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ELRISK: ELICITING BERNOULLIAN UTILITY FUNCTIONS

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Halter and Mason outlined a method for elicitation of Bernoullian utility curves with the intent of making it applicable to nearly all problems. They used an "equally likely risky outcome" method developed by Ramsey. Their major contribution was in seeding the games based on the user's expected income. They went on to show three producer examples with 5 points on each utility function. In total, eleven producers are analyzed and their Arrow-Pratt measures of risk aversions were estimated.

This paper will outline further improvements that will make the Ramsey method more adapted to general problem solving. These improvements include the use of elicitation in the context of solving an actual problem, use of the first and second moments of the uncertain income in seeding the elicitation process and the development of computer software that simplifies elicitation, at least for the researcher and likely for the user. Results from 29 producers and 18 non producers will be summarized and examined.

The Context of the Elicitation

Prior to crop harvest, yield and commodity price (futures + basis) are uncertain, but acreage, costs per acre and costs per bushel are known with reasonable certainty. If fixed costs are ignored, then performance can be measured by the distribution of the gross margin (gross income minus variable costs) of the crop. This gives mostly non-negative values in the gross margin distribution. Pre-harvest commodity marketing can be used to impact the gross margin distribution, depending on how many bushels are priced and how they are priced. ELRISK has been used in this context. It should be possible to elicit monthly or quarterly performance and wealth or total income, but, for the remainder of the paper, income will be annual pre-harvest gross margins for a single crop.

Seeding ELRISK and Rounding Inputs

The uncertain income around which ELRISK will elicit can be represented by a triangular distribution (low, mode, high) or by approximating the distribution using the mean and standard deviation. For the triangular distribution, the standard deviation is estimated to be one fourth of the high minus the low and the mode is used to seed the start of elicitation. The low and high define stopping points in the elicitation process. If the mean and standard deviation are used to represent the uncertain income, the stopping points are the mean minus two standard deviations on the low end and the mean plus two standard deviations on the upper end.

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Once endpoints for elicitation have been defined, values for the mean (mode for triangular distribution) and the standard deviation are rounded. Means (and modes) are rounded to two significant digits with the second digit being even or zero. Thus, a mean of $5501 would be rounded to $5600, and $64999 would be rounded to $64000. Very large means of more than $100,000 are truncated at the nearest thousands and producers are reminded that all values are in thousands of dollars. Standard deviations are rounded to the nearest even significant digit. Thus a standard deviation of $1501 is rounded to $2000. Details of rounding methods are presented in Alderfer (1991). Halter and Mason used a different method of rounding the income, and were unconcerned about the standard deviation of income (in the elicitation process).

Means (modes) and standard deviations were rounded to help decision makers make quicker and more consistent analysis. Unrounded numbers would require more mathematical and mental processes and would likely slow the elicitation process.

Setting up Situations and Running ELRISK

ELRISK sequentially presents the producer with two marketing plans to be analyzed. Each plan has two possible outcomes of equal probability. The producer is reminded that the four outcomes should be considered gross margins for the crop analyzed and for the same time period (annual). Three of the four possible outcomes in the first two plans are a function of the rounded mean (or mode) and rounded standard deviation of gross margin. A risk-neutral value for the fourth outcome, giving equal expected outcomes for the two plans, is suggested to the user for revision. ELRISK seeks a revised fourth outcome that makes the user indifferent between plan A and B. Expert system rules ensure that one plan does not dominate the other and they establish new situations. Elicitation continues until utilities have been elicited for incomes that are more than two standard deviations above and below the expected gross margin. Elicitation is also halted if the sixth situation is reached, either above or below the starting values.

With the modifications to the Halter and Mason methods, ELRISK establishes roughly 10 to 14 points on the utility curve. No specific form of the utility function is assumed.

Figure 1 shows an example of situations one and two. Every situation is shown to the producer on a separate computer screen in ELRISK. In Figure 1 this is not the case because we are focusing on the principles of the game. In situation one, the decision maker must enter an income level in the quadrant marketed by "?", that makes him or her indifferent between marketing plans A and B. In subsequent situations, the values of the other quadrants are varied and a new indifference level is sought.

Situation 1

PROB	Marketing Plans A	Marketing Plans B
.5 Bad Year	Mean - Stand. Dev.	?
.5 Good Year	Mean + Stand. Dev.	Mean + 1/2 * StDev.

Situation 2

PROB	Marketing Plans A	Marketing Plans B
.5 Bad Year	? from Game 1	??
.5 Good Year	Mean + Stand. Dev.	Mean + 1/2 * StDev.

Figure 1. Structure of Situations 1 and 2 in ELRISK

Halter and Mason showed that if the response to the second situation (indicated by "??") is arbitrarily given a utility of 200, and the mean minus one standard deviation is given a utility of zero, then the response to situation one (indicated by "?") must have a utility of 100. To demonstrate why:

$$\begin{aligned} &\textit{Game 1: } .5(0) + .5U(k_1) = .5U(?) + .5U(k_2) \\ &\textit{Game 2: } .5U(?) + .5U(k_1) = .5(200) + .5U(k_2) \end{aligned} \qquad (1)$$

Since all probabilities are equal in Equation 1...

$$\begin{aligned} &\textit{Game 1. } 0 + U(k_1) = U(?) + U(k_2) \\ &\textit{Game 2. } U(?) + U(k_1) = 200 + U(k_2) \end{aligned} \qquad (2)$$

Solving for $U(k_1)$ in game 2 of equation 2,

$U(k_1) = 200 + U(k_2) - U(?)$ *subbing this into* $U(k_1)$ *of game* 1 *equation* 2...

$0 + 200 + U(k_2) - U(?) = U(?) + U(k_2)$ *combining terms gives...*

$200 = 2U(?)$ { ? = *response to first game in* \$}

$U(?) = 100$

The previous condition holds, if the respondent's behavior satisfies the principle axioms of expected utility hypothesis. After the first 2 games there are dollar values for three utility levels. These three levels 0, 100, 200, can be used to sequentially build the remaining situations.

If plan A is a choice between U($)=100 and U($)=200, while plan B is a choice between U($)=0 and a response field, the user response, in dollars will be the dollar value for U($)=300. This process continues upward until the last point surpasses the mean plus two standard deviations. The process can be reversed with the dollars for U=(0,100) in plan A, and plan B is U=(?,200). The user response to ? in this case is U=(-100).

One caveat of this improved method of seeding ELRISK, is that solution space for maximizing expected utility generally is in a similar range of utility for a similar problem involving producers of quite different sizes (means). This aids in seeding non-linear optimization routines and helps troubleshoot problems if they were to occur. If most of the producers facing a common set of prices are getting expected utilities of 100 to 200 and one producer gets results substantially higher or lower, then perhaps this latter producer should reexamine the inputs to ELRISK.

Results

Twenty nine soybean producers have used ELRISK in 4 workshops and 15 extension educators have also used ELRISK in the same context of marketing soybeans. Most of the extension educators had no soybean production and had difficulty in some way with ELRISK. Producers seemed to do better at examining the tradeoffs in the situations, since their soybean production was "real." Both audiences were told that quality responses to the situation would impact subsequent program recommendations. The extension educator data from ELRISK has been sorted by risk aversion coefficient (b) and listed in Table 1. The negative exponential utility function was fitted to all but the perfectly linear cases. The producer data are likewise sorted and listed in Table 2.

The importance of context was stressed earlier and is supported by the data in Tables 1 and 2. Producers in Table 2 were analyzing their own farm gross margins, and show a significant relationship between gross margin and risk coefficient (b). That is:

$$\begin{array}{ll} b = .000164 - 1.9E\text{-}9[g.m.] & \\ \quad (5.9E\text{-}5) \quad (4.2E\text{-}10) & \textit{st.error is in parenthesis} \end{array} \tag{3}$$

For the non-producers in Table 1:

$$\begin{array}{l} b = .000345 - 6.1E\text{-}9[g.m.] \\ \quad (.0004566) \quad (4.4E\text{-}9) \end{array} \tag{4}$$

The non-farmers had a much lower r^2 between mean gross margin and the risk coefficient (.092), compared to the farm group (.442). Since the risk coefficient (b) and gross margin are co-factors in the negative exponential utility function, it is expected that the two would be related. Intuition also suggests that if a producer takes on a very large and uncertain income distribution, they are likely less risk averse than a smaller producer, who has chosen a smaller income distribution (ceteris paribus).

Table 1. NonFarm ELRISK Results.

NON FARM I.D.	Acres	Mean G.M.	StDev G.M.	risk b	R^2
JAT	100	12000	4400	0.0	0.8016
Fales	100	14000	4200	0.0	1.0
MIK	100	76000	6000	0.0	1.0
Ful	500	80000	12000	0.0	1.0
Gros	100	12000	4400	1.53E-5	0.9983
TFK	160	31754	10019	2.52E-5	0.9838
HAM	150	5419	2014	2.57E-5	0.9877
Stens	500	60000	10000	2.98E-5	0.9875
Schop	100	50000	14000	3.69E-5	0.9872
LAZ	500	56000	18000	5.29E-5	0.9894
Siff	100	10000	4600	7.01E-5	0.9878
ELZ	500	39468	21776	7.34E-5	0.9811
HLJ	50	22000	5600	8.61E-5	0.9913
Nord	100	12000	4000	9.03E-5	0.9868
WSP	100	12000	500	9.87E-5	0.9986
REC	130	18225	5729	1.8E-4	0.9270
BIS	50	5800	2600	2.58E-4	0.9946
PEY	20	1241	1112	2.03E-3	0.9505

Table 2. Farm ELRISK Results.

FARMS I.D.	Acres	Mean G.M.	S.Dev G.M.	risk b	R^2
MON13	500	81756	25653	0.00	1.0000
FRA8	700	88000	38240	1.6E-6	0.9722
FRA6	350	57698	24999	3.2E-6	0.9819
MON12	225	38934	9737	8.7E-6	0.9875
MON3	605	81269	31319	1.7E-5	0.9899
FRA1	810	99589	37178	1.83E-5	0.9962
MON1	400	32086	28949	2.36E-5	0.9619
FRA2	420	65342	24514	2.38E-5	0.9970
MON4	350	52708	18256	2.39E-5	0.9934
SHI1	629	76354	30307	2.84E-5	0.9887
MON2	480	75532	25943	3.06E-5	0.9812
MON14	347	47551	18888	3.11E-5	0.9891
MON9	575	100000	36709	3.50E-5	0.9838
CAL3	250	38122	11280	4.15E-5	0.9944
MON6	500	44997	27767	4.2E-5	0.9676
FRA5	300	41729	17012	4.79E-5	0.9746
MON8	300	44054	1345	5.26E-5	0.9946
MON7	630	72383	26095	5.71E-5	0.9765
FRA7	420	62226	21402	6.4E-5	0.9630
CAL2	320	46180	15451	7.09E-5	0.9522
CAL4	160	27337	8550	7.09E-5	0.9898
MON11	120	19394	5563	8.68E-5	0.9656
SHI6	190	22113	8063	9.47E-5	0.9738
SHI3	162	18638	6966	1.15E-4	0.9747
SHI2	187	23666	7486	1.19E-4	0.8692
MON5	98	20146	4280	1.28E-4	0.9567
SHI7	200	22140	8004	1.45E-4	0.9124
FRA4	77	10035	4096	2.67E-4	0.9837
FRA3	85	10632	4723	3.52E-4	0.9859

Further differences in the producer and educator groups also occurred. A risk neutral attitude about thousands of dollars of uncertain income, is either a sign that the person may have: (1) other income (both certain and uncertain) of larger proportion, (2) other risks with offsetting effects, (3) made a mistake in understanding the frequency of the situation, or (4) not adequately consider the consequences of the situations presented. Only 1 of 29 farmers had perfectly linear risk preferences, where 3 of 18 educators were perfectly linear.

Producers had more soybean acres (358) compared to educators (194), which led to larger expected gross margins (48986 versus 27112), both statistically significant differences at .95. Standard deviations of gross margins were higher for the producers, but when weighted by the expected gross margins to compute the coefficient of variation, producers had lower gross margin C.V.'s than the educators. Statistically there was no difference in the r^2 of the utility functions for either group, when evaluated with the negative exponential utility function.

As a group, farmers where less risk averse (b= .000069) compared to educators (b=.000175), but part of this would be due to the larger soybean acreage and larger expected gross margins of the farmers.

Conclusions

ELRISK succeeds at better representing the problem space for the uncertain income compared to the Halter and Mason method, due to including the second moment of the income distribution. When used with soybean farmers and extension educators there were several significant differences in risk aversion, and particularly the relation of gross margin to risk aversion coefficient within each group. These and other measures support the importance of context dependance, and suggest that when the problem is real for the decision-maker, the results will likely differ from simulated situations.

The literature in risk elicitation is rich with discussion of human biases grounded in the cognitive sciences including limitations of human cognitive process. These biases involve the subject, the elicitation design and most would believe, the quality of the utility based solutions, leading to the "futility of utility" mentioned by Cochran et al. and Dillon.

While it is important to consider these biases, there is merit to using utility functions to solve complex problems and help provide post-optimal tradeoffs. Direct elicitation of expected utility may have limitations, but its discriminatory power and simplicity in calculations should not be overlooked. Further, for applied risk analysis to succeed, a variety of risk related representations, will need to coexist for purposes of validation and comparison, as well as providing a variety of approaches for the user, to keep from developing input "habits."

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