environment. Sources of uncertainty considered by the models must be explicitly described and compared to the entire range of sources that may be relevant to the decision maker. Rankings of risk management strategies may change dependent upon how complete the total uncertainty of the operation has been simulated. Perhaps sensitivity analysis could prove useful in establishing the credibiltiy of the distributions.

The final point in this section is a reiteration of one of the conclusions of Pope and Ziemer (1984). That is, it appears that empirical functions may have advantages over "plug-in" estimational procedures. Given that most stochastic dominance algorithms have the capacity to estimate the empirical distributions readily, we should probably continue to use this procedure, when data are available.

Given the results of the studies examining the implications of sampling errors (Pope and Ziemer, 1984; Kroll and Levy, 1979; and Johnson and Burgess, 1975), additional care should be placed on the construction and interpretation of the probability distributions. This point will resurface in the discussion of the lack of statistical tests for differences in expected utility. Perhaps, more work needs to be done on representing probabilities inexactly like we have done with the preference intervals (Watson, Weiss and Donnell, 1979).

Selection of Preference Intervals and Scalings of the Outcome Variables

Two points of great concern must be discussed in relation with the selection of appropriate risk aversion coefficients. They are the definition of the preference interval and the scaling of outcome variables. It is becoming more and more common to select risk aversion coefficients based on secondary data available from studies that have actually elicited risk attitudes from farmers. Differences in decision environments may produce different trade-offs of Type I and Type II errors than what were originally intended by the researchers doing the elicitations. Particular concern needs to be expressed when coefficients from one study are used in a new setting where outcome variables have been measured in different terms (i.e. conversions from per acre returns to annual whole farm income to 10 year net present value). Raskin and Cochran (1986) provide a rule to guide such transformations which should approximate the same degree of risk aversion as the original coefficient. Their procedure is as follows: If $r(x) = -U''(x)/U'(x)$ and a transformation of x is such that $w = x/c$ then $r(w) = c * r(x)$.

McCarl and Bessler (1986) suggest a series of rules which may be used to identify an upper bound on appropriate risk aversion coefficients. Their three rules are: 1) $r(x) \leq 10/\sigma$; 2) $r(x) \leq 14/\sigma$; and 3) $r(x) \leq 3/\sigma$. The three alternative rules were derived in different ways -- rule 1 is based on a standardized Z table, rule 2 is based on Chebyshev's inequality, and rule 3 comes from MOTAD analysis.

It appears that most of the preferences elicited with the interval procedure of King and Robison (1981) produce coefficients that fall within the range of -.0002 and .0015, when measured at whole farm annual income levels. Distribution of preferences within the range is unclear. For some policy analyses it would be useful to know such a distribution for both predictive and prescriptive purposes. Perhaps the transformation process of Meyer and/or the break even aversion introduced by Hammond may prove to be ways around our inability to precisely represent preferences.

Diversification

Until the work of McCarl and Knight (1986) and McCamley and Kliebenstein (1986a), one of the disadvantages of stochastic dominance was its inability to build portfolios. The cumulative probability distributions used by stochastic dominance do not take into account any covariances and hence cannot investigate any optimal combinations of strategies.

Even with the McCarl and Knight guidelines, the choice set has to be articulated "a prior" to the analysis and probability distributions constructed before the rankings can be completed. Caution arises since even with FSD dominance since it is not assured that the preferred strategy will outperform the dominated one in all states of nature (and hence some combination of the two might be preferred). This is one of the advantages of mathematical programming models, as used by McCamley and Kliebenstein. Perhaps some of the stochastic dominance algorithms empoloying mathematical models (Lodwick, 1986; Bawa, et.al., 1979 and Bawa et. al., 1985) can incorporate the McCarl and Knight guides to overcome this deficiency.

Lack of Statistical Tests of Expected Utlity

Despite the suggestions of Whitmore (1978), there are still no valid procedures to test if differences recognized in the expected utilities of alternative strategies with SSD, TSD or SDWRF are statistically significant (Stein and Pfaffenberger, 1983). Given the relative high incidence of sampling errors and other problems incurred in constructing the probability distributions, we should be more cautious of our interpretations of stochastic dominance results.

Particularly when distributions are very similar, we risk a significant Type I error if rankings are always accepted without further attention to these problems. For distributions constructed with simulation models these problems may be really acute. Checks should be made, either through tests of means and variances or simplying plotting the distributions to develop an intuitive crediabiltiy in the rankings. Collins and Nelson (1986) propose a method to provide some crediability in the differences in expected utility. They propose the construction of a super distribution from all alternatives and then through random draws identify how far in the tails of this distribution the individual distributions would fall. This is a test to check if there is reason to believe that individual distributions were not all derived from the same population.

Validity of the EUH

Since stochastic dominance is based on the EUH, problems with the foundation must translate into concerns for the procedure. Attacks have been made on the axioms of the EUH (which were discussed in an earlier section). It should be noted that Machina (1982) and Fishburn (1982) have offered theoretical stochastic models which can be implemented without the independence or transitivity axioms. To my knowledge, however, no working algorithms for either model have been developed.

These concerns on these axioms arrive because occasional but systematic violations of the EUH have been observed. It is reasonable to question whether stochastic dominance techniques will be viable as the EUH is modified. To show that this is not likely to be the case, two of the major new developments in extending the EUH will be briefly described. The corresponding changes in stochstic dominance methodology that will result will be hypothesized.

The EUH states that to evaluate a particular distribution of outcomes f(x), only a weighting function u(x) must be known. That is, expected utility $[f(x)u(x)]$ is known entirely through a linear function of the probabilities, where the utility values provide the needed factors. It is worth noting that this linearity property is a direct consequence of the independence axiom of the EUH. It might be argued that a more complex functional relationship would be needed to explain the systematic violations of the EUH.

Fortunately, any (reasonably behaved) functional relationship can be approximated by the sum of constant, linear, quadratic, and higher order terms (this is the basis of the Taylor expansion). The greater the number of terms, the closer the approximation is to its true value. That the EUH has had such a strong success record is evidence that preferences are at least approximately linear in the probabilities; therefore, higher order terms would add little to the explanatory power of the model. However, in light of the occasional deviations observed from true EUH behavior, it is worthwhile to investigate the significance of a second order (quadratic) term applied to the probabilities.

It was the contribution of Machina to prove that the inclusion of this second term can explain virtually all known violations of the EUM. Thus a sum such as

$$11.) \sum [u(x)f(x)] + [\sum v(x)f(x)]^2$$

can describe preferences in a more precise manner than considering only the first term alone; however, the second term is negligible in most instances. Machina has furthermore shown that the negative of the second derivative over the first derivation of the above expression retains its meaning as a measure of aversion to risk. While no one has yet provided an interpretation of v(x), the estimation and bounding of parameters of the expression may provide a key to refining the present stochastic dominance techniques.

Fishburn takes a different approach in explaining the observed EUH violations. In redefining the concept of utility itself, preferences are defined on pairs of outcomes rather than on single outcomes. That is, preferences are descirbed by a function of the form w(x,Y) and not u(x). The preferred distribution is the one with highest expected value of w.

Fishburn's methods can also explain most of the EUH violations. The corresponding risk aversion index in this context is now a 2 x 2 symmetric matrix (as derivatives can be taken with respect to two different variables)! Nevertheless, as with Machina's methodology, careful estimation and interpretation of parameters can potentially provide a basis for improvements in current stochastic dominance techniques.

Further concerns center on the very nature of the measure of risk aversion employed by stochastic dominance -- the Pratt/Arrow absolute risk aversion coefficient. Critics charge that there is something more to risk aversion than changes in the marginal utility of money. This is referred to by Glenn Johnson as the "perversion of risk aversion." Schoemaker (1982) answers these critics, but in my view additonal attention is merited.

SUMMARY

As can be seen the field of stochastic dominance has come a long way in the short 15 years or so of its existence. However, many problems do arise that hinder its application in risk analysis. It will probably remain a dominant technique for applied work in agricultural economics, but as new developments in the theory of risky decision making become available, adjustments will have to be made to incorprorate them into the technology or the profession will need to look at alternative techniques. It is quite likely that the next ten years will see more advances in this area than the last and we as practioners must stay abreast of the changes.

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