Stochastic Microeconomic Production Modelling: Foundation for Policy Analysis Under Risk*

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Introduction

Policy analysis for government programs concerned with price and income stabilization has been performed using macroeconomic analysis (for example, see Just, 1981); traditionally such analyses use a supply equation with output related to price to describe production effects. However, as we demonstrate here, in the case of production under risk such supply models may not adequately describe output effects resulting from policies. As in other macroeconomic situations (Sargent), there may be a need to develop a microeconomic foundation for policy analysis in which policy effects on input use, output, and government cost would be considered at the individual producer level.

There are two reasons for developing such a microeconomic framework. First, government policies may produce unintended effects due to incentive problems and potential incentive problems can be identified by microeconomic modelling. For example, moral hazard is a recognized incentive problem of insurance; this problem occurs because the probability of loss after introduction of the insurance is different from what was anticipated. Similarly, price stabilization and other types of government programs may cause unanticipated shifts in resource allocation; for example, a price stabilization policy may result in a surplus of the commodity with the stabilized price. Shifts in both output and input mix may occur.

Furthermore, microeconomic modelling can also be useful to suggest ways to design policies to reduce incentive problems. Economic theory of contract design (Harris and Raviv) suggests that moral hazard problems can be reduced by monitoring input use. Following this suggestion, potential incentive effects of government programs could be mitigated by defining constraints on input use coupled with inspection to obtain compliance.

This paper demonstrates the above two uses of microeconomic modelling. We analyze some alternative types of insurance policies which might be applied by a government to stabilize price or income for farmers or to increase output of risky crops. The types of policies considered here are

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price, yield, and income insurance. (Price insurance is analogous to "target prices" or "price supports".)

Below, we also present the basic model for production with risk and show how to modify it to analyze alternative policies. Alternatives are compared using a hypothetical example of production under risk. The example is used both to demonstrate the incentive effects which could occur and to illustrate how behavior could be affected by constraints on input use.

Microeconomic Modelling of Production Decisions and Policy Effects

Production in our model is stochastic (rather than certain). The randomness in production is caused by a random state of nature. The effects on output due to the random state of nature and interactions among inputs and the state of nature are explicitly specified by the production function. Producers may produce several outputs which differ in terms of "riskiness"; the resulting level of risk depends on the types of interactions of inputs and the state of nature. The basic model for stochastic production explains how producers choose inputs (x) and acres planted (A) for each output in response to given "initial risk" (the distribution of the state of nature) and input and output prices. Output prices are also random variables when decisions about input use are made. Here, we will not assume that input prices are random; they are assumed to be known at the time that input decisions are made.

The basic microeconomic model of production under risk is described as follows:

a. Distributions of output prices ($F(p^j)$), the distribution of the state of nature ($F(\theta)$), and input prices (w), are taken as given.

b. The stochastic production function for yield (y^j) per acre for each crop is taken as given:

$$y^j = f^j(\{x_i^j\}, \theta).$$

This function expresses the relation of output to use of each input (x_i^j) and the realization of the state of nature. Total output of crop j (Y^j) is acres planted (A^j) times yield.

c. Income (Π) is the value of total output for each crop minus input costs. Inputs used and acres planted for each crop determine income for the farmer in each state of nature and price state. There is a utility function $u(\Pi)$ defined over income in each state. Inputs and acres planted are chosen according to expected utility maximization with the given distribution and production functions:

$$\underset{x_i^j,\ A^j}{\text{Max}} \int \ldots \int u\left(\sum_j A^j \left(p^j f^j (x_i^j, \theta) - \sum_i w_i\, x_i^j\right)\right) F'(p^1)..F'(p^n)\; F'(\theta)\; dp^1..dp^n\; d\theta.$$

(Each random variable is integrated over the domain of its distribution function.)

d. The resulting total output and income <u>distributions</u> $(F(Y^j;\{x_i^j\}, A^j)$ and $F(\Pi;\{x_i^j\}, A^j))$ are then determined by transformations of the given distribution functions from the stochastic production functions and input use and acres planted.

Thus, from solving the above optimization problem, optimal input mix and acres planted will vary depending on risk preferences, the nature of the production relation, input prices, the distributions of output prices, and the distribution of the state of nature.

Numerical computer methods are used here to solve the maximization problem (MINOS on a CYBER 205 was used). Since comparative static comparisons are difficult to make analytically, the use of computer methods allows comparative static analysis of alternative policy and preference scenarios. Rather than simply giving a sign to derivatives as is usually done in such analyses, we compare alternative cases exactly.

<u>Microeconomic Analysis of Policies</u>

Some examples of alternative policies are price insurance, yield insurance, revenue insurance, and combined price/yield insurance. Each of these alters the distribution of income for the farmer in a different way as discussed below.

To analyze the effects of a policy, the distribution of income associated with the policy is used in the expected utility maximization problem to solve for resulting input choice. In this analysis, we use a constant risk aversion preference model:

$$u(\Pi) = -e^{-r\Pi}.$$

One advantage of use of this form is that the optimal level of input use is independent of any potential charge for participating in an insurance program and also independent of initial wealth. Otherwise, in general, levels of wealth and insurance payments will affect the optimal input use.

In addition to input effects, effects of interest to policy makers include output effects, cost of insurance, and measures of benefits such as net farmer income. The introduction of a policy or change in a policy will, in general, cause changes in the acres planted and input mix and thus alter the distribution of output.

This in turn will affect government costs. Since the resulting government cost is determined by the mix of inputs <u>after</u> the policy change, the calculations of program and insurance premiums costs must take such changes into account; otherwise unintended effects such as "moral hazard" problems will affect the viability of government programs. Once identified, unintended effects can be reduced through appropriate redesign of insurance programs. Acreage and input expenditure constraints are potential design alterations which we study below.

To the individual producer, a measure of benefit of a policy is given in terms of the value of the change in expected utility with and without the policy. The money metric value of a policy (Pope, Chavas, and Just) can be computed as the change in the certainty equivalents before and after the policy. With a constant risk aversion utility function, the certainty equivalent (CE) is given by

$$CE = (1/r) \ln (1/Eu)$$

where Eu denotes the expected utility with optimum input choice. After a policy change, the change in certainty equivalent for a base Eu^o is

$$\Delta CE = (1/r) \ln (Eu^o/Eu).$$

Here, we solve for the certainty equivalents from expected utility values associated with each policy. Obviously, this value will vary with risk aversion, size of farm, natural risk, etc. and the type of program.

With price insurance, the farmer is given the maximum of the insured price or the actual price. Price insurance for crop j causes income for the farmer for the insured crop to be defined by

$$(\mathrm{Max} (\bar{p}^j, p^j) y^j - w.x^j)A^j$$

where $\bar{p}^j$ is the guarantee level of insurance and x^j is the vector of inputs used for crop j.

For yield insurance, the farmer is guaranteed at least a certain level of yield ($\bar{y}^j$) but faces the market price distribution. Income for the insured crop for the farmer is then

$$(p^j \mathrm{Max}(\bar{y}^j, y^j) - w.x^j) A^j.$$

With revenue insurance, income for the insured crop becomes

$$(\mathrm{Max}(\bar{R}^j, p^j y^j) - w.x^j) A^j;$$

that is, the farmer gets the larger of a guaranteed revenue ($\bar{R}^j$) or the actual revenue.

For combined price/yield insurance the resulting income for the insured crop is

$$(\text{Max }(\bar{p}^j,\ p^j) \times \text{Max }(\bar{y}^j,\ y^j) - w.x^j)\ A^j.$$

Because of the random state of nature, the cost associated with an insurance policy is a random variable. For price insurance with a guarantee level of $\bar{p}^j$, insurance cost is given by the distribution of

$$(\bar{p}^j - \text{Min }(p^j,\ \bar{p}^j))\ y^j\ A^j$$

and for yield insurance with a guarantee level of $\bar{y}^j$ the cost is defined by

$$p^j\ (\ \bar{y}^j - \text{Min }(y^j,\ \bar{y}^j))\ A^j.$$

For revenue insurance, the cost for a guaranteed revenue level of $\bar{R}^j$ is the distribution of

$$(\bar{R}^j - \text{Min }(p^j y^j,\ \bar{R}^j))\ A^j.$$

For combined price/yield insurance with guarantee levels of $\bar{p}^j$ and $\bar{y}^j$, the cost is defined by

$$(\text{Max }(\bar{p}^j, p^j) \times \text{Max }(\bar{y}^j, y^j) - p^j y^j)\ A^j.$$

Example

We will use a hypothetical production function to illustrate analysis of micro level effects. In this case, a farmer can produce two types of crops with two types of inputs in addition to land (acres planted). To interpret our example in "real world" terms, output one could be considered to be a subsistence food crop and output two could be an export crop. Some possible objectives of government policies with respect to these crops could be to increase total output of crop two for the purpose of increasing exports, to stabilize production, or to stabilize income of farmers. Four types of policies will be compared - price, yield, revenue, and combined price/yield insurance - with varying guarantee levels. In each case, we will apply the policy only to the riskier output (y^2).

Although generally inputs can not be globally classified as "risk increasing" or "risk decreasing", in the discussion below we will term x_1 the "risk reducing" input and x_2 the "risk increasing" input because of their properties in a base case; land (acres planted) is also a risk increasing input. (See the Appendix for definitions of input properties with respect to risk.

The form of the yield production function for the first crop is

$$y^1 = h(x_1^1, x_2^1) + g(x_1^2, x_2^2)\ \theta$$

and, for the second crop, is

$$y^2 = 1.2\ h(x_1^2, x_2^2) + 2\ g(x_1^2, x_2^2)\ \theta$$

Here, we have chosen the functions h(.) and g(.) to be quadratic. (More details about the production functions are given in Appendix 1.)

Natural risk is represented by θ, a random variable with a skewed distribution with mean zero. Thus, the term h(.) represents the mean yield; for a fixed level of input use the second crop has a mean yield 1.2 times the first crop. The variance of yield for each crop depends on input use and is given by

$$\text{Var } y^1 = [g(.)]^2 \text{ Var } \theta$$

$$\text{Var } y^2 = 4\ [g(.)]^2 \text{ Var } \theta;$$

thus, for a fixed level of input use, the second crop has a yield variance which is four times that of the first crop.

The farmer chooses what fraction of his acres (A^1 and A^2) to plant in each crop. Total output for each is given by

$$Y^1 = A^1 \times y^1$$

$$Y^2 = A^2 \times y^2,$$

the variance of the total output for crop j is the variance of its yield times $(A^j)^2$.

Output price risk is also a feature of this problem. The coefficient of variation of price is .08 for the first crop and 4.1 for the second crop. Thus, both in terms of natural risk and price, output two is "riskier" than output one.

First, we study how input use and output are affected by the level of risk aversion (r). Table 1 shows the effects of varying risk aversion in the "no policy" case. In the case of risk neutrality (r=0), the farmer uses inputs in such a way that yield is nearly maximized for output two and all acres are used for crop two. As risk aversion increases, the farmer switches more and more of the acres planted to the less risky crop; inputs are used in such a way that variance of yield for both outputs decreases as r increases. As r increases, the expected income decreases and the variance of income decreases.

In the comparison of policies below, we will use a base case (with r=1) to compare policy effects.

Policy Analyses

The analyses below demonstrate potential differences in alternative types of insurance policies in terms of use of "risk reducing" relative to "risk increasing" inputs and fraction of acres planted; also considered are the effects on yield and yield variance, total output and output variance, income to the farmer, farmer benefit of risk reduction (change in the certainty equivalent compared to the base case), and insurance cost.

The analyses are "partial" in nature since we do not take into account subsequent effects on the output price distribution which might result from production shifts. Of course, our specific conclusions are also only relevant for the price-production scenarios used here. Thus, some care should be used in making generalizations about the relative merits of alternative policies based on the analysis given here. However, the methodology is generally relevant and the types of behavior and "unintended effects" which are identified are generally possible.

Price Insurance (Table 2)

The guarantee level of price insurance is varied as a fraction of the mean price (μ_p) for output two; results are shown in Table 2. For a high enough guarantee level, the farmer switches from a mix of acres planted to produce only output two. For low levels of insurance (less than or equal to $.5\mu_p$), the main effect is a shift in acreage from crop 1 to crop 2. For high levels of price insurance (greater than or equal to $.9\mu_p$), a shift in the input mix occurs, resulting in little change in the expected yield for crop 2 but a decline in the variance of this yield.

The farmer's increased use of the risk reducing input (compared to decreasing use for other types of insurance) may be viewed as "self-insurance" on the output side. Apparently, the hypothetical farmer is willing to trade off a slight reduction in his expected income for a lower variance of income. For example, with an insurance level of μ_p, the farmer could have used the same levels of inputs as for the insurance level of $.75\mu_p$; his expected yield would have been essentially unchanged and his costs of production would have decreased (since x_1 is more expensive than x_2), so his expected income would then have been higher but the input choice indicates that the reduction in variance of yield and resulting reduction in income variance were preferred.

Yield Insurance (Table 3)

The guarantee level for yield is varied as a fraction of mean yield (μ_y) in the base case. Considering the results in Table 3, insurance levels up to .75 μ_y have no effect on decisions because the probability of output below this level is very low. For insurance levels between .75μ_y and μ_y, less of the risk decreasing input and more of the risk increasing input are used as the guarantee level increases. The expected yield changes only slightly, while the yield variance increases dramatically.

This situation demonstrates the "moral hazard" problem. Little or no gain in expected yield is traded for high variance. Since the farmer is protected from the consequences of low yield levels through insurance, the increased yield variance is attractive to him.

Finally, for a high enough level of yield insurance, the farmer's behavior exhibits extreme "moral hazard"; he claims to be producing output two on .66 of the acres planted but in fact does not produce output two at all (since no inputs are used)! The mean and variance of net income and insurance cost increase with the level of insurance.

<u>Revenue Insurance</u> (Table 4)

Here, we study revenue guarantee levels which are fractions of the mean price times mean yield for the base case. For some guarantee level greater than .5 of the base expected revenue, similar to price insurance, the farmer switches from producing both crops to producing only crop two. Prior to this switch, as the guarantee level increases, use of the risk reducing input for crop two decreases and more of the risk increasing input is used. The net effect is a slight increase in expected yield and a large increase in yield variance. The shift to production of only crop 2 (at a program level of 0.75) is accompanied by a large decline in usage of the risk reducing input and a smaller decline in usage of the risk increasing input. The net effects are only a small loss in expected yield and a substantial increase in yield variance.

Higher insurance levels result in only slight input changes with usage of the risk reducing input declining further and usage of the risk increasing input increasing; a slight increase in the variance of yield results.

<u>Summary Table</u> (Table 5)

To compare the alternative policies discussed above, the mean guarantee level is a reasonable basis. In Table 5, the policies are listed in increasing order of risk reduction benefit for the farmer and insurance cost (mean and variance). The most important point to note is how different the policies are in terms of input mix.

Note that price, revenue, and combined price/yield insurance all result in a complete shift in acres planted to crop two when the guarantees are set at the mean levels. The yield insurance policy is the only one for which crop one is still produced; total output is thus least drastically affected by this type of insurance and this program really affects farm income only in the worst states of nature. (Recall however that guarantee levels above the mean yield may cause extreme moral hazard.) Except for price insurance, all policies result in variances of yield for output two which are greater than in the base case. With yield insurance, yield variance for crop two is higher than for price or revenue insurance.

Combined price/yield insurance causes the greatest variance effect for crop two. Although this policy was not analyzed in detail here, logic indicates that when protection against both low price and low output is available the farmer will opt for increasing the variance of output in hopes of getting as large as possible an output level when times are good.

Implications for Policy Design

Governments might wish to increase risky export crops and thus want to use some of incentive programs described above. However, if food security and self-sufficiency are also relevant goals, governments would not find it desirable to use incentive programs which would switch output totally from food crops to an export crop. Thus price or revenue insurance for export crops would not be desirable. However, the proper mix of government objectives may be achieved by using an insurance policy combined with some constraints on input use. (Of course, monitoring of input use, with some economic costs incurred for monitoring, would be required to ensure that constraints are met.) Here, we demonstrate the effects of two types of such constrained insurance policies by continuing the examples above.

Price Insurance with Acre Limits (Table 6)

The form of the constraint used is

$$A^2 \leq \gamma$$

where γ is a fraction of acres planted which may be planted in crop two. Here, we perform comparative static analysis of the effect of varying this constraint with a price guarantee level fixed at the mean price for crop two (program level 1 in Table 2). For each limit, the constraint is binding. As γ is decreased toward the "no price insurance" base case, the input use per acre is hardly affected. Because of the acreage change, the mean and variance of income are reduced as the constraint is tightened. Thus, an acreage limit together with price insurance is a way of obtaining production of a mix of both crops with reduced risk for the government (variance in insurance cost).

Revenue Insurance with Acre Limits (Table 7)

The form of the acre constraint is the same as in the price case above. The acreage constraint has a greater effect on input use than in the price insurance case. For both crops, use of the "risk reducing" input increases and use of the "risk increasing" input decreases toward the no insurance base case as the constraint is tightening (ignoring the case of no production of crop one). Note that (except for when crop one is not produced) the mean output per acre is not affected for both crops but the variance of yield is reduced as the constraint is tightened.

As in the price insurance case, insurance cost (mean and variance) decreases as the constraint is tightened. With the same acreage constraint on production, both mean and variance are higher with revenue insurance than for price insurance. Farmer benefits are higher for this policy than for constrained price insurance.

<u>Yield Insurance with Expenditure Limits</u> (Table 8)

Here, yield guarantees are fixed at the base mean yield level without insurance but expenditure per acre for inputs for crop two is constrained; however, the mix of inputs per acre is not constrained. The cost is constrained to be at least that in the base case with no insurance. Compared to unconstrained yield insurance, this combined policy increases use of <u>both</u> the risk reducing and risk increasing inputs for crop two so that expenditure per acre for inputs is increased. As a result, the mean yield is increased and the variance is decreased. Surprisingly, use of the constraint did not affect the acres planted compared to yield insurance with no expenditure constraint. Total output is hardly affected because of no difference in acreage planted but the variance of output is lower with the constraint than without. Although we did not test it, this type of constraint could be used to avoid the "extreme moral hazard" which occurred with yield insurance set at higher guarantee levels than the mean case with no actual production of crop two.

This program has lower government costs (mean and variance) than the other constrained policies but also produces less benefit for the farmer.

<u>Conclusions</u>

Choice of a policy to alleviate problems of production under risk will depend on the relative values of joint social goals of income stabilization, price stabilization, and output mix (food security versus exports). The benefits and costs of any policy will depend on associated "moral hazard" and other less extreme incentive effects. Obviously, microeconomic analysis - such as that demonstrated here - is useful to identify and evaluate the effects of policies (including unintended effects) and to design policies to balance desired output and income effects with government costs. Of course, when extreme shifts in production technologies and output mix occur, price distributions will be affected. Thus, the type of analysis presented here gives only a partial picture of potential effects. The ultimate effects of policies can only be determined considering market level (macroeconomic) responses.

In traditional macroeconomic models, effects of policies are studied using a supply equation which relates outputs directly to prices and policies; that is, input effects may not be explicitly considered. Although rational expectations models have considered price to be a random variable, only expected price is modelled in a supply-demand equilibrium model.

In comparison, the features of the microeconomic model studied here suggest that the design of traditional macroeconomic models needs to be altered to analyze effects of government policies for production under risk. First, there is the need to focus explicitly on input use and interactions of inputs with sources of risk. Inputs (not outputs) are directly affected by government policies. Second, the full probability distributions of prices (not just expected prices and variances of prices) determine the choice of inputs. Finally, output is explicitly a random variable dependent on resulting input choices; output levels follow from realizations of the state of nature given input choices. Future work should study how to incorporate these elements into econometric, macroeconomic models for policy analysis.

Although the approach demonstrated here is useful to identify the nature of potential effects of policies, the analysis was hypothetical. To make the approach more relevant for "real world" situations, functions need to be specified: namely, probability distributions for the state of nature and prices, stochastic production functions, and preference functions. Many issues of empirical measurement need to be resolved in order to obtain such specifications.

Table 1
The Effect of Risk Aversion on Production

Risk Aversion	Fraction of Acre	Input Use				Expected Yield[a]		Expected Total Output[a]		Farmer Net Income[a]
	Crop 1	Crop 1		Crop 2		Crop 1	Crop 2	Crop 1	Crop 2	
		X_1	X_2	X_1	X_2					
0.	0.	0.	0.	1.61	1.65	0.	1.99 (.208)	0.	1.99 (.208)	9.58 (27.63)
.5	.61	1.80	1.42	2.00	1.21	1.66 (.029)	1.95 (.051)	1.01 (.011)	.76 (.008)	7.45 (4.22)
1.	.77	1.89	1.32	2.15	1.06	1.65 (.021)	1.93 (.019)	1.28 (.013)	0.43 (.001)	6.86 (1.93)
2.	.86	1.97	1.21	2.32	.91	1.63 (.014)	1.88 (.002)	1.41 (.010)	.25 (.00003)	6.50 (1.19)

[a]Variance given in parentheses

Table 2

Price Insurance for Crop 2, Guarantee $\bar{p}$[a]
(r=1)

Program Level	Fraction of Acre	Input Use				Expected Yield[b]		Expected Total Output[b]		Expected Farmer Net Income[b]	Farmer Risk Reduct. Benefit[c]	Expected Insurance Cost[b]
	Crop 1	Crop 1		Crop 2		Crop 1	Crop 2	Crop 1	Crop 2			
		X_1	X_2	X_1	X_2							
0. (base)	.77	1.89	1.32	2.15	1.06	1.65 (.021)	1.93 (.019)	1.28 (.013)	.43 (.001)	6.86 (1.93)	0.	0.
.5	.64	1.89	1.32	2.14	1.07	1.65 (.021)	1.93 (.021)	1.05 (.008)	.70 (.003)	7.36 (3.04)	.23	.08 (.07)
.75	0.	0.	0.	2.16	1.07	0.	1.93 (.019)	0.	1.93 (.019)	10.13 (11.95)	1.51	.93 (2.32)
.9	0.	0.	0.	2.20	1.05	0.	1.92 (.015)	0.	1.92 (.015)	10.60 (8.73)	2.71	1.44 (4.55)
1.0	1.92	0.	0.	2.22	1.04	0.	1.92 (0.12)	0.	1.92	11.05 (6.57)	3.55	1.91 (6.14)

[a] $\bar{p}$ = Program level times mean price for Crop 2.

[b] Variance given in parentheses.

[c] Benefit = change in certainty equivalent due to program as compared to base.

Table 3

Yield Insurance for Crop 2, Guarantee $\bar{y}$[a]

(r=1)

Program Level	Fraction of Acre Crop 1	Input Use Crop 1 X_1	Input Use Crop 1 X_2	Input Use Crop 2 X_1	Input Use Crop 2 X_2	Expected Yield[b] Crop 1	Expected Yield[b] Crop 2	Expected Total Output[b] Crop 1	Expected Total Output[b] Crop 2	Expected Farmer Net Income[b]	Farmer Risk Reduct. Benefit[c]	Expected Insurance Cost[b]
0 (base)	.77	1.89	1.32	2.15	1.06	1.65 (.021)	1.93 (.019)	1.28 (.013)	.43 (.001)	6.86 (1.93)	0.	0.
.75	.77	1.89	1.32	2.15	1.06	1.65 (.021)	1.93 (.019)	1.28 (.013)	.43 (.001)	6.86 (1.93)	0.	0.
.9	.77	1.93	1.28	1.25	1.81	1.64 (.018)	1.97 (.34)	1.26 (.010)	.46 (.019)	7.08 (2.74)	.04	.14 (.08)
1	.75	1.91	1.30	1.00	2.01	1.65 (.019)	1.92 (.485)	1.23 (.010)	.48 (.030)	7.32 (3.15)	.14	.37 (.28)
2	.34	1.75	1.47	0.	0.	1.66 (.034)	0.	.56 (.004)	0.	14.83 (34.66)	1.64	12.70 (34.48)

[a] $\bar{y}$ = Program level times base mean yield for Crop 2.

[b] Variance given in parentheses.

[c] Benefit = change in certainty equivalent due to program as compared to base.

Table 4

Revenue Insurance for Crop 2, Guarantee $\bar{R}$[a]
(r=1)

Program Level	Fraction of Acre	Input Use				Expected Yield[b]		Expected Total Output[b]		Expected Farmer Net Income[b]	Farmer Risk Reduct. Benefit[c]	Expected Insurance Cost[b]
	Crop 1	Crop 1		Crop 2		Crop 1	Crop 2	Crop 1	Crop 2			
		X_1	X_2	X_1	X_2							
0. (base)	.77	1.89	1.32	2.15	1.06	1.65 (.021)	1.93 (.019)	1.28 (.013)	.43 (.001)	6.86 (1.93)	0.	0.
0.5	.59	1.89	1.32	1.95	1.20	1.65 (.021)	1.96 (.055)	0.97 (.007)	.81 (.009)	7.60 (3.92)	0.23	0.11 (.08)
0.75	0.	0.	0.	1.66	1.15	0.	1.94 (.087)	0.	1.94 (.087)	10.26 (13.77)	1.61	0.94 (2.48)
0.9	0.	0.	0.	1.65	1.18	0.	1.95 (.094)	0.	1.95 (.094)	10.86 (10.64)	2.91	1.52 (4.58)
1.0	0.	0.	0.	1.61	1.23	0.	1.95 (.110)	0.	1.95 (.110)	11.35 (8.95)	3.77	1.98 (6.33)

[a] $\bar{R}$ = Program level times base mean yield times mean price for Crop 2.

[b] Variance given in parentheses.

[c] Benefit = change in certainty equivalent due to program as compared to base.

Table 5
Summary: Comparison of Policies for Crop 2 at Mean Insurance Levels
(r=1)

Insurance Type	Fraction of Acre	Input Use				Expected Yield[a]		Expected Total Output[a]		Expected Farmer Net Income[b]	Farmer Risk Reduct. Benefit[b]	Expected Insurance Cost[a]
	Crop 1	Crop 1 X_1	Crop 1 X_2	Crop 2 X_1	Crop 2 X_2	Crop 1	Crop 2	Crop 1	Crop 2			
None	.77	1.89	1.32	2.15	1.06	1.65 (.021)	1.93 (.019)	1.28 (.013)	.43 (.001)	6.86 (1.93)	0.	0.
Yield	.75	1.91	1.30	1.00	2.01	1.65 (.019)	1.92 (.485)	1.23 (.010)	.48 (.030)	7.32 (3.15)	0.14	0.37 (.28)
Price	0.	0.	0.	2.22	1.04	0.	1.92 (.012)	0.	1.92 (.012)	11.05 (6.57)	3.55	1.91 (6.14)
Revenue	0.	0.	0.	1.61	1.23	0.	1.95 (.110)	0.	1.95 (.110)	11.35 (8.95)	3.77	1.98 (6.33)
Price/ Yield	0.	0.	0.	0.48	2.06	0.	1.74 (.660)	0.	1.74 (.660)	12.96 (15.29)	4.42	4.56 (9.78)

[a] Variance given in parentheses.
[b] Benefit = change in certainty equivalent due to program as compared to base.

Table 6
Price Insurance at Mean Price Level with Fraction Acre Limits for Crop Two
(r=1)

Limit[a] Fraction of Acres, Crop 2[a]	Fraction of Acre Crop 1	Input Use Crop 1 X_1	Input Use Crop 1 X_2	Input Use Crop 2 X_1	Input Use Crop 2 X_2	Expected Yield[b] Crop 1	Expected Yield[b] Crop 2	Expected Total Output[b] Crop 1	Expected Total Output[b] Crop 2	Expected Farmer Net Income[b]	Farmer Risk Reduct. Benefit[c]	Expected Insurance Cost[b]
.23 (no ins. base)	.77	1.89	1.32	2.15	1.06	1.65 (.021)	1.93 (.019)	1.28 (.013)	.43 (.001)	6.86 (1.93)	0	0
.25	.75	1.91	1.30	2.20	1.06	1.65 (.019)	1.92 (.015)	1.23 (.011)	.48 (.011)	7.46 (1.26)	.87	.48 (.39)
.5	.50	1.92	1.30	2.21	1.05	1.65 (.019)	1.92 (.013)	.82 (.005)	.96 (.003)	8.62 (2.15)	1.84	.96 (1.54)
.75	.25	1.93	1.30	2.22	1.04	1.65 (.018)	1.92 (.012)	.41 (.001)	1.44 (.007)	9.83 (3.92)	2.70	1.44 (3.46)
1 (no constraint)	0.	0.	0.	2.22	1.04	0.	1.92 (.012)	0.	1.92 (.012)	11.05 (6.58)	3.55	1.91 (6.14)

[a] Fraction of Acres in Crop 2 ≤ Fraction limit.

[b] Variance given in parentheses.

[c] Benefit = change in certainty equivalent due to program as compared to base.

Table 8

Yield Insurance at Mean Base Yield Level with Expenditure Limit for Crop Two

(r=1)

Expenditure	Fraction of Acre	Input Use				Expected Yield[b]		Expected Total Output[b]		Expected Farmer Net Income[b]	Farmer Risk Reduct. Benefit[c]	Expected Insurance Cost[b]
	Crop 1	Crop 1		Crop 2		Crop 1	Crop 2	Crop 1	Crop 2			
		X_1	X_2	X_1	X_2							
Base (no ins.)	.77	1.89	1.32	2.15	1.06	1.65 (.021)	1.93 (.019)	1.28 (.013)	.43 (.001)	6.86 (1.93)	0	0
Limit[a] to base exp. per acre	.75	1.91	1.30	1.43	2.13	1.65 (.019)	1.96 (.39)	1.24 (.011)	.49 (.024)	7.27 (2.98)	.13	.30 (.19)
No constraint	.75	1.91	1.30	1.00	2.01	1.65 (.019)	1.92 (.485)	1.23 (.010)	.48 (.031)	7.32 (3.15)	.14	.37 (.28)

[a] Expenditure per acre $\geq$ expenditure per acre in base case.

[b] Variance given in parentheses.

[c] Benefit = change in certainty equivalent due to program as compared to base.

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Mathematical Appendix

1. First order conditions for the one output case

Define: $y = f(x,\theta)$ = yield per acre

$Y = f(x,\theta)A$ = total output

$\Pi(x,\theta,p) = (py - wx)A$ = production income

x = vector of inputs

θ = state of nature

$F'(p)$, $F'(\theta)$ = density functions.

Optimum x:

$$\max_x \int_D\!\int u(\Pi(x,\theta,p))\, F'(p)F'(\theta)\,dp\,d\theta;$$

$x_i > 0$ implies the first order condition

$$\iint u'(\Pi)\frac{\partial \Pi}{\partial x_i} F'(p)\,F'(\theta)\,dp\,d\theta = 0.$$

Note by definition

$$\text{Cov}\left(u', \frac{\partial \Pi}{\partial x_i}\right) = \iint u'(\Pi)\frac{\partial \Pi}{\partial x_i} F'(p)\,F'(\theta)\,dp\,d\theta - E\Pi_{x_i} Eu'$$

and by the first order condition,

$$\text{Cov}\left(u', \frac{\partial \Pi}{\partial x_i}\right) = -\,E\Pi_{x_i} Eu'.$$

(The first order conditions are the same with multiple outputs.)

2. Definition of input risk properties

Define the "marginal risk premium" for input x_i as:

$$MRP_{x_i} = \text{Cov}\left(u', \frac{\partial \Pi}{\partial x_i}\right)/Eu'.$$

At the optimum input use, define an input to be:

"risk increasing" if $MRP_{x_i} < 0$;

"risk decreasing" if $MRP_{x_i} > 0$.

3. Interpretation of input risk properties

From the first order condition

$$E\Pi_{x_i} + \text{Cov}\ (u', \frac{\partial \Pi}{\partial x_i})/Eu' = 0.$$

Risk increasing (decreasing) inputs may be thought of as imposing a marginal risk cost (benefit) on the marginal expected income from production. Note that if x_i is "risk increasing" (reducing"), the implication from the first order condition is

$$E\Pi_{x_i} = \frac{-\text{Cov}(u', \frac{\partial \pi}{\partial x_i})}{Eu'} > 0\ (< 0)$$

at the optimum.

4. Determination of risk properties of inputs

Assume independence between p and y and $\frac{\partial y}{\partial x}$ and use Taylor series for $u'(\Pi)$; then

$$\frac{\text{Cov}(u', \frac{\partial \Pi}{\partial x_i})}{Eu'} \approx -r\ [\text{Var}\ p\ Ef_{x_i} + \frac{E(p^2)}{2} \frac{\partial\ \text{Var}\ y}{\partial x_i}](A)^2.$$

where r is the risk aversion coefficient. Therefore MRP_x can be associated with the relative sizes of mean and variance effects of an input weighted by variance effects of price. Note that the sign of the covariance term is a "local" property, i.e., a given sign may hold at a specific point but may not hold globally. Thus, risk effects of inputs are local, rather than global, properties.

Sufficient conditions to determine input risk properties may be given for $u''' = 0$; then the above Taylor series approximation is exact. If

$\mathrm{Var}\ p \neq 0$, $Ef_x > 0$ and $\frac{\partial\ \mathrm{Var}\ y}{\partial x} > 0$ are sufficient conditions for an input to be risk increasing at a point; $Ef_x < 0$ and $\frac{\partial\ \mathrm{Var}\ y}{\partial x} < 0$ are sufficient conditions for an input to be a risk decreasing at a point. If $\mathrm{Var}\ p = 0$, sufficient conditions can be given in terms of yield variance only.

If the sufficient conditions do not hold and an optimum solution is known, then the risk properties of inputs may be inferred from the sign of $E\Pi_{x_i}$ at the optimum solution.

Note that for acres planted, the marginal risk premium is

$$MRP_A = \frac{\mathrm{Cov}(u', py-wx)}{Eu'} = -\ r[\mathrm{Var}\ p\ Ey + \frac{E(p^2)}{2}\ \mathrm{Var}\ y] < 0$$

so that "acres planted" is a risk increasing input.

5. Sufficient conditions for determining variance effects and their relation to risk properties of inputs for a specific production function

Consider the stochastic production function

$$y = f(x,\theta) = h(x) + g(x)\ \theta$$

where $E\theta = 0$. For this function,

$$\mathrm{Var}\ y = [g(x)]^2\ \mathrm{Var}\ \theta$$

$$\frac{\partial\ \mathrm{Var}\ y}{\partial x_i} = 2\ g(x)\ \frac{\partial g}{\partial x_i}\ \mathrm{Var}\ \theta.$$

Thus if $g(x)$ and $\frac{\partial g(x)}{\partial x_i}$ have the same sign, x_i is variance increasing at x; if $g(x)$ and $\frac{\partial g(x)}{\partial x_i}$ have opposite signs, x_i is variance reducing at x.

If also $\frac{\partial h(x)}{\partial x_i} > 0$, then $Ef_x > 0$. Thus for $u''' = 0$, sufficient conditions for an input x_i to be "risk increasing" at a point x are that $g(x)$ and $\frac{\partial g(x)}{\partial x_i}$ have the same signs and $\frac{\partial h(x)}{\partial x_i} > 0$ at x.

For input x_i to be "risk reducing" at a point x, sufficient conditions are $\frac{\partial h}{\partial x_i} \leq 0$ and g and $\frac{\partial g}{\partial x_i}$ have opposite signs.

6. Example used in this paper (a special case of (5))

Yield per acre production functions:

$$y^1 = x_1 + x_2 - .2x_1{}^2 - .2x_1x_2 - .2x_2{}^2 + (-x_2 + .1x_1{}^2 + .1x_1x_2 + .1x_2{}^2)\ (\Omega - 1)$$

$$y^2 = 1.2(x_1 + x_2 - .2x_1{}^2 - .2x_1x_2 - .2x_2{}^2) + 2(-x_2 + .1x_1{}^2 + .1x_1x_2 + .1x_2{}^2)\ (\Omega - 1)$$

Prices:

$w_1 = .15$, $w_2 = .10$;
p_1 = uniform; mean 4, variance .33;
p_2 = uniform; mean 5, variance 20.5

State of nature:

Ω has a beta distribution with mean 1, variance .0714

Solution $(\hat{x})$ for base case (r=1):

<u>Crop 1</u>	<u>Crop 2</u>
$x_1^1 = 1.88$	$x_1^2 = 2.14$
$x_2^1 = 1.33$	$x_2^2 = 1.06$

Tests of input properties for base case:

(i) At the optimum solution x_1 is variance reducing and x_2 is variance increasing for both crops.

<u>Crop 1</u>	<u>Crop 2</u>
$g^1(\hat{x}) = -.22 < 0$	$g^2(\hat{x}) = -.26 < 0$
$\dfrac{\partial g^1(\hat{x})}{\partial x_1} = .51 > 0$	$\dfrac{\partial g^2(\hat{x})}{\partial x_1} = .53 > 0$
$\dfrac{\partial g^1(\hat{x})}{\partial x_2} = -.55 < 0$	$\dfrac{\partial g^2(\hat{x})}{\partial x_2} = -.57 < 0$

Since $g_1^i(\hat{x})$ and $g^i(\hat{x})$ $(i = 1,2)$ have opposite signs and $g_2^i(\hat{x})$ and $g^i(\hat{x})$ $(i = 1,2)$ have the same signs then x_1 is variance reducing and x_2 is variance increasing at $\hat{x}$.

(ii) x_1 is risk reducing and x_2 is risk increasing at x for both crops.

<u>Crop 1</u>

$$Ep^1\ Ef^1_{x_1} - w_1 = 4(-.01) - .15 < 0$$

$$Ep^1\ Ef^1_{x_2} - w_2 = 4(.094) - .1 > 0$$

<u>Crop 2</u>

$$Ep^2\ Ef^2_{x_1} - w_1 = 5(-.08) - .15 < 0$$

$$Ep^2\ Ef^2_{x_2} - w_2 = 5(.18) - .10 > 0$$