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DEVELOPING OPTIMUM CROPPING PLANS FOR A TYPICAL PUNJAB FARM WITH MULTIPLE OBJECTIVES BY USING COMPROMISE PROGRAMMING

The traditional linear programming (LP) technique, which has become very popular with the agricultural economists after 1960 for preparing farm plans, seeks to maximise or minimise only a single objective function. The limitations of this approach are, therefore, too obvious as a decision-maker on a farm firm is seldom confronted with a single objective. In reality, while multiple objectives are a rule in farm planning, single objective is only an exception. For quite some time, it is very well established that the decision-maker in agriculture often seeks the simultaneous optimisation of several of his objectives which are invariably in conflict with each other. Since all the objectives may not be optimised together, hence the decision-maker often seeks to find the best compromise among different objectives rather than optimising only one of them. Such a goal has now luckily become a reality for the decision-maker on a farm firm with the development of multiple objectives programming (MOP) algorithms.

MOP or vector optimisation techniques seek to optimise simultaneously a number of objectives subject to a given set of constraints. The aim is to find out a sub-set of efficient solutions, also called the Pareto-optimal or non-dominated solutions, from amongst the set of feasible solutions. According to Romero and Rehman (1985), a set of efficient solutions are feasible solutions that can achieve the same or better performance for all the objectives and strictly better for at least one objective.

The popularity of MOP techniques with the agricultural economists is likely to revolutionise the entire farm palnning process. Because of the great potential of MOP in handling a more realistic and complete farming system, its acceptability for choosing optimum farm plans is quite imminent. Farm management experts would find it much easier to work out the trade-offs between various pairs of objectives faced by a decision-maker on a farm firm, like that between net incomes and employment, net incomes and stable use of human labour, net income and borrowing of short-term capital, net incomes and risk aversion, etc.

Earlier Attempts

Applications of MOP have been very rare in agricultural planning. A land allocation problem in New Southwales (Australia) with two conflicting objectives, nêt money income and net environmental benefits, was solved by Hitchens, Thampapillai and Sinden (1978), and Thampapillai and Sinden (1979). A similar problem was attempted in India by Vedula and Rogers (1981) by considering economic benefits and total irrigated cropped area as the two conflicting objectives. The same technique was used by Apland, Barnes and Justus (1984) for examining various leasing alternatives under risk for owner-tenant and a landlord in Kentucky, U.S.A. It, however, goes to the credit of Romero and Rehman (1985) to give a theoretical introduction to compromise planning (CP) in the context of agricultural planning. Finally, Romero *et al.* (1987) are the only ones so far to make use of CP in solving an MOP farm planning problem, with three conflicting objectives of a co-operative in Andalusia, Spain. They attempted to find a best compromise among gross margins, seasonal labour and employment while choosing from a large set of Pareto-optimal farm plans.

The Study

This study aims at moving a step ahead of Romero *et al.* (1987) by using CP to handle a more complete model of MOP to suggest a farm plan for a typical farm in the Punjab. We have provided for human labour hiring activity in each of the four quarters of the year, capital transfer activity from *kharif* to *rabi* season and working capital borrowing activities in the *kharif* and *rabi* seasons at the current rate of interest of 12 per cent per annum. Besides, five objectives, some of them conflicting, are sought to be optimised simultaneously in the MOP farm planning model. While gross margins and family labour employment objectives are to be maximised, borrowing of working capital, human labour hiring and labour use variability over different quarters of the year are sought to be minimised by the decision-maker. Above all, we have developed a set of farm plans by giving varying weights to different objectives while using CP as a technique of multiple objectives farm palnning. The present MOP model, therefore, provides a closer and better approximation to the real world conditions as compared to that given by Romere *et al.* (1987). The main emphasis of the present study is to demonstrate the use of MOP by using CP in the context of farm planning under Indian conditions and thereby encouraging the farm management experts to make use of this technique for suggesting more realistic farm plans in the future.

The Economic Model

Compromise programming as proposed by Zeleny (1973, 1976) was used to select a sub-set of optimum farm plans from the set of efficient ones. The first step in this constituted in establishing the 'ideal point', the co-ordinates of which represented the optimum values of the five objectives considered in this study. As the ideal point so established was not feasible, the efficient farm plan closest to it was obtained by using CP. Such a farm plan was designated as the best compromise or optimum farm plan. By using different measures of distances and different measures of importance to measure the discrepancy between the jth objective and its ideal value, a sub-set of ten compromise solutions were obtained.

The necessary details about the CP model in this study can be summarised as follows.

First of all, the degree of closeness as relative deviations d_j between the jth objective and its ideal value was defined by

$$d_j = \frac{Z_j^* - Z_j(\underline{x})}{Z_j^* - Z_{*j}},$$

where Z_j^* and Z_{*j} were the ideal and anti-ideal values for the jth objective. Relative rather than absolute deviations had to be used as the units of measurement of the different objectives were not the same. $\underline{x}$ is the vector of the decision variables and $Z_j(\underline{x})$ represents the jth objective function sought to be optimised.

In order to measure the distances between each solution and the ideal point, following distance function was used:

$$L_p(\delta, K) = [\sum_{j=i}^{k} (\delta_j d_j)^p]^{1/p},$$

where p was taken as 1 and ∞, representing the 'longest' and the 'chebysev' distances in the geometric sense. The factor p in the above expression weights the deviations according to their magnitudes. Greater weight is given to the largest deviation as the magnitude of p increases. Thus with $p = \infty$, the maximum of the individual deviations is minimised. δ_j represented the weights to d_j signifying the importance of the discrepancy between the jth objective and its ideal value. In this study five sets of δ_j were considered to obtain the different compromise solutions under the assumption of varying weights for the discrepancies. The magnitude of K in the present case was also 5, the number of objectives considered for optimisation.

L_1, representing the longest distance geometrically, was minimised by using the following LP problem for obtaining the best compromise farm plan. That is,

$$\text{Min } L_1 = \sum \frac{\delta_j [Z_j^* - Z_j(\underline{x})]}{Z_j^* - Z_{*j}}$$

Subject to $\underline{x} \in \underline{F}$, where $\underline{F}$ is the set of all feasible farm plans and $\underline{x}$ is a vector of the decision variables. $\underline{x} \in \underline{F}$ thus denotes the linear constraints and non-negativity restrictions component of a standard LP problem.

For L_∞, where the maximum of the individual deviations is minimised. the best compromise farm plan was obtained by solving the following LP problem:

$$\text{Min } L_\infty = d_\infty$$

Subject to

$$\frac{\delta_1[Z_1^* - Z_1(\underline{x})]}{Z_1^* - Z*_1} \leq d_\infty$$

$$\frac{\delta_2[Z_2^* - Z_2(\underline{x})]}{Z_2^* - Z*_2} \leq d_\infty$$

$$\vdots$$

$$\frac{\delta_5[Z_5^* - Z_5(\underline{x})]}{Z_5^* - Z*_5} \leq d_\infty, \text{ and}$$

$$\underline{x} \in \underline{F}$$

It is well established by now that L_1 and L_∞ metrics define the two bounds of all the compromise solutions. Thus all other best compromise solutions fall between them.

Setting the Matrix

The MOP model for a typical farm in the Punjab is presented in Table I. It clearly shows all the details in respect of objectives, constraints and activities considered for farm planning. The first five rows of the matrix represent the mathematical expressions in respect of the five objectives considered for optimisation, namely, maximisation of gross margins and family labour employment and minimisation of borrowing for working capital, human labour hiring and variability of labour use over different quarters in a year. Land, crop maximum/minimum, tractor labour and working capital constraints are self-explanatory. Four constraints were used for human labour according to different quarters in a year. It would have been certainly better to incorporate 12 human labour constraints according to months, but we could not succeed in doing so due to non-availability of labour use data according to months. As one of the objectives was to

TABLE 1. MULTIPLE OBJECTIVE LINEAR PROGRAMMING MODEL FOR A TYPICAL FARM IN THE PUNJAB

Sr. No.	Objectives and constraints	Crop production activities						
		Paddy x_1	Maize x_2	*Kharif* fodder x_3	*Arhar* x_4	Wheat x_5	*Rabi* fodder x_6	Sugarcane x_7
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Objectives							
R_1	Maximise gross margins, $Z_1(\underline{x})$ (Rs.)	2,728	425	1,205	2,357	2,701	2,860	5,598
R_2	Maximise H.L. employment, $Z_2(\underline{x})$ (hrs.)	308	226	140	193	127	440	662
R_3	Minimise working capital borrowing, $Z_3(\underline{x})$ (Rs.)							
R_4	Minimise H.L. hiring, $Z_4(\underline{x})$ (hrs.)							
R_5	Minimise H.L. use variability, $Z_5(\underline{x})$ (hrs.)							
	Constraints							
R_6	*Kharif* land (acres)	1	1	1	1			1
R_7	*Rabi* land (acres)					1	1	1
R_8	Sugarcane maximum (acres)							1
R_9	*Kharif* fodder minimum (acres)			1				
R_{10}	*Rabi* fodder minimum (acres)						1	
R_{11}	Human labour period I (hrs.)	0	0	0	0	10	240	225
R_{12}	Human labour period II (hrs.)	86	10	10	5	105	0	152
R_{13}	Human labour period III (hrs.)	120	76	50	40	0	20	20
R_{14}	Human labour period IV (hrs.)	102	140	80	148	12	180	265
R_{15}	Tractor labour (hrs.)	6	4	5	5	15	2	0
R_{16}	*Kharif* working capital (Rs.)	1,247	320	280	115	0	0	626
R_{17}	*Rabi* working capital (Rs.)	0	0	0	0	1,274	380	360
R_{18}	Human labour deviational activities Period I (hrs.)	− 77	− 56.50	− 35	− 48.25	− 21.75	130	59.50
R_{19}	Human labour deviational activities Period II (hrs.)	9	− 46.50	− 25	− 43.25	73.25	− 110	− 13.50
R_{20}	Human labour deviational activities Period III (hrs.)	43	19.50	15	− 8.25	− 31.75	− 90	− 145.50
R_{21}	Human labour deviational activities Period IV (hrs.)	25	83.50	45	99.75	− 19.75	70	99.50

(Contd.)

TABLE I (*Concld.*)

Sr. No.	Labour hiring activities				Capital activities			Human labour deviational activities								Constraints	
	Period I	Period II	Period III	Period IV	Transfer	Borrowing		Period I		Period II		Period III		Period IV		Type	Value
						Kharif	*Rabi*										
	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}		
(1)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
R_1	− 2	− 3	− 2	− 2		− 0.06	− 0.06										
R_2	− 1	− 1	− 1	− 1													
R_3						1	1										
R_4	1	1	1	1													
R_5								1	1	1	1	1	1	1	1		
R_6																≤	12
R_7																≤	12
R_8																≤	0.5
R_9																≤	2.4
R_{10}																≤	0.95
R_{11}	− 1															≤	1,152
R_{12}		− 1														≤	1,164
R_{13}			− 1													≤	1,178
R_{14}				− 1												≤	1,178
R_{15}																≤	800
R_{16}					1	− 1										≤	8,400
R_{17}					− 1		− 1									≤	10,910
R_{18}								1	− 1							=	0
R_{19}										1	− 1					=	0
R_{20}												1	− 1			=	0
R_{21}														1	− 1	=	0

minimise the human labour use variability over different quarters, it was, therefore, essential to incorporate four constraints — rows 18 to 21 — on human labour deviational activities corresponding to each of the four labour use periods (Hazell, 1971). The coefficients in these rows, therefore, represented the deviations between labour utilisation in each of the four periods (quarters) and the per acre average labour utilisation for each crop activity.

Seven crop production activities were considered for planning. A labour hiring activity was provided in each of the four periods. A capital transfer activity from *kharif* to *rabi* season and capital borrowing activities for *kharif* and *rabi* seasons at the current rate of interest of 12 per cent per annum were also included in the farm planning model. Eight human labour deviational activities were finally incorporated, *i.e.*, two corresponding to each labour use period.

MOP Pay-Off Matrix and the Ideal Farm Plan

The pay-off matrix for this MOP problem with five objectives is given in Table II.

TABLE II. PAY-OFF MATRIX FOR THE FIVE OBJECTIVES AND THE IDEAL POINT

Objective optimised	Corresponding values of the objectives				
	Gross margins (Rs.) Z_1 (x)	Labour employment (hrs.) Z_2 (x)	Capital borrowing (Rs.) Z_3 (x)	Labour hiring (hrs.) Z_4 (x)	Labour use variability (hrs.) Z_5 (x)
Gross margins (Z_1 (x))	*58,436.22*	4,672.00	2,500.07	1,337.61	1,349.61
Labour employment (Z_2 (x))	48,115.42	*4,672.00*	0.00	1,409.37	1,734.23
Capital borrowing (Z_3 (x))	17,846.41	1,792.33	*0.00*	0.00	401.58
Labour hiring (Z_4 (x))	17,846.41	1,792.33	0.00	*0.00*	401.58
Labour use variability (Z_5 (x))	11,674.73	1,039.21	0.00	0.00	*260.29*

The elements of the matrix were arrived at by optimising one objective in a row and then working out the corresponding magnitudes of the rest of the objectives in that optimum plan. For example, the elements in the first row mean that to the maximum gross margins of Rs. 58,436.22 correspond an empoyment of 4,672 hours of human labour, Rs. 2500.07 of borrowed capital and 1,337.61 and 1,349.61 hours of hired labour and labour use variability over different quarters. The degree of conflict or otherwise between different objectives can also be studied from the same table. It is quite obvious that the first two objectives, namely, maximisation of gross margins and maximisation of family labour employment are complementary to each other. This is clear from the fact that since no imputed cost for the family labour was accounted for in arriving at the gross margin figures, hence increased family labour use would always result in increased gross margins. The last three objectives show some degree of conflict in the first two farm plans as given in the first two rows of the matrix.

Objectives $Z_1(\underline{x})$ or $Z_2(\underline{x})$ clearly show the degree of conflict with the remaining three objectives, $Z_3(\underline{x})$, $Z_4(\underline{x})$ and $Z_5(\underline{x})$. Thus it can be seen from the pay-off matrix that the desired decline in either capital borrowing, labour hiring or labour use variability can only be brought about by settling in for lower gross margins/family labour employment which the decision-maker seeks to maximise. Minimisation of $Z_3(\underline{x})$ or $Z_4(\underline{x})$ gives identical results. Therefore, one should consider only one of the two objectives in the MOP farm planning problem. They are retained in this problem just to demonstrate that optimisation of a certain number of objectives individually may not result in different optimum farm plans. It would present no difficulties for the farm planning results except that the computer would unnecessarily be loaded.

The elements in the main diagonal of the pay-off matrix in Table II give the 'ideal point' or 'ideal farm plan' as all the stated five objectives stand optimised here. This means that an ideal farm plan would be one which results in Rs. 58,436.22 worth of gross margins and 4,672 hours of farm family labour employment while requiring the decision-maker not to borrow any working capital or hire any human labour and at the same time ensuring lowest variability of 260.29 hours in human labour use over the four quarters during a year. Unfortunately, such an ideal farm plan was clearly infeasible because of conflict among the objectives.

Compromise Set of Farm Plans

The way out of the situation was either to select any one of the five farm plans corresponding to any row in Table II or to arrive at a compromise farm plan. This type of plan ensured that the absolute deviations between each solution and its ideal point were minimised. As already explained, we minimised L_1 and L_∞ metrics by giving five different sets of weights, δ_j, to the various objectives in the MOP problem at hand. Thus a set of ten different efficient compromise farm plans were generated as presented in Table III. Within a given set of δ_j weights, L_1 and L_∞ compromise farm plans constituted the two extreme bounds of all the possible set of compromise farm plans.

Farm plans 9 and 10 where $\delta_1 = 1$ and $\delta_j = 0$, $j = 2, 3, 4, 5$ are just identical for $p=1$ and $p=\infty$ metrics. This means that when only the single objective of gross margin

TABLE III. EFFICIENT COMPROMISE SET OF FARM PLANS FOR A TYPICAL FARM IN THE PUNJAB

Farm plan no. (1)	Magnitudes of p and δ_j (2)	Objective functions: Gross margins (Rs.) $Z_1(\underline{x})$ (3)	Labour employment (hrs.) $Z_2(\underline{x})$ (4)	Capital borrowing (Rs.) $Z_3(\underline{x})$ (5)	Labour hiring (hrs.) $Z_4(\underline{x})$ (6)	Labour use variability (hrs.) $(Z_5(\underline{x})$ (7)	Decision variables: Paddy (acres) x_1 (8)	Maize (acres) x_2 (9)
1.	L_1 } $\delta j = 1$, $j = 1, 2, \ldots 5$	47,112.85	4,241.11	0.00	366.89	809.79	4.48	
2.	$L\infty$	43,735.81	4,012.20	0.00	264.07	746.00	4.10	
3.	L_1 } $\delta_1 = 5$, $\delta_2 = \delta_3$, $\delta_4 = 1$, $\delta_5 = 2$	51,404.04	4,452.18	0.00	780.45	904.83	6.20	
4.	$L\infty$	50,946.61	4,414.02	0.00	744.32	889.72	5.87	
5.	L_1 } $\delta_1 = 5$, $\delta j = 1$, $j \# 1$	57,781.20	4,672.00	2,249.44	1,213.14	1,225.14	7.57	
6.	$L\infty$	53,288.14	4,370.75	0.00	817.90	1,131.15	5.33	
7.	L_1 } $\delta_1 = 10$, $\delta j = 1$, $j \# 1$	58,436,22	4,643.58	2,237.18	1,253.81	1,294.23	7.43	
8.	$L\infty$	55,463.54	4,363.85	61.89	972.15	1,292.30	5.52	
9.	L_1 } $\delta_1 = 1$, $\delta j = 0$, $j \# 1$	58,436.22	4,613.28	2,500.00	1,337,61	1,349.61	7.63	
10.	$L\infty$	58,436.22	4,631.28	2,500.00	1,337.61	1,349.61	7.63	

(Contd.)

TABLE III (*Concld.*)

Farm plan no.	Decision variables												
	Kharif fodder (acres)	*Arhar* (acres)	Wheat (acres)	*Rabi* fodder (acres)	Sugarcane (acres)	Labour hiring (hrs.)				Capital transfer (Rs.)	Capital borrowing (Rs.)		Min L_1 and Min L_∞
						Period I	Period II	Period III	Period IV		Kharif	Rabi	
	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	
(1)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
1.	2.40		7.19	4.50					366.89				1.0176
2.	2.40		6.59	4.18					264.07				0.3226
3.	2.40		7.15	4.73		53.67	144.14		582.57				1.3135
4.	2.40		7.24	4.76		62.29	125.59		556.45	124.54			0.8406
5.	2.40	1.50	7.51	4.49			307.29		905.86		1,882.78	366.61	1.7040
6.	2.40	1.74	7.68	4.32			133.16		684.74	516.05			0.3845
7.	2.40	1.99	7.64	4.36			310.35		943.46	477.12	2,237.18		1.7414
8.	2.40	2.73	7.91	4.03			170.00		802.15	659.45		61.89	0.6941
9.	2.40	1.47	7.44	3.92	0.50		396.17	4.47	936.97		2,264.53	235.54	0.0039
10.	2.40	1.47	7.44	3.92	0.50		396.17	4.47	936.97		2,264.53	235.54	0.0039

maximisation is considered, ignored all the rest of the objectives, the set of compromise farm plans consists of a unique farm plan just identical with the one obtained by using the usual LP model for optimising a single objective.

As expected, it may be seen from Table III that the higher the relative weight given to Z_1 ($\underset{\sim}{x}$) objective in relation to other objectives, the higher the magnitude of its maximised value. Also, the lower relative weight given to $Z_5(\underset{\sim}{x})$ resulted in higher value for it in the compromise farm plans. Sensitivity analysis by assigning varying weights to δ_j can generate a set of efficient compromise farm plans like those presented in Table III to enable the decision-maker to choose the one consistent with his mental make up.

The different trade-offs between the five objectives for each set of δ_j between the two extreme boundaries of the set of compromise farm plans represented by L_1 and L_∞ can also be seen from Table III. For example, when $\delta_1 = 5$ and $\delta_j = 1$, $j = 2, 3, 4, 5$, selection of compromise plan 6 instead of 5 means that the decision-maker considers gains of Rs. 4,493.06 worth of gross margins and 301.25 hours of family labour employment more important than decreases of Rs. 2,249.44 of capital borrowing and 395.24 and 93.99 hours of labour hiring and labour use variability, respectively.

Conclusions

It is possible to generate a set of compromise farm plans with L_1 and L_∞ and by assigning different weights δ_j to the different objectives in the MOP farm planning problem. This certainly gives more freedom to the decision-maker to choose an optimum farm plan in the light of the weights that he wishes to assign to different objectives. This type of a choice of a farm plan is bound to be closer in reality than an optimum farm plan obtained just by optimising a single objective as is normally done by a majority of farm management experts at present. In terms of amount of gross margins generated, the compromise plan 6 was found to be very close to the existing farm plan being currently followed by the farmer. In comparison to the gross margins of Rs. 53,288.14 generated by this farm plan, the farmer obtained Rs.52,553 through his existing farm plan. The cropping patterns in the two plans were, however, at somewhat variance.

Though it may appear that the farmer as a decision-maker is irrational when looked from the traditional LP model point of view, yet our analysis indicated that he followed a compromise farm plan 6 or a farm plan close to it. Thus the decision-maker appears to have considered optimisation of gross margins approximately five times more important than optimisation of other objectives. Since weightage given to different objectives varies from one decision-maker to another, hence the departures between the existing and the optimum farm plans as obtained by using the traditional LP models may also vary.

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