A REVIEW OF ASSET PRICING THEORIES UNDER UNCERTAINTY

by

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The capital asset pricing model (CAPM) has been central to modern financial economic theory as it applies to corporation valuation and investment analysis. An alternative model which is somewhat more general in nature but serves the same role is the abitrage pricing theory (APT). Both of these models are based on the concept that the asset markets establish a determinable price of risk which helps determine an asset's value.

The conceptual framework and potential empirical applications of CAPM and APT have received limited attention in the agricultural finance and risk literature. The basic CAPM framework was applied to farm real estate by both Barry and White and Ziemer. Beyond these two studies, however, little has been done to apply, modify, or refine the models for other agricultural situations. This paper is a review of the basic CAPM and APT including current developments and critiques. The purpose of this review is to stimulate interest in theoretical and empirical research that potentially could prove beneficial in understanding asset valuation and aid in investment decisions for farms and agribusiness firms.

ASSUMPTIONS AND DERIVATION OF CAPM

Although the assumptions of CAPM appear to be quite limiting, they are substantially unchanged from the premises necessary to develop the standard economic theory of the firm in a purely competitive environment. The usual assumptions for the development of CAPM are:

1) All investors are risk-averse, single-period expected utility maxmizers of terminal wealth.

2) All investors have homogeneous expectations concerning alternative assets which have a joint normal distribution. Also, all investors are price takers.

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3) There exists a risk-free asset such that all investors can lend and/or borrow at the risk-free rate, R_f.

4) Short sales of any asset exist.

5) All asset quantities are fixed. Furthermore, all assets are marketable and perfectly divisible.

6) All asset markets are frictionless and characterized by costless, instantenous, and simultaneously available information.

7) No market imperfections exist such as institutional regulations and restrictions, or taxes.

Derivation and proof of the CAPM requires the existence of an expected value-variance (E-V) efficient market portfolio. Since all investors have homogeneous expectations, a single opportunity set is present. Furthermore, regardless of individual risk preferences all individuals will select efficient portfolios. The market portfolio is now necessarily efficient because it is the sum of all individual portfolios which are efficient (Fama, 1965a).

Consider Figure 1 which illustrates an opportunity set available to all investors in a market characterized by the CAPM assumptions. M is the efficient market portfolio in a market containing two risky assets, A and B, and a risk-free asset, with rate of return R_f. The market portfolio has expected return, $E(R_m)$ and risk, σ_m. The expected returns of the risky assets A and B are respectively, $E(R_a)$ and $E(R_b)$, while the estimates of risk for each asset are σ_a and σ_b.

A portfolio containing X percent of risky asset A and (1-X) percent of the efficient market portfolio M has the following expected value and standard deviation:

$$E(R_p) = XE(R_a) + (1-X)E(R_m) \tag{1}$$

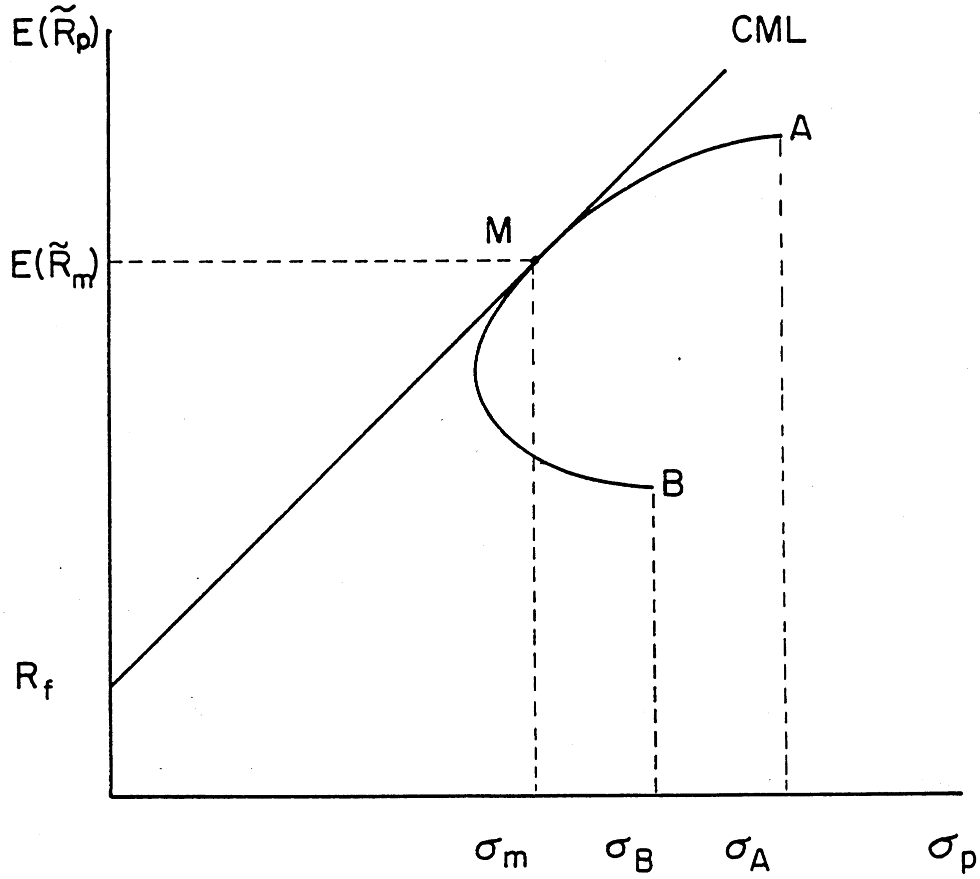
$$\sigma_p = [X^2\sigma_a{}^2 + (1-X)^2 \sigma_m{}^2 + 2X(1-X)\sigma_{am}]^{1/2} \tag{2}$$

where σ_{am} is the covariance between the risky asset A and the efficient market portfolio M.

Recall that the market portfolio already contains risky asset A and, because the market is assumed to be in equilibrium, all marketable assets, A and B, are held in a value- weight proportion. That proportion is defined as:

$$VW = \frac{\text{market value of A}}{\text{market value of A and B}} \tag{3}$$

Figure I. The Efficient Market Portfolio and a Risk - Free Asset



The change in the expected value of the portfolio, $E(R_p)$, with respect to the proportion invested in the risky asset A can be represented as:

$$\frac{\partial E(R_p)}{\partial X} = E(R_a) - E(R_m) \qquad (4)$$

Similarly, a change in the risk of the portfolio is determined to be:

$$\frac{\partial \sigma_p}{\partial X} = 1/2\ [\ X^2 \sigma_a{}^2 + (1-X)^2 \sigma_m{}^2 + 2X(1-X)\sigma_{am}]^{-1/2}$$

$$\cdot\ [2X\sigma_a{}^2 - 2\sigma_m{}^2 + 2X\sigma_m{}^2 + 2\sigma_{am} - 4X\sigma_{am}] \qquad (5)$$

With the market in equilibrium the quantity of asset A invested in the portfolio, represents excess demand. By the definition of equilibrium, excess demand must equal zero. Substituting $X=0$ into equations (4) and (5) yields, respectively:

$$\left.\frac{\partial E(R_p)}{\partial X}\right|_{X=0} = E(R_a) - E(R_m) \qquad (6)$$

and

$$\left.\frac{\partial \sigma_p}{\partial X}\right|_{X=0} = \frac{\sigma_{am} - \sigma_m{}^2}{\sigma_m} \qquad (7)$$

The slope of line AMB in Figure 1 evaluated at the market portfolio M is determined by dividing (6) by (7). The result is:

$$\left.\frac{\partial E(R_p)/\partial X}{\partial \sigma_p/\partial X}\right|_{X=0} = \frac{E(R_a) - E(R_m)}{(\sigma_{am} - \sigma_m{}^2)/\sigma_m}\ . \qquad (8)$$

The slope of the capital market line CML in Figure 1 is:

$$\frac{\partial E(R_p)/\partial X}{\partial \sigma_p / \partial X} = \frac{E(R_m) - R_f}{\sigma_m} \quad (9)$$

Therefore, at the tangency point, M, the slopes of the CML and line AMB are necessarily equivalent given market efficiency. The expression in (8) and (9) can be now be equated forming:

$$\frac{E(R_m) - R_f}{\sigma_m} = \frac{E(R_a) - E(R_m)}{(\sigma_{am} - \sigma_m^2)/\sigma_m} \quad (10)$$

The usual form of the capital asset pricing model or CAPM can now be obtained by solving (10) for the expected value of the risky asset A. The result is:

$$E(R_a) = R_f + [E(R_m) - R_f] \beta \quad (11)$$

where $\beta = \frac{\sigma_{am}}{\sigma_m^2}$

PROPERTIES OF THE CAPM

Several properties of the CAPM are conceptually important. However, two properties are particularly relevant and have received much attention in the financial economic literature. First, every asset in the market must be priced such that its rate of return equals the risk-free rate plus a risk premium. The risk premium for an asset is the price of risk multiplied by the quantity of risk in the market. In terms of the CAPM the price of risk is:

$$[E(R_m) - R_f] \quad (12)$$

and the quantity risk is:

$$\frac{\sigma_{am}}{\sigma_m^2} \quad (13)$$

Now consider pricing an asset using the CAPM when the asset being considered is the market portfolio. Equation (11) reduces to:

$$E(R_m) = R_f + [(E(R_m) - R_f)] \quad (14)$$

because the risk of the market equals 1. Figure 2 graphically illustrates the relationship in equation (14) which is called the Security Market Line, SML. Since all assets are priced in the previously mentioned form, an asset's risk-adjusted equilibrium rate of return is exactly located on the SML.

The total risk of an investment is composed of systematic and unsystematic risk. The CAPM implies that an individual can eliminate all risk through diversification except the risk associted with the covariance between the asset and the market--its systematic risk. Consequently, the only risk a rational risk-averse individual would pay to avoid is the systematic risk. A mathematical characterization of this relationship is a linear function of the market return plus a random error term.

$$R_a = \alpha + \beta R_m + \varepsilon . \quad (15)$$

From the properties of random variables it follows that:

$$\sigma_a^2 = \varphi\sigma_m^2 + \sigma_e^2 \quad (16)$$

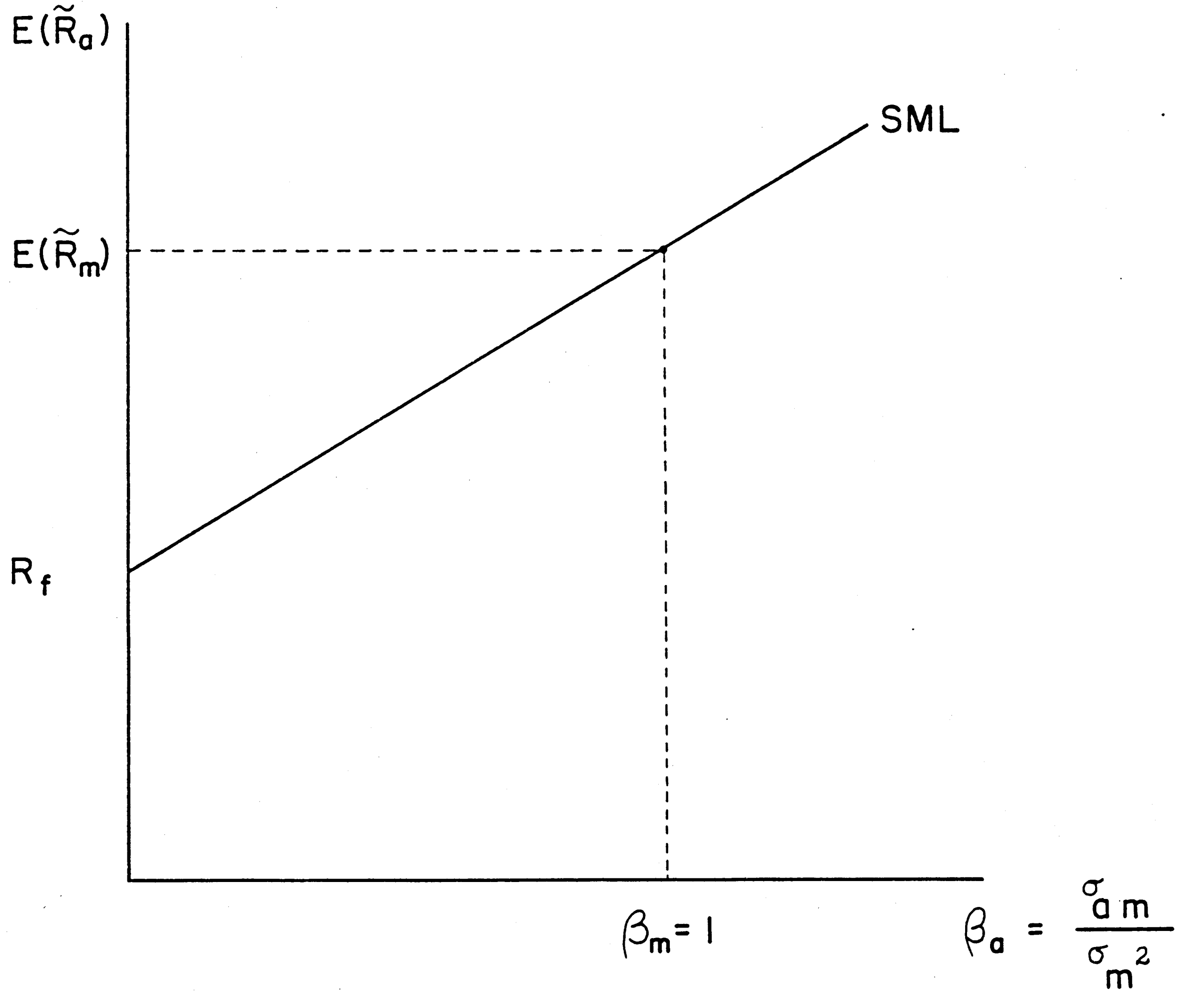
where σ_m^2 is the systematic risk and σ_e^2 is the unsystematic risk. The φ term is equivalent to the β in equation 15.

The second important property of the CAPM is that the measure of risk for all assets is linearly additive when portfolios are formed. For example, again consider risky assets, A and B, with portfolio composition of X and (1-X), respectively. The systematic risk of each asset can be represented as β_a and β_b. The systematic risk of a portfolio containing these two assets would be:

$$\beta_p = X\beta_a + (1-X)\,\beta_b \quad (17)$$

The proof of equation (17) is obtained from the definition of covariance and the properties of expected values and variances. The definition of β_p is:

Figure 2. A Graphical Representation of the Capital Asset Pricing Model



$$\beta_p = \frac{E\{[XR_a + (1-X)R_b - XE(R_a) - (1-X)E(R_b)]\,[R_m - E(R_m)]\}}{\sigma_m^2} \quad (18)$$

The following expression results by factoring equation (18):

$$\beta_p = \frac{X[E\{[R_a-E(R_a)]\,[R_m-E(R_m)]\}]}{\sigma_m^2} + \frac{(1-X)[E\{[R_b-E(R_b)]\,[R_m-E(R_m)]\}]}{\sigma_m^2} \quad (19)$$

Again, employing the definition of β, the terms following X and (1-X) are β_a and β_b, respectively. Thus, equation (19) reduces to the identity in equation (17). The implication of this property is that the β's of individual assets are all that is needed to estimate the systematic risk in the portfolio.

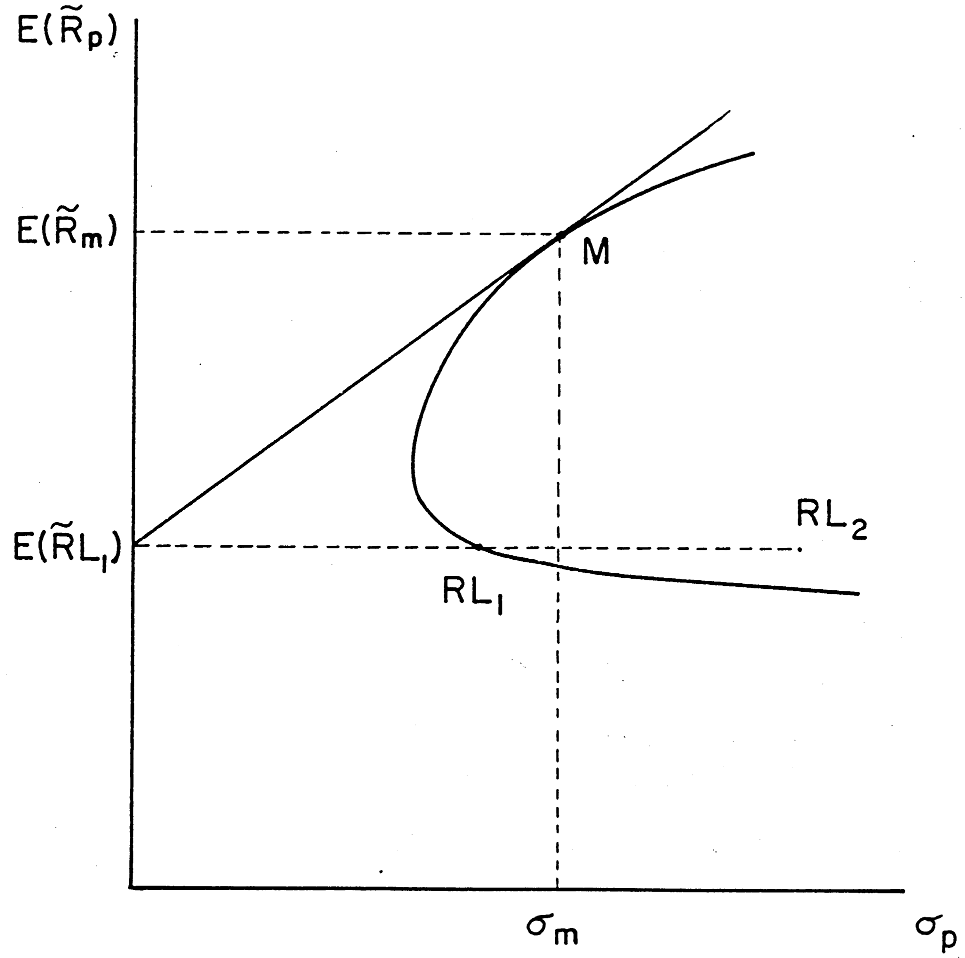
EXTENSIONS OF THE CAPM

The assumptions that were used in the previous section to derive the CAPM are all extremely unrealistic. Consequently, concern about the model's empirical validity and/or the model's general applicability seem relevant. A number of articles have focused on the formulation and applicability of the model by relaxing one or more of the basic assumptions. A few of the more important investigations include the cases when no riskless asset exists (Black), when a nonmarketable asset exists (Mayers), and when heterogeneous expectations prevail (Lintner, 1969). Two derivations are given here as examples of CAPM extensions.

Consider the situation where no riskless asset exists as first examined by Black. Figure 3 illustrates the argument. Portfolio M is again the efficient market portfolio necessary for the development of the CAPM. Black assumes that an investor can identify two portfolios that have zero covariance with M. These portfolios are identified as RL_1 and RL_2. Both portfolios are uncorrelated with M and each portfolio contains the same level of systematic risk. In addition, both portfolios have equivalent expected returns. One of the portfolios must be the minimum risk uncorrelated portfolio and be unique. In Figure 3, RL_1 is such a portfolio.

Consider a line $(E(R_{RL1}), M)$ representing an infinite number of portfolios formed from combinations of the uncorrelated portfolio and the market portfolio. If X represents the portion of the investor's wealth invested in M and (1-X) represents that portion invested in RL_1, the expected return and risk of such a portfolio become respectively:

Figure 3. An Illustration of the CAPM Without a Risk-Free Asset



$$E(R_p) = XE(R_m) + (1-X)E(R_{RL1}) \quad (20)$$

$$\sigma_p = [X^2\sigma_m{}^2 + (1-X)^2\sigma_{RL1} + 2X(1-X)\,\rho_{m,RL1}\,\sigma_{RL1}\,\sigma_m]^{1/2} \quad (21)$$

The slope of the line can be determined by dividing the partial derivatives of these equations. The derivatives are:

$$\frac{\partial E(R_p)}{\partial X} = E(R_m) - E(RL_1) \quad (22)$$

$$\frac{\partial \sigma_p}{\partial X} = 1/2[X^2\sigma_m{}^2+(1-X)^2\sigma_{RL1}{}^2]^{-1/2}\;[2X\sigma_m{}^2-2\;{}_{RL1}{}^2+2X\sigma_{RL1}{}^2] \quad (23)$$

Since, the intercept of the line is $E(RL_1)$ and the line must intersect the market portfolio, M, the equation representing the line becomes:

$$E(R_p) = E(RL_1) + \frac{E(R_m) - E(RL_1)}{\sigma_m}\,\sigma_p \quad (24)$$

Equation (24) is the CAPM in the same formulation as equation (11).

Black further extended the results expressed in equation (24) by proving that the expected rate of return on any risky asset must be a linear combination of the rate of return on the uncorrelated portfolio and the market portfolio. The proof is not dependent on the assumption that the uncorrelated portfolio be on the efficient set. This version of Black's extension is referred to as the two-factor model. An interesting caveat of the two-factor model is that the principal implications of the CAPM do not require the existence of a truly risk-free asset and the β value remains an appropriate measure of systematic risk. The outstanding limitation of this extension is the reliance on short sales to form uncorrelated portfolios. The existence of short sales was necessary because, empirically, most if not all securities have positive correlations.

Investors can often be faced with the dilemma of valuing an asset that is not marketable. Such situations can result from institutional barriers, infinite transactions costs, or any other impediments to well functioning markets. An early work by Mayers examined this violation of the standard CAPM assumption. Consider the situation when investors are constrained to maintain a portion of their portfolios in nonmarketable assets. Although many examples of nonmarketable assets exist, Mayers used human capital to illustrate his derivation. Consider the expected return and variance of a portfolio containing a portfolio of risky marketable assets and a risky nonmarketable asset defined as follows:

$$E(R_j) = E(R_m) + E(R_h) \qquad (25)$$

$$V_{Rj} = \sigma_m^2 + \sigma_h^2 + 2\sigma_{mh} \qquad (26)$$

where $E(R_h)$ is the expected return on the risky nonmarketable asset, and σ_h^2 is the variance of returns of the risky nonmarketable asset.

Furthermore, assume the investor seeks to maximize some function of current consumption, expected returns, and variance of the investor's portfolio of assets subject to a budget constraint. Conceptually, the Lagrangian function becomes:

$$L = f(C_i, E(R_i), \sigma_i^2) + \quad (W_i - C_i - \sum_{j=1}^{n} X_{ij}V_j) \qquad (27)$$

where C_i = the ith individual's consumption,

$E(R_i)$ = the expected rate of return on the ith individual's portfolio,

σ_i^2 = the variance of rate of return on the ith individual's portfolio,

W_i = the ith individual's total marketable wealth at the beginning of the period, and

V_j = the total market value of jth firm.

Subsequently Mayers aggregated the first-order conditions to obtain an equilibrium relationship:

$$\frac{\partial \sigma_i^2}{\partial E(R_i)} E(R_j) + 2[\sum_{j}^{k} X_{ij}\ \rho_{jk}\ \sigma_j\ \sigma_k + \sigma_{hj}] + \frac{\partial \sigma_i^2}{\partial C_i} V_j = 0 \qquad (28)$$

where $E(R_j)$ = the expected rate of return for the jth firm,

ρ_{jk} = the correlation coefficient between the rates of return for the jth and kth firms,

σ_j = the standard deviation of the rate of return for the jth firm,

σ_{hj} = the covariance between the rates of return for the ith individual's portfolio including the nonmarketable asset and the jth firm,

Employing the assumption of homogeneous expectations and aggregating over all individuals Mayers reformulated (28) to become:

$$2(\sigma^2_{jm} - \sigma_{jh}) + \sum_{i=1}^{n} \frac{\partial \sigma_i^2}{\partial C_i} V_j - \sum_{i=1}^{n} \frac{\partial \sigma_i^2}{\partial E(R_i)} E(R_m) = 0 \qquad (29)$$

Under certainty the summation preceding the V_j term divided by the summation preceding the $E(R_m)$ term equals R_f. A similar expression can be obtained for the market portfolio and the familiar tangency requirement yields:

$$E(R_j) = R_f + [V_m \sigma_{jm} + \sigma_{jh}] \qquad (30)$$

where $\lambda = \dfrac{E(R_m) - R_f}{V_m \sigma_m^2 + \quad mh}$

The λ term in equation (30) can be interpreted to be the market price per unit of risk. However, under this extension of the CAPM risk contains market variance and the covariance between market and nonmarket returns. Among the obvious implications are that individuals hold different portfolios of risky assets because their nonmarketable assets have different value and risk and the equilibrium price of a risky asset remains independent of individual indifference curves (the seperation theorem holds).

The third extension of the CAPM that has received noteable attention in the literature is Lintner's (1969) the investigation of the assumption of homogeneous expectations. Basically, Lintner shows that the assumption of homogeneous expectations is not critical to the derivation of the CAPM. Using heterogeneous expectations the variance, covariance, and expected return terms become complex weighted sums of all individual expectations. A very important implication of Lintner's work is that given heterogeneous expectations the market portfolio may not be E-V efficient.

ROLL'S CRITIQUE OF THE CAPM EMPIRICAL TESTS

The development of the CAPM produced a conceptual framework for risk-return tradeoffs and asset valuation based on capital market equilibrium and efficiency. The empirical validity of the CAPM concepts has been of great interest to financial economists. The CAPM has been subjected to many empirical tests, including tests by Friend and Blume; Blume and Friend; Fama and MacBeth; Basu; Reinganum; and Banz. The general procedure usually followed in testing the CAPM is to estimate the individual betas of common stock based on a market index. The betas are used to place the securities into various groups to form portfolios which have a high degree of dispersion of systematic risk. Grouping stocks in this manner to form portfolios for empirically testing the CAPM helps reduce measurement error that is associated with betas of individual securities. The portfolio betas are calculated and used to explain the portfolio returns using a regression model of the following form:

$$R'_{pt} = \gamma_0 + \gamma_1 \beta_p + \varepsilon_t \tag{31}$$

where R'_{pt} = the return on the portfolio above the risk-free rate, i.e. $R_{pt} - R_{ft}$,

γ_1 = parameter estimate of $R_{mt} - R_{ft}$,

γ_0 = the intercept term which should be zero.

Based on the formulation of the empirical CAPM model given above, researchers generally agree that γ_0 is significantly different from zero and that γ_1 is less than $R_{mt} - R_{ft}$. Factors have been found which are successful in explaining returns not captured by the beta. For example, both Reinganum and Banz found that smaller firms tend to have abnormally high rates of return. Generally, researchers attempting to test the empirical validity of the CAPM model agree that the theoretical form must be rejected. However, the empirical form, which has a positive intercept, adequately explains security returns.

Roll has written an extensive critique on the empirical tests of the CAPM. Roll's major contention is that the CAPM is not testable unless the composition of the true market portfolio (the portfolio containing all assets) is known exactly (p. 130). Even so, the only testable hypothesis associated with the CAPM model is that of E-V efficiency of the market portfolio. Linearity between returns and beta result if the market portfolio is E-V efficient, and if linearity exists between returns and beta measured on the true market portfolio, the market portfolio is E-V efficient. That is, linearity between returns and beta measured on the market portfolio is both a necessary and sufficient condition for E-V efficiency of the market portfolio. Roll's proof is based on Black's two-factor model which shows a risk-free asset is not necessary for linearity properties of CAPM. The rate of return on an asset can be written as a linear combination of the market portfolio and a zero-beta portfolio (a portfolio with returns orthogonal to the market portfolio),

$$R_j = R_z + (R_m - R_z)\ \beta_j \qquad (32)$$

where R_j = returns on asset j,

R_z = returns on a zero-beta portfolio,

R_m = returns on the market portfolio.

Roll uses this basic result to make related arguments regarding the use of proxies to represent the market portfolio in testing the CAPM. A market portfolio proxy (such as a market index) may be E-V efficient, and given an orthogonal portfolio, linearity in the measured beta will occur. If the proxy and the true market are not the same, or are not perfectly correlated, the measured beta (β) and the true beta (β*) will differ. Thus, testing γ_0 as being significantly different from zero is meaningless as a test of the empirical validity of the CAPM.

Figure 4 illustrates the foregoing discussion. Suppose M is chosen as a market portfolio proxy when M* is the true market portfolio. Given that both are E-V efficient and have orthogonal portfolios Z and Z*, respectively, then γ_0 will be the intercept rather than γ_0^*. This also implies $\gamma_1 < \gamma_1^*$.

Furthermore, it should be noted that a market portfolio proxy may be E-V efficient even if the market portfolio is not, or vice versa. Therefore, the CAPM can be subjected to a valid test only when the market portfolio is known, and the only hypothesis which can be tested is whether the market portfolio is E-V efficient. However, this in no way implies that the CAPM is invalid; it implies only that empirical tests of the CAPM are difficult if not impossible.

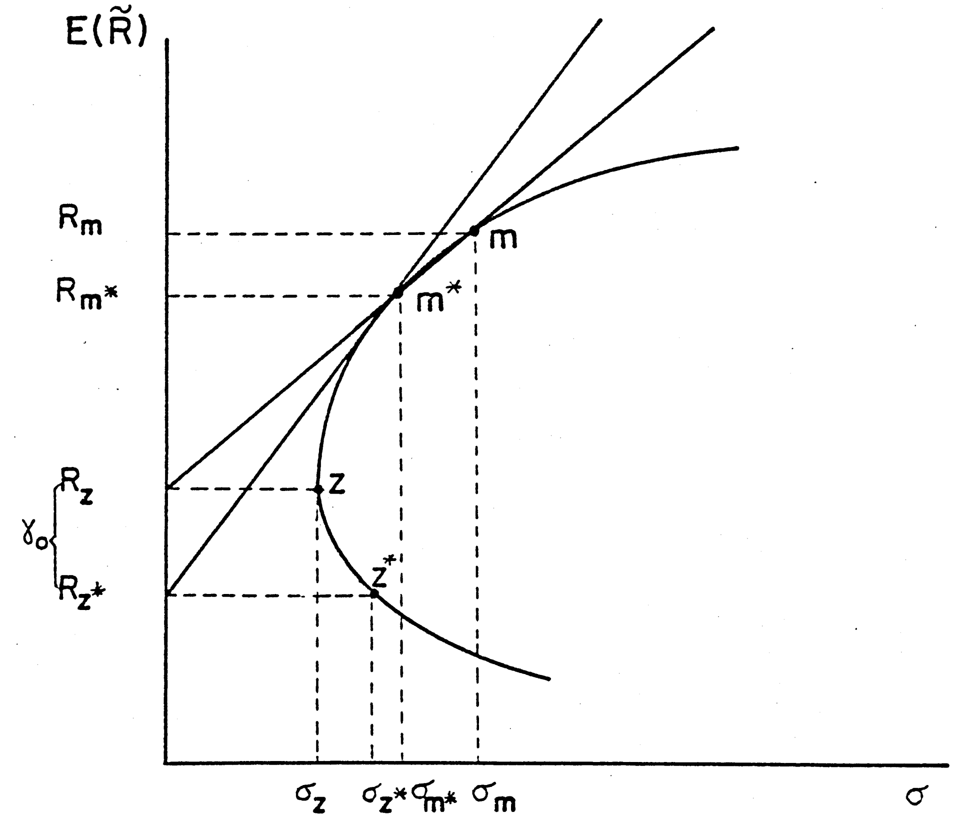
ASSUMPTIONS AND DERIVATION OF THE ARBITRAGE PRICING THEORY

The arbitrage pricing model of capital asset pricing (APT) was developed by Ross as an alternative to the CAPM. Ross' primary motivation appears to stem from his dissatisfaction with the CAPM assumption of either normality of returns or quadratic utility to ensure efficiency. While CAPM and APT are both based on an efficiency assumption, APT takes quite a different approach by employing the concept of arbitrage. This allows APT to be based on less restrictive assumptions than those of CAPM, while the results retain some of the intuitive appeal of the CAPM.

Like CAPM, the APT model is rooted in the belief that the rate of return of an asset is determined by a linear function. However, the APT model is more general in that it assumes that the return is governed by k factors,

$$\tilde{R}_i = E(\tilde{R}_i) + b_{i1}\tilde{F}_1 + b_{i2}\tilde{F}_2 + \dots + b_{ik}\tilde{F}_k + \tilde{\varepsilon}_i \qquad (33)$$

Figure 4. Illustration That Efficient Market Proxy Gives CAPM Properties But Incorrect Intercept



where $\tilde{R}_i$ = random rate of return on the ith asset,

$E(\tilde{R}_i)$ = expected rate of return of the ith asset,

b_{ik} = sensitivity of the ith asset's rate of return to the kth factor,

$\tilde{F}_k$ = the kth factor common to the returns of all assets under consideration, with a mean-zero effect on expected returns,

$\tilde{\varepsilon}_i$ = a random zero mean noise term for the ith asset.

The APT model can be developed from equation (33) given the following assumptions:

(1) Capital markets are perfectly competitive and frictionless.

(2) Individuals have homogeneous beliefs that the random return for assets, $\tilde{R}_i$, is governed by the linear k-factor model given by equation (33). The individuals are not required to agree about the distribution of returns.

(3) The number of assets being considered, n, must be much larger than the number of factors, k.

(4) The $\tilde{\varepsilon}_i$ is the unsystematic risk of the ith asset and it must be independent of all of the k factors and $\tilde{\varepsilon}_j$ where $i \neq j$.

The central theme of the APT is that only a few systematic risk components exist in nature. Therefore, many portfolios are close substitutes in terms of risk and return and, thus, must be valued similarly (Roll and Ross). The frustrating aspect of APT at the current stage of its development is that the fundamental economic factors underlying the k-factors which explain returns are unknown. Roll and Ross recognized this problem by stating (p. 1077):

> "...the return generating process is taken as one of the primitive assumptions of the theory. We do consider the basic underlying causes of the generating process of returns to be a potentially important area of research, but we think it is an area that can be investigated separately from testing asset pricing theories."

Hence, APT is developed from the generating process given in equation (33) without knowing the fundamental economic forces involved.

Given the assumptions, APT is further developed on the concept that in equilibrium all new portfolios which can be formed from the set of assets under consideration earn no additional return above old portfolios, if (a) no additional wealth is used and (b) no additional risk is taken. The portfolio formed from the changes in assets with the properties of no (extra) wealth and no (extra) risk are called arbitrage portfolios.

If n assets exist for forming an arbitrage portfolio, then additional portfolio return is:

$$\tilde{R}_p = \sum_{i=1}^{n} w_i R_i \tag{34}$$

where $\tilde{R}_p$ = the additional portfolio return due to forming the arbritage portfolio,

$\tilde{R}_i$ = the rate of return on the ith asset,

w_i = the change in dollar amount invested in the ith asset as a percentage of the total invested wealth.

By substituting from equation (33), equation (34) becomes:

$$R_p = \sum_{i=1}^{n} w_i [E(R_i)\ b_{i1}\tilde{F}_1 + \ldots + b_{ik}\tilde{F}_k + \tilde{\varepsilon}_i \tag{35a}$$

$$= \sum_{i=1}^{n} w_i E(R_i) + \sum_{i=1}^{n} \sum_{j=1}^{k} w_i b_{ij} + \sum_{i=1}^{n} w_i \tilde{\varepsilon}_i \tag{35b}$$

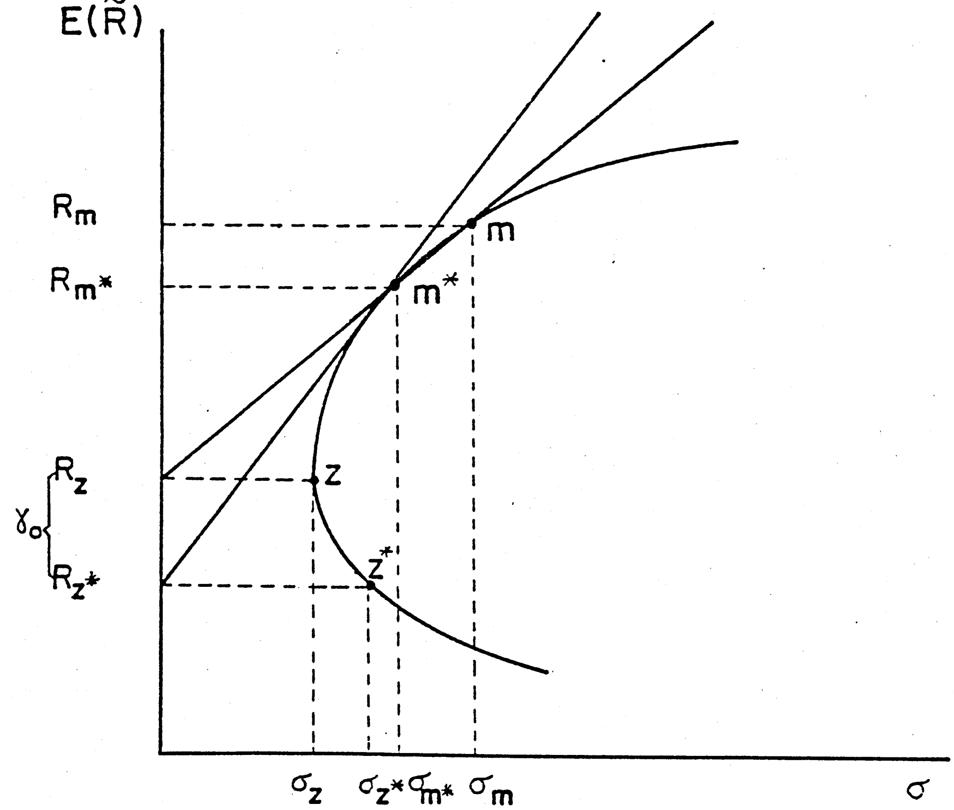
Furthermore the risk factors and noise term of (35b) can be eliminated by assuming that (a) the number of assets n, in the arbritrage portfolio is large, (b) the w_i's chosen are small, and (c) the w_i's are chosen such that the weighted sum of the systematic risk factor b_{ik} is zero for each of the k factors. The risk factors are eliminated because for each of the k factors:

$$\sum_{i=1}^{n} w_i b_{ij} = 0 \tag{36}$$

The error term is eliminated because the b_i's are independent and the law of large numbers insures that as n becomes very large:

$$\sum_{i=1}^{n} w_i \varepsilon_i = 0. \tag{37}$$

Figure 4. Illustration That Efficient Market Proxy Gives CAPM Properties But Incorrect Intercept



Now equation (35b) can be rewritten as:

$$R_p = \sum_{i=1}^{n} w_i E(R_i) \tag{38}$$

Note that R_p is now a certain amount because the arbitrage portfolio is formed in a manner which eliminates risk. That is, changing the original portfolio to the new portfolio occurs such that no additional risk of any kind is taken. Therefore, if the markets are in equilibrium, then for any arbitrage portfolio:

$$R_p = \sum_i w_i E(R_i) = 0 \tag{39}$$

A $R_p > 0$ for any riskless arbitrage portfolio requiring no additional wealth indicates that additional return on assets is possible, meaning that asset values are not efficient.

Equations (36), (37), and (39) indicate that the vector of w_i's is orthogonal to a unit vector, to the vector of coefficients, and thus, to the vector of expected returns. Algebraically this condition implies the expected returns vector must be a linear combination of the constant vector and the coefficient vectors of the k factors.

$$E(R_i) = \lambda_0 + \lambda_1 b_{i1} + \ldots + \lambda_k b_{ik} \tag{40}$$

Assuming that a risk-free asset exists with a risk-free rate, R_f, then:

$$\lambda_0 = R_f \tag{41}$$

With R_f as the intercept and λ_k as the slope for the kth factor, λ_k can be interpreted as a risk premium for the kth factor:

$$\lambda_k = \delta_k - R_f \tag{42}$$

where δ_k is the expected return of a portfolio with unit response to factor k. Now the arbitrage pricing model can be written in excess returns form as:

$$E(R_i) - R_f = (\delta_1 - R_f)b_{i1} + (\delta_2 - R_f)b_{i2} + \ldots + (\delta_k - R_f)b_{ik} \tag{43}$$

The b_{ik}'s can be defined the same way as betas from the CAPM model if the vectors of returns have joint normal distribution and the derived factors have been linearly transformed so that their vectors are orthonormal. Therefore, the CAPM model can be considered a special case of the APT in which only one factor, the market portfolio, explains the returns on an asset and the returns are assumed to be joint normal.

A CRITIQUE OF APT BY PHOEBUS J. DHRYMES

The original CAPM model was used extensively for pricing securities during the 1960's and 1970's. The model is still widely used; however, the development of the APT has recently generated much controversy. The proponents of APT rely on several important advantages relative to the CAPM. First, derivation of the APT requires very simple assumptions. Second, the APT does not suffer from the need to explicitly define the market portfolio, and supporters of the APT refer to its ability to explain rates of return.

Friedman, in his essays on economic methodology, states that the validity of a model cannot be determined by an assessment of the reasonableness of its assumptions. Moreover, highly unrealistic assumptions can yield profound theoretical implicatons. Given a theory does not contain errors in logic a more important question is: Can any hypotheses of a model be subjected to falsification by empirical testing? If such potential falsification does not exist the theory need not be taken seriously. Furthermore, the adequacy of a theory is determined by its ability to predict.

Dhrymes considers several implications of the APT which are potentially falsifiable. First, the original theoretical work by Ross hypothesized the existence of a small number of common risk factors. Second, the theory's usefulness could be established if reliable estimates of the various risk premiums could be obtained. Finally and perhaps most importantly, Dhrymes considers the extent to which the APT "explains" variation in the rate of returns.

Empirical work by Roll and Ross found support for the hypothesis that a small number of risk factors exist. However, two comprehensive articles have produced results directly contradicting the empirical conclusions of Roll and Ross. In 1984, Dhrymes, Friend and Gultekin determined that the number of factors obtained from a group of securities increases as the size of the group increases. In addition, an article by Dhrymes, et. al. in 1985 determined that increasing the number of observations on each security in the group caused the number of factors to increase. As a result, Dhrymes considers the APT simply a data transformation scheme. In a further attempt to test the validity of APT, Dhrymes found that with only one factor 15% of the variation in rates of returns can be "explained." Furthermore, when Dhrymes uses all five factors postulated by Roll and Ross only 30% of the variation is "explained."

Certainly the CAPM has several difficuties many of which are correctly identified and discussed in the critique by Roll. However, by providing empirical observations that falsify the implications of the theory, Dhrymes has shown that the APT initially appears to be a poor "replacement" and using it as a paradigm is highly premature.

DOES CAPM AND/OR APT HAVE POTENTIAL FOR AGRICULTURAL APPLICATIONS?

The CAPM and APT were originally specified based on characteristics of the financial securities market. Several modifications to the original CAPM specification have occurred in an effort to explain observed limited diversification, or to examine model characteristics when various assumptions are relaxed. Some of important modifications were discussed earlier. (Modifications to the APT have not been found by the authors perhaps because of its generality.)

While the original specification of CAPM seems reasonable for application to publicly traded agribusiness firms, it seems that modifications are needed for other applications. Some of the modified specifications found in the literature may be directly applicable to a situation, or form the basis for incorporating assumptions deemed strategic to explaining an empirical situation. Two modifications that appear to have promise in agriculturally related applications are those which relate to incomplete markets and imperfect markets.

The version of the CAPM model that assumes incomplete markets (investors hold nonmarketable assets) originally formulated by Mayers seems to be applicable to situations such as valuing nonmarketable equity shares in farmers cooperatives or valuing farmer claims to commodity program payments. This approach has already received attention from the accounting profession as a possible approach to determining "fair value" for nonmarketable assets such as capital leases (Boatsman and Baskin).

The imperfect market model developed by Levy (referred to as generalized capital asset pricing model--GCAPM) also has good potential for some farm-level applications. The GCAPM is based on the concept that an asset value in equilibrium results from the weighted average of systematic risk it contributes to portfolios of investors who hold it. Thus, this model may have application for farmers in a fairly homogeneous resource region with homogeneous expectations, and who have limited diversification (such as holding most of their assets in farm assets). The implication is that the risk premium will be determined relative to the common asset investments (regional farm asset portfolio) rather than to the market portfolio of all assets. In this type of application, however, spatial arbitrage may be a problem to confront.

Other potential agricultural applications of CAPM-type models seemingly exist. The fruitfulness of employing the CAPM framework in agricultural situations depends on imagination and insight, theoretical developments, and empirical tests. On the other hand, APT may be general enough to apply to any problem because of its lack of a priori specified factors. Certainly, the CAPM and APT have enough potential in various areas of the agricultural sector to warrant serious consideration by agricultural economists.

REFERENCES

Banz, R.W. "The Relationship Between Return and Market Value of Common Stocks." **J. of Fin. Econ.** 9(1981): 3-18.

Barry, Peter J. "Capital Asset Pricing and Farm Real Estate." **Amer. J. Agr. Econ.** 62(1980): 549-53.

Basu, S. "Investment Performance of Common Stocks in Relation to Their Price-Earnings Ratios: A Test of the Efficients Markets Hypothesis." **J. of Fin.** 32(1977): 663-82.

Black, F. "Capital Market Equilibrium with Restricted Borrowing." **J. of Bus.** 45(1972): 444-55.

Blume, M. and I. Friend. "A New Look at the Capital Asset Pricing Model." **J. of Fin.** 46(1973): 19-34.

Boatman, James R. and Elba F. Baskin. "Asset Valuation with Incomplete Markets." **The Accounting Review** 56(1981): 38-53.

Dhrymes, Phoebus J. "The Empirical Relevance of Arbitrage Pricing Models." **J. of Port. Mgt.** 10(1984): 35-44.

Dhrymes, P.J., I Friend and N.B. Gultekin. "A Critical Re-Examination of the Empirical Evidence on the Arbitrage Pricing Theory." **J. of Fin.** 39(1984): 323-46.

Dhrymes, P.J., I. Friend, M.N. Gultekin, and N.B. Gultekin. "An Empirical Examination of the Implications of Arbitrage Pricing Theory." **J. Banking and Fin.** 9(1985): 73-99.

Fama, Eugene F. **Foundation of Finance.** Basic Books, Inc., New York, New York, 1976.

Fama, Eugene. "The Behavior of Stock Market Prices." **J. of Bus.** 38(1965a): 34-105.

Fama, Eugene. "Portfolio Analysis in a Stable Paretian Market." **Mgt. S.** 11(1965b): 404-19.

Fama, E.G. and J. MacBeth. "Risk Return and Equilibrium: Empirical Test." **J. of Pol. Econ.** (1973): 607-36.

Friedman, Milton. "The Methodology of Positive Economics" in **Essays in Positive Economics**, University of Chicago Press, 1953, pp. 3-43.

Friend, I. and M. Blume. "Measurement of Portfolio Performance Under Uncertainty." **Amer. Econ. Rev.** 60(1970): 561-75.

Jensen, M.C. "Capital Markets: Theory and Evidence." **Bell Journal of Economics.** (1972): 357-98.

Levy, Haim. "Equilibrium in an Imperfect Market." **Amer. Econ. Rev.** 86(1978): 643-58.

Lintner, John. "The Aggregation of Investor's Diverse Judgements and Preferences in Purely Competitive Security Markets." **J. of Fin. and Quant. Anal.** 4(1969): 347-400.

Lintner, John. "Security Prices and Maximal Gains from Diversification." **J. of Fin.** 20(1965): 587-616.

Mayers, D. "Non-Marketable Assets and the Capital Market Equilibrium under Uncertainty." **Studies in the Theory of Capital Markets.** Praeger Publishing. New York, New York, 1972.

Merton, R. "An Intertemporal Capital Asset Pricing Model." **Econometrica.** 41(1973): 867-88.

Reinganum, M.R. "Misspecification of Capital Asset Pricing: Empirical Anomolies Based on Earnings Yields and Market Values." **J. of Fin. Econ.** 9(1981): 19-46.

Roll, Richard. "A Critique of the Asset Pricing Theory's Tests." **J. of Fin. Econ.** 4(1977): 129-76.

Roll, Richard and Steven A. Ross. "An Empirical Investigation of the Arbitrage Pricing Theory." **J. of Fin.** 35(1980): 1073-1103.

Ross, Steven A. "The Arbitrage Theory of Capital Asset Pricing." **J. of Econ. Theory.** 13(1976): 341-60.

Sharpe, William F. "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk." **J. of Fin.** 19(1964): 425-42.

White, Fred C. and Rod F. Ziemer. "Farm Real Estate Pricing under Risk: An Empirical Investigation." **So. Econ. J.** 49(1982): 77-87.