INNOVATIONS IN PROGRAMMING TECHNIQUES FOR RISK ANALYSIS

by

Bruce A. McCarl
with contributions by Donald W. Reid and Bernard V. Tew*

Several key words appear in the title we were given for this paper which influence its content. These words and their influence are

<u>Programming Techniques</u>: This paper focuses on the technique aspect of mathematical programming, and does not examine new mathematical programming applications.

<u>For Risk Analysis</u>: This phrase implies a focus on programming techniques as applied to risk analysis. This permits general mathematical programming techniques to be examined as long as they may have some use in the risk arena. The phrase also permits an examination of new ways that mathematical programming techniques have been applied to risk analysis.

<u>Innovation</u>: Innovations will be defined as those approaches known to the first author which were developed after 1978 as well as approaches developed earlier which have not come into wide use, but should be used more.

Given this preamble, then, this paper attempts to expose readers to potential mathematical programming technique innovations that will influence or potentially will influence future risk modeling.

The paper contains three sections. The first deals with general innovations in mathematical programming solution techniques. The second deals with innovations in risk modeling using mathematical programming, including various formulations and findings which may improve our ability to do risk modeling. The third contains several ideas for possible innovations regarding the mathematical programming risk literature.

INNOVATIONS IN SOLUTION TECHNIQUES

There have been a number of recent innovations in mathematical programming solution techniques. These include the ellipsoid method (Khachiyan), the projection method (Karmarkar), the ability to solve non-linear programs, and the capabilities of microcomputers. Interest in the ellipsoid method has waned; however, interest in the others remains high. These will be discussed in order of their apparent importance to risk analysis, followed by a brief section on other developments.

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*Bruce McCarl is Professor of Agricultural Economics, Texas A&M University
Donald Reid is Associate Professor of Agricultural Economics, Uni. of Georgia
Bernard Tew is Assistant Professor of Agricultural Economics, Uni. of Kentucky

Ability to Solve Non-linear Programming Problems

Many risk formulations contain nonlinear objective functions; some contain non-linear constraint terms. An important new development involves the expanded availability and capability of non-linear programming software. This has largely been manifest in the release of software such as MINOS (Murtagh and Saunders) or GRG2 as imbedded in GINO (Liebman, Lasdon, Schrage and Waren). These algorithms, especially MINOS, permit solution of much larger non-linear programming problems than has heretofore been the case; Lambert solved an E-V type problem with over 2000 variables and 1000 constraints using MINOS. The realized capabilities of the available non-linear programming refutes McCarl and Tice's argument that approximations (e.g., MOTAD) should be used when problems have more than 100 variables. Rather, approximations do not appear necessary with problems up to 1500 variables. This removes much of the motivation for MOTAD-type models as approximations; E-V nonlinear models of very substantial size may be solved and need not be approximated. MOTAD should be re-examined, not as an approximation, but as to whether it a logical risk model in its own right. However, it is still a valid approximation where there is limited computer capacity, very large problems, or where nonlinear algorithms are inaccessible (MINOS' $300 cost makes it rather accessible).

Microcomputers

A second major innovation involves the development of microcomputer mathematical programming software. Recently, powerful optimizers such as GAMS (Kendrick and Meeraus), which contains MINOS; LINDO (Schrage) and GINO (Liebman et al) have been released for microcomputers. The IBM PC 640K barrier appears virtually broken. IBM compatible microcomputers will soon be able to address megabytes of memory. This will greatly expand the capability of microcomputers for solving mathematical programming problems, particularly considering that most microcomputers are idle during evenings, nights and weekends.

Processing speeds on micros are also becoming relatively more acceptable. For assessment relative to processing speed and ability to solve linear and non-linear programming problems, see the piece by Sharda and Harrison.

Karmarkar's Method

Approximately a year ago, Karmarkar released a new linear programming method with information on some rather spectacular results. There was widespread hope that this method would make linear and other programming problems much simpler to solve. Unfortunately, this has not come to pass. Many management scientists have studied and implemented Karmarkar's algorithm. The apparent group consensus, reflected in papers prepared for the recent ORSA/TIMS meetings, is that Karmarkar's algorithm is inferior to the simplex method in both reliability and solution time performance. It also has been discovered that Karmarkar's spectacular results may have been due to a special problem structure (i.e., a band matrix) within his test problems, which can also be exploited with the simplex method. Thus, the Karmarkar method does not appear to be an innovation which will change the face of risk analysis.

Other Solution Techniques

The management science literature is rich with investigations of risk problems. A quick perusal of the library revealed some other potential innovations: the literature review on "fuzzy" optimization (Llena), the piece on objective function coefficients which fall in a range (Bard and Chatterjee), the piece on the entropic penalty approach (Ben-Tal) and the literature review on uncertainty in multi-objective programming (Rakes and Reeves).

INNOVATIONS IN MATHEMATICAL PROGRAMMING RISK MODELING

Attention is now turned to innovations in the application of programming techniques to risk problems, referred to hereafter as risk modeling. In considering the risk modeling literature, two classifications are useful. First, risk models may be either sequential or non-sequential. Non-sequential models depict a "decide now, find out later with no intermediate information" process. Virtually all of the objective function uncertainty models we now work with are non-sequential as well as the majority of the right-hand-side uncertainty models. The sequential models are the alternative model form wherein decisions are made now, information is gained, then decisions may be altered, more information gained, etc. Agricultural economists commonly call this type of model a discrete stochastic program (e.g., Boisvert; Cocks; Rae (1971a,b); Apland and Kaiser). The second major risk model distinction made here involves the type of risk handled, whether it be objective function, right-hand side, or technical coefficient risk, either independently or jointly.

Using these classifications, there are several innovations which merit discussion. These innovations will be discussed in the order they arose.

Stochastic Programming with Recourse

In the 1950s, both Dantzig and Beale developed what are now called two-stage linear programming models.[1] In the late 1960s and early 1970s, this was extended to N stages and re-christened "discrete stochastic programming" by Cocks. Soon thereafter, Rae (1971a,b) discussed it in the agricultural economics journals. In addition the models, renamed stochastic programming with recourse (SPR), have been the subject of considerable research in the management science arena (see the literature review by Hansotia; or Hogan, Morris and Thompson). This remains an active management science research area, as perusal of either the Management Science/Operations Research or International Abstracts in Operations Research abstracts reveals.

Considerable potential exists for use of such models within agricultural economics; for example, considering a wheat marketing problem, the decision to store or sell wheat is made virtually daily and as time passes information is obtained on market movements and developments which can stimulate alterations in storage plans. Furthermore, the cash price on the day of a sell-or-store decision is not a stochastic variable. Consequently, one compares the

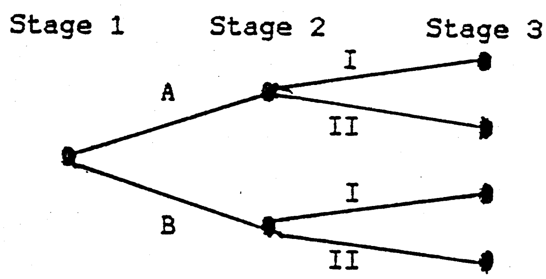
1) The stages in these models refer to the number of points at which decision are made in a sequential decision making environment with information received between stages.

known price today against a future unknown price. There are many similar sequential situations within agriculture. Perhaps expanded use of SPR models under under recourse could enhance our risk modeling ability.

The SPR model, while well known, is under-exploited in the agricultural economics literature. There have been few applications (see Apland and Kaiser for a partial review). Within its framework, one may handle objective function, right-hand-side, or technical coefficient uncertainty, flushing them all into the objective function. The model also can include risk averse utility functions, as mentioned originally in Cocks, or a MOTAD type model as developed in O'Brien. A more general risk averse type model may also be used, as explained in Lambert and McCarl, or as developed in Lambert.

There have been repeated calls for SPR model use (Boisvert gave one at last year's meeting) but few seem to have been heeded. Perhaps the biggest obstacle to its use involves the complexity of the presentations of it. A simpler explanation may help. The basic SPR model involves decisions made now that proceed a number of states of nature into the future. Consider the case depicted in Figure 1. Suppose we have three decision stages: current decisions, those made after one round of uncertainty is resolved and those made after a second round is resolved. Further suppose that between stages 1 and 2 either event A or event B might occur and between stages 2 and 3 event I or Event II can occur.

Figure 1



The model then becomes as depicted in Figure 2. This figure illustrates several important elements of the structure of an SPR model. Note the following:

1) There are as many sets of activities and resources for each stage as there are events leading to that stage in the decision tree. Thus, there is one set of stage 1 activities, two of stage 2 (one for event A, one for B) and four of stage 3 (one for each of events AI, AII, BI and BII).

2) The stage 1 activities precede both of the stage two activities which in turn precede the stage 3 activities. This shows both the sequential and recourse aspects of the model. The stage 1 activities are set initially preceding whatever stage 2 might occur. The stage 2 activities then provide additional decision making given knowledge of whether event A or B occurred and the Stage 1 decisions. Thus in our wheat example what is sold or stored at Stage 1 conditions decision making at Stage 2 regardless of whether events A or B occurs. Similarly, Stage 2 decisions condition what can be done at Stage 3.

Figure 2: Tableau of Three-Stage SPR Example

	Stage 1 Activities	Stage 2 Activities A	Stage 2 Activities B	Stage 3 Activities AI	Stage 3 Activities AII	Stage 3 Activities BI	Stage 3 Activities BII	RHS
Stage 1 Resources	G							$\leq b$
Stage 1 Link to Stage 2: State of Nature A	-1	1						≤ 0
Stage 1 Link to Stage 2: State of Nature B	-1		1					≤ 0
Stage 2: State of Nature A Resources		H_A						$\leq d_A$
Stage 2: State of Nature B Resources			H_B					$\leq d_B$
Stage 2: State of Nature A Link to Stage 3: State of Nature I		-1		1				≤ 0
Stage 2: State of Nature A Link to Stage 3: Stage of Nature II		-1			1			≤ 0
Stage 2: State of Nature B Link to Stage 3: Stage of Nature I			-1			1		≤ 0
Stage 2: State of Nature B Link to Stage 3: Stage Nature II			-1				1	≤ 0
Stage 3 Resources given Stage 2, State of Nature A and Stage 3 I				K_{AI}				$\leq e_{AI}$
Stage 3 Resources given Stage 2, State of Nature A and Stage 3 II					K_{AII}			$\leq e_{AII}$
Stage 3 Resources given Stage 2, State of Nature B and Stage 3 I						K_{BI}		$\leq e_{BI}$
Stage 3 Resources given Stage 2, State of Nature B and Stage 3 II							K_{BII}	$\leq e_{BII}$

3) The two sets of stage 2 activities use independent resources, reflecting two mutually exclusive events (A and B). Further, the stage 3 activities constitute 4 sets of mutually exclusive events depending on stage 2 and stage 3 state of nature. Thus in our wheat example, decision making at stage 2 is done with perfect knowledge of price movements in the time period between stages 1 and 2. Furthermore, the price movements are mutually exclusive and resources are not shared between alternative events at stage 2.

4) The resource usage, resource endowment and objective function coefficient are provisional on state of nature. All these may be uncertain and their joint distribution reflected.

5) This example shows the size explosion of SPR models but the sparsity of nonzeros and repeated structure make the problems somewhat easier than their size would imply (see Lambert or Birge [1985] for computational results).

6) Finally, let us examine the objective function, the relevant objective function is

$$C_1 X_1 + P_A C_{2A} X_{2A} + P_B C_{2B} X_{2B} + P_{AI} C_{3AI} X_{3AI} + P_{AII} C_{3AII} X_{3AII}$$
$$+ P_{BI} C_{3BI} X_{3BI} + P_{BII} C_{3BII} X_{3BII}$$

where P_i and P_{ij} are respectively the probabilities of state i in stage 2 and the joint probability of state i in stage 2 along with state j in stage 3. This objective function simply multiplies each income outcome by its probability, and thus represents expected income. The objective function may also be depicted under way by entering a new variable Y (with subscripts defined as above) which equals the income under stage 3, event ij, the constraint

$$Y_{ij} = C_1 X_1 + C_{2i} X_{2i} + C_{3ij} X_{3ij}$$

and modifying the objective to be

$$\text{Maximize} \sum_i \sum_j P_{ij} Y_{ij}$$

This can also be extended to an E-V model, following Cocks, by changing the objective function to

$$\text{Max} \sum_i \sum_j P_{ij} Y_{ij} - \emptyset \sum_i \sum_j \sum_k \sum_l \sigma_{ijkl} Y_{ij} Y_{kl}$$

where $\sigma_{ijkl} = (1 - P_{ij}) P_{ij}$ when i=k and j=l

$= - P_{ij} P_{kl}$ otherwise

or the MOTAD (O'Brien) model by changing the objective function to

$$\text{Max } \bar{Y} - \psi \sum_i \sum_j d_{ij}$$

and adding the constraints

$$\bar{Y} - \sum_i \sum_j P_{ij} Y_{ij} = 0$$

$$\bar{Y} - Y_{ij} + d_{ij} \leq 0 \quad \text{for all } i$$

<u>Wicks and Guise aij Uncertainty Method</u>

A second innovation is Wicks and Guise's formulation, which handles technical coefficient uncertainty in a MOTAD-like framework. This formulation provides a linearization of the risk within sums of uncertain parameters in the constraints. There is an associated nonlinear formulation (Merrill) which given the nonlinear constraint features of MINOS, may now be solvable. Nevertheless, the Wicks and Guise formulation is covered herein and is developed as follows:

Given that technical coefficients are uncertain in a row (i), then one is uncertain of the sum,

$$\sum_{j=1}^{N} a_{ij} X_{ij}$$

which should be no greater than the right hand side. So, if one decomposes the sum into its mean and deviation from the mean, one gets the mean inequality

$$\sum_{j=1}^{N} \overline{a_{ij}} X_{ij} \leq b_i$$

and some deviation equations

$$\sum_{j=1}^{N} d_{sij} X_{ij} + e_{si} \leq 0 \quad \text{for all } s$$

where d_{sij} is the deviation of the s^{th} observation on a_{ij} from the mean; (i.e., $d_{sij} = a_{sij} - \overline{a_{ij}}$) and e_{si} is the s^{th} observation on the deviation that the s^{th} version of the sum exhibits from the mean resource within the i^{th} constraint. These deviations are summed and a term added to the mean constraint, leading to the version of the uncertain which is put into the model

$$\sum_{j=1}^{N} \overline{a_{ij}} X_{ij} + \psi \sum_{s=1}^{S} e_{si} \leq b_i$$

where the term involving the sum of the e's is the total negative deviation of resource usage in the ith constraint from average resource usage and Ψ is the risk aversion parameter. The Wicks and Guise formulation consists of adding the set of deviation equations appearing above into the formulation and modifying the uncertain constraint as above. This formulation has not been widely used since its publication; it does deserve more use, although it is a non-sequential formulation and does not deal with joint probabilities among among technical coefficients across constraints. Paris and Easter also give a related non-linear formulation.

Paris' E-V Model Generalization

In 1979 Paris presented an E-V model generalization which simultaneously depicts right-hand side and objective function coefficient risk. The formulation of Paris' model is

$$\text{Maximize} \quad CX - \Phi X' \Sigma_c X - \theta Y' \Sigma_b Y$$

$$AX - \theta \Sigma_b \bar{Y} \leq b$$

$$X, Y \geq 0$$

where the term Σb is the variance-covariance matrix of the right-hand sides, Y the vector of dual variables and Σc the variance covariance matrix of the objective function coefficients. This particular formulation has only been applied and published by Kramer, McSweeny and Stavros. It merits further investigation, although in my numerical trials I noticed that as θ (risk aversion) increased, so did the objective function. This is because the θ term impacts the right-hand sides as well as the objective function, and the effect of more resources outweighed the effect of increased risk aversion.

Risk Based on Target Income

Recently, Tauer introduced Target MOTAD, showing that it was related to the Second Degree Stochastic Dominant Set. Tauer followed the earlier target models such as that of Roy; and Boussard and Petit. Subsequently, Atwood, Watts and Helmers looked at lower partial moment models. All of these models look at uncertainty in the objective function coefficients and produce infinite sized efficient solution sets. I have very little experience with these models but I believe there are some interesting questions that need to be looked into. One question is: How well can these models work in a policy setting where one does not have a lot of contact with the decision maker and targets are therefore hard to obtain? Second: How well, in the decision maker context, does the Target MOTAD model work when one makes changes in prices, resources available, etc., thus changing the capability of meeting the target and changing the efficient set? Third, how can the number of elements in the "efficient" set be reduced? There have been a number of applications of these kinds of techniques, as indicated by the programs of the latest Western, American or Southern meetings.

Development of More General Risk Aversion Parameter Formulations

Harvey; Lambert and McCarl; Kroll, Levy and Markowitz; Collander and Zilberman; and Tew and Reid among others have investigated the consequences of more general risk aversion formulations than that in the E-V model, such as decreasing absolute risk aversion. This section discusses the DEMP formulation of Lambert and McCarl, which is as follows:

$$\text{Maximize} \quad \sum_{s=1}^{S} P_s \, U\,(W_s)$$

$$AX \leq b$$

$$C_s X - W_s = -W_0 \qquad \text{for all } s$$

$$X,\ W_s \geq 0$$

This formulation contains a linear programming problem with decision variables X. However, in addition, the variables Ws are added - total wealth under the sth state of nature. This addition also requires the parameters Cs - the objective function values under the sth state of nature; Ps the probability of the sth state of nature; and WO - initial wealth. Ws equals initial wealth plus the increment in wealth caused by X. The objective function then contains the sum of the utility of wealth terms under each state of nature multiplied by their probability, which is by definition expected utility. Thus the overall formulation maximizes expected utility, setting the X variables so that the resultant expected utility of wealth is at a maximum. Such a formulation is nonlinear and for global optimality to be found, the objective function must be concave or quasi-concave. The function is obviously concave whenever the U(Ws) terms are concave, which involves an everywhere risk averse utility function but one which can exhibit constant, increasing, or decreasing risk aversion with wealth. DEMP will also work with a quasi-concave objective function. Unfortunately, this does not mean that one can have a general Friedman-Savage type quasi-concave utility function. The objective function involves a positive weighted sum of quasi-concave functions which is not necessarily quasi-concave (as explained in McCarl and Lambert). However, this may work in cases.

Thus, DEMP permits one to avoid the traditional criticism of mathematical tical programming models that they can only handle functions with constant or increasing absolute risk aversion. Furthermore, the use of the explicit Cs terms in the model does not embody distributional assumptions (normality, etc.) other than the assumption that the distribution of outcomes is fully represented by the empirical distribution (Cs) contained in the model.

The DEMP and related formulations are non-linear programming formulations. However, as discussed in the Innovations in Mathematical Programming section, these problems are relatively easy to solve using software such as MINOS (Murtagh and Saunders) now becoming commonly available.

The DEMP formulation can handle either sequential or nonsequential mathematical programming problems. The sequential nature is handled by using the Cocks E-V formulation where explicit income variables are entered in the model, then forming the utility of wealth objective function (for a simple example see Lambert and McCarl). Finally, since DEMP handles sequential formulations, it can also handle uncertainty in of the technical parameters independently and/or jointly within the model structure.

Developing Appropriate Magnitudes of and Developing Pratt Risk Aversion Coefficients

An important factor in the specification of many mathematical programming problems, especially those of the E-V and DEMP types, involves specification of the Pratt risk aversion coefficient. Recently there have been developments about appropriate magnitudes of the Pratt risk aversion coefficient (McCarl and Bessler) and transferring Pratt risk aversion coefficients between studies (Raskin and Cochran, McCarl and Bessler). The McCarl and Bessler results indicate that the Pratt risk aversion coefficient should fall where between 0 and 10 divided by the standard error of income, with the more likely range being between 0 and 3 divided by the standard error of income. In this case the standard error of income should be set based on the size of the gamble undertaken; this could could be set a priori using, for example, an existing crop plan or some magnitude estimation of the size of the risk in the potential solution.

Consideration of the results in McCarl and Bessler or Raskin and Cochran indicates that different but related Pratt risk aversion coefficients should be used when different-sized gambles are being considered. For example, different Pratt risk aversion coefficients should be used when one considers decisions on a per acre basis as opposed to a per farm basis; on a one-year basis as opposed to a net present value basis, or as enterprises shift, since in all these cases the relevant standard error is different (Raskin and Cochran illustrate these points through a number of examples). McCarl and Bessler also argue that Pratt risk aversion coefficients can be transferred from one study to another simply by taking the Pratt risk aversion from the reference study, multiplying it by the standard error of the risky process in that study, then dividing it by the standard error of the risky prospect in the other study. Such a result also shows the alterations in the magnitude that are necessary when going from one study to another. This may invalidate a number of studies that have, for example, used Wilson and Eidman's risk aversion parameters in their E-V programming or stochastic dominance efforts.

Appropriateness of E-V Analysis[2]

E-V analysis has been subjected to considerable criticism. Recently there have been efforts to study the appropriateness E-V models. An early test of the appropriateness of the criterion was performed by Porter and Gaumnitz (PG). PG concluded that generally the choice between the E-V efficient set and the theoretically superior stochastic dominance model is not critical for all but the very risk averse individual.

This finding has been amplified on in a number of recent studies. Kroll, Levy and Markowitz (KLM) examined this question in a portfolio setting involving a general nonlinear utility maximizing formulation extending the earlier results in Levy and Markowitz. The KLM study compared the portfolios under several different types of utility functions. The main result is that the E-V criterion is shown to be empirically valid.

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[2] This section is largely the work of Bernard V. Tew and Donald W. Reid.

Tew and Reid did a similar study to that of KLM but in a farm modeling setting. Their results strongly support the effectiveness of the E-V criterion in generating portfolios like those of more general utility function models in the utility function cases they considered. Tew and Reid also tested the criterion's sensitivity to severely skewed hypothetical yield data. The strength of the results remained unchanged. In a related study Reid and Tew approached the problem in a setting with only capital constraints. Again the results strongly support the use of the E-V criterion.

On the other hand, Lambert and McCarl's hypothetical trials with the DEMP model show some differerences in the portfolios, but Lambert's follow-up work does not (all solutions were the same as the profit maximizing one).

Other Developments

There have been a number of other developments which merit brief mention. These involve alternative explanations to risk and a number of miscellaneous topics which have appeared in the literature. Taking these in reverse order,

the miscellaneous topics involve aggregation within the variables and constraints in a risk programming model (Birge 1985a); extensions to the E-V model which consider skewness and the testing of the effects of this on portfolios (Park and Yeh); multi-objective programming theories under risk (Rakes and Reeves); integer stochastic programming formulations wherein zero-one variables are included in conjunction with risk (Wollmer; Duran and Grossman; Hatch et al; or Perry); and, finally, Sengupta's presentation of a review of several other developments.

There have also been a number of developments showing that mathematical programming models may bias the importance of risk unless they are properly specified in terms of constraints (Baker and McCarl; Musser, McCarl and Smith), dynamics (Antle; Gray and Furtan), rotations (El-Nazer and McCarl; Musser et al), dispersion of price expectations (Pope), and transactions costs (Roumasset), as well as sequential decisions within a year (Lambert).

POTENTIAL OTHER INNOVATIONS

At the end of a paper on innovations, it seems natural to suggest a few other innovations which probably could or should be looked into. A list of these follows:

Do Risk Models Help?

It appears that more comprehensive studies are needed on how risk programming models improve our ability to predict behavior, especially after a number of studies have come up with things that could be confused with risk. This is an old topic for me, as it goes back to the validation paper I gave a couple of years ago (McCarl).

The Comparability of Risk Aversion Parameters

It appears that the paper by McCarl and Bessler discussed above provides the opportunity to study the comparability of risk aversion parameters across studies. I think the formula multiplying the Pratt risk aversion parameter by by a base study's standard error and dividing it by a second study's standard error provides a basis for comparison of risk aversion coefficients across studies, much as has been done in the MOTAD literature (Apland, McCarl and Miller). Such a drawing together of the body of empirical findings on risk aversion parameters would provide greater guidance to those doing E-V modeling, stochastic dominance with respect to a function, and other applications where risk aversion parameters are needed.

Role of Sequential Models

Fundamentally, much of the risk in agriculture is sequential. This implies that sequential risk models are important when modeling agricultural decisions. I believe we should research the quality of predictions obtained from non-sequential versus sequential models. Sequential models are the topic of a research project at Texas A&M by a number of the A&M S-180 participants. Other such efforts are needed.

Joint Sources of Risk

It appears that the vast majority of applications which have been done only consider objective function risk. One also faces uncertainty of resource availability, working rates, and other technical coefficients. It appears that further investigations all three sources of risk are merited. Perhaps such things as SPR are the answer to doing this; on the other hand, the joint use of the Wicks and Guise, Paris, or old chance-constrained programming (Charnes and Cooper) methods perhaps should be considered.

When is Modeling Risk Important?

When a linear programming model is formulated, every parameter in that model usually has some degree of uncertainty about it. For example, consider the land available for farming. I remember a case when a torrential rainstorm inundated half of a farm two years in a row. This seems to be a manifestation of risk in acreage available, which we normally think of as absolutely certain. This shows that even those things which we think are certain may actually be uncertain. Two research needs and potential innovations in this area immediately come to mind. First, some systematic investigation is needed of decision rules as to when risk in the parameters should be modeled. Second, investigations could be done on the effects of the various types of risk modeling methods on the model predictions. There are findings (Kallberg, White and Ziemba) where it is argued that including risk is cost effective.

Sample Information

A long discussed but unresolved topic in mathematical programming has involved the consequences of sample information for the distribution of answers. Discrete stochastic programming allows some of these sorts of things to be addressed, as they are in the recent paper by Jagannathan. Drynan also presents an investigation.

Policy Consequences of Risk Averse Behavior

Interesting work might be done on the setting of policies considering that people behave in a risk-averse fashion. This might follow multi-level programming (Candler, Fortuny and McCarl) but use a more complex behavioral model as a risk model. Another interesting possibility would be studies using models such as Hazell and Scandizzo's, and comparing the consequences of implementing policies without considering the risk averse behavior within agriculture versus policies considering such behavior.

Interval Analysis

E-V analysis in intimately linked with the efficient frontier concept. A research need involves whether one can derive a reliable and complete efficient frontier using, for example, the DEMP model with knowledge of only the interval within which the Pratt risk aversion falls. The paper by Hammond may provide some guidance in such and endeavor.

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