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EXPANDING THE SET OF EXPECTED UTILITY AND MEAN STANDARD DEVIATION CONSISTENT MODELS

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Introduction

Confusing many students of risk are the competing decision models for choosing between risky alternatives. The generally accepted risk model is expected utility maximization. This model usually assumes diminishing marginal utility for continuously divisible levels of wealth. The utility function is at least twice differentiable and concave. Inconsistent with this view of preferences are most ad hoc decision models including safety-first models, jump discontinuous functions, maximin, minimax, and versions of mean-variance or mean-standard deviation models.

Common to most of these ad hoc models is an implied utility of wealth function that is not twice differentiable and concave as economists most often assume.

Indirect and Direct Outcome Variables

The question is how to rationalize these ad hoc models with what economists assume about preferences. To begin, consider a direct outcome variable w and a twice differentiable concave function $U(w)$ such that $U'(w)>0$ and $U''(w)<0$. Next, identify an indirect outcome variable where the dependency of w on y is described by the function $w=h(y)$ where $a<y<b$.

In the discussion that follows, the transformation between y and w is not, unless specified, the transformation between income and wealth. Instead, the transformation function h represents the effects of institutions and other risk altering arrangements that transform the probability distribution of y, an indirect outcome variable, to the probability distribution of w, a direct outcome variable and the argument of utility.

The transformation function h(y) might represent the effects of insurance, hedging, transactions costs, taxes, bankruptcy provisions, liquidation fees, and win-lose outcomes on the indirect outcome variable y. As a result, the transformation function h(y) might convert uninsured, before tax, unhedged, before transactions cost income or wealth, for example, to insured, after tax, hedged, transactions cost deduced income or wealth.

The reason ad hoc functions defined over indirect outcome variable y are popular is that we often only observe y. Or, we fail to recognize the transformation h(y) and make no distinction between the indirect outcome variable y and the direct outcome variable w.

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The function U(w) defines the decision maker's risk attitudes. But we can just as easily define risk attitudes over y. Beginning with the preference function U(w), a function V(y) can be obtained that orders risky choices consistent with the orderings produced by U(w). The function V(y) satisfies: U(w)=U[h(y)]=V(y). Moreover, an absolute risk aversion function can be defined over y equal to:

$$\begin{aligned} R[U(h(y))] &= R[V(y)] \\ &= -h''(y)/h'(y) + -U''[h(y)]h'(y)/U'[h(y)] \\ &= h''(y)/h'(y) + R[Uh(y)]h'(y) \end{aligned} \quad (1)$$

as long as h(y) is a twice differentiable function. At y values where h(y) is not differentiable, there will exist undefined values of R(y).

If h(y) is not continuous or differentiable or concave, then the function V(y) need not be continuous or twice differentiable, or even concave down even though U(w) demonstrates all of these properties. Moreover, if one observed V(y) without recognizing that it was obtained from U(w) and the transformation of w=h(y), one might infer unusual risk attitudes about the decision maker. Thus, it is reasonable to hypothesize that some ad hoc decision rules for ordering risky choices are inferred from measures of V(y) defined over y and should not be confused with risk attitudes as we usually define them over U(w).

Larry Lev and I once compared the indirect outcome variable y to yards gained in a football game and the direct outcome variable w to winning or losing. Suppose we inferred the coach's risk attitude from the choice of plays resulting in yards gained y. If we made such an inference, we might obtain incorrect impressions about the preferences of the decision maker for winning because the play selection depends on time remaining in the game, the score, and the team's distance from the goal line. Time remaining, the score, and distance from the goal line act as transformations on the indirect outcome variable y.

Some possible relationships between y and w, and V(y) and U(w) are now illustrated.

Linear Transformations: Negative Income Tax

Suppose w is related to y by a linear transformation described in Figure 1a, such that:

$$w = \alpha + \beta y \qquad \text{where} \quad \beta > 0. \quad (2)$$

The transformation is consistent with a negative income tax such that if one's income falls below a certain level the decision maker receives income instead of paying taxes. Thus, one might define y to be before-tax income and w to be after-tax income.

For the linear transformation defined above, $R[U(w)] = \beta R[U(\alpha+\beta y)]$. As a result, risk attitude measures R(w) and R(y) compared at points y=w will show R(y) as less (more) risk averse than R(w) depending on $\beta>1$ ($\beta<1$). This last result is the point made by Raskin and Cochran, that the absolute risk aversion function is defined over units of income and changing the income levels at which risk aversion is measured changes the risk aversion measure.

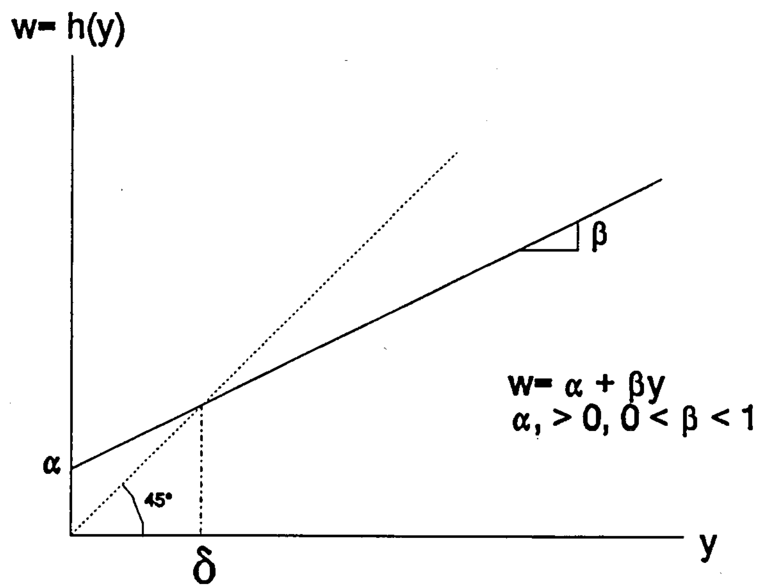

Figure 1a. Linear Transformation (e.g., Negative Income Tax)

For the linear transformation and other transformations to follow, we introduce the relationships between U(w) and V(y) and R[U(w)] and R[V(y)] graphically in Figure 1b. In the northwest quadrant of our six quadrant graphs is drawn U(w), smooth and concave down. In the northeast quadrant is drawn V(y) whose shape relative to U(w) depends on the transformation w=h(w), the inverse of which is drawn in the west central quadrant. The east central quadrant is a 45 degree line that reflects y values from the transformation described in the west central quadrant to the horizontal axis over which V(y) is defined. In the southwest quadrant is drawn an assumed absolute risk aversion function associated with U(w). In this case, we draw it downward sloping consistent with Decreasing Absolute Risk Averse (DARA) risk attitudes. Finally, in the southeast quadrant, we draw R[V(y)] also related to R[U(w)] by the transformations defined in the west central quadrant.

Suppose a transformation w=y. Then, U(w) would equal V(y) whenever w=y. Moreover, R[U(w)] would equal R[V(y)]. Now compare these results to the linear transformation. In Figure 1b, the function V(y) is flattened or stretched relative to U(w). Correspondingly, R(y)<R(w) when measured at points where w=y.

Jump Discontinuities: Transactions Costs

Suppose that w=y except for a transaction cost that occurs when outcome $y=y_d$ or less. One may relate to such a discontinuity by establishing a threshold level of income y_d such that if income falls below y_d, a transaction is required that costs δ. For example, if income falls below y_d, a firm may be required to take out a loan or liquidate assets. Both of these actions incur transactions costs and are not undertaken except of necessity when income falls below y_d.

The relationship between w and y described in Figure 2a is:

$$w = \begin{cases} y-\delta & \text{for } y \le y_d \\ y & \text{for } y > y_d \end{cases} \tag{3}$$

The relationship between U(w) and V(y) is described graphically in Figures 2a and 2b. The function V(y) is jump discontinuous at y_d. The jump discontinuity occurs because only portions of U(w) for $w=y>y_d$ and $w=y-\delta<y_d$ are mapped to V(y). Absolute risk aversion is undefined at y_d and remains the same as R(w) for w=y for $y>y_d$. However, for DARA decision makers, the transformation has increased risk attitudes measured locally over $y<y_d$.

Masson claims that peasant farmers in Mexico reflect jump discontinuous preferences. We wonder if, in fact, their preferences were measured over indirect outcome variables instead of a direct outcome variable. Jump discontinuous preferences for peasant farmers could exist if there was an outcome y_d such that income below this level meant they defaulted on their loans threatening their continued operations.

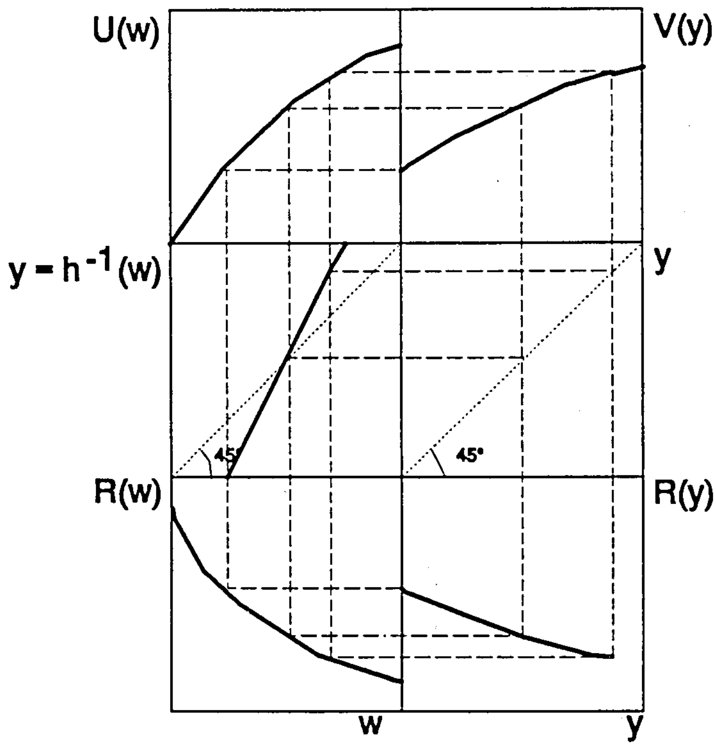

Figure 1b. Linear Transformation

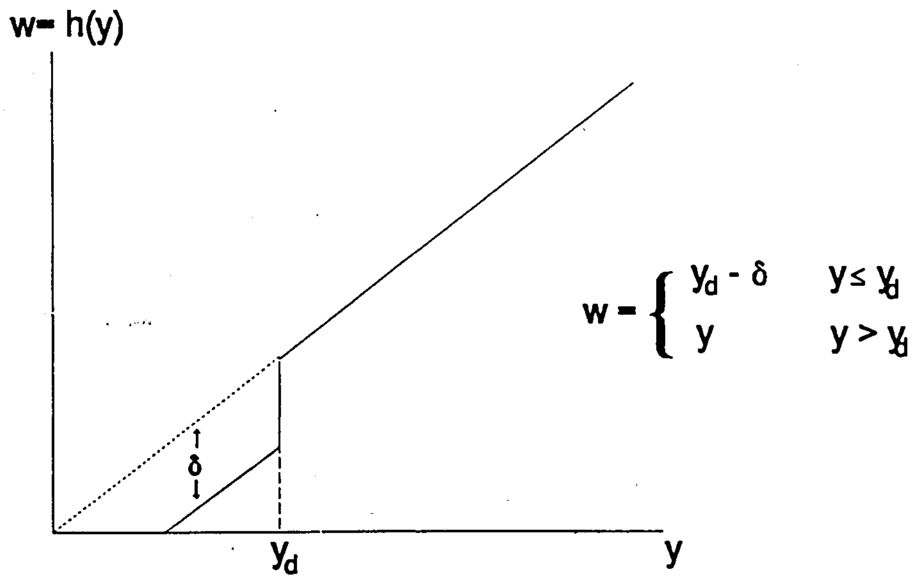

Figure 2a. Liquidation Costs at y_d

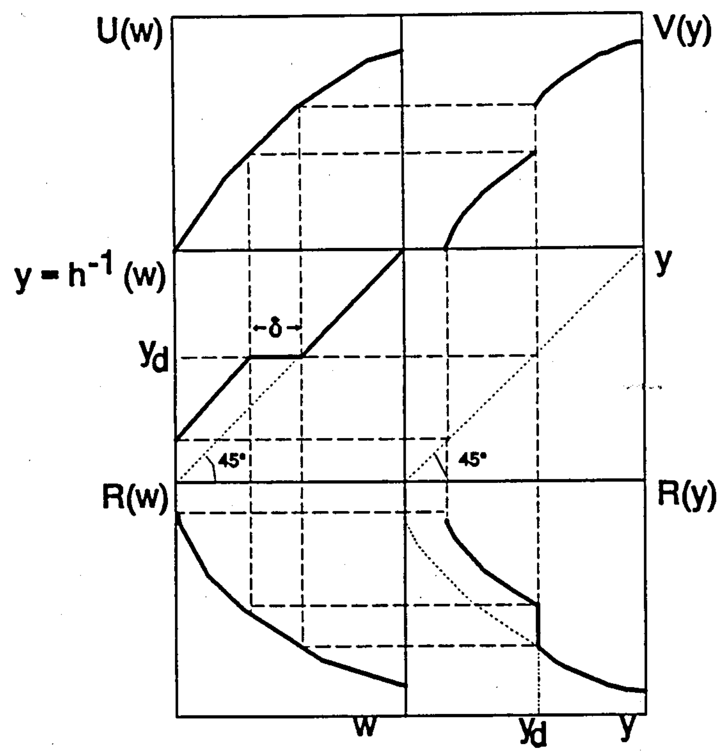

Figure 2b. Liquidation Costs at y_d

Limited Liability: Complete Insurance

The next transformation can be typified by a comprehensive insurance plan financed by the government such as disaster insurance such that for $y<y_d$, $y=y_d$. The relationship between w and y is graphed in Figures 3a and 3b and is expressed mathematically as:

$$w = \begin{cases} y_d & \text{for } y \leq y_d \\ y & \text{for } y > y_d \end{cases} \tag{4}$$

Under the influence of the transformation of limited liability, the function V(y) views all y values less than y_d as equal in value to $U(w=y_d)$. Thus, V(y) has a horizontal tail to the left of y_d and is equal to U(w) for w=y for $y>y_d$. Consistent with the transformation of limited liability is R(w)=R(y) for $y>y_d$ and R[V(y)]=0 for $y<y_d$ described graphically in the southeast quadrant.

If risk preferences are measured over y, the decision maker will appear much less risk averse than if measured over w. Thus, we might even find persons willing to take unfair gambles if their responses are measured over y. However, these risk preferences are not those we usually associate with U(w).

The function V(y) described in Figure 3b implies a particular kind of ad hoc decision model. For U(w)=w, the model has the flavor, though not the precise form, of Kataoka's safety-first rule which maximizes mean income subject to the requirement that $F(y<y_d)<\alpha$ where α is some established parameter. The difficulty with Kataoka's rule is establishing α, which prevents establishing a precise equivalence between Kataoka's rule and the case of limited liability. Models with limited liability properties have been examined by Robison, Barry, and Burghardt and Collins and Gbur.

Limited Winnings

Earning opportunities for lenders are frequently limited to recovering their loans plus interest. Their losses, however, can equal the amount of the loan plus the earnings they might have received in an alternative investment. In this case, let y=rL where r is the borrower's rate of return on loan amount L. Let y_d=iL where i is the interest rate the lender charged the borrower. If r>i, the lender still earns y_d because the borrower is obligated to repay only the agreed on interest. If r<i, the lender may earn less than y_d because the borrower may be unable to repay interest and may be unable to repay principal as well. Under these circumstances when r<i, y may be negative. This relationship, between the borrower's income y and the lender's income w, can be written as:

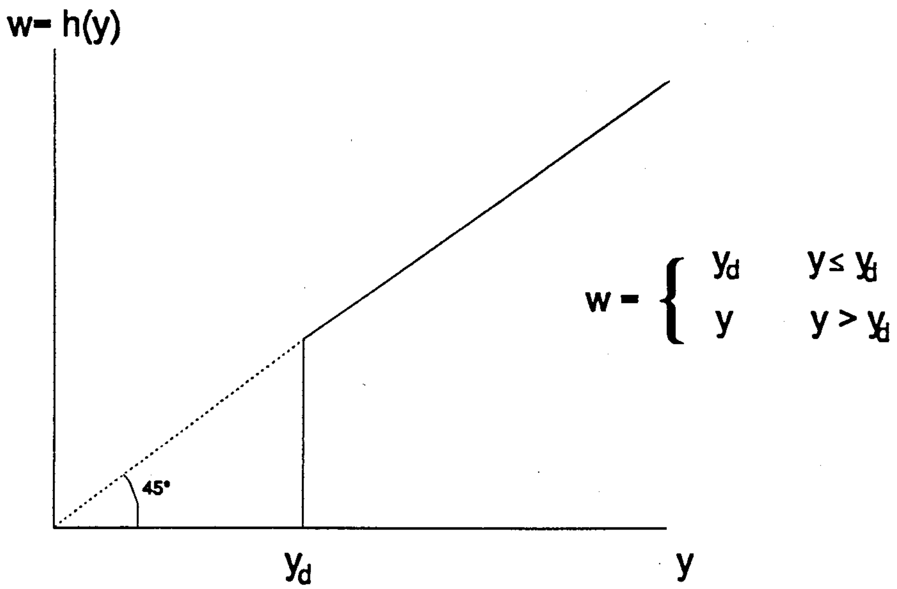

Figure 3a. Limited Liability

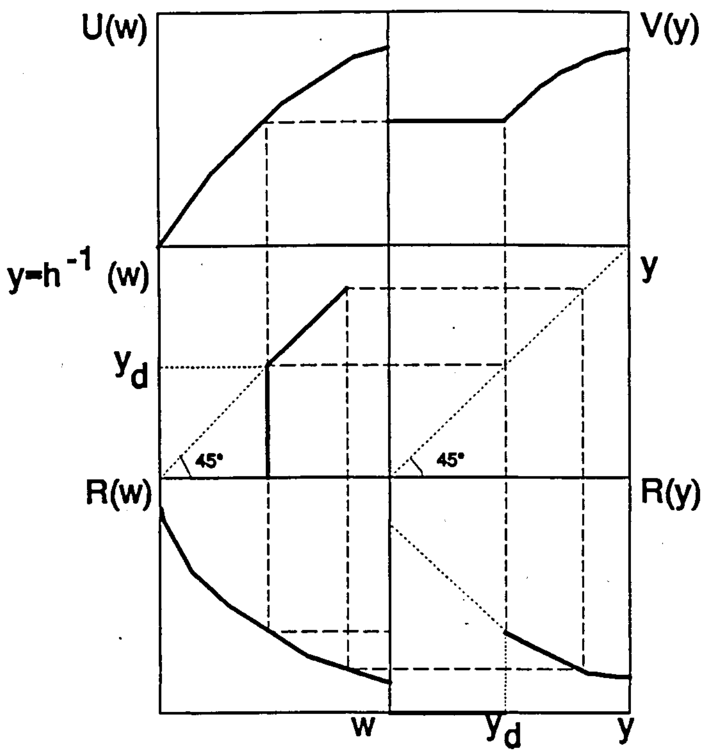

Figure 3b. Limited Liability

$$w = \begin{cases} y & \text{for } y \leq y_d = iL \\ y_d & \text{for } y > y_d = iL \end{cases} \quad (5)$$

The relationship between y and w is described graphically in Figure 4.

Note that in this model, earnings $y > y_d$ yield the same return for the bank. Also note that the borrower's earnings and the lender's are not directly comparable because the amount they have at risk differs.

Win/Lose Transformation

Suppose the outcomes of an activity can be defined as win or lose. Athletic and other contests are often characterized in this manner. Other win/lose outcomes are getting the job, promotion, contract, sale, or failing to achieve these goals.

These discrete outcomes of win or lose most often depend on an indirect outcome variable y exceeding some threshold y_d but yielding the same outcome regardless of by how much y exceeds y_d. So, the relationship between y and w is:

$$w = \begin{cases} 0 & \text{for } y \leq y_d \\ 1 & \text{for } y > y_d \end{cases} \quad (6)$$

The relationship between w and y is described graphically in Figures 5a and 5b. The obvious decision rule consistent with V(y) is to minimize the probability that y less than y_d occurs. This rule is most frequently referred to as Roy's safety-first rule. The advantage of such a rule is its ease in application. Roumasset notes that peasant rice farmers in the Philippines persistently display safety-first behavior. His observations would be consistent with a win/lose transformation in which farmers have identified a survival level of crop yields and their outcomes are essentially to survive or not.

Expected Utility and Mean Standard Deviation Consistency

The ability to analyze risk problems using MS models has several advantages. First, as Meyer and Robison showed, the theoretical results for the hedging model are more easily derived using an MS model. Moreover, they also showed that theoretical results in MS space can often be represented graphically. From a pedagogical point of view, teaching risk theory using MS models has much to recommend it.

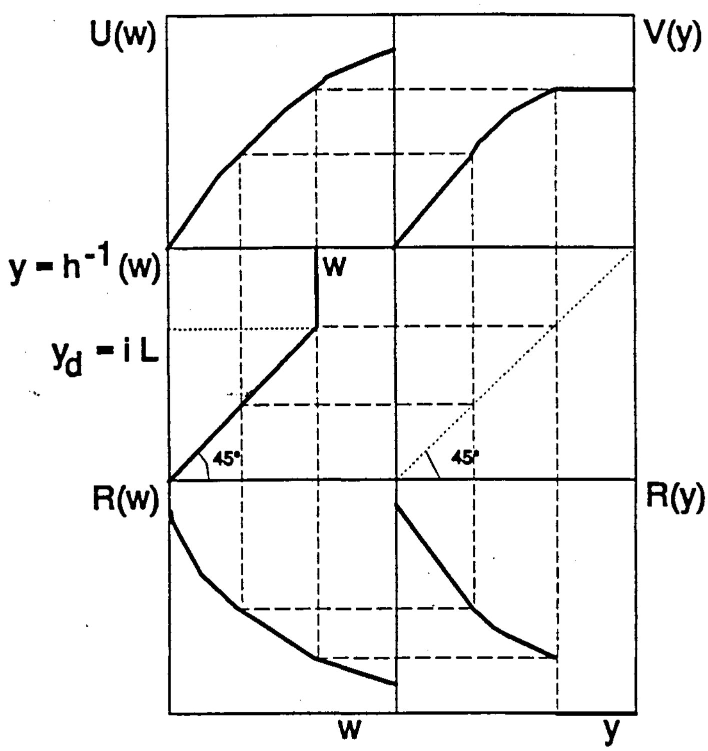

Figure 4. Limited Earnings

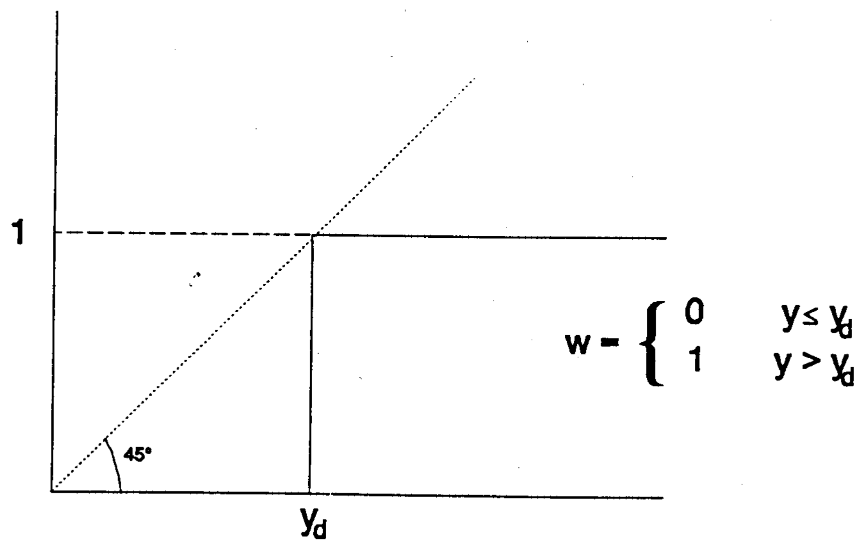

Figure 5a. Win / Lose

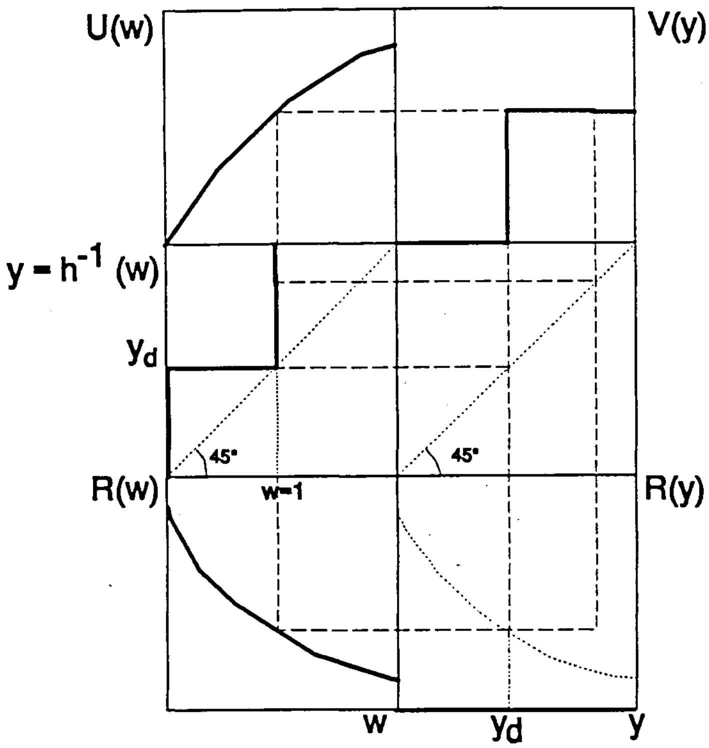

Figure 5b. Win ($y \geq y_d$) versus Lose ($y < y_d$)

Economists have long debated the conditions required for consistency between mean standard deviation (MS) models and expected utility maximization (EU) models. Early efforts established that quadratic utility functions or normal distributions would lead to MS and EU consistency. Independently, Sinn and Meyer produced a more general result. Meyer showed that when random variables are related to each other by location and scale, MS and EU models ranked risky choices consistently. Meyer's other important contribution on this subject was to derive general comparative static results for MS models.

Despite its offering more general consistency conditions, so far few general theoretical models (and perhaps some minor variations of these models) satisfy location-scale. These models include Sandmo's competitive firm, Feder, Just, and Schmitz's hedging model, and Tobin's portfolio model.

Obviously, there are more MS models than the three cited above. These models, however, have not been shown to be consistent with EU models. The view often expressed is that MS models are useful even if they are not always consistent with EU models (Chavas and Pope). Robison and Barry's *The Competitive Firm's Response to Risk* contains many MS models, many of which fail location-scale conditions. Their approach was much like that taken in econometrics that assumes small sample model properties from large sample property models. Robison and Barry assumed properties obtained from EU and MS consistent models would also apply to MS models even when consistency was not assured.

There is a fundamental problem with MS models that are not consistent with EU models. It is that we have no carefully derived theory of choice except that derived for EU models. For example, risk attitudes are derived in EU space. Income and substitution effects under risk are derived for the portfolio model that is MS and EU consistent (Cass and Stiglitz). Stochastic dominance efficiency criteria are also derived for EU models.

The next section expands the set of MS and EU consistent models. The approach used to expand this set of consistent models focuses on the distinction between direct and indirect outcome variables already described. To begin, allow that random variables y_1, y_2, y_3,... are related to each other by location-scale. It is allowed that the transformation of y to w may create random variables that no longer satisfy location-scale conditions.

The importance of this approach is two-fold. First, many important risk problems can be described as transformations from y to w. Some examples have already been given. The second advantage of this approach is that the approach followed in Meyer to find property in w space for EU and MS consistent models can be employed in y space even though location-scale fails to hold in w space. Then, the issue is not can we write MS and EU consistent models, but can we derive meaningful comparative static results in y space?

We begin by writing MS models that obey location-scale by following Meyer. That is, let all y_i be distributed equal to $\mu_i + \sigma_i x$ where $\mu_x = 0$ and $\sigma_x = 1$.. Furthermore, let μ_i and σ_i be the mean and standard deviation of y_i. It follows that:

$$EU(w) = EU[h(y)] = EV(y) = EV(\mu_i + \sigma_i x) = \hat{V}(\mu_i, \sigma_i) \quad (7)$$

Consistent with Meyer, it is assumed that the integral converges which places some boundedness restrictions on V(y). Besides boundedness, we have no other restrictions on the form of V(y) and only require location-scale on the random variables y_1, y_2, ... Thus, there is considerable flexibility on the form V(y) can take while still allowing us to expand the set of models that can be described in MS space and still be consistent with EU.

To illustrate, let EU(w)=E(w) and let the relationship between w and y be that described in (4). Then, we can write V(y) as:

$$V(y) = \begin{cases} y_d & \text{for } y \le y_d \\ y & \text{for } y > y_d \end{cases}$$

We can write the model in y space as:

$$EV(y) = y_d\, F\left(\frac{y_d - \mu_i}{\sigma_i}\right) + \int_{(y_d-\mu_i)/\sigma_i}^{\infty} (\mu_i + \sigma_i x)\, dF(x) \quad (8)$$

$$= \mu_i + (y_d - \mu_i)\, F\left(\frac{y_d - \mu_i}{\sigma_i}\right) - \sigma_i\, \bar{\varepsilon}$$

where:

$$\bar{\varepsilon} = \int_{-\infty}^{(y_d-\mu_i)/\sigma_i} x\, dF(x) < 0$$

If we write $EV(y) = \hat{V}(\mu_i, \sigma_i)$, then:

$$\hat{V}_{\sigma_i} = -\bar{\varepsilon} > 0 \quad (9)$$

Furthermore:

$$\hat{V}_{\mu_i} = 1 - F\left(\frac{y_d - \mu_i}{\sigma_i}\right) > 0 \tag{10}$$

Using Meyer's notation for the slope of the indifference curve, we write:

$$S(\sigma_i, \mu_i) = \frac{-\hat{V}_\sigma(\sigma,\mu)}{\hat{V}_\mu(\sigma,\mu)} = \frac{\bar{\varepsilon}}{1 - F\left(\frac{y_d - \mu_i}{\sigma_i}\right)} < 0$$

Thus, isoexpected utility lines in (μ_i, σ_i) space are downward sloping. If the MS choice set is upward sloping, the preferred choice would be the one with the greatest expected value and standard deviation. Furthermore, increases in σ_i increase expected utility.

Although the form of the above model is somewhat unusual and contains an important parameter y_d, it is clearly an MS model that ranks risky choices consistent with the EU(w) model.

It is unlikely that the properties of the models in the expanded set of EV and EU consistent models will be the same as those deduced by Meyer when U(w) was well behaved.

Consider the model described in (3) that involves transactions costs. For this case, U(w) is not given a specific form except those properties assumed by Meyer; namely, U'(w)>0 and U"(w)<0. In y space, we can write:

$$\hat{V}(\sigma_i, \mu_i) = \int_{-\infty}^{\infty} U(\mu_i + \sigma_i x)\, dF(x) + \int_{-\infty}^{(y_d - \mu_i)/\sigma_i} [U(\mu_i + \sigma_i x) - U(\mu_i + \sigma_i x - \delta)]\, dF(x) \tag{11}$$

$$= \int_{-\infty}^{\infty} U(\mu_i + \sigma_i x)\, dF(x) - \gamma F\left(\frac{y_d - \mu_i}{\sigma_i}\right)$$

In the equation above, the difference between two integrals measured at y and y-δ is assumed to be a constant γ. Furthermore, because the first integral has all of the properties associated with EU(w) all of the static results derived by Meyer hold for the first integral as well. The only question remains: under what conditions will $\gamma F\left(\frac{y_d - \mu_i}{\sigma_i}\right)$ lead to a violation of those conditions?

Let

$$V^*(\sigma_i, \mu_i) = -\gamma F\left(\frac{y_d - \mu_i}{\sigma_i}\right).$$

It follows that $V^*_{\mu_i} > 0$, $V^*_{\sigma_i} < 0$. These signs are consistent with the effects of increases in μ_i and σ_i on $EU(\mu_i + \sigma_i x)$.

Obviously, as $\gamma F\left(\frac{y_d - \mu_i}{\sigma_i}\right)$ becomes smaller, it becomes less likely to alter comparative statics associated with the first integral.

The conditions under which the transactions cost model loses the properties described by Meyer — leading to corner solutions — are important properties of this model. Examining specific model properties appears to be the next stage of research of MS and EU consistent models.

Conclusions

The main findings of this paper are described next. First, we began with a set of indirect outcome random variables related to each other by location-scale. It followed that one can construct an MS model defined over the means and standard deviations of the indirect random variables that produce EU consistent rankings. This finding increases significantly the number and classes of models that can be studied using MS models.

This finding is important because we infrequently observe final outcome variables. Instead, we mostly observe indirect outcome variables. Furthermore, the MS model offers some pedagogical advantages (e.g., the models can be represented graphically and they usually are simpler to work with analytically).

A caution about the new MS approach is that it still requires location-scale conditions on the indirect outcome variables. Furthermore, each model depends on the transformation function h leading to $\hat{V}(\sigma_i, \mu_i)$ models that may exhibit unique properties. Thus, the general prospecties described by Meyer for well-behaved functions will likely not exist for the various $\hat{V}(\sigma_i, \mu_i)$ models.

This last observation, the uniqueness of the expanded MS models, offers a rich opportunity set for researchers to mine.

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